TT TEXTBOOK TESTBANKS

Solutions Manual for Trigonometry 12th Lial

SOLUTIONS MANUAL SAMPLE PREVIEW

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Chapter 1

TRIGONOMETRIC FUNCTIONS

Section 1.1 Angles

1. One degree, written 1°, represents $\frac{1}{360}$ of a complete rotation.

2. If the measure of an angle is $x°$, its complement can be expressed as $\underline{90°} - x°$.

3. If the measure of an angle is $x°$, its supplement can be expressed as $\underline{180°} - x°$.

4. The measure of an angle that is its own complement is $\underline{45°}$.

5. The measure of an angle that is its own supplement is $\underline{90°}$.

6. One minute, written 1′ is $\frac{1}{60}$ of a degree.

7. One second, written 1″, is $\frac{1}{60}$ of a minute.

8. $12°30'$ written in decimal degrees is $\underline{12.5°}$.

9. 55.25° written in degrees and minutes is $\underline{55°15'}$.

10. If n represents any integer, then an expression representing all angles coterminal with 45° is $45° + \underline{n \cdot 360°}$.

11. 30°
    (a) $90° - 30° = 60°$
    (b) $180° - 30° = 150°$

12. 60°
    (a) $90° - 60° = 30°$
    (b) $180° - 60° = 120°$

13. 45°
    (a) $90° - 45° = 45°$
    (b) $180° - 45° = 135°$

14. 90°
    (a) $90° - 90° = 0°$
    (b) $180° - 90° = 90°$

15. 54°
    (a) $90° - 54° = 36°$
    (b) $180° - 54° = 126°$

16. 10°
    (a) $90° - 10° = 80°$
    (b) $180° - 10° = 170°$

17. 1°
    (a) $90° - 1° = 89°$
    (b) $180° - 1° = 179°$

18. 89°
    (a) $90° - 89° = 1°$
    (b) $180° - 89° = 91°$

19. $14°20'$
    (a) $90° - 14°20' = 89°60' - 14°20' = 75°40'$
    (b) $180° - 14°20' = 179°60' - 14°20' = 165°40'$

20. $39°50'$
    (a) $90° - 39°50' = 89°60' - 39°50' = 50°10'$
    (b) $180° - 39°50' = 179°60' - 39°50' = 140°10'$

21. $20°10'30''$
    (a) $90° - 20°10'30'' = 89°59'60'' - 20°10'30'' = 69°49'30''$
    (b) $180° - 20°10'30'' = 179°59'60'' - 20°10'30'' = 159°49'30''$

22. $50°40'50''$
    (a) $90° - 50°40'50'' = 89°59'60'' - 50°40'50'' = 39°19'10''$
    (b) $180° - 50°40'50'' = 179°59'60'' - 50°40'50'' = 129°19'10''$

23. The two angles form a straight angle.
$7x+11x=180 \Rightarrow 18x=180 \Rightarrow x=10$
The measures of the two angles are
$(7x)° = [7(10)]° = 70°$ and
$(11x)° = [11(10)]° = 110°$.

24. The two angles form a straight angle.
$$(20x+10)+(3x+9)=180$$
$$23x+19=180$$
$$23x=161 \Rightarrow x=7$$
The measures of the two angles are
$(20x+10)° = [20(7)+10]° = 150°$ and
$(3x+9)° = [3(7)+9]° = 30°$.

25. The two angles form a right angle.
$4x+2x=90 \Rightarrow 6x=90 \Rightarrow x=15$
The two angles have measures of
$(4x)° = [4(15)]° = 60°$ and
$(2x)° = [2(15)]° = 30°$.

26. The two angles form a right angle.
$$(5x+5)+(3x+5)=90 \Rightarrow 8x+10=90 \Rightarrow$$
$$8x=80 \Rightarrow x=10$$
The measures of the two angles are
$(5x+5)° = [5(10)+5]° = (50+5)° = 55°$ and
$(3x+5)° = [3(10)+5]° = (30+5)° = 35°$.

27. The two angles form a straight angle.
$$(-4x)+(-14x)=180 \Rightarrow -18x=180 \Rightarrow$$
$$x=-10$$
The measures of the two angles are
$(-4x)° = [-4(-10)]° = 40°$ and
$(-14x)° = [-14(-10)]° = 140°$.

28. The two angles form a straight angle.
$$9x+9x=180$$
$$18x=180$$
$$x=10$$
The measure of each of the angles is
$9 \cdot 10° = 90°$.

29. The sum of the measures of two supplementary angles is 180°.
$$(10x+7)+(7x+3)=180$$
$$17x+10=180$$
$$17x=170 \Rightarrow x=10$$
The measures of the two angles are
$(10x+7)° = [10(10)+7]° = (100+7)° = 107°$
and $(7x+3)° = [7(10)+3]° = (70+3)° = 73°$.

30. The sum of the measures of two supplementary angles is 180°.
$$(6x-4)+(8x-12)=180 \Rightarrow 14x-16=180 \Rightarrow$$
$$14x=196 \Rightarrow x=14$$
The measures of the two angles are
$(6x-4)° = [6(14)-4]° = (84-4)° = 80°$
and $(8x-12)° = [8(14)-12]°$
$= (112-12)° = 100°$.

31. The sum of the measures of two complementary angles is 90°.
$$(9x+6)+3x=90 \Rightarrow 12x+6=90 \Rightarrow$$
$$12x=84 \Rightarrow x=7$$
The measures of the two angles are
$(9x+6)° = [9(7)+6]° = (63+6)° = 69°$ and
$(3x)° = [3(7)]° = 21°$.

32. The sum of the measures of two complementary angles is 90°.
$$(3x-5)+(6x-40)=90 \Rightarrow 9x-45=90 \Rightarrow$$
$$9x=135 \Rightarrow x=15$$
The measures of the two angles are
$(3x-5)° = [3(15)-5]° = (45-5)° = 40°$ and
$(6x-40)° = [6(15)-40]° = (90-40)° = 50°$.

33. $$\frac{25 \text{ minutes}}{60 \text{ minutes}} = \frac{x}{360°}$$
$$x = \frac{25}{60}(360) = 25(6) = 150°$$

34. The minute hand is $\frac{3}{4}$ the way around, so the hour hand is $\frac{3}{4}$ of the way between the 1 and 2. Thus, the hour hand is located 8.75 minutes past 12. The minute hand is 15 minutes before the 12. The smaller angle formed by the hands of the clock can be found by solving the proportion $\frac{(15+8.75) \text{ minutes}}{60 \text{ minutes}} = \frac{x}{360°}$.
$$\frac{(15+8.75) \text{ minutes}}{60 \text{ minutes}} = \frac{x}{360°} \Rightarrow \frac{23.75}{60} = \frac{x}{360} \Rightarrow$$
$$x = \frac{23.75}{60}(360) = 23.75(6) = 142.5° = 142°30'$$

35. At 15 minutes after the hour, the minute hand is $\frac{1}{4}$ the way around, so the hour hand is $\frac{1}{4}$ of the way between the 3 and 4. Thus, the hour hand is located 16.25 minutes past 12. The minute hand is 15 minutes after the 12. The smaller angle formed by the hands of the clock can be found by solving the proportion $\frac{(16.25-15)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°}$.

$$\frac{(16.25-15)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°} \Rightarrow \frac{1.25}{60} = \frac{x}{360} \Rightarrow$$
$$x = \frac{1.25}{60}(360) = 1.25(6) = 7.5° = 7°30'$$

36. At 45 minutes after the hour, the minute hand is $\frac{3}{4}$ the way around, so the hour hand is $\frac{3}{4}$ of the way between the 9 and 10. Thus, the hour hand is located 48.75 minutes past 12. The minute hand is 45 minutes after the 12. The smaller angle formed by the hands of the clock can be found by solving the proportion $\frac{(48.75-45)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°}$.

$$\frac{(48.75-45)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°} \Rightarrow \frac{3.75}{60} = \frac{x}{360} \Rightarrow$$
$$x = \frac{3.75}{60}(360) = 3.75(6) = 22.5° = 22°30'$$

37. At 20 minutes after the hour, the minute hand is $\frac{1}{3}$ the way around, so the hour hand is $\frac{1}{3}$ of the way between the 8 and 9. Thus, the hour hand is located $41\frac{2}{3}$ minutes past 12. The minute hand is 20 minutes after the 12. The smaller angle formed by the hands of the clock can be found by solving the proportion $\frac{(41\frac{2}{3}-20)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°}$.

$$\frac{(41\frac{2}{3}-20)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°} \Rightarrow \frac{21\frac{2}{3}}{60} = \frac{x}{360} \Rightarrow$$
$$x = \frac{21\frac{2}{3}}{60}(360) = \left(21\tfrac{2}{3}\right)(6) = 130°$$

38. At 6:10, the minute hand is $\frac{1}{6}$ the way around, so the hour hand is $\frac{1}{6}$ of the way between the 6 and 7. Thus, the hour hand is located $30\frac{5}{6}$ minutes past 12.

The smaller angle formed by the hands of the clock can be found by solving the proportion $\frac{(30\frac{5}{6}-10)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°}$.

$$\frac{(30\frac{5}{6}-10)\text{ minutes}}{60\text{ minutes}} = \frac{x}{360°} \Rightarrow \frac{20\frac{5}{6}}{60} = \frac{x}{360} \Rightarrow$$
$$x = \frac{20\frac{5}{6}}{60}(360) = 20\tfrac{5}{6}(6) = 125°$$

39.
$$\begin{array}{r} 62°\ 18' \\ +21°\ 41' \\ \hline 83°\ 59' \end{array}$$

40.
$$\begin{array}{r} 75°\ 15' \\ +\ 83°\ 32' \\ \hline 158°\ 47' \end{array}$$

41. $97°42' + 81°37' = 178°79' = 179°19'$

42. $110°25' + 32°55' = 142°80' = 143°20'$

43. $47°\ 29' - 71°18' = -(71°18' - 47°\ 29')$
$= -(70°\ 78' - 47°\ 29')$
$= -23°\ 49'$

44. $47°\ 23' - 73°\ 48' = -(73°\ 48' - 47°\ 23')$
$73°\ 48' - 47°\ 23' = 26°\ 25'$, so
$47°\ 23' - 73°\ 48' = -(73°\ 48' - 47°\ 23')$
$= -26°\ 25'$

45. $90° - 51°\ 28' = 89°\ 60' - 51°\ 28' = 38°32'$

46. $90° - 17°\ 13' = 89°\ 60' - 17°\ 13' = 72°\ 47'$

47. $180° - 119°\ 26' = 179°\ 60' - 119°\ 26' = 60°\ 34'$

48. $180° - 124°\ 51' = 179°\ 60' - 124°\ 51' = 55°09'$

49. $90° - 72°\ 58'\ 11'' = 89°\ 59'\ 60'' - 72°\ 58'\ 11''$
$= 17°\ 01'\ 49''$

50. $90° - 36°18'47'' = 89°59'60'' - 36°18'47''$
$= 53°41'13''$

51. $26°20' + 18°17' - 14°10' = 44°37' - 14°10'$
$= 30°27'$

52. $55°30' + 12°44' - 8°15' = 67°74' - 8°15'$
$= 59°59'$

53. $35°30' = 35° + \frac{30}{60}° = 35° + 0.5° = 35.5°$

54. $82°30' = 82° + \frac{30}{60}° = 82° + 0.5° = 82.5°$

55. $112°15' = 112° + \frac{15}{60}° = 112° + 0.25° = 112.25°$

56. $133°45' = 133° + \frac{45}{60}° = 133° + 0.75° = 133.75°$

57. $-60°12' = -\left(60° + \frac{12}{60}°\right) = -(60° + 0.2°)$
$= -60.2°$

58. $-70°48' = -\left(70° + \frac{48}{60}°\right)$
$= -(70° + 0.8°) = -70.8°$

59. $20°54'36'' = 20° + \frac{54}{60}° + \frac{36}{3600}°$
$= 20° + 0.900° + 0.01° = 20.91°$

60. $38°\,42'18'' = 38° + \frac{42}{60}° + \frac{18}{3600}°$
$= 38° + 0.7° + 0.005° = 38.705°$

61. $91°35'54'' = 91° + \frac{35}{60}° + \frac{54}{3600}°$
$\approx 91° + 0.5833° + 0.0150°$
$\approx 91.598°$

62. $34°\ 51'\ 35'' = 34° + \frac{51}{60}° + \frac{35}{3600}°$
$\approx 34° + 0.8500° + 0.0097°$
$\approx 34.860°$

63. $274°18'59'' = 274° + \frac{18}{60}° + \frac{59}{3600}°$
$\approx 274° + 0.3000° + 0.0164°$
$\approx 274.316°$

64. $165°\ 51'\ 09'' = 165° + \frac{51}{60}° + \frac{9}{3600}°$
$= 165° + 0.8500° + 0.0025°$
$= 165.853°$

65. $39.25° = 39° + 0.25° = 39° + 0.25(60')$
$= 39° + 15' + 0'' = 39°15'00''$

66. $46.75° = 46° + 0.75° = 46° + 0.75(60')$
$= 46° + 45' + 0'' = 46°45'00''$

67. $126.76° = 126° + 0.76° = 126° + 0.76(60')$
$= 126° + 45.6' = 126° + 45' + 0.6'$
$= 126° + 45' + 0.6(60'')$
$= 126° + 45' + 36'' = 126°45'36''$

68. $174.255° = 174° + 0.255° = 174° + 0.255(60')$
$= 174° + 15.3' = 174° + 15' + 0.3'$
$= 174° + 15' + 0.3(60'')$
$= 174° + 15' + 18'' = 174°15'18''$

69. $-18.515° = -(18° + 0.515°)$
$= -(18° + 0.515(60'))$
$= -(18° + 30.9') = -(18° + 30' + 0.9')$
$= -(18° + 30' + 0.9(60''))$
$= -(18° + 30' + 54'') = -18°30'54''$

70. $-25.485° = -(25° + 0.485°)$
$= -(25° + 0.485(60'))$
$= -(25° + 29.1') = -(25° + 29' + 0.1')$
$= -(25° + 29' + 0.1(60''))$
$= -(25° + 29' + 6'') = -25°29'06''$

71. $31.4296° = 31° + 0.4296° = 31° + 0.4296(60')$
$= 31° + 25.776' = 31° + 25' + 0.776'$
$= 31° + 25' + 0.776(60'')$
$= 31°25'46.56'' \approx 31°\,25'\,47''$

72. $59.0854° = 59° + 0.0854° = 59° + 0.0854(60')$
$= 59° + 5.124' = 59° + 5' + 0.124'$
$= 59° + 5' + 0.124(60'')$
$= 59°05'7.44'' \approx 59°05'07''$

73. $89.9004° = 89° + 0.9004° = 89° + 0.9004(60')$
$= 89° + 54.024' = 89° + 54' + 0.024'$
$= 89° + 54' + 0.024(60'')$
$= 89°54'1.44'' \approx 89°54'01''$

74. $102.3771° = 102° + 0.3771°$
$= 102° + 0.3771(60')$
$= 102° + 22.626'$
$= 102° + 22' + 0.626'$
$= 120° + 22' + 0.626(60'')$
$= 102°\ 22'\ 37.56'' \approx 102°\ 22'\ 38''$

75. $178.5994° = 178° + 0.5994°$
$= 178° + 0.5994(60')$
$= 178° + 35.964'$
$= 178° + 35' + 0.964'$
$= 178° + 35' + 0.964(60'')$
$= 178°35'57.84'' \approx 178°35'58''$

76. $122.6853° = 122° + 0.6853°$
$= 122° + 0.6853(60')$
$= 122° + 41.118'$
$= 122° + 41' + 0.118'$
$= 122° + 41' + 0.118(60'')$
$= 122°\ 41'\ 7.08'' \approx 122°\ 41'\ 07''$

77. 32° is coterminal with $360° + 32° = 392°$.

78. 86° is coterminal with $360° + 86° = 446°$.

79. $26°30'$ is coterminal with $360° + 26°30' = 386°30'$.

80. $58°40'$ is coterminal with $360° + 58°40' = 418°40'$.

81. –40° is coterminal with 360° + (–40°) = 320°

82. $-98°$ is coterminal with $360° + (-98°) = 262°$.

83. $-125°30'$ is coterminal with $360° + (-125°30') = 359°60' - 125°30' = 234°30'$.

84. $-203°20'$ is coterminal with $360° + (-203°20') = 359°60' - 203°20' = 156°40'$.

85. $361°$ is coterminal with $361° - 360° = 1°$.

86. $541°$ is coterminal with $541° - 360° = 181°$.

87. $-361°$ is coterminal with $-361° + 2(360°) = 359°$.

88. $-541°$ is coterminal with $-541° + 2(360°) = 179°$.

89. $539°$ is coterminal with $539° - 360° = 179°$.

90. $699°$ is coterminal with $699° - 360° = 339°$.

91. $850°$ is coterminal with $850° - 2(360°) = 850° - 720° = 130°$.

92. $1000°$ is coterminal with $1000° - 2 \cdot 360° = 1000° - 720° = 280°$.

93. $5280°$ is coterminal with $5280° - 14 \cdot 360° = 5280° - 5040° = 240°$.

94. $8440°$ is coterminal with $8440° - 23 \cdot 360° = 8440° - 8280° = 160°$.

95. $-5280°$ is coterminal with $-5280° + 15 \cdot 360° = -5280° + 5400° = 120°$.

96. $-8440°$ is coterminal with $-8440° + 24 \cdot 360° = -8440° + 8640° = 200°$.

In exercises 97−100, answers may vary.

97. $90°$ is coterminal with
$$90° + 360° = 450°$$
$$90° + 2(360°) = 810°$$
$$90° - 360° = -270°$$
$$90° - 2(360°) = -630°$$

98. $180°$ is coterminal with
$$180° + 360° = 540°$$
$$180° + 2(360°) = 900°$$
$$180° - 360° = -180°$$
$$180° - 2(360°) = -540°$$

99. $0°$ is coterminal with
$$0° + 360° = 360°$$
$$0° + 2(360°) = 720°$$
$$0° - 360° = -360°$$
$$0° - 2(360°) = -720°$$

100. $270°$ is coterminal with
$$270° + 360° = 630°$$
$$270° + 2(360°) = 990°$$
$$270° - 360° = -90°$$
$$270° - 2(360°) = -450°$$

101. $30°$
A coterminal angle can be obtained by adding an integer multiple of $360°$.
$30° + n \cdot 360°$

102. $45°$
A coterminal angle can be obtained by adding an integer multiple of $360°$.
$45° + n \cdot 360°$

103. $135°$
A coterminal angle can be obtained by adding an integer multiple of $360°$.
$135° + n \cdot 360°$

104. $225°$
A coterminal angle can be obtained by adding an integer multiple of $360°$.
$225° + n \cdot 360°$

105. $-90°$
A coterminal angle can be obtained by adding an integer multiple of $360°$.
$-90° + n \cdot 360°$

106. $-180°$
A coterminal angle can be obtained by adding integer multiple of $360°$.
$-180° + n \cdot 360°$

107. $0°$
A coterminal angle can be obtained by adding integer multiple of $360°$.
$0° + n \cdot 360° = n \cdot 360°$

108. $360°$
A coterminal angle can be obtained by adding integer multiple of $360°$.
$360° + n \cdot 360°$, or $n \cdot 360°$

109. The answers to Exercises 107 and 108 give the same set of angles because $0°$ is coterminal with $360°$.

110. **A.** $360° + r°$ is coterminal with $r°$ because you are adding an integer multiple of $360°$ to $r°$, $r° + 1 \cdot 360°$.

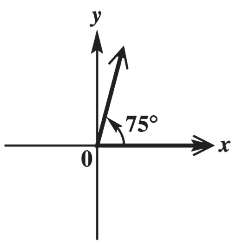
B. $r° - 360°$ is coterminal with $r°$ because you are adding an integer multiple of $360°$ to $r°$, $r° + (-1) \cdot 360°$.

C. $360° - r°$ is not coterminal with $r°$ because you are not adding an integer multiple of 360° to $r°$.
$360° - r° \neq r° + n \cdot 360°$ for an integer value n.

D. $r° + 180°$ is not coterminal with $r°$ because you are not adding an integer multiple of 360° to $r°$.
$r° + 180° \neq r° + n \cdot 360°$ for an integer value n. You are adding $\frac{1}{2}(360°)$.
Thus, choices C and D are not coterminal with $r°$.

For Exercises 111−122, angles other than those given are possible.

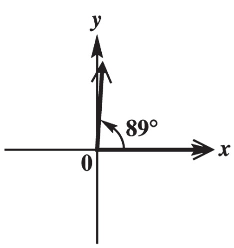
111.



435°; –285°; quadrant I

75° is coterminal with 75° + 360° = 435° and 75° – 360° = –285°. These angles are in quadrant I.

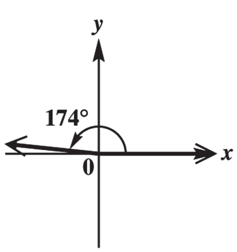
112.



449°; –271°; quadrant I

89° is coterminal with 89° + 360° = 449° and 89° – 360° = –271°. These angles are in quadrant I.

