TT

TEXTBOOK TESTBANKS

Solutions Manual for Mathematics for Elementary and Middle School Teachers with Activities 6th Beckmann

SOLUTIONS MANUAL SAMPLE PREVIEW

PUBLISHER

Pearson

COPYRIGHT

2022

RESOURCE TYPE

Solutions Manual

ISBN

9780136922001

Chapter 1
Numbers and the Base-Ten System

1.1 The Counting Numbers

1. Answers will vary. For example, when connecting the counting numbers as a list view of numbers with the number of objects in a set view of numbers, a child must learn to associate each number in the list in a one to one correspondence with each object in the set, starting with one. Also, the child must be able to learn that the last number from the list, used to connect with the last object in the set, is the number of objects in the set.

2. Yes, there is a better way to respond. For instance, you could group the beads into sets of 10 beads in each group. Then you would have 3 groups of 10 beads in each group and there would be 5 left over beads. This grouping would facilitate a discussion about place value and allow the conversation to focus on 3 tens.

3. a. You could group the beads into sets of 10 beads in each group. Then you would have 4 groups of 10 beads in each group and there would be 7 left over beads. Using the place value system of representing numbers, 4 tens and 7 ones is 47. Figure 1.1 shows a simple math drawing that could be drawn.

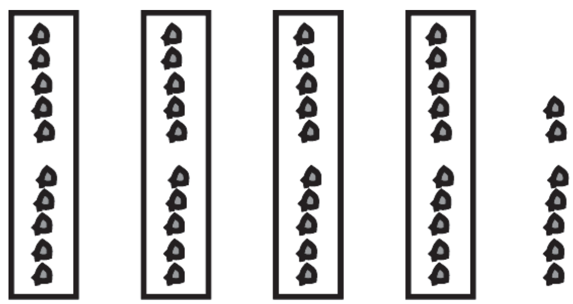


Figure 1.1: Representation of 47

   b. You could bag the toothpicks into sets of 10 toothpicks in each bag. Then when you get 10 bags of 10 toothpicks in each, you could bundle, with a rubber band, 10 bags of 10 toothpicks to make sets of 100 toothpicks in each bundle. Then you would have 3 bundles of 100 toothpicks in each bundle (or 3 hundreds) and you would have 2 bags of 10 toothpicks in each bag (or 2 tens) and there would be 8 left over toothpicks. Using the place value system of representing numbers, 3 hundreds, 2 tens, and 8 ones is 328. Figure 1.2 shows a simple math drawing that could be drawn.

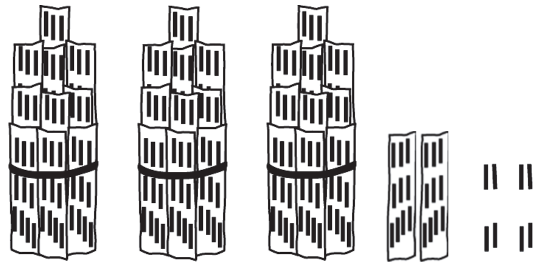


Figure 1.2: Representation of 328

c. You could bag the toothpicks into sets of 10 toothpicks in each bag. Then when you get 10 bags of 10 toothpicks in each, you could bundle, with a rubber band, 10 bags of 10 toothpicks to make sets of 100 toothpicks in each bundle. Then when you have 10 bundles of 100 toothpicks, you could get a giant gallon sized plastic bag and put them into it and group these 10 sets of 100 toothpicks into 1 set of 1000 toothpicks. Using the place value system of representing numbers, 1 thousand is represented as 1000. Figure 1.3 shows a simple math drawing that could be drawn.