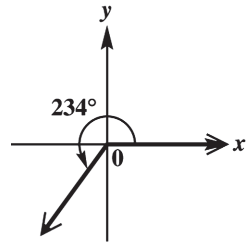
113.



534°; –186°; quadrant II

174° is coterminal with 174° + 360° = 534° and 174° – 360° = –186°. These angles are in quadrant II.

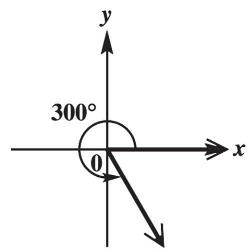
114.



594°; –126°; quadrant III

234° is coterminal with 234° + 360° = 594° and 234° – 360° = –126°. These angles are in quadrant III.

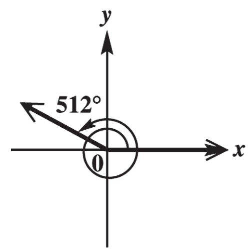
115.



660°; –60°; quadrant IV

300° is coterminal with 300° + 360° = 660° and 300° – 360° = –60°. These angles are in quadrant IV.

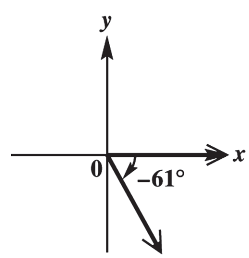
116.



152°; –208°; quadrant II

512° is coterminal with $512° - 360° = 152°$ and $512° - 2 \cdot 360° = -208°$. These angles are in quadrant II.

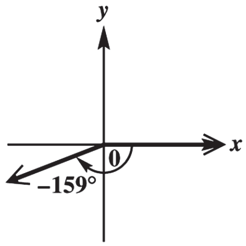
117.



299°; –421°; quadrant IV

–61° is coterminal with $-61° + 360° = 299°$ and $-61° - 360° = -421°$. These angles are in quadrant IV.

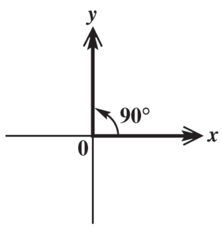
118.



201°; –519°; quadrant III

$-159°$ is coterminal with $-159° + 360° = 201°$ and $-159° - 360° = -519°$. These angles are in quadrant III.

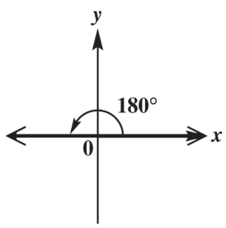
119.



450°; –270°; no quadrant

$90°$ is coterminal with $90° + 360° = 450°$ and $90° - 360° = -270°$. These angles are not in a quadrant.

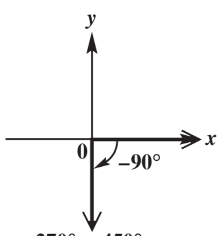
120.



540°; –180°; no quadrant

$180°$ is coterminal with $180° + 360° = 540°$ and $180° - 360° = -180°$. These angles are not in a quadrant.

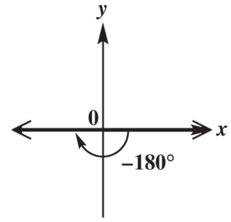
121.



270°; –450°; no quadrant

$-90°$ is coterminal with $-90° + 360° = 270°$ and $-90° - 360° = -450°$. These angles are not in a quadrant.

122.



180°; –540°; no quadrant

$-180°$ is coterminal with $-180° + 360° = 180°$ and $-180° - 360° = -540°$. These angles are not in a quadrant.

123. 45 revolutions per min $= \frac{45}{60}$ revolution per sec

$= \frac{3}{4}$ revolution per sec

A turntable will make $\frac{3}{4}$ revolution in 1 sec.

124. 90 revolutions per min $= \frac{90}{60}$ revolutions per min = 1.5 revolutions per sec

A windmill will make 1.5 revolutions in 1 sec.

125. 600 rotations per min $= \frac{600}{60}$ rotations per sec

$= 10$ rotations per sec

$= 5$ rotations per $\frac{1}{2}$ sec

$= 5(360°)$ per $\frac{1}{2}$ sec

$= 1800°$ per $\frac{1}{2}$ sec

A point on the edge of the tire will move $1800°$ in $\frac{1}{2}$ sec.

126. If the propeller rotates 1000 times per minute, then it rotates $\frac{1000}{60} = 16\frac{2}{3}$ times per sec. Each rotation is 360°, so the total number of degrees a point rotates in 1 sec is $(360°)\left(16\frac{2}{3}\right) = 6000°$. So the propeller rotates 12,000° in 2 sec.

127. $75°$ per min $= 75°(60)$ per hr $= 4500°$ per hr

$= \frac{4500°}{360°}$ rotations per hr

$= 12.5$ rotations per hr

The pulley makes 12.5 rotations in 1 hour.

128. First, convert 74.25° to degrees and minutes. Find the difference between this measurement and 74°20′.

$74.25° = 74° + 0.25(60') = 74°15'$, so

$74°20' - 74°15' = 5'$

Next, convert 74°20′ to decimal degrees. Find the difference between this measurement and 74.25° rounded to the nearest hundredth of a degree.

(*continued on next page*)

(*continued*)

$74° \, 20' = 74° + \frac{20}{60}° \approx 74.333°$, so

$74.333° - 74.250° = 0.083° \approx 0.08°$

The difference in measurements is 5′ to the nearest minute or 0.08° to the nearest hundredth of a degree.

129. Earth rotates 360° in 24 hr. 360° is equal to $360(60') = 21,600'$.

$$\frac{24 \text{ hr}}{21,600'} = \frac{x}{1'} \Rightarrow$$
$$x = \frac{24}{21,600} \text{hr} = \frac{24}{21,600}(60 \text{ min})$$
$$= \frac{1}{15} \text{ min} = \frac{1}{15}(60 \text{ sec}) = 4 \text{ sec}$$

It should take the motor 4 sec to rotate the telescope through an angle of 1 min.

130. Because we have five central angles that comprise a full circle, we have $5(2\theta) = 360° \Rightarrow 10\theta = 360° \Rightarrow \theta = 36°$

The angle of each point of the five-pointed star measures 36°.

Section 1.2 Angle Relationships and Similar Triangles

1. The sum of the measures of the angles of any triangle is <u>180°</u>.

2. An isosceles right triangle has one <u>right</u> angle and <u>two</u> equal sides.

3. An equilateral triangle has <u>three</u> equal sides.

4. If two triangles are similar, then their corresponding <u>sides</u> are proportional and their corresponding <u>angles</u> have equal measure.

5. In the figure, there are two parallel lines and a transversal, so the measures of angles 1, 2, 5 and 6 are all the same. Also, the measures of the angle marked 131° and angles 3, 4, and 7 are the same. Angle 1 is supplementary to the angle marked 131°, so the measure of angle 1 is 49°, as are the measures of angles 2, 5, and 6.

6. $m\angle 1 = 180° - 120° = 60°$
$m\angle 2 = 120°$
$m\angle 3 = m\angle 1 = 60°$
$m\angle 4 = m\angle 3 = 60°$
$m\angle 5 = 180° - 55° - m\angle 4$
$= 180° - 55° - 60° = 65°$
$m\angle 6 = \angle 4 = 60°$
$m\angle 7 = m\angle 5 = 65°$
$m\angle 8 = 55°$; $m\angle 9 = 55°$
$m\angle 10 = m\angle 9 = 55°$

7. Corresponding angles are *A* and *P*, *B* and *Q*, *C* and *R*. Corresponding sides are *AC* and *PR*, *BC* and *QR*, *AB* and *PQ*.

8. Corresponding angles are *A* and *P*, *C* and *R*, *B* and *Q*. Corresponding sides are *AC* and *PR*, *CB* and *RQ*, *AB* and *PQ*.

9. Corresponding angles are *A* and *C*, *E* and *D*, *ABE* and *CBD*. Corresponding sides are *EB* and *DB*, *AB* and *CB*, *AE* and *CD*

10. Corresponding angles are *H* and *F*, *K* and *E*, *HGK* and *FGE*. Corresponding sides are *HK* and *FE*, *GK* and *GE*, *HG* and *FG*.

11. The two indicated angles are vertical angles, so their measures are equal.
$5x - 129 = 2x - 21 \Rightarrow 3x = 108 \Rightarrow x = 36$
$5(36) - 129 = 51$ and $2(36) - 21 = 51$, so both angles measure 51°.

12. The two indicated angles are vertical angles, so their measures are equal.
$11x - 37 = 7x + 27 \Rightarrow 4x = 64 \Rightarrow x = 16$
$11(16) - 37 = 139$ and $7(16) + 27 = 139$, so both angles measure 139°.

13. The three angles are the interior angles of a triangle, so the sum of their measures is 180°.
$x + (x + 20) + (210 - 3x) = 180 \Rightarrow$
$230 - x = 180 \Rightarrow x = 50$
50 + 20 = 70 and 210 – 3(50) = 60, so the three angles measure 50°, 60°, and 70°.

14. The three angles are the interior angles of a triangle, so the sum of their measures is 180°.
$(x + 15) + (x + 5) + (10x - 20) = 180 \Rightarrow$
$12x = 180 \Rightarrow x = 15$
15 + 15 = 30, 15 + 5 = 20, and 10(15) – 20 = 130, so the three angles measure 20°, 30°, and 130°.

15. The three angles are the interior angles of a triangle, so the sum of their measures is 180°.

$$(2x - 120) + \left(\tfrac{1}{2}x + 15\right) + (x - 30) = 180$$
$$\tfrac{7}{2}x - 135 = 180$$
$$\tfrac{7}{2}x = 315$$
$$x = 90$$

$2(90) - 120 = 60, \frac{1}{2}(90) + 15 = 60$, and 90 – 30 = 60, so the three angles each measure 60°.

16. The three angles are the interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned}(2x+16)+(5x-50)+(3x-6)&=180\\ 10x-40&=180\\ 10x&=220\\ x&=22\end{aligned}$$

$2(22)+16=60, 5(22)-50=60,$ and $3(22)-6=60$, so the three angles each measure 60°.

17. In a triangle, the measure of an exterior angle equals the sum of the measures of the non-adjacent interior angles. Thus,

$$\begin{aligned}(6x+3)+(4x-3)&=9x+12\\ 10x&=9x+12\\ x&=12\end{aligned}$$

$6(12)+3=75, 4(12)-3=45,$ and $9(12)+12=120$, so the three angles measure 45°, 75°, and 120°.

18. In a triangle, the measure of an exterior angle equals the sum of the measures of the non-adjacent interior angles. Thus,

$$\begin{aligned}(-8x+3)+(-5x)&=7-12x\\ -13x+3&=7-12x\\ -4&=x\end{aligned}$$

$-8(-4)+3=35, -5(-4)=20,$ and $7-12(-4)=55$, so the three angles measure 20°, 35°, and 55°.

19. The two angles are alternate interior angles, so their measures are equal.

$2x-5=x+22 \Rightarrow x=27$

$2(27)-5=49$ and $27+22=49$, so both angles measure 49°.

20. The two angles are alternate exterior angles, so their measures are equal.

$2x+61=6x-51 \Rightarrow 112=4x \Rightarrow 28=x$

$2(28)+61=117$ and $6(28)-51=117$, so both angles measure 117°.

21. The two angles are interior angles on the same side of the transversal, so the sum of their measures is 180°.

$(x+1)+(4x-56)=180 \Rightarrow 5x-55=180 \Rightarrow$
$5x=235 \Rightarrow x=47$

$47+1=48$ and $4(47)-56=132$, so the angles measure 48° and 132°.

22. The two angles are alternate exterior angles, so their measures are equal.

$15x-54=10x+11 \Rightarrow 5x=65 \Rightarrow x=13$

$15(13)-54=141$ and $10(13)+11=141,$ so both angles measure 141°.

23. Let x = the measure of the third angle. Then

$37+52+x=180 \Rightarrow x=91$

The third angle of the triangle measures 91°.

24. Let x = the measure of the third angle. Then

$29+104+x=180 \Rightarrow x=47$

The third angle of the triangle measures 47°.

25. Let x = the measure of the third angle. Then

$$\begin{aligned}147°12'+30°19'+x&=180°\\ 177°31'+x&=180°\\ 177°31'+x&=179°60' \Rightarrow x=2°29'\end{aligned}$$

The third angle of the triangle measures $2°29'$.

26. Let x = the measure of the third angle. Then

$$\begin{aligned}136°50'+41°38'+x&=180°\\ 177°88'+x&=180°\\ 178°28'+x&=179°60' \Rightarrow x=1°32'\end{aligned}$$

The third angle of the triangle measures $1°32'$.

27. Let x = the measure of the third angle. Then

$74.2°+80.4°+x=180° \Rightarrow x=25.4°$

The third angle of the triangle measures 25.4°.

28. Let x = the measure of the third angle. Then

$29.6°+49.7°+x=180° \Rightarrow x=100.7°$

The third angle of the triangle measures 100.7°.

29. Let x = the measure of the third angle. Then

$$\begin{aligned}51°20'14''+106°10'12''+x&=180°\\ 157°30'26''+x&=180°\\ 157°30'26''+x&=179°59'60''\\ x&=22°29'34''\end{aligned}$$

The third angle of the triangle measures $22°29'34''$.

30. Let x = the measure of the third angle. Then

$$\begin{aligned}17°41'13''+96°12'10''+x&=180°\\ 113°53'23''+x&=180°\\ 113°53'23''+x&=179°59'60''\\ x&=66°06'37''\end{aligned}$$

The third angle of the triangle measures $66°06'37''$.

31. No, a triangle cannot have angles of measures 85° and 100°. The sum of the measures of these two angles is 85° + 100°=185°, which exceeds 180°.

32. No, a triangle cannot have two obtuse angles. An obtuse angle measures between 90° and 180°, so the sum of two obtuse angles would be between 180° and 360°, which exceeds 180°.

33. The triangle has a right angle, but each side has a different measure. The triangle is a right triangle and a scalene triangle.

34. The triangle has one obtuse angle and three unequal sides, so it is obtuse and scalene.

35. The triangle has three acute angles and three equal sides, so it is acute and equilateral.

36. The triangle has two equal sides and all angles are acute, so it is acute and isosceles.

37. The triangle has a right angle and three unequal sides, so it is right and scalene.

38. The triangle has one obtuse angle and two equal sides, so it is obtuse and isosceles.

39. The triangle has a right angle and two equal sides, so it is right and isosceles.

40. The triangle has a right angle with three unequal sides, so it is right and scalene.

41. The triangle has one obtuse angle and three unequal sides, so it is obtuse and scalene.

42. This triangle has three equal sides and all angles are acute, so it is acute and equilateral.

43. The triangle has three acute angles and two equal sides, so it is acute and isosceles.

44. This triangle has a right angle with three unequal sides, so it is right and scalene.

45. Angles 1, 2, and 3 form a straight angle on line *m* and, therefore, sum to 180°. It follows that the sum of the measures of the angles of triangle *PQR* is 180°, because the angles marked 1 are alternate interior angles whose measures are equal, as are the angles marked 2.

46. Connect the right end of the semicircle to the point where the arc crosses the semicircle. The setting of the compass has never changed, so the triangle is equilateral. Therefore, each of its angles measures 60°.

47. Angle Q corresponds to angle A, so the measure of angle Q is 42°. Angles A, B, and C are interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ \\ 42^\circ + m\angle B + 90^\circ &= 180^\circ \\ 132^\circ + m\angle B &= 180^\circ \\ m\angle B &= 48^\circ \end{aligned}$$

Angle R corresponds to angle B, so the measure of angle R is 48°.

48. Angle M corresponds to angle B, so the measure of angle M is 46°. Angle P corresponds to angle C, so the measure of angle P is 78°. Angles A, B, and C are interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ \\ m\angle A + 46^\circ + 78^\circ &= 180^\circ \\ m\angle A + 124^\circ &= 180^\circ \Rightarrow m\angle A = 56^\circ \end{aligned}$$

Angle N corresponds to angle A, so the measure of angle N is 56°.

49. Angle B corresponds to angle K, so the measure of angle B is 106°. Angles A, B, and C are interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned} m\angle A + m\angle B + m\angle C &= 180^\circ \\ m\angle A + 106^\circ + 30^\circ &= 180^\circ \\ m\angle A + 136^\circ &= 180^\circ \\ m\angle A &= 44^\circ \end{aligned}$$

Angle M corresponds to angle A, so the measure of angle M is 44°.

50. Angle Y corresponds to angle V, so the measure of angle Y is 28°. Angle T corresponds to angle X, so the measure of angle T is 74°. Angles X, Y, and Z are interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned} m\angle X + m\angle Y + m\angle Z &= 180^\circ \\ 74^\circ + 28^\circ + m\angle Z &= 180^\circ \\ 102^\circ + m\angle Z &= 180^\circ \\ m\angle Z &= 78^\circ \end{aligned}$$

Angle W corresponds to angle Z, so the measure of angle W is 78°.

51. Angles X, Y, and Z are interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned} m\angle X + m\angle Y + m\angle Z &= 180^\circ \\ m\angle X + 90^\circ + 38^\circ &= 180^\circ \\ m\angle X + 128^\circ &= 180^\circ \\ m\angle X &= 52^\circ \end{aligned}$$

Angle M corresponds to angle X, so the measure of angle M is 52°.

52. Angle T corresponds to angle P, so the measure of angle T is 20°. Angle V corresponds to angle Q, so the measure of angle V is 64°. Angles P, Q, and R are interior angles of a triangle, so the sum of their measures is 180°.

$$m\angle P + m\angle Q + m\angle R = 180°$$
$$20° + 64° + m\angle R = 180°$$
$$84° + m\angle R = 180° \Rightarrow m\angle R = 96°$$

Because angle U corresponds to angle R, the measure of angle U is 96°.

In Exercises 53−58, corresponding sides of similar triangles are proportional. Other proportions are possible in solving these exercises.

53. $\frac{25}{10} = \frac{a}{8} \Rightarrow 8(25) = 10a \Rightarrow 200 = 10a \Rightarrow 20 = a$

$\frac{25}{10} = \frac{b}{6} \Rightarrow 6(25) = 10b \Rightarrow 150 = 10b \Rightarrow 15 = b$

54. $\frac{a}{10} = \frac{75}{25} \Rightarrow 25a = 75(10) \Rightarrow a = 30$

$\frac{b}{20} = \frac{75}{25} \Rightarrow 25b = 75(20) \Rightarrow b = 60$

55. $\frac{6}{12} = \frac{a}{12} \Rightarrow a = 6$

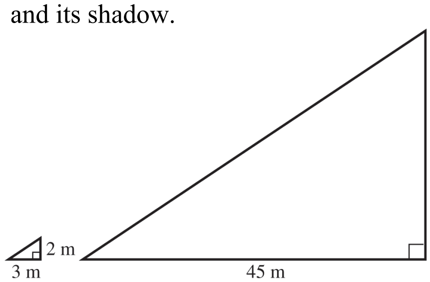
$\frac{6}{12} = \frac{b}{15} \Rightarrow \frac{1}{2} = \frac{b}{15} \Rightarrow b = \frac{15}{2} = 7\frac{1}{2}$

56. $\frac{a}{6} = \frac{3}{9} \Rightarrow \frac{a}{6} = \frac{1}{3} \Rightarrow 3a = 6 \Rightarrow a = 2$

57. $\frac{4}{6} = \frac{x}{9} \Rightarrow 6x = 36 \Rightarrow x = 6$

58. $\frac{m}{21} = \frac{12}{14} \Rightarrow 14m = 252 \Rightarrow m = 18$

59. Let x = the height of the tree.
The triangle formed by the tree and its shadow is similar to the triangle formed by the stick and its shadow.

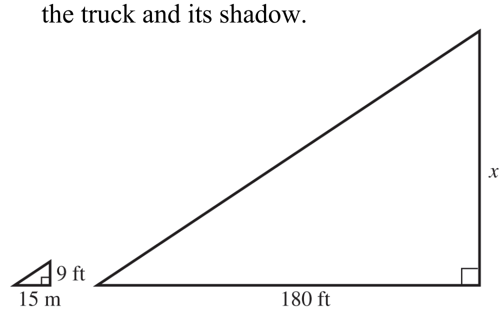


not drawn to scale

$\frac{x}{2} = \frac{45}{3} \Rightarrow \frac{x}{2} = \frac{15}{1} \Rightarrow x = 30$

The tree is 30 m high.

60. Let x = the height of the tower.
The triangle formed by the lookout tower and its shadow is similar to the triangle formed by the truck and its shadow.



not drawn to scale

$\frac{x}{180} = \frac{9}{15} \Rightarrow \frac{x}{180} = \frac{3}{5} \Rightarrow 5x = 540 \Rightarrow x = 108$

The tower is 108 ft tall.

61. Let x = the middle side of the actual triangle (in meters); y = the longest side of the actual triangle (in meters).
The triangles in the photograph and the piece of land are similar. The shortest side on the land corresponds to the shortest side on the photograph.

$\frac{400 \text{ m}}{4 \text{ cm}} = \frac{x}{5 \text{ cm}} \Rightarrow \frac{100}{1} = \frac{x}{5} \Rightarrow x = 500$ and

$\frac{400 \text{ m}}{4 \text{ cm}} = \frac{y}{7 \text{ cm}} \Rightarrow \frac{100}{1} = \frac{y}{7} \Rightarrow y = 700$

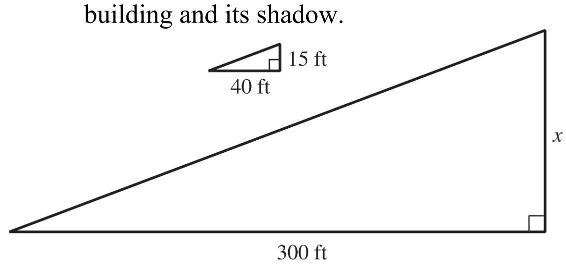
The other two sides are 500 m and 700 m long.

62. Let x = the height of the lighthouse.

$\frac{x}{1.75} = \frac{28}{3.5} \Rightarrow 3.5x = 1.75(28) \Rightarrow$
$3.5x = 49 \Rightarrow x = 14$

The lighthouse is 14 m tall.

63. Let x = the height of the building.
The triangle formed by the house and its shadow is similar to the triangle formed by the building and its shadow.



$\frac{15}{40} = \frac{x}{300} \Rightarrow 40x = 4500 \Rightarrow x = 112.5$

The building is 112.5 ft tall.

64. Let x = length of the entire body carved into the mountain.

$$\frac{\frac{3}{4}}{60} = \frac{6\frac{1}{3}}{x} \Rightarrow \frac{3}{4}x = 60\left(\frac{19}{3}\right) \Rightarrow \frac{3}{4}x = 380 \Rightarrow$$
$$x = \frac{1520}{3} = 506\tfrac{2}{3}$$

Abraham Lincoln's body would be $506\frac{2}{3}$ ft tall.

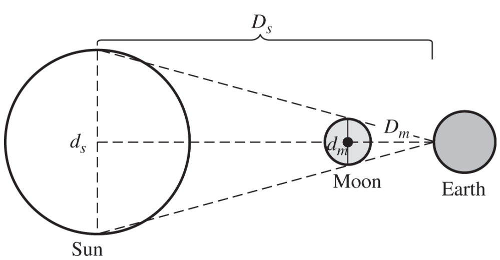
65. $$\frac{x}{50} = \frac{100+120}{100} \Rightarrow \frac{x}{50} = \frac{220}{100} \Rightarrow \frac{x}{50} = \frac{11}{5} \Rightarrow$$
$$5x = 550 \Rightarrow x = 110$$

66. $$\frac{y}{60} = \frac{40}{160+40} \Rightarrow \frac{y}{60} = \frac{40}{200} \Rightarrow \frac{y}{60} = \frac{1}{5} \Rightarrow$$
$$5y = 60 \Rightarrow y = 12$$

67. $$\frac{c}{100} = \frac{10+90}{90} \Rightarrow \frac{c}{100} = \frac{100}{90} \Rightarrow \frac{c}{100} = \frac{10}{9} \Rightarrow$$
$$9c = 1000 \Rightarrow c = \frac{1000}{9} \approx 111.1$$

68. $$\frac{m}{80} = \frac{75+5}{75} \Rightarrow \frac{m}{80} = \frac{80}{75} \Rightarrow 75m = 6400 \Rightarrow$$
$$m = \frac{6400}{75} = \frac{256}{3} \approx 85.3$$

69. **(a)** Let D_s = the distance from the Earth to the sun; d_s = the diameter of the sun, D_m = the distance from the Earth to the moon; and d_m = the diameter of the moon.



$$\frac{D_s}{D_m} = \frac{d_s}{d_m}$$
$$\frac{94,500,000}{D_m} = \frac{865,000}{2159}$$
$$865,000D_m = 94,500,000 \cdot 2159$$
$$D_m = \frac{94,500,000 \cdot 2159}{865,000}$$
$$\approx 236,000 \text{ mi}$$

(b) No, a total solar eclipse cannot occur every time. The moon must be less than 236,000 miles away from Earth for an eclipse to occur, and sometimes it is farther than this.

70. **(a)** Let D_s = the distance from Neptune to the sun; d_s = the diameter of the sun, D_m = the distance from Neptune to Triton; and d_m = the diameter of Triton.

$$\frac{D_s}{D_m} = \frac{d_s}{d_m}$$
$$\frac{2,800,000,000}{D_m} = \frac{865,000}{1680}$$
$$865,000D_m = 2,800,000,000 \cdot 1680$$
$$D_m = \frac{2,800,000,000 \cdot 1680}{865,000}$$
$$\approx 5,438,000 \text{ mi}$$

(b) Triton is approximately 220,000 miles from Neptune, so it is possible for Triton to cause a total eclipse on Neptune.

71. **(a)** Let D_s = the distance from Mars to the sun; d_s = the diameter of the sun, D_m = the distance from Mars to Phobos; and d_m = the diameter of Phobos.

$$\frac{D_s}{D_m} = \frac{d_s}{d_m}$$
$$\frac{142,000,000}{D_m} = \frac{865,000}{17.4}$$
$$865,000D_m = 142,000,000 \cdot 17.4$$
$$D_m = \frac{142,000,000 \cdot 17.4}{865,000}$$
$$\approx 2900 \text{ mi}$$

(b) No, Phobos does not come close enough to the surface of Mars.

72. **(a)** Let D_s = the distance from Jupiter to the sun; d_s = the diameter of the sun, D_m = the distance from Jupiter to Ganymede; and d_m = the diameter of Ganymede.

$$\frac{D_s}{D_m} = \frac{d_s}{d_m}$$
$$\frac{484,000,000}{D_m} = \frac{865,000}{3270}$$
$$865,000D_m = 484,000,000 \cdot 3270$$
$$D_m = \frac{484,000,000 \cdot 3270}{865,000}$$
$$\approx 1,830,000 \text{ mi}$$

(b) Yes, Ganymede can cause a total eclipse on Jupiter.

73. **(a)** The thumb covers about 2 arc degrees or about 120 arc minutes. This is $\frac{31}{120}$ or approximately $\frac{1}{4}$ of the thumb would cover the moon.

(b) 20° + 10° = 30°
The stars are 30 arc degrees apart.

74. **(a)** The ratios of corresponding sides of similar triangles *CAG* and *HAD* are equal and *HD* = 1.

(b) The ratios of corresponding sides of similar triangles *AGE* and *ADB* are equal.

(c) *EF* = *BD* = 1 pace

(d) From parts (a)−(c),
$$CG = \frac{AG}{AD} = \frac{EG}{BD} = \frac{EG}{1} = EG.$$

(e) The height of the tree (in feet) is (approximately) the number of paces.

Chapter 1 Quiz
(Sections 1.1−1.2)

1. 19°

(a) complement: 90° − 19° = 71°

(b) supplement: 180° − 19° = 161°

2. The two angles form a straight angle.
$$(3x+5)+(5x+15)=180 \Rightarrow 8x+20=180 \Rightarrow$$
$$8x=160 \Rightarrow x=20$$
The measures of the two angles are
$(3x+5)° = [3(20)+5]° = 65°$ and
$(5x+15)° = [5(20)+15]° = 115°$.

3. The two angles form a right angle.
$$(5x-1)+2x=90 \Rightarrow 7x-1=90 \Rightarrow$$
$$7x=91 \Rightarrow x=13$$
The measures of the two angles are
$(5x-1)° = [5(13)-1] = 64°$ and
$2x = 2(13) = 26°$.

4. The three angles are the interior angles of a triangle, so the sum of their measures is 180°.
$$\begin{aligned}(3x+3)+(13x+45)+(4x-8)&=180\\ 20x+40&=180\\ 20x&=140\\ x&=7\end{aligned}$$
The measures of the three angles are
$3(7)+3=24°, 13(7)+45=136°,$ and
$4(7)-8=20°$.

5. The two marked angles are supplements, so their sum is 180°.
$$\begin{aligned}(-14x+18)+(-6x+2)&=180\\ -20x+20&=180\\ -20x&=160 \Rightarrow x=-8\end{aligned}$$
The measures of the angles are
$-14(-8)+18=130°$ and $-6(-8)+2=50°$.

6. **(a)** $$\begin{aligned}77°12'09'' &= 77° + \tfrac{12}{60}° + \tfrac{9}{3600}°\\ &= 77° + 0.2° + 0.0025°\\ &= 77.2025°\end{aligned}$$

(b) $$\begin{aligned}22.0250° &= 22° + 0.0250(60')\\ &= 22° + 1.5' = 22° + 01' + 0.5(60'')\\ &= 22° + 01' + 30'' = 22°01'30''\end{aligned}$$

7. **(a)** 410° is coterminal with
410° − 360° = 50°.

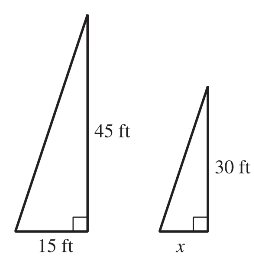
(b) −60° is coterminal with
−60° + 360° = 300°.

(c) 890° is coterminal with
890° − 2(360°) = 890° − 720° = 170°.

(d) 57º is coterminal with 57º + 360º = 417º.

8. 300 rotations per min = $\frac{300}{60} = 5$ rotations per sec = 5(360°) per sec = 1800° per sec
The edge of the flywheel will move 1800° in 1 second.

9.



Using similar triangles, we have
$$\frac{45}{15} = \frac{30}{x} \Rightarrow \frac{3}{1} = \frac{30}{x} \Rightarrow 3x = 30 \Rightarrow x = 10.$$
The pole's shadow is 10 ft.

10. **(a)** $$\frac{6}{9} = \frac{8}{x} \Rightarrow \frac{2}{3} = \frac{8}{x} \Rightarrow 2x = 24 \Rightarrow x = 12$$
$$\frac{6}{9} = \frac{y}{15} \Rightarrow \frac{2}{3} = \frac{y}{15} \Rightarrow 30 = 3y \Rightarrow y = 10$$

(b) The two triangles are similar, so the corresponding angles have the same measure. Note that the angle with measure (10*x* + 8)° and the angle with measure 58° are vertical angles, and thus, are equal.
$10x + 8 = 58 \Rightarrow 10x = 50 \Rightarrow x = 5$.

Section 1.3 Trigonometric Functions

1. The Pythagorean theorem for right triangles states that the sum of the squares of the lengths of the legs is equal to the square of the hypotenuse.

2. In the definitions of the sine, cosine, secant, and cosecant functions, r is interpreted geometrically as the distance from a given point (x, y) on the terminal side of an angle θ in standard position to the origin.

3. For any nonquadrantal angle θ, $\sin\theta$ and $\csc\theta$ will have the same sign.

4. If $\cot\theta$ is undefined, then $\tan\theta = 0$.

5. If the terminal side of an angle θ lies in quadrant III, then the value of $\tan\theta$ and $\cot\theta$ are positive, and all other trigonometric function values are negative.

6. If a quadrantal angle θ is coterminal with 0° or 180° then the trigonometric functions $\cot\theta$ and $\csc\theta$ are undefined.

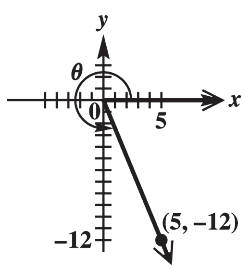
7. $r = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

8. $\sin\theta = \dfrac{y}{r} = \dfrac{-3}{3\sqrt{2}} = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

9. $\cos\theta = \dfrac{x}{r} = \dfrac{-3}{3\sqrt{2}} = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$

10. $\tan\theta = \dfrac{y}{x} = \dfrac{-3}{-3} = 1$

11.



$x = 5,\ y = -12$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-12)^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$\sin\theta = \frac{y}{r} = \frac{-12}{13} = -\frac{12}{13}$$

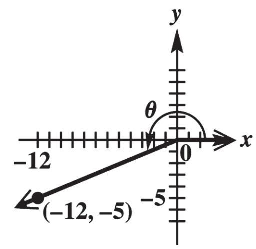
$$\cos\theta = \frac{x}{r} = \frac{5}{13}$$

$$\tan\theta = \frac{y}{x} = \frac{-12}{5} = -\frac{12}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{5}{-12} = -\frac{5}{12}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{5};\ \csc\theta = \frac{r}{y} = \frac{13}{-12} = -\frac{13}{12}$$

12.



$x = -12,\ y = -5$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$\sin\theta = \frac{y}{r} = \frac{-5}{13} = -\frac{5}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}$$

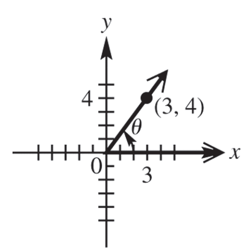
$$\tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$$

$$\cot\theta = \frac{x}{y} = \frac{-12}{-5} = \frac{12}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{13}{-12} = -\frac{13}{12}$$

$$\csc\theta = \frac{r}{y} = \frac{13}{-5} = -\frac{13}{5}$$

13.



$x = 3,\ y = 4$ and

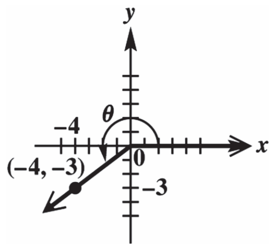
$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\sin\theta = \frac{y}{r} = \frac{4}{5};\ \cos\theta = \frac{x}{r} = \frac{3}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{4}{3};\ \cot\theta = \frac{x}{y} = \frac{3}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{3};\ \csc\theta = \frac{r}{y} = \frac{5}{4}$$

14.



$x = -4$, $y = -3$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$$

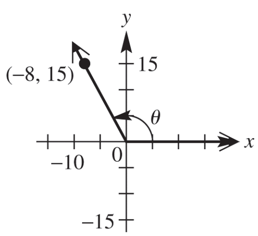
$$\tan\theta = \frac{y}{x} = \frac{-3}{-4} = \frac{3}{4}$$

$$\cot\theta = \frac{x}{y} = \frac{-4}{-3} = \frac{4}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-4} = -\frac{5}{4}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$$

15.



$x = -8$, $y = 15$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-8)^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17$$

$$\sin\theta = \frac{y}{r} = \frac{15}{17}$$

$$\cos\theta = \frac{x}{r} = \frac{-8}{17} = -\frac{8}{17}$$

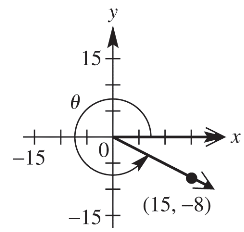
$$\tan\theta = \frac{y}{x} = \frac{15}{-8} = -\frac{15}{8}$$

$$\cot\theta = \frac{x}{y} = \frac{-8}{15} = -\frac{8}{15}$$

$$\sec\theta = \frac{r}{x} = \frac{17}{-8} = -\frac{17}{8}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{15}$$

16.



$x = 15$, $y = -8$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{15^2 + (-8)^2} = \sqrt{225 + 64} = \sqrt{289} = 17$$

$$\sin\theta = \frac{y}{r} = \frac{-8}{17} = -\frac{8}{17}$$

$$\cos\theta = \frac{x}{r} = \frac{15}{17}$$

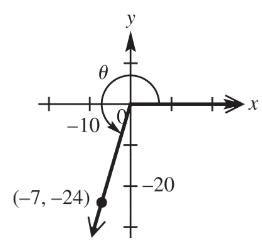
$$\tan\theta = \frac{y}{x} = \frac{-8}{15} = -\frac{8}{15}$$

$$\cot\theta = \frac{x}{y} = \frac{15}{-8} = -\frac{15}{8}$$

$$\sec\theta = \frac{r}{x} = \frac{17}{15}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{-8} = -\frac{17}{8}$$

17.



$x = -7$, $y = -24$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-7)^2 + (-24)^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

$$\sin\theta = \frac{y}{r} = \frac{-24}{25} = -\frac{24}{25}$$

$$\cos\theta = \frac{x}{r} = \frac{-7}{25} = -\frac{7}{25}$$

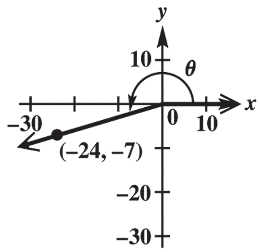
$$\tan\theta = \frac{y}{x} = \frac{-24}{-7} = \frac{24}{7}$$

$$\cot\theta = \frac{x}{y} = \frac{-7}{-24} = \frac{7}{24}$$

$$\sec\theta = \frac{r}{x} = \frac{25}{-7} = -\frac{25}{7}$$

$$\csc\theta = \frac{r}{y} = \frac{25}{-24} = -\frac{25}{24}$$

18.

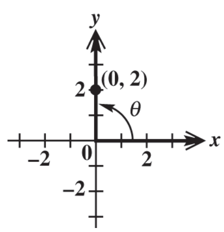


$x = -24,\ y = -7$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-24)^2 + (-7)^2}$$
$$= \sqrt{576 + 49} = \sqrt{625} = 25$$

$$\sin\theta = \frac{y}{r} = \frac{-7}{25} = -\frac{7}{25}$$
$$\cos\theta = \frac{x}{r} = \frac{-24}{25} = -\frac{24}{25}$$
$$\tan\theta = \frac{y}{x} = \frac{-7}{-24} = \frac{7}{24}$$
$$\cot\theta = \frac{x}{y} = \frac{-24}{-7} = \frac{24}{7}$$
$$\sec\theta = \frac{r}{x} = \frac{25}{-24} = -\frac{25}{24}$$
$$\csc\theta = \frac{r}{y} = \frac{25}{-7} = -\frac{25}{7}$$

19.

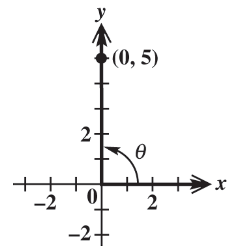


$x = 0,\ y = 2$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + 2^2} = \sqrt{0 + 4} = \sqrt{4} = 2$$

$$\sin\theta = \frac{y}{r} = \frac{2}{2} = 1$$
$$\cos\theta = \frac{x}{r} = \frac{0}{2} = 0$$
$$\tan\theta = \frac{y}{x} = \frac{2}{0} \text{ undefined}$$
$$\cot\theta = \frac{x}{y} = \frac{0}{2} = 0$$
$$\sec\theta = \frac{r}{x} = \frac{2}{0} \text{ undefined}$$
$$\csc\theta = \frac{r}{y} = \frac{2}{2} = 1$$

20.

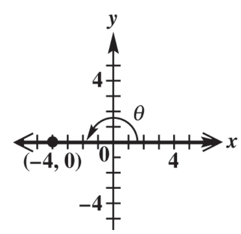


$x = 0,\ y = 5$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + 5^2} = \sqrt{0 + 5} = \sqrt{25} = 5$$

$$\sin\theta = \frac{y}{r} = \frac{5}{5} = 1$$
$$\cos\theta = \frac{x}{r} = \frac{0}{5} = 0$$
$$\tan\theta = \frac{y}{x} = \frac{5}{0} \text{ undefined}$$
$$\cot\theta = \frac{x}{y} = \frac{0}{5} = 0$$
$$\sec\theta = \frac{r}{x} = \frac{5}{0} \text{ undefined}$$
$$\csc\theta = \frac{r}{y} = \frac{5}{5} = 1$$

21.

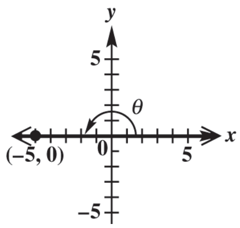


$x = 4,\ y = 0$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + 0^2}$$
$$= \sqrt{16 + 0} = \sqrt{16} = 4$$

$$\sin\theta = \frac{y}{r} = \frac{0}{4} = 0$$
$$\cos\theta = \frac{x}{r} = \frac{-4}{4} = -1$$
$$\tan\theta = \frac{y}{x} = \frac{0}{-4} = 0$$
$$\cot\theta = \frac{x}{y} = \frac{-4}{0} \text{ undefined}$$
$$\sec\theta = \frac{r}{x} = \frac{4}{-4} = -1$$
$$\csc\theta = \frac{r}{y} = \frac{4}{0} \text{ undefined}$$

22.

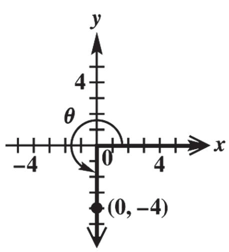


$x = -5, y = 0$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + 0^2} = \sqrt{25 + 0} = \sqrt{25} = 5$$

$$\sin\theta = \frac{y}{r} = \frac{0}{5} = 0$$
$$\cos\theta = \frac{x}{r} = \frac{-5}{5} = -1$$
$$\tan\theta = \frac{y}{x} = \frac{0}{-5} = 0$$
$$\cot\theta = \frac{x}{y} = \frac{-5}{0} \text{ undefined}$$
$$\sec\theta = \frac{r}{x} = \frac{5}{-5} = -1$$
$$\csc\theta = \frac{r}{y} = \frac{5}{0} \text{ undefined}$$

23.

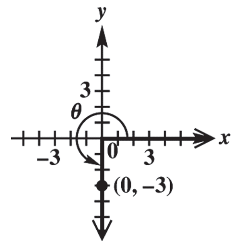


$x = 0, y = -4$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-4)^2} = \sqrt{0 + 16} = \sqrt{16} = 4$$

$$\sin\theta = \frac{y}{r} = \frac{-4}{4} = -1$$
$$\cos\theta = \frac{x}{r} = \frac{0}{4} = 0$$
$$\tan\theta = \frac{y}{x} = \frac{-4}{0} \text{ undefined}$$
$$\cot\theta = \frac{x}{y} = \frac{0}{-4} = 0$$
$$\sec\theta = \frac{r}{x} = \frac{4}{0} \text{ undefined}$$
$$\csc\theta = \frac{r}{y} = \frac{4}{-4} = -1$$

24.

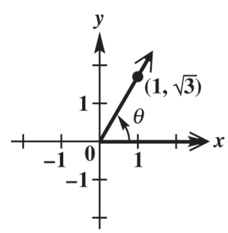


$x = 0, y = -3$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-3)^2} = \sqrt{0 + 9} = \sqrt{9} = 3$$

$$\sin\theta = \frac{y}{r} = \frac{-3}{3} = -1$$
$$\cos\theta = \frac{x}{r} = \frac{0}{3} = 0$$
$$\tan\theta = \frac{y}{x} = \frac{-3}{0} \text{ undefined}$$
$$\cot\theta = \frac{x}{y} = \frac{0}{-3} = 0$$
$$\sec\theta = \frac{r}{x} = \frac{3}{0} \text{ undefined}$$
$$\csc\theta = \frac{r}{y} = \frac{3}{-3} = -1$$

25.

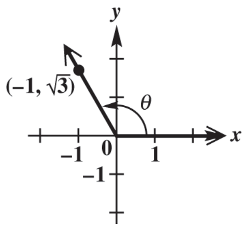


$x = 1,\ y = \sqrt{3}$, and

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = \sqrt{1 + 3} = \sqrt{4} = 2$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2};\ \cos\theta = \frac{x}{r} = \frac{1}{2}$$
$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{1} = \sqrt{3}$$
$$\cot\theta = \frac{x}{y} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\sec\theta = \frac{r}{x} = \frac{2}{1} = 2$$
$$\csc\theta = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

26.



$x = -1,\ y = \sqrt{3},$ and

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{2} = -\frac{1}{2}$$

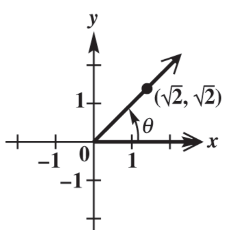
$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{2}{-1} = -2$$

$$\csc\theta = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

27.



$x = \sqrt{2}, y = \sqrt{2},$ and

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(\sqrt{2}\right)^2 + \left(\sqrt{2}\right)^2} = \sqrt{2+2} = \sqrt{4} = 2$$

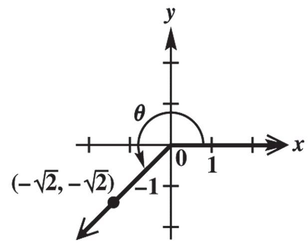
$$\sin\theta = \frac{y}{r} = \frac{\sqrt{2}}{2};\ \cos\theta = \frac{x}{r} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{2}}{\sqrt{2}} = 1;\ \cot\theta = \frac{x}{y} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$\sec\theta = \frac{r}{x} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

28.



$x = -\sqrt{2}, y = -\sqrt{2},$ and

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(-\sqrt{2}\right)^2 + \left(-\sqrt{2}\right)^2} = \sqrt{2+2} = \sqrt{4} = 2$$

$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$

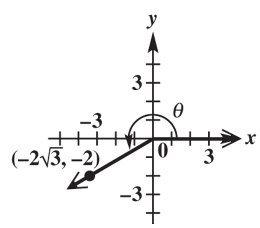
$$\cos\theta = \frac{x}{r} = \frac{-\sqrt{2}}{2} = -\frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{2}}{-\sqrt{2}} = 1;\ \cot\theta = \frac{x}{y} = \frac{-\sqrt{2}}{-\sqrt{2}} = 1$$

$$\sec\theta = \frac{r}{x} = \frac{2}{-\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \frac{2}{-\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$$

29.



$x = -2\sqrt{3},\ y = -2,$ and

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(-2\sqrt{3}\right)^2 + (-2)^2} = \sqrt{12+4} = \sqrt{16} = 4$$

$$\sin\theta = \frac{y}{r} = \frac{-2}{4} = -\frac{1}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$$

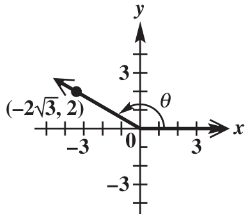
$$\tan\theta = \frac{y}{x} = \frac{-2}{-2\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

$$\sec\theta = \frac{r}{x} = \frac{4}{-2\sqrt{3}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{4}{-2} = -2$$

30.



$x = -2\sqrt{3},\ y = 2,$ and

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(-2\sqrt{3}\right)^2 + (2)^2} = \sqrt{12+4} = \sqrt{16} = 4$$

$$\sin\theta = \frac{y}{r} = \frac{2}{4} = \frac{1}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-2\sqrt{3}}{4} = -\frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{2}{-2\sqrt{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-2\sqrt{3}}{2} = -\sqrt{3}$$

$$\sec\theta = \frac{r}{x} = \frac{4}{-2\sqrt{3}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{4}{2} = 2$$

In Exercises 31−50, $r = \sqrt{x^2 + y^2}$, which is positive.

31. In quadrant II, x is negative, so $\frac{x}{r}$ is negative.

32. In quadrant III, y is negative, so $\frac{y}{r}$ is negative.

33. In quadrant IV, x is positive and y is negative, so $\frac{y}{x}$ is negative.

34. In quadrant IV, x is positive and y is negative, so $\frac{x}{y}$ is negative.

35. In quadrant II, y is positive, so $\frac{y}{r}$ is positive.

36. In quadrant III, x is negative, so $\frac{x}{r}$ is negative.

37. In quadrant IV, x is positive, so $\frac{x}{r}$ is positive.

38. In quadrant IV, y is negative, so $\frac{y}{r}$ is negative.

39. In quadrant II, x is negative and y is positive, so $\frac{x}{y}$ is negative.

40. In quadrant II, x is negative and y is positive, so $\frac{y}{x}$ is negative.

41. In quadrant III, x is negative and y is negative, so $\frac{y}{x}$ is positive.

42. In quadrant III, x is negative and y is negative, so $\frac{x}{y}$ is positive.

43. In quadrant III, x is negative, so $\frac{r}{x}$ is negative.

44. In quadrant III, y is negative, so $\frac{r}{y}$ is negative.

45. In quadrant I, x is positive and y is positive, so $\frac{x}{y}$ is positive.

46. In quadrant I, x is positive and y is positive, so $\frac{y}{x}$ is positive.

47. In quadrant I, y is positive, so $\frac{y}{r}$ is positive.

48. In quadrant I, x is positive, so $\frac{x}{r}$ is positive.

49. In quadrant I, x is positive, so $\frac{r}{x}$ is positive.

50. In quadrant I, y is positive, so $\frac{r}{y}$ is positive.

51. Because $x \ge 0$, the graph of the line $2x + y = 0$ is shown to the right of the y-axis. A point on this line is $(1, -2)$ because $2(1) + (-2) = 0$. The corresponding value of r is $r = \sqrt{1^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$.

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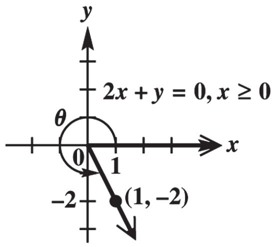


$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{5}} = -\frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

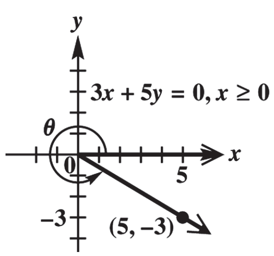
$$\tan\theta = \frac{y}{x} = \frac{-2}{1} = -2$$

$$\cot\theta = \frac{x}{y} = \frac{1}{-2} = -\frac{1}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$$

52. Because $x \ge 0$, the graph of the line $3x + 5y = 0$ is shown to the right of the y-axis. A point on this graph is $(5,-3)$ because $3(5)+5(-3)=0$. The corresponding value of r is $r = \sqrt{5^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$.



$$\sin\theta = \frac{y}{r} = \frac{-3}{\sqrt{34}} = -\frac{3}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}} = -\frac{3\sqrt{34}}{34}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{34}} = \frac{5}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

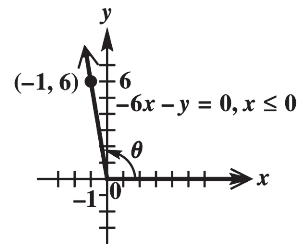
$$\tan\theta = \frac{y}{x} = \frac{-3}{5} = -\frac{3}{5}$$

$$\cot\theta = \frac{x}{y} = \frac{5}{-3} = -\frac{5}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{34}}{5}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{34}}{-3} = -\frac{\sqrt{34}}{3}$$

53. Because $x \le 0$, the graph of the line $-6x - y = 0$ is shown to the left of the y-axis. A point on this graph is $(-1,6)$ because $-6(-1)-6=0$. The corresponding value of r is $r = \sqrt{(-1)^2+6^2} = \sqrt{1+36} = \sqrt{37}$.



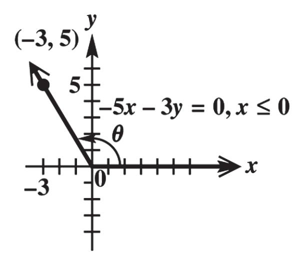
$$\sin\theta = \frac{y}{r} = \frac{6}{\sqrt{37}} = \frac{6}{\sqrt{37}}\cdot\frac{\sqrt{37}}{\sqrt{37}} = \frac{6\sqrt{37}}{37}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{37}} = -\frac{1}{\sqrt{37}}\cdot\frac{\sqrt{37}}{\sqrt{37}} = -\frac{\sqrt{37}}{37}$$

$$\tan\theta = \frac{y}{x} = \frac{6}{-1} = -6;\quad \cot\theta = \frac{x}{y} = \frac{-1}{6} = -\frac{1}{6}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{37}}{-1} = -\sqrt{37};\quad \csc\theta = \frac{r}{y} = \frac{\sqrt{37}}{6}$$

54. Because $x \le 0$, the graph of the line $-5x - 3y = 0$ is shown to the left of the y-axis. A point on this line is $(-3,5)$ because $-5(-3)-3(5)=0$. The corresponding value of r is $r = \sqrt{(-3)^2+5^2} = \sqrt{9+25} = \sqrt{34}$.



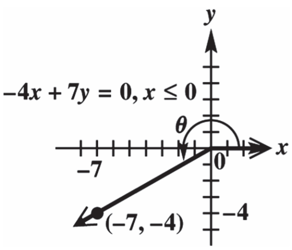
$$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{34}} = \frac{5}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{34}} = -\frac{3}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}} = -\frac{3\sqrt{34}}{34}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{-3} = -\frac{5}{3};\quad \cot\theta = \frac{x}{y} = \frac{-3}{5} = -\frac{3}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{34}}{-3} = -\frac{\sqrt{34}}{3};\quad \csc\theta = \frac{r}{y} = \frac{\sqrt{34}}{5}$$

55. Because $x \le 0$, the graph of the line $-4x + 7y = 0$ is shown to the left of the y-axis. A point on this line is $(-7, -4)$ because $-4(-7) + 7(-4) = 0$. The corresponding value of r is $r = \sqrt{(-7)^2 + (-4)^2} = \sqrt{49 + 16} = \sqrt{65}$.



$$\sin\theta = \frac{y}{r} = \frac{-4}{\sqrt{65}} = -\frac{4}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}} = -\frac{4\sqrt{65}}{65}$$

$$\cos\theta = \frac{x}{r} = \frac{-7}{\sqrt{65}} = -\frac{7}{\sqrt{65}} \cdot \frac{\sqrt{65}}{\sqrt{65}} = -\frac{7\sqrt{65}}{65}$$

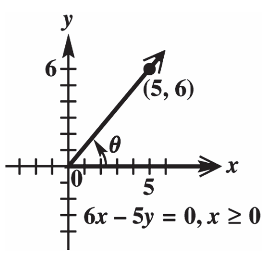
$$\tan\theta = \frac{y}{x} = \frac{-4}{-7} = \frac{4}{7}$$

$$\cot\theta = \frac{x}{y} = \frac{-7}{-4} = \frac{7}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{65}}{-7} = -\frac{\sqrt{65}}{7}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{65}}{-4} = -\frac{\sqrt{65}}{4}$$

56. Because $x \ge 0$, the graph of the line $6x - 5y = 0$ is shown to the right of the y-axis. A point on this line is $(5, 6)$ because $6(5) - 5(6) = 0$. The corresponding value of r is $r = \sqrt{5^2 + 6^2} = \sqrt{25 + 36} = \sqrt{61}$.



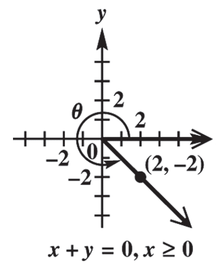
$$\sin\theta = \frac{y}{r} = \frac{6}{\sqrt{61}} = \frac{6}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = \frac{6\sqrt{61}}{61}$$

$$\cos\theta = \frac{x}{r} = \frac{5}{\sqrt{61}} = \frac{5}{\sqrt{61}} \cdot \frac{\sqrt{61}}{\sqrt{61}} = \frac{5\sqrt{61}}{61}$$

$$\tan\theta = \frac{y}{x} = \frac{6}{5}; \ \cot\theta = \frac{x}{y} = \frac{5}{6}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{61}}{5}; \ \csc\theta = \frac{r}{y} = \frac{\sqrt{61}}{6}$$

57. Because $x \ge 0$, the graph of the line $x + y = 0$ is shown to the right of the y-axis. A point on this line is $(2, -2)$ because $2 + (-2) = 0$. The corresponding value of r is $r = \sqrt{2^2 + (-2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$.



$$\sin\theta = \frac{y}{r} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

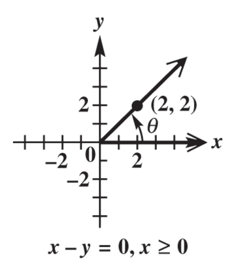
$$\cos\theta = \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{2} = -1; \ \cot\theta = \frac{x}{y} = \frac{2}{-2} = -1$$

$$\sec\theta = \frac{r}{x} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \frac{2\sqrt{2}}{-2} = -\sqrt{2}$$

58. Because $x \ge 0$, the graph of the line $x - y = 0$ is shown to the right of the y-axis. A point on this line is $(2, 2)$ because $2 - 2 = 0$. The corresponding value of r is $r = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$.



$$\sin\theta = \frac{y}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

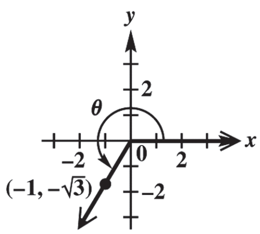
$$\tan\theta = \frac{y}{x} = \frac{2}{2} = 1; \ \cot\theta = \frac{x}{y} = \frac{2}{2} = 1$$

$$\sec\theta = \frac{r}{x} = \frac{2\sqrt{2}}{2} = \sqrt{2}; \ \csc\theta = \frac{r}{y} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

59. Because $x \le 0$, the graph of the line $-\sqrt{3}x + y = 0$ is shown to the left of the y-axis. A point on this line is $\left(-1, -\sqrt{3}\right)$ because $-\sqrt{3}\left(-1\right) - \sqrt{3} = \sqrt{3} - \sqrt{3} = 0$

The corresponding value of r is

$$r = \sqrt{\left(-1\right)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2.$$



$-\sqrt{3}x + y = 0, x \le 0$

$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{2} = -\frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

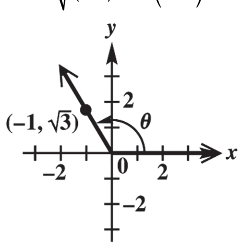
$$\cot\theta = \frac{x}{y} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{2}{-1} = -2$$

$$\csc\theta = \frac{r}{y} = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

60. Because $x \le 0$, the graph of the line $\sqrt{3}x + y = 0$ is shown to the left of the y-axis. A point on this line is $\left(-1, \sqrt{3}\right)$ because $\sqrt{3}\left(-1\right) + \sqrt{3} = -\sqrt{3} + \sqrt{3} = 0$ The corresponding value of r is

$$r = \sqrt{\left(-1\right)^2 + \left(\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2.$$



$\sqrt{3}x + y = 0, x \le 0$

$$\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}; \ \cos\theta = \frac{x}{r} = \frac{-1}{2} = -\frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

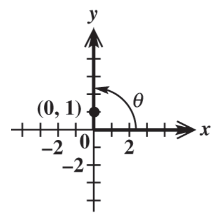
$$\cot\theta = \frac{x}{y} = \frac{-1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{2}{-1} = -2$$

$$\csc\theta = \frac{r}{y} = \frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

61. Because $x = 0$ and $y \ge 0$, the graph is the positive portion of the y-axis. A point on this line is (0, 1). The corresponding value of r is

$$r = \sqrt{\left(0\right)^2 + \left(1\right)^2} = \sqrt{1} = 1.$$



$x = 0, y \ge 0$

$$\sin\theta = \frac{y}{r} = \frac{1}{1} = 1; \ \cos\theta = \frac{x}{r} = \frac{0}{1} = 0$$

$$\tan\theta = \frac{y}{x} = \frac{1}{0}, \text{ undefined}$$

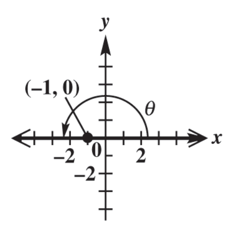
$$\cot\theta = \frac{x}{y} = \frac{0}{1} = 0$$

$$\sec\theta = \frac{r}{x} = \frac{1}{0}, \text{ undefined}$$

$$\csc\theta = \frac{r}{y} = \frac{1}{1} = 1$$

62. Because $y = 0$ and $x \le 0$, the graph is the negative portion of the x-axis. A point on this line is (−1, 0). The corresponding value of r is

$$r = \sqrt{\left(-1\right)^2 + \left(0\right)^2} = \sqrt{1} = 1.$$



$y = 0, x \le 0$

$$\sin\theta = \frac{y}{r} = \frac{0}{1} = 0; \ \cos\theta = \frac{x}{r} = \frac{-1}{1} = -1$$

$$\tan\theta = \frac{y}{x} = \frac{0}{-1} = 0$$

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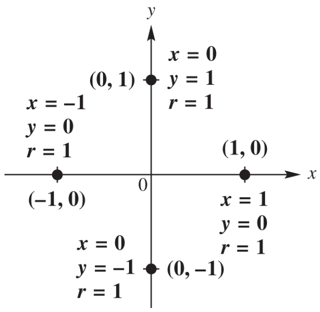
$$\cot\theta = \frac{x}{y} = \frac{-1}{0}, \text{ undefined}$$

$$\sec\theta = \frac{r}{x} = \frac{1}{-1} = -1$$

$$\csc\theta = \frac{r}{y} = \frac{1}{0}, \text{ undefined}$$

Use the figure below to help solve exercises 63−98.



63. $\cos 90°$

$$\cos 90° = \frac{x}{r} = \frac{0}{1} = 0$$

64. $\sin 90°$

$$\sin 90° = \frac{y}{r} = \frac{1}{1} = 1$$

65. $\tan 180°$

$$\tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$$

66. $\cot 90°$

$$\cot 90° = \frac{x}{y} = \frac{0}{1} = 0$$

67. $\sec 180°$

$$\sec 180° = \frac{r}{x} = \frac{1}{-1} = -1$$

68. $\csc 270°$

$$\csc 270° = \frac{r}{y} = \frac{1}{-1} = -1$$

69. $\sin(-270°)$

The quadrantal angle $\theta = -270°$ is coterminal with $-270° + 360° = 90°$.

$$\sin(-270°) = \sin 90 = \frac{y}{r} = \frac{1}{1} = 1$$

70. $\cos(-90°)$

The quadrantal angle $\theta = -90°$ is coterminal with $-90° + 360° = 270°$.

$$\cos(-90°) = \cos 270° = \frac{x}{r} = \frac{0}{1} = 0$$

71. $\cot 540°$

The quadrantal angle $\theta = 540°$ is coterminal with $540° - 360° = 180°$.

$$\cot 540° = \cot 180° = \frac{x}{y} = \frac{-1}{0} \text{ undefined}$$

72. $\tan 450°$

The quadrantal angle $\theta = 450°$ is coterminal with $450° - 360° = 90°$.

$$\tan 450° = \tan 90° = \frac{y}{x} = \frac{1}{0} \text{ undefined}$$

73. $\csc(-450°)$

The quadrantal angle $\theta = -450°$ is coterminal with $720° - 450° = 270°$.

$$\csc(-450°) = \csc 270° = \frac{r}{y} = \frac{1}{-1} = -1$$

74. $\sec(-540°)$

The quadrantal angle $\theta = -540°$ is coterminal with $720° - 540° = 180°$.

$$\sec(-540°) = \sec 180° = \frac{r}{x} = \frac{1}{-1} = -1$$

75. $\sin 1800°$

The quadrantal angle $\theta = 1800°$ is coterminal with $1800° - 5(360°) = 1800° - 1800° = 0°$

$$\sin 1800° = \sin 0° = \frac{y}{r} = \frac{0}{1} = 0$$

76. $\cos 1800°$

The quadrantal angle $\theta = 1800°$ is coterminal with $1800° - 5(360°) = 1800° - 1800° = 0°$

$$\cos 1800° = \cos 0° = \frac{x}{r} = \frac{1}{1} = 1$$

77. $\csc 1800°$

The quadrantal angle $\theta = 1800°$ is coterminal with $1800° - 5(360°) = 1800° - 1800° = 0°$

$$\csc 1800° = \csc 0° = \frac{r}{y} = \frac{1}{0} \text{ undefined}$$

78. $\cot 1800°$
The quadrantal angle $\theta = 1800°$ is coterminal with $1800° - 5(360°) = 1800° - 1800° = 0°$

$\cot 1800° = \cot 0° = \frac{x}{y} = \frac{1}{0}$ undefined

79. $\sec 1800°$
The quadrantal angle $\theta = 1800°$ is coterminal with $1800° - 5(360°) = 1800° - 1800° = 0°$

$\sec 1800° = \sec 0° = \frac{r}{x} = \frac{1}{1} = 1$

80. $\tan 1800°$
The quadrantal angle $\theta = 1800°$ is coterminal with $1800° - 5(360°) = 1800° - 1800° = 0°$

$\tan 1800° = \tan 0° = \frac{y}{x} = \frac{0}{1} = 0$

81. $\cos(-900°)$
The quadrantal angle $\theta = -900°$ is coterminal with $-900° + 3(360°) = -900° + 1080° = 180°$.

$\cos(-900°) = \cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

82. $\sin(-900°)$
The quadrantal angle $\theta = -900°$ is coterminal with $-900° + 3(360°) = -900° + 1080° = 180°$.

$\sin(-900°) = \sin 180° = \frac{y}{r} = \frac{0}{1} = 0$

83. $\tan(-900°)$
The quadrantal angle $\theta = -900°$ is coterminal with $-900° + 3(360°) = -900° + 1080° = 180°$.

$\tan(-900°) = \tan 180° = \frac{y}{x} = \frac{0}{-1} = 0$

84. Answers will vary. Sample answer: Once $\sin(-900°)$ and $\cos(-900°)$ are found,

$\tan(-900°) = \frac{\sin(-900°)}{\cos(-900°)} = \frac{0}{-1} = 0.$

85. $\cos 90° + 3\sin 270°$

$\cos 90° = \frac{x}{r} = \frac{0}{1} = 0$

$\sin 270° = \frac{y}{r} = \frac{-1}{1} = -1$

$\cos 90° + 3\sin 270° = 0 + 3(-1) = -3$

86. $\tan 0° - 6\sin 90°$

$\tan 0° = \frac{y}{x} = \frac{0}{1} = 0$

$\sin 90° = \frac{y}{r} = \frac{1}{1} = 1$

$\tan 0° - 6\sin 90° = 0 - 6(1) = -6$

87. $3\sec 180° - 5\tan 360°$

$\sec 180° = \frac{r}{x} = \frac{1}{-1} = -1$ and

$\tan 360° = \tan 0° = \frac{y}{x} = \frac{0}{1} = 0$

$$\begin{aligned} 3\sec 180° - 5\tan 360° &= 3(-1) - 5(0) \\ &= -3 - 0 = -3 \end{aligned}$$

88. $4\csc 270° + 3\cos 180°$

$\csc 270° = \frac{r}{y} = \frac{1}{-1} = -1$ and

$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$$\begin{aligned} 4\csc 270° + 3\cos 180° &= 4(-1) + 3(-1) \\ &= -4 + (-3) = -7 \end{aligned}$$

89. $\tan 360° + 4\sin 180° + 5\cos^2 180°$

$\tan 360° = \tan 0° = \frac{y}{x} = \frac{0}{1} = 0,$

$\sin 180° = \frac{y}{r} = \frac{0}{1} = 0,$ and

$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$$\begin{aligned} &\tan 360° + 4\sin 180° + 5\cos^2 180° \\ &\quad = 0 + 4(0) + 5(-1)^2 = 0 + 0 + 5(1) = 5 \end{aligned}$$

90. $5\sin^2 90° + 2\cos^2 270° - \tan 360°$

$\sin 90° = \frac{y}{r} = \frac{1}{1} = 1$, $\cos 270° = \frac{x}{r} = \frac{0}{1} = 0$,

and $\tan 360° = \tan 0° = \frac{y}{x} = \frac{0}{1} = 0$

$$\begin{aligned} &5\sin^2 90° + 2\cos^2 270° - \tan 360° \\ &\quad = 5(1)^2 + 2(0)^2 - 0 = 5 \end{aligned}$$

91. $\sin^2 180° + \cos^2 180°$

$\sin 180° = \frac{y}{r} = \frac{0}{1} = 0$ and

$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$\sin^2 180° + \cos^2 180° = 0^2 + (-1)^2 = 0 + 1 = 1$

92. $\sin^2 360° + \cos^2 360°$

$\sin 360° = \sin 0° = \frac{y}{r} = \frac{0}{1} = 0$ and

$\cos 360° = \cos 0° = \frac{x}{r} = \frac{1}{1} = 1$

$\sin^2 360° + \cos^2 360° = 0^2 + 1^2 = 0 + 1 = 1$

93. $\sec^2 180° - 3\sin^2 360° + \cos 180°$

$\sec 180° = \frac{r}{x} = \frac{1}{-1} = -1$,

$\sin 360° = \sin 0° = \frac{y}{r} = \frac{0}{1} = 0$ and

$\cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$\sec^2 180° - 3\sin^2 360° + \cos 180°$
$= (-1)^2 - 3(0)^2 + (-1) = 1 - 1 = 0$

94. $2\sec 0° + 4\cot^2 90° + \cos 360°$

$\sec 0° = \frac{r}{x} = \frac{1}{1} = 1,\ \cot 90° = \frac{x}{y} = \frac{0}{1} = 0,$ and

$\cos 360° = \cos 0° = \frac{x}{r} = \frac{1}{1} = 1$

$2\sec 0° + 4\cot^2 90° + \cos 360°$
$= 2(1) + 4(0)^2 + 1 = 2 + 4(0) + 1$
$= 2 + 0 + 1 = 3$

95. $-2\sin^4 0° + 3\tan^2 0°$

$\sin 0° = \frac{y}{r} = \frac{0}{1} = 0; \tan 0° = \frac{y}{x} = \frac{0}{1} = 0$

$-2\sin^4 0° + 3\tan^2 0° = -2(0)^4 + 3(0)^2 = 0$

96. $-3\sin^4 90° + 4\cos^3 180°$

$\sin 90° = \frac{y}{r} = \frac{1}{1} = 1; \cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$-3\sin^4 90° + 4\cos^3 180°$
$= -3(1)^4 + 4(-1)^3 = -3 - 4 = -7$

97. $\sin^2(-90°) + \cos^2(-90°)$

$\sin(-90°) = \sin(-90° + 360°)$
$= \sin 270° = \frac{y}{r} = \frac{-1}{1} = -1$

$\cos(-90°) = \cos(-90° + 360°)$
$= \cos 270° = \frac{x}{r} = \frac{0}{1} = 0$

$\sin^2(-90°) + \cos^2(-90°) = (-1)^2 + 0^2 = 1$

98. $\sin^2(-180°) + \cos^2(-180°)$

$\sin(-180°) = \sin(-180° + 360°)$
$= \sin 180° = \frac{y}{r} = \frac{0}{1} = 0$

$\cos(-180°) = \cos(-180° + 360°)$
$= \cos 180° = \frac{x}{r} = \frac{-1}{1} = -1$

$\sin^2(-180°) + \cos^2(-180°) = 0^2 + (-1)^2 = 1$

99. $\cos[(2n+1)\cdot 90°]$

This angle is a quadrantal angle whose terminal side lies on either the positive part of the y-axis or the negative part of the y-axis. Any point on these terminal sides would have the form $(0, k)$, where k is any real number, $k \neq 0$.

$$\cos[(2n+1)\cdot 90°] = \frac{x}{r} = \frac{0}{\sqrt{0^2 + k^2}} = \frac{0}{\sqrt{k^2}} = \frac{0}{|k|} = 0$$

100. $\sin[n \cdot 180°]$

This angle is a quadrantal angle whose terminal side lies on either the positive part of the x-axis or the negative part of the x-axis. Any point on these terminal sides would have the form $(k, 0)$, where k is any real number, $k \neq 0$.

$$\sin[n \cdot 180°] = \frac{y}{r} = \frac{0}{\sqrt{k^2 + 0^2}} = \frac{0}{\sqrt{k^2}} = \frac{0}{|k|} = 0$$

101. $\tan[n \cdot 180°]$

The angle is a quadrantal angle whose terminal side lies on either the positive part of the x-axis or the negative part of the x-axis. Any point on these terminal sides would have the form $(k, 0)$, where k is any real number, $k \neq 0$.

$$\tan[n \cdot 180°] = \frac{y}{x} = \frac{0}{k} = 0$$

102. $\sin[270° + n \cdot 360°]$

This angle is a quadrantal angle that is coterminal with $\theta = 270°$. $\sin 270° = -1$, so $\sin[270° + n \cdot 360°] = -1$.

103. $\tan\left[(2n+1)\cdot 90°\right]$

This angle is a quadrantal angle whose terminal side lies on either the positive part of the y-axis or the negative part of the y-axis. Any point on these terminal sides would have the form $(0,k)$, where k is any real number, $k \neq 0$.

$$\tan\left[(2n+1)\cdot 90°\right]=\frac{y}{x}=\frac{k}{0} \text{ undefined}$$

104. $\cot\left[n\cdot 180°\right]$

The angle is a quadrantal angle whose terminal side lies on either the positive part of the x-axis or the negative part of the x-axis. Any point on these terminal sides would have the form $(k, 0)$, where k is any real number, $k \neq 0$.

$$\cot\left[n\cdot 180°\right]=\frac{x}{y}=\frac{k}{0} \text{ undefined}$$

105. $\cot\left[(2n+1)\cdot 90°\right]$

This angle is a quadrantal angle whose terminal side lies on either the positive part of the y-axis or the negative part of the y-axis. Any point on these terminal sides would have the form $(0,k)$, where k is any real number, $k \neq 0$.

$$\cot\left[(2n+1)\cdot 90°\right]=\frac{x}{y}=\frac{0}{k}=0$$

106. $\cos\left[n\cdot 360°\right]$

This angle is a quadrantal angle that is coterminal with $\theta = 0°$. $\cos 0° = 1$, so $\cos\left[n\cdot 360°\right]=1$.

107. $\sec\left[(2n+1)\cdot 90°\right]$

This angle is a quadrantal angle whose terminal side lies on either the positive part of the y-axis or the negative part of the y-axis. Any point on these terminal sides would have the form $(0,k)$, where k is any real number, $k \neq 0$.

$$\sec\left[(2n+1)\cdot 90°\right]=\frac{r}{x}=\frac{\sqrt{0^2+k^2}}{0} \text{ undefined}$$

108. $\csc\left[n\cdot 180°\right]$

This angle is a quadrantal angle whose terminal side lies on either the positive part of the x-axis or the negative part of the x-axis. Any point on these terminal sides would have the form $(k,0)$, where k is any real number, $k \neq 0$.

$$\csc\left[n\cdot 180°\right]=\frac{r}{y}=\frac{\sqrt{k^2+0^2}}{0} \text{ undefined}$$

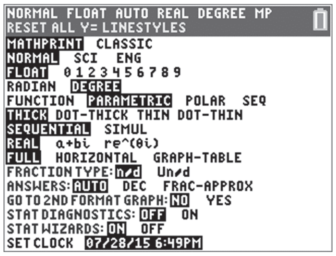
109. Using a calculator, sin 15° = 0.258819045 and cos 75° = 0.258819045. We can conjecture that the sines and cosines of complementary angles are equal. Trying another pair of complementary angles, we obtain $\sin 30° = \cos 60° = 0.5$. Therefore, our conjecture appears to be true.

110. Using a calculator, tan 25° = 0.466307658 and cot 65° = 0.466307658. We can conjecture that the tangent and cotangent of complementary angles are equal. Trying another pair of complementary angles, we obtain $\tan 45° = \cot 45° = 1$. Therefore, our conjecture appears to be true.

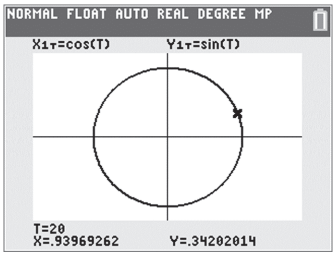
111. Using a calculator, sin 10° = 0.173648178 and sin(–10°) = −0.173648178. We can conjecture that the sines of an angle and its negative are opposites of each other. Using a circle, an angle θ having the point (x, y) on its terminal side has a corresponding angle $-\theta$ with a point $(x, -y)$ on its terminal side. From the definition of sine, $\sin(-\theta)=\frac{-y}{r}=-\frac{y}{r}$ and $\sin\theta=\frac{y}{r}$. The sines are negatives of each other.

112. Using a calculator, cos 20° = 0.93969262 and cos (–20°) = 0.93969262. We can conjecture that the cosines of an angle and its negative are equal. Using a circle, and angle θ having the point (x, y) on its terminal side has a corresponding angle $-\theta$ with point $(x, -y)$ on its terminal side. From the definition of cosine, $\cos(-\theta)=\frac{x}{r}$ and $\cos\theta=\frac{x}{r}$. The cosines are equal.

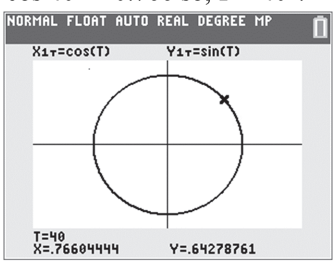
In Exercises 113−118, make sure your calculator is in the modes indicated in the instructions.



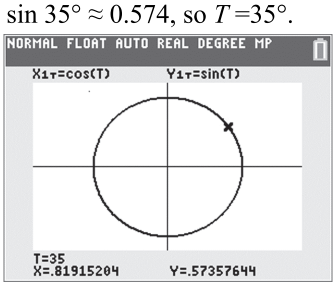
113. Use the TRACE feature to move around the circle in quadrant I to find $T = 20$. We see that $\cos 20° \approx 0.940$, and $\sin 20° \approx 0.342$.



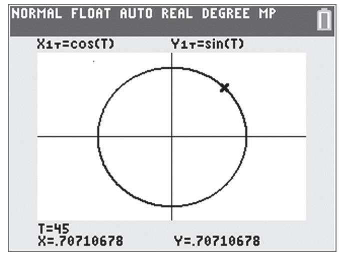
114. Use the TRACE feature to move around the circle in quadrant I. We see that $\cos 40° \approx 0.766$ so, $T = 40°$.



115. Use the TRACE feature to move around the circle in quadrant I. We see that $\sin 35° \approx 0.574$, so $T = 35°$.



116. Use the TRACE feature to move around the circle in quadrant I. We see that $\cos 45° = \sin 45°$, so $T = 45°$.



117. As T increases from 0° to 90°, the cosine decreases and the sine increases.

118. As T increases from 90° to 180°, the cosine decreases and the sine decreases.

Section 1.4 Using the Definitions of the Trigonometric Functions

1. Given $\cos\theta = \frac{1}{\sec\theta}$, two equivalent forms of this identity are $\sec\theta = \frac{1}{\cos\theta}$ and $\cos\theta \cdot \sec\theta = 1$.

2. Given $\tan\theta = \frac{1}{\cot\theta}$, two equivalent forms of this identity are $\cot\theta = \frac{1}{\tan\theta}$ and $\tan\theta \cdot \cot\theta = 1$.

3. For an angle θ measuring 105°, the trigonometric functions $\sin\theta$ and $\csc\theta$ are positive, and the remaining trigonometric functions are negative.

4. If $\sin\theta > 0$ and $\tan\theta > 0$, then θ is in quadrant I.

5. $\sin\theta = \dfrac{1}{2}$ is possible because the range of $\sin\theta$ is [−1, 1]. Furthermore, when $\sin\theta = \dfrac{1}{2}$, $\csc\theta = \dfrac{1}{\frac{1}{2}} = 2$. Thus, $\sin\theta = \dfrac{1}{2}$ and $\csc\theta = 2$ is possible.

6. $\tan\theta = 2$ is possible because the range of $\tan\theta$ is $(-\infty, \infty)$. However, when $\tan\theta = 2$, $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{2}$. Thus, $\tan\theta = 2$ and $\cot\theta = -2$ is impossible.

7. $\sin\theta > 0,\ \csc\theta < 0$ is impossible because $\csc\theta = \frac{1}{\sin\theta}$, so both functions must have the same sign.

8. $\cos\theta = 1.5$ is impossible because the range of $\cos\theta$ is $[-1, 1]$.

9. $\cot\theta = -1.5$ is possible because the range of $\cot\theta$ is $(-\infty, \infty)$.

10. $\sin^2\theta + \cos^2\theta = 2$
impossible because $\sin^2\theta + \cos^2\theta = 1$

11. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{2}{3}} = \dfrac{3}{2}$

12. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{\frac{5}{8}} = \dfrac{8}{5}$

13. $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\frac{3}{7}} = -\dfrac{7}{3}$

14. $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\frac{8}{43}} = -\dfrac{43}{8}$

15. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{5}$

16. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{18}$

17. $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-\frac{5}{2}} = -\dfrac{2}{5}$

18. $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-\frac{11}{7}} = -\dfrac{7}{11}$

19. $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{\frac{\sqrt{8}}{2}} = \dfrac{2}{\sqrt{8}} = \dfrac{2}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

20. $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{\frac{\sqrt{24}}{3}} = \dfrac{3}{\sqrt{24}} = \dfrac{3}{2\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}}$
$= \dfrac{3\sqrt{6}}{12} = \dfrac{\sqrt{6}}{4}$

21. $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{-2.5} = -0.4$

22. $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{1}{-0.01} = -100$

23. $\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{1.25} = 0.8$

24. $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{8} = 0.125$

25. The value of $\cos\theta$ cannot exceed 1.

26. Because tan 90° is undefined, it does not have a reciprocal.

27. A 74° angle in standard position lies in quadrant I, so all its trigonometric functions are positive.

28. An 84° angle in standard position lies in quadrant I, so all its trigonometric functions are positive.

29. A 218° angle in standard position lies in quadrant III, so its sine, cosine, secant, and cosecant are negative, while its tangent and cotangent are positive.

30. A 195° angle in standard position lies in quadrant III, so its sine, cosine, secant, and cosecant are negative, while its tangent and cotangent are positive.

31. A 178° angle in standard position lies in quadrant II, so its sine and cosecant are positive, while its cosine, secant, tangent and cotangent are negative.

32. A 125° angle in standard position lies in quadrant II, so its sine and cosecant are positive, while its cosine, secant, tangent and cotangent are negative.

33. A −80° angle in standard position lies in quadrant IV, so its cosine and secant are positive, while its sine, cosecant, tangent and cotangent are negative.

34. A −15° angle in standard position lies in quadrant IV, so its cosine and secant are positive, while its sine, cosecant, tangent and cotangent are negative.

35. An 855° angle in standard position is coterminal with a 135° angle, and thus, lies in quadrant II. So its sine and cosecant are positive, while its cosine, secant, tangent and cotangent are negative.

36. A 1005° angle in standard position is coterminal with a 285° angle, and thus lies in quadrant IV, so its sine, cosecant, tangent and cotangent are negative, while its cosine and secant are positive.

37. A −345° angle in standard position lies in quadrant I, so all its trigonometric functions are positive.

38. A −640° angle in standard position is coterminal with an 80° angle, and thus lies in quadrant I. So all its trigonometric functions are positive.

39. $\sin\theta > 0$, so $\csc\theta$ is also greater than 0. The functions are greater than 0 (positive) in quadrants I and II.

40. $\cos\theta > 0$, so $\sec\theta$ is also greater than 0. The functions are greater than 0 (positive) in quadrants I and IV.

41. $\cos\theta > 0$ in quadrants I and IV, while $\sin\theta > 0$ in quadrants I and II. Both conditions are met only in quadrant I.

42. $\sin\theta > 0$ in quadrants I and II, while $\tan\theta > 0$ in quadrants I and III. Both conditions are met only in quadrant I.

43. $\tan\theta < 0$ in quadrants II and IV, while $\cos\theta < 0$ in quadrants II and III. Both conditions are met only in quadrant II.

44. $\cos\theta < 0$ in quadrants II and III, while $\sin\theta < 0$ in quadrants III and IV. Both conditions are met only in quadrant III.

45. $\sec\theta > 0$ in quadrants I and IV, while $\csc\theta > 0$ in quadrants I and II. Both conditions are met only in quadrant I.

46. $\csc\theta > 0$ in quadrants I and II, while $\cot\theta > 0$ in quadrants I and III. Both conditions are met only in quadrant I.

47. $\sec\theta < 0$ in quadrants II and III, while $\csc\theta < 0$ in quadrants III and IV. Both conditions are met only in quadrant III.

48. $\cot\theta < 0$ in quadrants II and IV, while $\sec\theta < 0$ in quadrants II and III. Both conditions are met only in quadrant II.

49. $\sin\theta < 0$, so $\csc\theta$ is also less than 0. The functions are less than 0 (negative) in quadrants III and IV.

50. $\tan\theta < 0$, so $\cot\theta$ is also less than 0. The functions are less that 0 (negative) in quadrants II and IV.

51. The answers to exercises 41 and 45 are the same because functions in exercise 45 are the reciprocals of the functions in exercise 41.

52. Because $\tan\theta = \dfrac{1}{\cot\theta}$, if $\tan\theta > 0$, then $\cot\theta > 0$.

53. Impossible because the range of $\sin\theta$ is $[-1, 1]$.

54. Impossible because the range of $\sin\theta$ is $[-1, 1]$.

55. Possible because the range of $\cos\theta$ is $[-1, 1]$.

56. Possible because the range of $\cos\theta$ is $[-1, 1]$.

57. Possible because the range of $\tan\theta$ is $(-\infty, \infty)$.

58. Possible because the range of $\cot\theta$ is $(-\infty, \infty)$.

59. Impossible because the range of $\sec\theta$ is $(-\infty, -1] \cup [1, \infty)$.

60. Impossible because the range of $\sec\theta$ is $(-\infty, -1] \cup [1, \infty)$.

61. Possible because the range of $\csc\theta$ is $(-\infty, -1] \cup [1, \infty)$.

62. Possible because the range of $\csc\theta$ is $(-\infty, -1] \cup [1, \infty)$.

63. Possible because the range of $\cot\theta$ is $(-\infty, \infty)$.

64. Possible because the range of $\cot\theta$ is $(-\infty, \infty)$.

65. If $\sin\theta = \dfrac{3}{5}$, then $y = 3$ and $r = 5$. So $r^2 = x^2 + y^2 \Rightarrow 5^2 = x^2 + 3^2 \Rightarrow 25 = x^2 + 9 \Rightarrow 16 = x^2 \Rightarrow \pm 4 = x$. θ is in quadrant II, so $x = -4$. Therefore, $\cos\theta = -\dfrac{4}{5}$.

Alternatively, use the identity

$\sin^2\theta + \cos^2\theta = 1$: $\left(\dfrac{3}{5}\right)^2 + \cos^2\theta = 1 \Rightarrow$

$\dfrac{9}{25} + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \dfrac{16}{25} \Rightarrow \cos\theta = \pm\dfrac{4}{5}$

θ is in quadrant II, so $\cos\theta$ is negative.

Thus, $\cos\theta = -\dfrac{4}{5}$.

66. If $\cos\theta = \frac{4}{5}$, then $x = 4$ and $r = 5$. So
$r^2 = x^2 + y^2 \Rightarrow 5^2 = 4^2 + y^2 \Rightarrow$
$25 = 16 + y^2 \Rightarrow 9 = y^2 \Rightarrow \pm 3 = y$
θ is in quadrant IV, so $y = -3$. Therefore, $\sin\theta = -\frac{3}{5}$. Alternatively, use the identity
$\sin^2\theta + \cos^2\theta = 1$: $\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1 \Rightarrow$
$\sin^2\theta + \frac{16}{25} = 1 \Rightarrow \sin^2\theta = \frac{9}{25} \Rightarrow \sin\theta = \pm\frac{3}{5}$
θ is in quadrant IV, so $\sin\theta$ is negative.
Thus, $\sin\theta = -\frac{3}{5}$.

67. If $\cot\theta = -\frac{1}{2}$ and θ is in quadrant IV, then $x = 1$ and $y = -2$. So
$r^2 = x^2 + y^2 \Rightarrow r^2 = 1^2 + (-2)^2 \Rightarrow$
$r^2 = 1 + 4 \Rightarrow r^2 = 5 \Rightarrow r = \sqrt{5}$
Therefore, $\csc\theta = \frac{r}{y} = -\frac{\sqrt{5}}{2}$. Alternatively, use the identity $1 + \cot^2\theta = \csc^2\theta$:
$1 + \left(-\frac{1}{2}\right)^2 = \csc^2\theta \Rightarrow 1 + \frac{1}{4} = \csc^2\theta \Rightarrow$
$\frac{5}{4} = \csc^2\theta \Rightarrow \pm\frac{\sqrt{5}}{2} = \csc\theta$. θ is in quadrant IV, so $\csc\theta$ is negative. Thus, $\csc\theta = -\frac{\sqrt{5}}{2}$

68. If $\tan\theta = \frac{\sqrt{7}}{3}$ and θ is in quadrant III, then $x = -3$ and $y = -\sqrt{7}$. So $r^2 = x^2 + y^2 \Rightarrow$
$r^2 = (-3)^2 + \left(-\sqrt{7}\right)^2 \Rightarrow r^2 = 9 + 7 \Rightarrow$
$r^2 = 16 \Rightarrow r = 4$. Therefore, $\sec\theta = \frac{r}{x} = -\frac{4}{3}$.
Alternatively, use the identity
$\tan^2\theta + 1 = \sec^2\theta$:
$\left(\frac{\sqrt{7}}{3}\right)^2 + 1 = \sec^2\theta \Rightarrow \frac{7}{9} + 1 = \sec^2\theta \Rightarrow$
$\frac{16}{9} = \sec^2\theta \Rightarrow \pm\frac{4}{3} = \sec\theta$
θ is in quadrant III, so $\sec\theta$ is negative.
Thus $\sec\theta = -\frac{4}{3}$

69. If $\sin\theta = \frac{1}{2}$, then $y = 1$ and $r = 2$. So
$r^2 = x^2 + y^2 \Rightarrow 2^2 = x^2 + 1^2 \Rightarrow 4 = x^2 + 1 \Rightarrow$
$3 = x^2 \Rightarrow \pm\sqrt{3} = x$
θ is in quadrant II, so $x = -\sqrt{3}$. Therefore,
$\tan\theta = -\frac{1}{\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$.
Alternatively, use the identity
$\sin^2\theta + \cos^2\theta = 1$ to obtain
$\left(\frac{1}{2}\right)^2 + \cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{3}{4} \Rightarrow$
$\cos\theta = \pm\frac{\sqrt{3}}{2}$.
θ is in quadrant II, so $\cos\theta = -\frac{\sqrt{3}}{2}$. Then,
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$
$$= -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

70. If $\csc\theta = -2$, then $r = 2$ and $y = -1$. So
$r^2 = x^2 + y^2 \Rightarrow 2^2 = x^2 + (-1)^2 \Rightarrow$
$4 = x^2 + 1 \Rightarrow 3 = x^2 \Rightarrow \pm\sqrt{3} = x$
θ is in quadrant III, so $x = -\sqrt{3}$. Therefore,
$\cot\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$.
Alternatively, using the identity
$1 + \cot^2\theta = \csc^2\theta$ gives
$1 + \cot^2\theta = (-2)^2 \Rightarrow \cot^2\theta = 3 \Rightarrow$
$\cot\theta = \pm\sqrt{3}$. Because θ is in quadrant III, $\cot\theta = \sqrt{3}$.

71. Using the identity $1 + \cot^2\theta = \csc^2\theta$ gives
$1 + \cot^2\theta = (-1.45)^2 \Rightarrow \cot^2\theta = 1.1025 \Rightarrow$
$\cot\theta = \pm 1.05$
Because θ is in quadrant III, $\cot\theta = 1.05$.

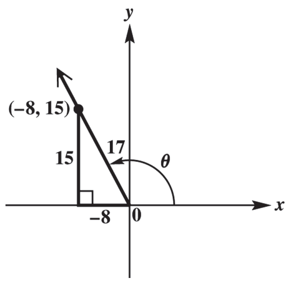
72. Using the identity $\sin^2\theta + \cos^2\theta = 1$. gives
$(0.6)^2 + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 0.64 \Rightarrow$
$\cos\theta = \pm 0.8$
Because θ is in quadrant II, $\cos\theta < 0$; therefore, $\cos\theta \approx -0.8$.
$\tan\theta = \frac{\sin\theta}{\cos\theta}$, so $\tan\theta \approx \frac{0.6}{-0.8} = -0.75$.

For Exercises 73−78, remember that r is always positive.

73. $\tan\theta = -\dfrac{15}{8} = \dfrac{15}{-8}$, with θ in quadrant II

$\tan\theta = \dfrac{y}{x}$ and θ is in quadrant II, so let $y = 15$ and $x = -8$.

$x^2 + y^2 = r^2 \Rightarrow (-8)^2 + 15^2 = r^2 \Rightarrow$
$64 + 225 = r^2 \Rightarrow 289 = r^2 \Rightarrow r = 17$



$\sin\theta = \dfrac{y}{r} = \dfrac{15}{17}$; $\cos\theta = \dfrac{x}{r} = \dfrac{-8}{17} = -\dfrac{8}{17}$

$\tan\theta = \dfrac{y}{x} = \dfrac{15}{-8} = -\dfrac{15}{8}$

$\cot\theta = \dfrac{x}{y} = \dfrac{-8}{15} = -\dfrac{8}{15}$

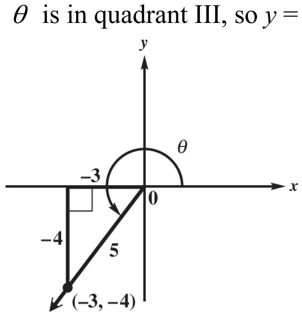
$\sec\theta = \dfrac{r}{x} = \dfrac{17}{-8} = -\dfrac{17}{8}$; $\csc\theta = \dfrac{r}{y} = \dfrac{17}{15}$

74. $\cos\theta = -\dfrac{3}{5} = \dfrac{-3}{5}$, with θ in quadrant III

$\cos\theta = \dfrac{x}{r}$ and θ in quadrant III, so let $x = -3$, $r = 5$.

$x^2 + y^2 = r^2 \Rightarrow (-3)^2 + y^2 = 5^2 \Rightarrow$
$9 + y^2 = 25 \Rightarrow y^2 = 16 \Rightarrow y = \pm\sqrt{16} \Rightarrow$
$y = \pm 4$

θ is in quadrant III, so $y = -4$.



$\sin\theta = \dfrac{y}{r} = \dfrac{-4}{5} = -\dfrac{4}{5}$; $\cos\theta = \dfrac{x}{r} = \dfrac{-3}{5} = -\dfrac{3}{5}$

$\tan\theta = \dfrac{y}{x} = \dfrac{-4}{-3} = \dfrac{4}{3}$; $\cot\theta = \dfrac{x}{y} = \dfrac{-3}{-4} = \dfrac{3}{4}$

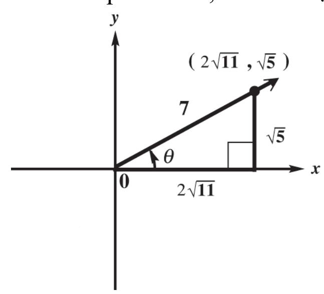
$\sec\theta = \dfrac{r}{x} = \dfrac{5}{-3} = -\dfrac{5}{3}$; $\csc\theta = \dfrac{r}{y} = \dfrac{5}{-4} = -\dfrac{5}{4}$

75. $\sin\theta = \dfrac{\sqrt{5}}{7}$, with θ in quadrant I

$\sin\theta = \dfrac{y}{r}$ and θ in quadrant I, so let $y = \sqrt{5}, r = 7$.

$x^2 + y^2 = r^2 \Rightarrow x^2 + \left(\sqrt{5}\right)^2 = 7^2 \Rightarrow$
$x^2 + 5 = 49 \Rightarrow x^2 = 44 \Rightarrow x = \pm\sqrt{44} \Rightarrow$
$x = \pm 2\sqrt{11}$

θ is in quadrant I, so $x = 2\sqrt{11}$.



Drawing not to scale

$\sin\theta = \dfrac{y}{r} = \dfrac{\sqrt{5}}{7}$

$\cos\theta = \dfrac{x}{r} = \dfrac{2\sqrt{11}}{7}$

$\tan\theta = \dfrac{y}{x} = \dfrac{\sqrt{5}}{2\sqrt{11}} = \dfrac{\sqrt{5}}{2\sqrt{11}} \cdot \dfrac{\sqrt{11}}{\sqrt{11}} = \dfrac{\sqrt{55}}{22}$

$\cot\theta = \dfrac{x}{y} = \dfrac{2\sqrt{11}}{\sqrt{5}} = \dfrac{2\sqrt{11}}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{2\sqrt{55}}{5}$

$\sec\theta = \dfrac{r}{x} = \dfrac{7}{2\sqrt{11}} = \dfrac{7}{2\sqrt{11}} \cdot \dfrac{\sqrt{11}}{\sqrt{11}} = \dfrac{7\sqrt{11}}{22}$

$\csc\theta = \dfrac{r}{y} = \dfrac{7}{\sqrt{5}} = \dfrac{7}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{7\sqrt{5}}{5}$

76. $\tan\theta = \sqrt{3} = \dfrac{-\sqrt{3}}{-1}$

$\tan\theta = \dfrac{y}{x}$ and θ is in quadrant III, so let $y = -\sqrt{3}$ and $x = -1$.

$x^2 + y^2 = r^2 \Rightarrow (-1)^2 + \left(-\sqrt{3}\right)^2 = r^2 \Rightarrow$
$1 + 3 = r^2 \Rightarrow 4 = r^2 \Rightarrow r = 2$

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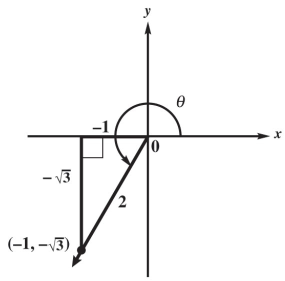


$$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{2} = -\frac{1}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

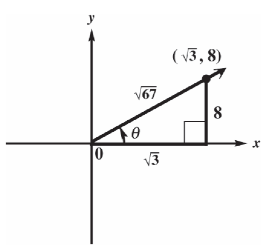
$$\sec\theta = \frac{r}{x} = \frac{2}{-1} = -2$$

$$\csc\theta = \frac{r}{y} = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

77. $\cot\theta = \frac{\sqrt{3}}{8}$, with θ in quadrant I

$\cot\theta = \frac{x}{y}$ and θ is in quadrant I, so let $x = \sqrt{3}$ and $y = 8$.

$$x^2 + y^2 = r^2 \Rightarrow \left(\sqrt{3}\right)^2 + (8)^2 = r^2 \Rightarrow$$
$$3 + 64 = r^2 \Rightarrow 67 = r^2 \Rightarrow \sqrt{67} = r$$



$$\sin\theta = \frac{y}{r} = \frac{8}{\sqrt{67}} = \frac{8}{\sqrt{67}} \cdot \frac{\sqrt{67}}{\sqrt{67}} = \frac{8\sqrt{67}}{67}$$

$$\cos\theta = \frac{x}{r} = \frac{\sqrt{3}}{\sqrt{67}} = \frac{\sqrt{3}}{\sqrt{67}} \cdot \frac{\sqrt{67}}{\sqrt{67}} = \frac{\sqrt{3}\sqrt{67}}{67} = \frac{\sqrt{201}}{67}$$

$$\tan\theta = \frac{y}{x} = \frac{8}{\sqrt{3}} = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{\sqrt{3}}{8}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{67}}{\sqrt{3}} = \frac{\sqrt{67}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{67}\sqrt{3}}{3} = \frac{\sqrt{201}}{3}$$

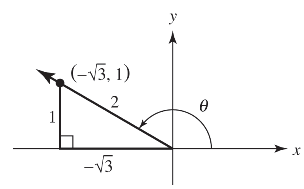
$$\csc\theta = \frac{r}{y} = \frac{\sqrt{67}}{8}$$

78. $\csc\theta = 2$, with θ in quadrant II

$\csc\theta = \frac{r}{y}$ and θ is in quadrant II, so let $r = 2$ and $y = 1$.

$$x^2 + y^2 = r^2 \Rightarrow x^2 + 1^2 = 2^2 \Rightarrow$$
$$x^2 + 1 = 4 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$$

θ is in quadrant II, so $x = -\sqrt{3}$



$$\sin\theta = \frac{y}{r} = \frac{1}{2};\ \cos\theta = \frac{x}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$$

$$\sec\theta = \frac{r}{x} = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{2}{1} = 2$$

79. $\sin\theta = \frac{\sqrt{2}}{6}$, given that $\cos\theta < 0$

$\sin\theta$ is positive and $\cos\theta$ is negative when θ is in quadrant II.

$\sin\theta = \frac{y}{r}$ and θ in quadrant II, so let $y = \sqrt{2}, r = 6$.

$$x^2 + y^2 = r^2 \Rightarrow x^2 + \left(\sqrt{2}\right)^2 = 6^2 \Rightarrow$$
$$x^2 + 2 = 36 \Rightarrow x^2 = 34 \Rightarrow$$
$$x = \pm\sqrt{34}$$

θ is in quadrant II, so $x = -\sqrt{34}$.

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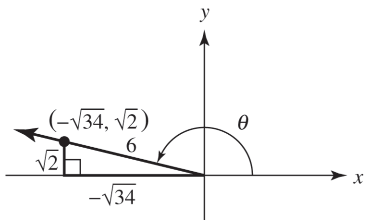


$$\sin\theta = \frac{y}{r} = \frac{\sqrt{2}}{6};\ \cos\theta = \frac{x}{r} = -\frac{\sqrt{34}}{6}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{2}}{\sqrt{34}} = -\frac{\sqrt{2}}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}} = -\frac{\sqrt{68}}{34} = -\frac{2\sqrt{17}}{34} = -\frac{\sqrt{17}}{17}$$

$$\cot\theta = \frac{x}{y} = -\frac{\sqrt{34}}{\sqrt{2}} = -\frac{\sqrt{34}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{68}}{2} = -\frac{2\sqrt{17}}{2} = -\sqrt{17}$$

$$\sec\theta = \frac{r}{x} = -\frac{6}{\sqrt{34}} = -\frac{6}{\sqrt{34}}\cdot\frac{\sqrt{34}}{\sqrt{34}} = -\frac{6\sqrt{34}}{34} = -\frac{3\sqrt{34}}{17}$$

$$\csc\theta = \frac{r}{y} = \frac{6}{\sqrt{2}} = \frac{6}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

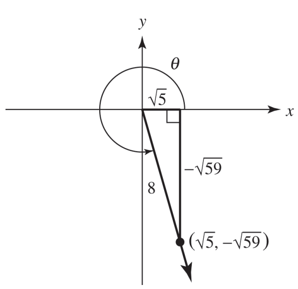
80. $\cos\theta = \frac{\sqrt{5}}{8}$, given that $\tan\theta < 0$

$\cos\theta$ is positive and $\tan\theta$ is negative when θ is in quadrant IV.

$\cos\theta = \frac{x}{r}$ and θ in quadrant IV, so let $x = \sqrt{5}, r = 8$.

$$x^2 + y^2 = r^2 \Rightarrow \left(\sqrt{5}\right)^2 + y^2 = 8^2 \Rightarrow 5 + y^2 = 64 \Rightarrow y^2 = 59 \Rightarrow y = \pm\sqrt{59}$$

θ is in quadrant IV, so $y = -\sqrt{59}$.



$$\sin\theta = \frac{y}{r} = -\frac{\sqrt{59}}{8};\ \cos\theta = \frac{x}{r} = \frac{\sqrt{5}}{8}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{59}}{\sqrt{5}} = -\frac{\sqrt{59}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{295}}{5}$$

$$\cot\theta = \frac{x}{y} = -\frac{\sqrt{5}}{\sqrt{59}} = -\frac{\sqrt{5}}{\sqrt{59}}\cdot\frac{\sqrt{59}}{\sqrt{59}} = -\frac{\sqrt{295}}{59}$$

$$\sec\theta = \frac{r}{x} = \frac{8}{\sqrt{5}} = \frac{8}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \frac{8\sqrt{5}}{5}$$

$$\csc\theta = \frac{r}{y} = -\frac{8}{\sqrt{59}} = -\frac{8}{\sqrt{59}}\cdot\frac{\sqrt{59}}{\sqrt{59}} = -\frac{8\sqrt{59}}{59}$$

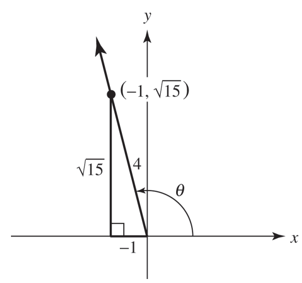
81. $\sec\theta = -4$, given that $\sin\theta > 0$

$\sec\theta$ is negative and $\sin\theta$ is positive when θ is in quadrant II.

$\sec\theta = \frac{r}{x}$ and θ in quadrant II, so let $x = -1, r = 4$.

$$x^2 + y^2 = r^2 \Rightarrow (-1)^2 + y^2 = 4^2 \Rightarrow 1 + y^2 = 16 \Rightarrow y^2 = 15 \Rightarrow y = \pm\sqrt{15}$$

θ is in quadrant II, so $y = \sqrt{15}$.



$$\sin\theta = \frac{y}{r} = \frac{\sqrt{15}}{4};\ \cos\theta = \frac{x}{r} = -\frac{1}{4}$$

$$\tan\theta = \frac{y}{x} = -\frac{\sqrt{15}}{1} = -\sqrt{15}$$

$$\cot\theta = \frac{x}{y} = -\frac{1}{\sqrt{15}} = -\frac{1}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = -\frac{\sqrt{15}}{15}$$

$$\sec\theta = \frac{r}{x} = -4$$

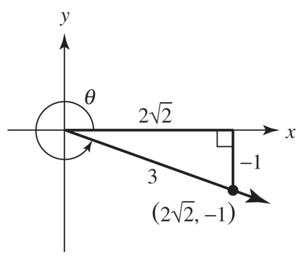
$$\csc\theta = \frac{r}{y} = \frac{4}{\sqrt{15}} = \frac{4}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

82. $\csc\theta = -3$, given that $\cos\theta > 0$
$\csc\theta$ is negative and $\cos\theta$ is positive when θ is in quadrant IV.

$\csc\theta = \frac{r}{y}$ and θ in quadrant IV, so let $y = -1, r = 3$.

$$x^2 + y^2 = r^2 \Rightarrow x^2 + (-1)^2 = 3^2 \Rightarrow$$
$$x^2 + 1 = 9 \Rightarrow x^2 = 8 \Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$$

θ is in quadrant IV, so $x = 2\sqrt{2}$.



$$\sin\theta = \frac{y}{r} = -\frac{1}{3}$$
$$\cos\theta = \frac{x}{r} = \frac{2\sqrt{2}}{3}$$
$$\tan\theta = \frac{y}{x} = -\frac{1}{2\sqrt{2}} = -\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{4}$$
$$\cot\theta = \frac{x}{y} = -\frac{2\sqrt{2}}{1} = -2\sqrt{2}$$
$$\sec\theta = \frac{r}{x} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{4}$$
$$\csc\theta = \frac{r}{y} = -3$$

83. $\sin\theta = 1$

$$\sin^2\theta + \cos^2\theta = 1^2 \Rightarrow 1^2 + \cos^2\theta = 1 \Rightarrow$$
$$\cos^2\theta = 0 \Rightarrow \cos\theta = 0$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{0} \Rightarrow \tan\theta \text{ is undefined}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{0}{1} = 0$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{0} \Rightarrow \sec\theta \text{ is undefined}$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1} = 1$$

84. $\cos\theta = 1$

$$\sin^2\theta + \cos^2\theta = 1^2 \Rightarrow \sin^2\theta + 1^2 = 1 \Rightarrow$$
$$\sin^2\theta = 0 \Rightarrow \sin\theta = 0$$
$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{0}{1} = 0$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{0} \Rightarrow \cot\theta \text{ is undefined}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{1} = 1$$
$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{0} \Rightarrow \text{ is undefined}$$

85. $$x^2 + y^2 = r^2 \Rightarrow \frac{x^2 + y^2}{y^2} = \frac{r^2}{y^2} \Rightarrow$$
$$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2} \Rightarrow \left(\frac{x}{y}\right)^2 + 1 = \left(\frac{r}{y}\right)^2 \Rightarrow$$
$$1 + \left(\frac{x}{y}\right)^2 = \left(\frac{r}{y}\right)^2$$

Because $\cot\theta = \frac{x}{y}$ and $\csc\theta = \frac{r}{y}$, we have $1 + (\cot\theta)^2 = (\csc\theta)^2$ or $1 + \cot^2\theta = \csc^2\theta$.

86. $$\frac{\cos\theta}{\sin\theta} = \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{x}{r} \div \frac{y}{r} = \frac{x}{r} \cdot \frac{r}{y} = \frac{x}{y} = \cot\theta$$

87. The statement is false. For example, $\sin 180° + \cos 180° = 0 + (-1) = -1 \neq 1$.

88. The statement is false because 2 is not in the range of the sine function.

89. $90° < \theta < 180° \Rightarrow 180° < 2\theta < 360°$, so 2θ lies in either quadrant III or IV. Thus, $\sin 2\theta$ is negative.

90. $90° < \theta < 180° \Rightarrow 180° < 2\theta < 360°$, so 2θ lies in either quadrant III or IV. Thus, $\csc 2\theta$ is negative.

91. $90° < \theta < 180° \Rightarrow 45° < \frac{\theta}{2} < 90°$, so $\frac{\theta}{2}$ lies in quadrant I. Thus, $\tan\frac{\theta}{2}$ is positive.

92. $90° < \theta < 180° \Rightarrow 45° < \frac{\theta}{2} < 90°$, so $\frac{\theta}{2}$ lies in quadrant I. Thus, $\cot\frac{\theta}{2}$ is positive.

93. $90° < \theta < 180° \Rightarrow 270° < \theta + 180° < 360°$, so $\theta + 180°$ lies in quadrant IV. Thus, $\cot(\theta + 180°)$ is negative.

94. $90° < \theta < 180° \Rightarrow 270° < \theta + 180° < 360°$, so $\theta + 180°$ lies in quadrant IV. Thus, $\tan(\theta + 180°)$ is negative.

95. $90° < \theta < 180° \Rightarrow -90° > -\theta > -180° \Rightarrow$ $-180° < \theta < -90°$, so $-\theta$ lies in quadrant III ($-180°$ is coterminal with 180°, and $-90°$ is coterminal with 270°.) Thus, $\cos(-\theta)$ is negative.

96. $90° < \theta < 180° \Rightarrow -90° > -\theta > -180° \Rightarrow$ $-180° < \theta < -90°$, so $-\theta$ lies in quadrant III ($-180°$ is coterminal with 180°, and $-90°$ is coterminal with 270°.) Thus, $\sec(-\theta)$ is negative.

97. $-90° < \theta < 90° \Rightarrow -45° < \frac{\theta}{2} < 45°$, so $\frac{\theta}{2}$ lies in either quadrant I or quadrant IV. Thus $\cos\frac{\theta}{2}$ is positive.

98. $-90° < \theta < 90° \Rightarrow -45° < \frac{\theta}{2} < 45°$, so $\frac{\theta}{2}$ lies in either quadrant I or quadrant IV. Thus $\sec\frac{\theta}{2}$ is positive.

99. $-90° < \theta < 90° \Rightarrow 90° < \theta + 180° < 270°$, so $\theta + 180°$ lies in either quadrant II or quadrant III. Thus $\sec(\theta + 180°)$ is negative.

100. $-90° < \theta < 90° \Rightarrow 90° < \theta + 180° < 270°$, so $\theta + 180°$ lies in either quadrant II or quadrant III. Thus $\cos(\theta + 180°)$ is negative.

101. $-90° < \theta < 90° \Rightarrow 90° > -\theta > -90° \Rightarrow$ $-90° < -\theta < 90°$, so $-\theta$ lies in either quadrant I or quadrant IV. Thus $\sec(-\theta)$ is positive.

102. $-90° < \theta < 90° \Rightarrow 90° > -\theta > -90° \Rightarrow$ $-90° < -\theta < 90°$, so $-\theta$ lies in either quadrant I or quadrant IV. Thus $\cos(-\theta)$ is positive.

103. $-90° < \theta < 90° \Rightarrow -270° < \theta - 180° < -90°$, so $\theta - 180°$ lies in either quadrant II or quadrant III. Thus $\cos(\theta - 180°)$ is negative.

104. $-90° < \theta < 90° \Rightarrow -270° < \theta - 180° < -90°$, so $\theta - 180°$ lies in either quadrant II or quadrant III. Thus $\sec(\theta - 180°)$ is negative.

105. $\tan(3\theta - 4°) = \frac{1}{\cot(5\theta - 8°)} \Rightarrow$
$\tan(3\theta - 4°) = \tan(5\theta - 8°)$
The second equation is true if $3\theta - 4° = 5\theta - 8°$, so solving this equation will give a value (but not the only value) for which the given equation is true.
$3\theta - 4° = 5\theta - 8° \Rightarrow 4° = 2\theta \Rightarrow \theta = 2°$

106. $\cos(6\theta + 5°) = \frac{1}{\sec(4\theta + 15°)} \Rightarrow$
$\cos(6\theta + 5°) = \cos(4\theta + 15°)$
The second equation is true if $6\theta + 5° = 4\theta + 15°$, so solving this equation will give a value (but not the only value) for which the given equation is true.
$6\theta + 5° = 4\theta + 15° \Rightarrow 2\theta = 10° \Rightarrow \theta = 5°$

107. $\sin(4\theta + 2°)\csc(3\theta + 5°) = 1 \Rightarrow$
$$\sin(4\theta + 2°) = \frac{1}{\csc(3\theta + 5°)} \Rightarrow$$
$$\sin(4\theta + 2°) = \sin(3\theta + 5°)$$
The third equation is true if $4\theta + 2° = 3\theta + 5°$, so solving this equation will give a value (but not the only value) for which the given equation is true.
$4\theta + 2° = 3\theta + 5° \Rightarrow \theta = 3°$

108. $\sec(2\theta + 6°)\cos(5\theta + 3°) = 1 \Rightarrow$
$$\cos(5\theta + 3°) = \frac{1}{\sec(2\theta + 6°)} \Rightarrow$$
$$\cos(5\theta + 3°) = \cos(2\theta + 6°)$$
The third equation is true if $5\theta + 3° = 2\theta + 6°$, so solving this equation will give a value (but not the only value) for which the given equation is true.
$5\theta + 3° = 2\theta + 6° \Rightarrow 3\theta = 3° \Rightarrow \theta = 1°$

109. In quadrant II, the cosine is negative and the sine is positive.

110. An angle of 1294° must lie in quadrant III because the sine is negative and the tangent is positive.

Chapter 1 Review Exercises

1. The complement of 35° is $90° - 35° = 55°$. The supplement of 35° is $180° - 35° = 145°$.

2. $-51°$ is coterminal with $360° + (-51°) = 309°$

3. $-174°$. is coterminal with $-174° + 360° = 186°$

4. 792° is coterminal with 792° − 2(360°) = 72°.

5. 650 rotations per min = $\frac{650}{60}$ rotations per sec

= $\frac{65}{6}$ rotations per sec

= 26 rotations per 2.4 sec

= $26(360°)$ per 2.4 sec = 9360° per 2.4 sec

A point of the edge of the propeller will move 9360° in 2.4 sec.

6. 320 rotations per min = $\frac{320}{60}$ rotations per sec

= $\frac{16}{3}$ rotations per sec

= $\frac{16}{3}\cdot\frac{2}{3}=\frac{32}{9}$ rotations per $\frac{2}{3}$ sec

= $\frac{32}{9}(360°)$ per $\frac{2}{3}$ sec = 1280° per $\frac{2}{3}$ sec

A point of the edge of the pulley will move 1280° in $\frac{2}{3}$ sec.

7. $$\begin{aligned}119°8'3'' &= 119° + \tfrac{8}{60}° + \tfrac{3}{3600}° \\ &\approx 119° + 0.1333° + 0.0008° \\ &\approx 119.134°\end{aligned}$$

8. $$\begin{aligned}47°25'11'' &= 47° + \tfrac{25}{60}° + \tfrac{11}{3600}° \\ &\approx 47° + 0.4167° + 0.0031° \\ &\approx 47.420°\end{aligned}$$

9. $$\begin{aligned}275.1005 &= 275° + 0.1005(60') = 275° + 6.03' \\ &= 275°06' + 0.03' \\ &= 275°06' + 0.03(60'') \\ &= 275°6'1.8'' \approx 275°06'02''\end{aligned}$$

10. $$\begin{aligned}-61.5034° &= -\left[61° + 0.5034(60')\right] \\ &= -\left[61° + 30.204'\right] \\ &= -\left[61° + 30' + 0.204(60'')\right] \\ &= -\left[61° + 30' + 12.24''\right] \\ &= -61°30'12.24'' \approx -61°30'12''\end{aligned}$$

11. The three angles are the interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned}4x + (5x+5) + (4x-20) &= 180 \\ 13x - 15 &= 180 \\ 13x = 195 &\Rightarrow x = 15\end{aligned}$$

4(15) = 60, 5(15) + 5 = 80, and 4(15) − 20 = 40, so the measures of the angles are 40°, 60°, and 80°.

12. The two indicated angles are vertical angles. Hence, their measures are equal.

$9x + 4 = 12x - 14 \Rightarrow 18 = 3x \Rightarrow x = 6$

9(6) + 4 = 58, and 12(6) − 14 = 58, so the measure of each of the two angles is 58°.

13. The two indicated angles are alternate exterior angles, so their measures are equal.

$4x + 5 = 6x - 45 \Rightarrow 50 = 2x \Rightarrow 25 = x$

$4(25) + 5 = 105$

The measure of each indicated angle is 105°.

14. The sum of the measures of the interior angles of a triangle is 180°.

$60° + y + 90° = 180° \Rightarrow 150° + y = 180° \Rightarrow$
$y = 30°$

Substitute 30° for y, and solve for x:

$$\begin{aligned}30° + (x+y) + 90° &= 180° \\ 30° + (x+30°) + 90° &= 180° \\ 150° + x = 180° &\Rightarrow x = 30°\end{aligned}$$

Thus, $x = y = 30°$.

15. Assuming PQ and BA are parallel, $\triangle PCQ$ is similar to $\triangle ACB$ because the measure of $\angle PCQ$ is equal to the measure of $\angle ACB$ (they are vertical angles). The corresponding sides of similar triangles are proportional, so $\frac{PQ}{BA}=\frac{PC}{AC}$. Thus, we have $\frac{PQ}{BA}=\frac{PC}{AC}\Rightarrow$

$\frac{1.25\text{ mm}}{BA}=\frac{150\text{ mm}}{30\text{ km}}\Rightarrow BA=\frac{1.25\cdot 30}{150}\Rightarrow$
$BA = 0.25$ km

16. Because a line has 180°, the angle supplementary to β is $180-\beta$. The sum of the angles of a triangle is 180°, so

$$\begin{aligned}\theta + \alpha + (180-\beta) = 180° &\Rightarrow \theta + \alpha - \beta = 0 \Rightarrow \\ \theta = -\alpha + \beta &= \beta - \alpha\end{aligned}$$

17. Angle R corresponds to angle P, so the measure of angle R is 82°. Angle M corresponds to angle S, so the measure of angle M is 86°. Angle N corresponds to angle Q, so the measure of angle N is 12°. Note: 12°+82°+86° =180°.

18. Angle Z corresponds to angle T, so the measure of angle Z is 32°. Angle V corresponds to angle X, so the measure of angle V is 41°. Angles X, Y, and Z are interior angles of a triangle, so the sum of their measures is 180°.

$$\begin{aligned}m\angle X + m\angle Y + m\angle Z &= 180° \\ 41° + m\angle Y + 32° = 180° &\Rightarrow m\angle Y = 107°\end{aligned}$$

Angle U corresponds to angle Y, so the measure of angle U is 107°.

19. The large triangle is equilateral, so the smaller triangle is also equilateral, and $p = q = 7$.

20. $\frac{m}{30} = \frac{75}{50} \Rightarrow m = \frac{75 \cdot 30}{50} = 45$

$\frac{n}{40} = \frac{75}{50} \Rightarrow n = \frac{75 \cdot 40}{50} = 60$

21. $\frac{k}{6} = \frac{12+9}{9} \Rightarrow k = \frac{6 \cdot 21}{9} = 14$

22. $\frac{6}{r} = \frac{7}{11+7} \Rightarrow \frac{6}{r} = \frac{7}{18} \Rightarrow 7r = 108 \Rightarrow r = 15.4$

23. Let x = the shadow of the 30-ft tree.

$\frac{20}{8} = \frac{30}{x} \Rightarrow 20x = 240 \Rightarrow x = 12$ ft

24. $x = -3, y = -3$ and

$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\sin\theta = \frac{y}{r} = -\frac{3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\cos\theta = \frac{x}{r} = \frac{-3}{3\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\tan\theta = \frac{y}{x} = \frac{-3}{-3} = 1$; $\cot\theta = \frac{x}{y} = \frac{-3}{-3} = 1$

$\sec\theta = \frac{r}{x} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$

$\csc\theta = \frac{r}{y} = \frac{3\sqrt{2}}{-3} = -\sqrt{2}$

25. $x = 1,\ y = -\sqrt{3}$ and

$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + \left(-\sqrt{3}\right)^2}$
$= \sqrt{1+3} = \sqrt{4} = 2$

$\sin\theta = \frac{y}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{2}$

$\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

$\cot\theta = \frac{x}{y} = \frac{1}{-\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\sec\theta = \frac{r}{x} = \frac{2}{1} = 2$

$\csc\theta = \frac{r}{y} = \frac{2}{-\sqrt{3}} = -\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$

26. $x = -2, y = 0$, and

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (0)^2} = \sqrt{4} = 2$

$\sin 180° = \frac{y}{r} = \frac{0}{2} = 0$

$\cos 180° = \frac{x}{r} = \frac{-2}{2} = -1$

$\tan 180° = \frac{y}{x} = \frac{0}{-2} = 0$

$\cot 180° = \frac{x}{y} = \frac{-2}{0}$, undefined

$\sec 180° = \frac{r}{x} = \frac{2}{-2} = -1$

$\csc 180° = \frac{r}{y} = \frac{2}{0}$, undefined

27. $x = 3, y = -4$ and $r = \sqrt{3^2 + (-4)^2} = \sqrt{25} = 5$

$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$; $\cos\theta = \frac{x}{r} = \frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{-4}{3} = -\frac{4}{3}$

$\cot\theta = \frac{x}{y} = \frac{3}{-4} = -\frac{3}{4}$

$\sec\theta = \frac{r}{x} = \frac{5}{3}$; $\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$

28. $x = 9, y = -2$, and $r = \sqrt{9^2 + (-2)^2} = \sqrt{85}$

$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{85}} = -\frac{2}{\sqrt{85}} \cdot \frac{\sqrt{85}}{\sqrt{85}} = -\frac{2\sqrt{85}}{85}$

$\cos\theta = \frac{x}{r} = \frac{9}{\sqrt{85}} = \frac{9}{\sqrt{85}} \cdot \frac{\sqrt{85}}{\sqrt{85}} = \frac{9\sqrt{85}}{85}$

$\tan\theta = \frac{y}{x} = \frac{-2}{9} = -\frac{2}{9}$; $\cot\theta = \frac{x}{y} = \frac{9}{-2} = -\frac{9}{2}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{85}}{9}$; $\csc\theta = \frac{r}{y} = \frac{\sqrt{85}}{-2} = -\frac{\sqrt{85}}{2}$

29. $x = -8, y = 15$, and

$r = \sqrt{(-8)^2 + 15^2} = \sqrt{289} = 17$

$\sin\theta = \frac{y}{r} = \frac{15}{17}$; $\cos\theta = \frac{x}{r} = \frac{-8}{17} = -\frac{8}{17}$

$\tan\theta = \frac{y}{x} = \frac{15}{-8} = -\frac{15}{8}$

$\cot\theta = \frac{x}{y} = \frac{-8}{15} = -\frac{8}{15}$

$\sec\theta = \frac{r}{x} = \frac{17}{-8} = -\frac{17}{8}$

$\csc\theta = \frac{r}{y} = \frac{17}{15}$

30. $x = 1, y = -5$, and $r = \sqrt{1^2 + (-5)^2} = \sqrt{26}$

$$\sin\theta = \frac{y}{r} = -\frac{5}{\sqrt{26}} = -\frac{5}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = -\frac{5\sqrt{26}}{26}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sqrt{26}} = \frac{1}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{\sqrt{26}}{26}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{1} = -5$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{5} = -\frac{1}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{26}}{1} = \sqrt{26}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{26}}{-5} = -\frac{\sqrt{26}}{5}$$

31. $x = 6\sqrt{3}$, $y = -6$, and

$$r = \sqrt{\left(6\sqrt{3}\right)^2 + (-6)^2} = \sqrt{108 + 36} = \sqrt{144} = 12$$

$$\sin\theta = \frac{y}{r} = \frac{-6}{12} = -\frac{1}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{6\sqrt{3}}{12} = \frac{\sqrt{3}}{2}$$

$$\tan\theta = \frac{y}{x} = \frac{-6}{6\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{6\sqrt{3}}{-6} = -\sqrt{3}$$

$$\sec\theta = \frac{r}{x} = \frac{12}{6\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{12}{-6} = -2$$

32. $x = -2\sqrt{2},\ y = 2\sqrt{2}$, and

$$r = \sqrt{\left(-2\sqrt{2}\right)^2 + \left(2\sqrt{2}\right)^2} = \sqrt{8+8} = \sqrt{16} = 4$$

$$\sin\theta = \frac{y}{r} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-2\sqrt{2}}{4} = -\frac{\sqrt{2}}{2}$$

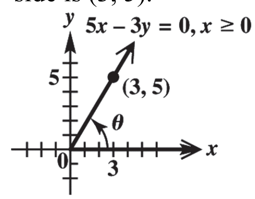
$$\tan\theta = \frac{y}{x} = \frac{2\sqrt{2}}{-2\sqrt{2}} = -1$$

$$\cot\theta = \frac{x}{y} = \frac{-2\sqrt{2}}{2\sqrt{2}} = -1$$

$$\sec\theta = \frac{r}{x} = \frac{4}{-2\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\sqrt{2}$$

$$\csc\theta = \frac{r}{y} = \frac{4}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

33. The terminal side of the angle is defined by $5x - 3y = 0, x \ge 0$, so a point on this terminal side is (3, 5).



$r = \sqrt{3^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$.

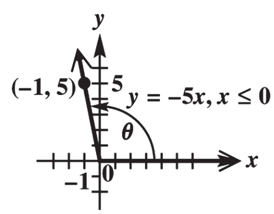
$$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{34}} = \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

$$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{34}} = \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{3}; \quad \cot\theta = \frac{x}{y} = \frac{3}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{34}}{3}; \quad \csc\theta = \frac{r}{y} = \frac{\sqrt{34}}{5}$$

34. The terminal side of the angle is defined by $y = -5x,\ x \le 0$, so a point on this terminal side is (–1, 5).



$r = \sqrt{(-1)^2 + (5)^2} = \sqrt{1 + 25} = \sqrt{26}$.

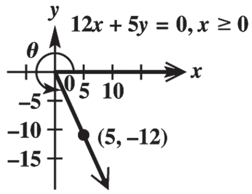
$$\sin\theta = \frac{y}{r} = \frac{5}{\sqrt{26}} = \frac{5}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{26}} = -\frac{1}{\sqrt{26}} \cdot \frac{\sqrt{26}}{\sqrt{26}} = -\frac{\sqrt{26}}{26}$$

$$\tan\theta = \frac{y}{x} = \frac{5}{-1} = -5; \ \cot\theta = \frac{x}{y} = \frac{-1}{5} = -\frac{1}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{26}}{-1} = -\sqrt{26}; \ \csc\theta = \frac{r}{y} = \frac{\sqrt{26}}{5}$$

35. The terminal side of the angle is defined by $12x+5y=0, x\ge 0,$ so a point on this terminal side is (5, –12).



$r=\sqrt{(-12)^2+5^2}=\sqrt{144+25}=\sqrt{169}=13$.

$\sin\theta=\frac{y}{r}=\frac{-12}{13}=-\frac{12}{13};\quad \cos\theta=\frac{x}{r}=\frac{5}{13}$

$\tan\theta=\frac{y}{x}=\frac{-12}{5}=-\frac{12}{5}$

$\cot\theta=\frac{x}{y}=\frac{5}{-12}=-\frac{5}{12}$

$\sec\theta=\frac{r}{x}=\frac{13}{5};\ \csc\theta=\frac{r}{y}=\frac{13}{-12}=-\frac{13}{12}$

36. $\sin 180°=0$
$\cos 180°=-1$
$\tan 180°=0$
cot 180° is undefined
$\sec 180°=-1$
csc 180° is undefined.

37. $\sin(-90)°=-1$
$\cos(-90)°=0$
tan (−90)° is undefined
$\cot(-90)°=0$
sec (−90)° is undefined
$\csc(-90)°=-1$

38. If the terminal side of a quadrantal angle lies along the y-axis, a point on the terminal side would be of the form (0, k), where k is a real number, $k\ne 0$.

$\sin\theta=\frac{y}{r}=\frac{k}{r}$

$\cos\theta=\frac{x}{r}=\frac{0}{r}=0$

$\tan\theta=\frac{y}{x}=\frac{k}{0}$ undefined

$\cot\theta=\frac{x}{y}=\frac{0}{k}=0$

$\sec\theta=\frac{r}{x}=\frac{r}{0}$ undefined

$\csc\theta=\frac{r}{y}=\frac{r}{k}$

The tangent and secant are undefined.

39. $\cos\theta=-\frac{5}{8}$ and θ is in quadrant III

$\cos\theta=\frac{x}{r}=\frac{-5}{8}$ and θ in quadrant III, so let $x=-5, r=8$.

$x^2+y^2=r^2\Rightarrow(-5)^2+y^2=8^2\Rightarrow$
$25+y^2=64\Rightarrow y^2=39\Rightarrow y=\pm\sqrt{39}$

θ is in quadrant III, so $y=-\sqrt{39}$.

$\sin\theta=\frac{y}{r}=\frac{-\sqrt{39}}{8}=-\frac{\sqrt{39}}{8}$

$\cos\theta=\frac{x}{r}=\frac{-5}{8}=-\frac{5}{8}$

$\tan\theta=\frac{y}{x}=\frac{-\sqrt{39}}{-5}=\frac{\sqrt{39}}{5}$

$\cot\theta=\frac{x}{y}=\frac{-5}{-\sqrt{39}}=\frac{5}{\sqrt{39}}\cdot\frac{\sqrt{39}}{\sqrt{39}}=\frac{5\sqrt{39}}{39}$

$\sec\theta=\frac{r}{x}=\frac{8}{-5}=-\frac{8}{5}$

$\csc\theta=\frac{r}{y}=\frac{8}{-\sqrt{39}}=-\frac{8}{\sqrt{39}}$
$=-\frac{8}{\sqrt{39}}\cdot\frac{\sqrt{39}}{\sqrt{39}}=-\frac{8\sqrt{39}}{39}$

40. $\sin\theta=\frac{\sqrt{3}}{5}$ and $\cos\theta<0$

$\sin\theta=\frac{y}{r}$, so let $y=\sqrt{3}, r=5$.

$x^2+y^2=r^2\Rightarrow x^2+\left(\sqrt{3}\right)^2=5^2\Rightarrow$
$x^2+3=25\Rightarrow x^2=22\Rightarrow x=\pm\sqrt{22}$

$\cos\theta<0$, so $x=-\sqrt{22}$.

$\sin\theta=\frac{y}{r}=\frac{\sqrt{3}}{5};\ \cos\theta=\frac{x}{r}=\frac{-\sqrt{22}}{5}=-\frac{\sqrt{22}}{5}$

$\tan\theta=\frac{y}{x}=\frac{\sqrt{3}}{-\sqrt{22}}=-\frac{\sqrt{3}}{\sqrt{22}}\cdot\frac{\sqrt{22}}{\sqrt{22}}=-\frac{\sqrt{66}}{22}$

$\cot\theta=\frac{x}{y}=\frac{-\sqrt{22}}{\sqrt{3}}=-\frac{\sqrt{22}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=-\frac{\sqrt{66}}{3}$

$\sec\theta=\frac{r}{x}=\frac{5}{-\sqrt{22}}=-\frac{5}{\sqrt{22}}\cdot\frac{\sqrt{22}}{\sqrt{22}}=-\frac{5\sqrt{22}}{22}$

$\csc\theta=\frac{r}{y}=\frac{5}{\sqrt{3}}=\frac{5}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{5\sqrt{3}}{3}$

41. $\sec\theta = -\sqrt{5}$ and θ is in quadrant II

θ in quadrant II $\Rightarrow x < 0, y > 0$, so

$$\sec\theta = -\sqrt{5} = \frac{r}{x} = \frac{\sqrt{5}}{-1} \Rightarrow x = -1, r = \sqrt{5}$$

$$x^2 + y^2 = r^2 \Rightarrow (-1)^2 + y^2 = \left(\sqrt{5}\right)^2 \Rightarrow$$
$$1 + y^2 = 5 \Rightarrow y^2 = 4 \Rightarrow y = 2$$

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{5}} = -\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{2}{-1} = -2; \ \cot\theta = \frac{x}{y} = \frac{-1}{2} = -\frac{1}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{-1} = -\sqrt{5}; \ \csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{2}$$

42. $\tan\theta = 2$ and θ is in quadrant III

θ in quadrant III $\Rightarrow x < 0, y < 0$, so

$$\tan\theta = \frac{y}{x} = 2 = \frac{-2}{-1} \Rightarrow x = -1, y = -2$$

$$x^2 + y^2 = r^2 \Rightarrow (-1)^2 + (-2)^2 = r^2 \Rightarrow$$

$r^2 = 5 \Rightarrow r = \sqrt{5}$, because r is positive.

$$\sin\theta = \frac{y}{r} = \frac{-2}{\sqrt{5}} = -\frac{2}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{5}} = -\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{-1} = 2; \quad \cot\theta = \frac{x}{y} = \frac{-1}{-2} = \frac{1}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{5}}{-1} = -\sqrt{5}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$$

43. $\sec\theta = \frac{5}{4}$ and θ is in quadrant IV

θ in quadrant IV $\Rightarrow x > 0, y < 0$, so

$$\sec\theta = \frac{r}{x} = \frac{5}{4} \Rightarrow x = 4, r = 5$$

$$x^2 + y^2 = r^2 \Rightarrow (4)^2 + y^2 = 5^2 \Rightarrow$$
$$16 + y^2 = 25 \Rightarrow y^2 = 9 \Rightarrow y = -3$$

$$\sin\theta = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}; \ \cos\theta = \frac{x}{r} = \frac{4}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-3}{4} = -\frac{3}{4}$$

$$\cot\theta = \frac{x}{y} = \frac{4}{-3} = -\frac{4}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{4}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-3} = -\frac{5}{3}$$

44. $\sin\theta = -\frac{2}{5}$ and θ is in quadrant III

θ in quadrant III $\Rightarrow x < 0, y < 0$, so

$$\sin\theta = \frac{y}{r} = -\frac{2}{5} = \frac{-2}{5} \Rightarrow y = -2, r = 5$$

$$x^2 + y^2 = r^2 \Rightarrow x^2 + (-2)^2 = 5^2 \Rightarrow$$
$$x^2 + 4 = 25 \Rightarrow x^2 = 21 \Rightarrow x = -\sqrt{21}$$

$$\sin\theta = \frac{y}{r} = -\frac{2}{5}; \ \cos\theta = \frac{x}{r} = \frac{-\sqrt{21}}{5} = -\frac{\sqrt{21}}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-2}{-\sqrt{21}} = \frac{2}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = \frac{2\sqrt{21}}{21}$$

$$\cot\theta = \frac{x}{y} = \frac{-\sqrt{21}}{-2} = \frac{\sqrt{21}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-\sqrt{21}} = -\frac{5}{\sqrt{21}} \cdot \frac{\sqrt{21}}{\sqrt{21}} = -\frac{5\sqrt{21}}{21}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-2} = -\frac{5}{2}$$

45. **(a)** Impossible because the range of $\sec\theta$ is $(-\infty, -1] \cup [1, \infty)$.

(b) Possible because the range of $\tan\theta$ is $(-\infty, \infty)$.

(c) Impossible because the range of $\cos\theta$ is $[-1, 1]$.

46. The sine is negative in quadrants III and IV. The cosine is positive in quadrants I and IV. $\sin\theta < 0$ and $\cos\theta > 0$, so θ lies in quadrant IV. $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin\theta < 0$ and $\cos\theta > 0$, so the sign of $\tan\theta$ is negative.

47. The triangles are similar, so

$$\frac{20}{30} = \frac{x}{100 - x} \Rightarrow 20(100 - x) = 30x \Rightarrow$$
$$2000 - 20x = 30x \Rightarrow 2000 = 50x \Rightarrow 40 = x$$

The lifeguard will enter the water 40 yards east of his original position.

48. $\dfrac{360°}{26,000\text{ yr}} = \dfrac{9°}{650\text{ yr}} = \dfrac{9}{650}(60)\dfrac{\text{min}}{\text{yr}} = \dfrac{54\text{ min}}{65\text{ yr}}$
$= \dfrac{54}{65}(60)\dfrac{\text{sec}}{\text{yr}} = \dfrac{648\text{ sec}}{13\text{ yr}} \approx 50\text{ sec/yr}$

49. Let x = the depth of the crater Autolycus. Then
$\dfrac{x}{11,000} = \dfrac{1.3}{1.5} \Rightarrow 1.5x = 1.3(11,000) \Rightarrow$
$1.5x = 14,300 \Rightarrow x \approx 9500$
Autoclycus is about 9500 feet deep.

50. Let x = the height of Bradley. Then
$\dfrac{x}{21,000} = \dfrac{1.8}{2.8} \Rightarrow 2.8x = 1.8(21,000) \Rightarrow$
$2.8x = 37,800 \Rightarrow x = 13,500$
Bradley is 13,500 feet tall.

Chapter 1 Test

1. 67°
complement: 90° − 67° = 23°
supplement: 180° − 67° = 113°

2. The two angles are supplements, so their sum is 180°.
$(7x+19)+(2x-1)=180 \Rightarrow 9x+18=180 \Rightarrow$
$9x=162 \Rightarrow x=18$
$7(18)+19=145; 2(18)-1=35$
The measures of the angles are 145° and 35°.

3. The two angles are complements, so their sum is 90°.
$(-8x+30)+(-3x+5)=90$
$-11x+35=90$
$-11x=55 \Rightarrow x=-5$
$-8(-5)+30=70;\ -3(-5)+5=20$
The angles measure 20° and 70°.

4. The angles are vertical angles, so their measures are equal.
$4x-30=5x-70 \Rightarrow 40=x$
$4(40)-30=130;\ 5(40)-70=130$
The angles each measure 130°.

5. The angles are alternate interior angles, so their measures are equal.
$8x+14=10x-10 \Rightarrow 24=2x \Rightarrow 12=x$
$8(12)+14=110;\ 10(12)-10=110$
The angles each measure 110°.

6. The angles are interior angles of a triangle, so the sum of the three angles is 180°.
$(2x+18)+(20x+10)+(32-2x)=180$
$20x+60=180$
$20x=120$
$x=6$
$2(6)+18=30; 20(6)+10=130; 32-2(6)=20$
The three angles measure 30°, 130°, and 20°.

7. Two of the angles are interior angles, but one is an exterior angle. From geometry, we know that the measure of an exterior angle is equal to the sum of the two nonadjacent interior angles.
$12x+40=12x+8x$
$40=8x$
$5=x$
$12(5)+40=100;\ 12(5)=60;\ 8(5)=40$
The three angles measure 100°, 60°, and 40°.

8. $74°18'36'' = 74° + \frac{18}{60}° + \frac{36}{3600}°$
$\approx 74° + 0.3° + 0.01° = 74.31°$

9. $45.2025° = 45° + 0.2025°$
$= 45° + 0.2025(60')$
$= 45° + 12.15'$
$= 45° + 12' + 0.15'$
$= 45° + 12' + 0.15(60'')$
$= 45° + 12' + 09'' = 45°12'09''$

10. **(a)** 390° is coterminal with 390° − 360° = 30°.

(b) −80° is coterminal with −80° + 360° = 280°.

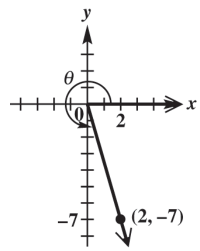
(c) 810° is coterminal with 810° − 2(360°) = 810° − 720° = 90°.

11. $\dfrac{450(360°)}{1\text{ min}} = \dfrac{450(360°)}{60\text{ sec}} = \dfrac{450(6°)}{\text{sec}}$
$= 2700°/\text{sec}$
A point on the tire rotates 2700° in one second.

12. $\dfrac{8}{30} = \dfrac{x}{40} \Rightarrow \dfrac{8 \cdot 40}{30} = x \Rightarrow x = 10\frac{2}{3}$
The shadow of the 40-ft pole is $10\frac{2}{3}$ ft, or 10 ft, 8 in.

13. $\dfrac{10}{25} = \dfrac{x}{20} \Rightarrow x = \dfrac{10 \cdot 20}{25} = 8$
$\dfrac{10}{25} = \dfrac{y}{15} \Rightarrow y = \dfrac{10 \cdot 15}{25} = 6$

14.



$x = 2, y = -7$

$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + (-7)^2} = \sqrt{4+49} = \sqrt{53}$

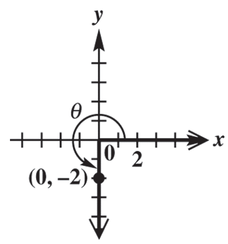
$\sin\theta = \frac{y}{r} = \frac{-7}{\sqrt{53}} = -\frac{7}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}} = -\frac{7\sqrt{53}}{53}$

$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{53}} = \frac{2}{\sqrt{53}} \cdot \frac{\sqrt{53}}{\sqrt{53}} = \frac{2\sqrt{53}}{53}$

$\tan\theta = \frac{y}{x} = \frac{-7}{2} = -\frac{7}{2}$; $\cot\theta = \frac{x}{y} = \frac{2}{-7} = -\frac{2}{7}$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{53}}{2}$; $\csc\theta = \frac{r}{y} = \frac{\sqrt{53}}{-7} = -\frac{\sqrt{53}}{7}$

15.



$x = 0, y = -2$

$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

$\sin\theta = \frac{y}{r} = \frac{-2}{2} = -1$; $\cos\theta = \frac{x}{r} = \frac{0}{2} = 0$

$\tan\theta = \frac{y}{x} = \frac{-2}{0}$ undefined

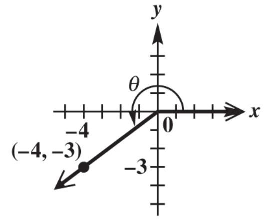
$\cot\theta = \frac{x}{y} = \frac{0}{-2} = 0$

$\sec\theta = \frac{r}{x} = \frac{2}{0}$ undefined

$\csc\theta = \frac{r}{y} = \frac{2}{-2} = -1$

16. Because $x \le 0$, the graph of the line $3x - 4y = 0$ is shown to the left of the y-axis. A point on this graph is $(-4, -3)$. The corresponding value of r is

$r = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5.$



3x − 4y = 0, x 0

$x = -4, y = -3$

$\sin\theta = \frac{y}{r} = -\frac{3}{5}$; $\cos\theta = \frac{x}{r} = -\frac{4}{5}$

$\tan\theta = \frac{y}{x} = \frac{3}{4}$; $\cot\theta = \frac{x}{y} = \frac{4}{3}$

$\sec\theta = \frac{r}{x} = -\frac{5}{4}$; $\csc\theta = \frac{r}{y} = -\frac{5}{3}$

17.

θ	90°	−360°	630°
$\sin\theta$	1	0	−1
$\cos\theta$	0	1	0
$\tan\theta$	undefined	0	undefined
$\cot\theta$	0	undefined	0
$\sec\theta$	undefined	1	undefined
$\csc\theta$	1	undefined	−1

18. If the terminal side of a quadrantal angle lies on the negative part of the x-axis, any point on the terminal side would have the form $(k, 0)$, where k is any real number < 0.

$\sin\theta = \frac{y}{r} = \frac{0}{r} = 0$; $\cos\theta = \frac{x}{r} = \frac{k}{r}$

$\tan\theta = \frac{y}{x} = \frac{0}{k} = 0$; $\cot\theta = \frac{x}{y} = \frac{k}{0}$ undefined

$\sec\theta = \frac{r}{x} = \frac{r}{k}$; $\csc\theta = \frac{r}{y} = \frac{r}{0}$ undefined

Thus, the cotangent and the cosecant are undefined.

19. **(a)** $\cos\theta > 0$ in quadrants I and IV, while $\tan\theta > 0$ in quadrants I and III. So, both conditions are met only in quadrant I.

(b) $\sin\theta < 0$ in quadrants III and IV. $\csc\theta$ is the reciprocal of $\sin\theta < 0$, so $\csc\theta < 0$ also in quadrants III and IV. Thus, both conditions are met in quadrants III and IV.

(c) $\cot\theta > 0$ in quadrants I and III, while $\cos < 0$ in quadrants II and III. Both conditions are met only in quadrant III.

20. **(a)** Impossible because the range of $\sin\theta$ is $[-1, 1]$.

(b) Possible because the range of $\sec\theta$ is $(-\infty, -1] \cup [1, \infty)$.

(c) Possible because the range of $\tan\theta$ is $(-\infty, \infty)$.

21. $\cos\theta = -\frac{7}{12} \Rightarrow \sec\theta = -\frac{12}{7}$

22. $\sin\theta = \frac{3}{7}$ with θ in quadrant II

θ in quadrant II $\Rightarrow x < 0, y > 0$

$$\sin\theta = \frac{y}{r} = \frac{3}{7} \Rightarrow y = 3, r = 7$$

$$r^2 = x^2 + y^2 \Rightarrow 7^2 = x^2 + 3^2 \Rightarrow 49 = x^2 + 9 \Rightarrow$$
$$x^2 = 40 \Rightarrow x = -\sqrt{40} = -2\sqrt{10}$$

$$\cos\theta = \frac{x}{r} = \frac{-2\sqrt{10}}{7} = -\frac{2\sqrt{10}}{7}$$

$$\tan\theta = \frac{y}{x} = \frac{3}{-2\sqrt{10}} = -\frac{3}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{3\sqrt{10}}{20}$$

$$\cot\theta = \frac{x}{y} = \frac{-2\sqrt{10}}{3}$$

$$\sec\theta = \frac{r}{x} = \frac{7}{-2\sqrt{10}} = -\frac{7}{2\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = -\frac{7\sqrt{10}}{20}$$

$$\csc\theta = \frac{r}{y} = \frac{7}{3}$$