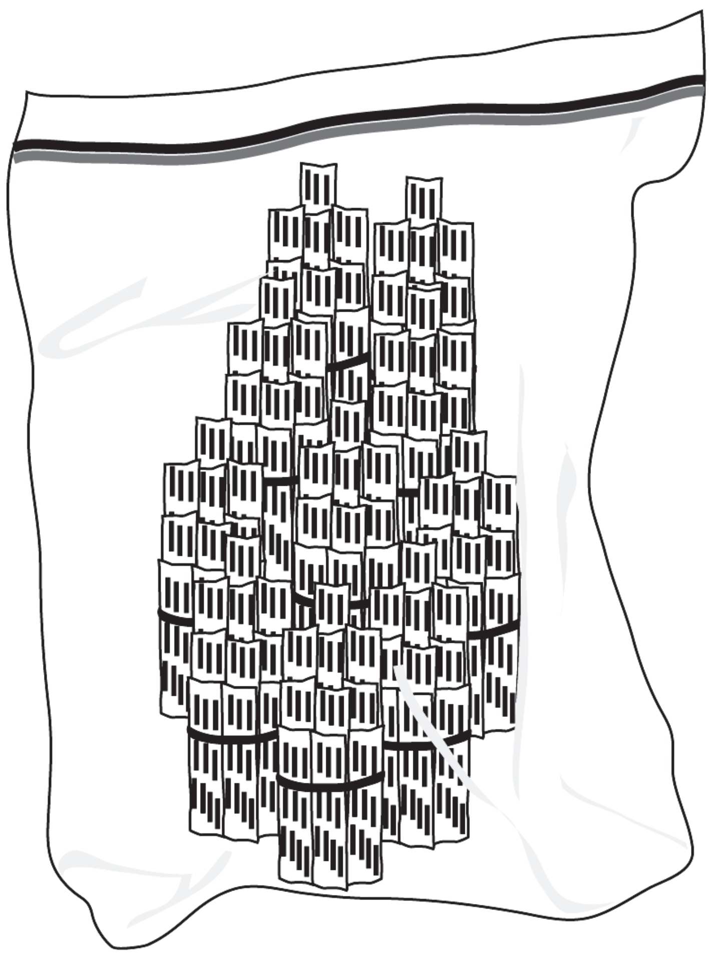


Figure 1.3: Representation of 1000

4.

a. Let's say you have a relatively unorganized collection of 62 toothpicks as shown below in Figure 1.4

Figure 1.4: Representation of 62 toothpicks

You could bag the toothpicks into sets of 10 toothpicks in each bag
See Figure 1.5.

Figure 1.5: Math drawing of 10 ones regrouped as 1 ten.

You could continue doing that with all 62 toothpicks, that is, you can continue collecting sets of ten toothpicks and regrouping them into tens until you can no longer do that. When you are done, you will be able to count the number of tens. If you look at the number 62, the digit in the tens place is a 6 and that corresponds with the number of tens that you find. Similarly, the digit is ones place is 2 and we have 2 single toothpicks not in bundles of ten. See Figure 1.6.

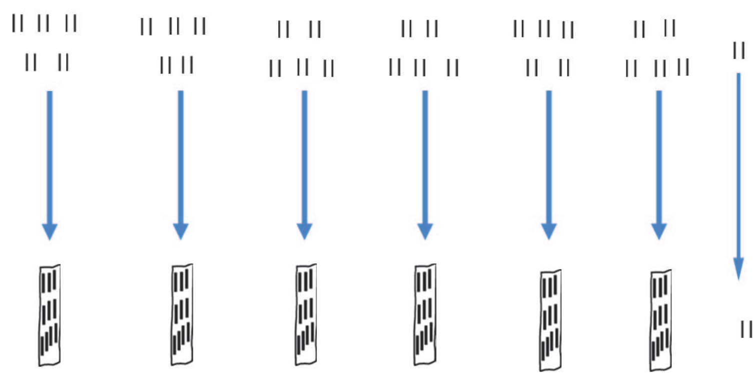


Figure 1.6: 62 represented in base-ten bundles.

b. The 6 in the number 62 could represent 6 tens, as shown in Figure 1.6. If we took the bundles apart, we would have 60 ones so 6 can stand for that as well. In summary the 6 stands for 6 tens or 60 ones.

5.

a.

You could bag the toothpicks into sets of 10 toothpicks in each bag. Then when you get 10 bags of 10 toothpicks in each, you could bundle, with a rubber band, 10 bags of 10 toothpicks to make sets of 100 toothpicks in each bundle. Then you would have 3 bundles of 100 toothpicks in each bundle (or 3 hundreds) and you would have 2 bags of 10 toothpicks in each bag (or 2 tens) and there would be 8 left over toothpicks. Using the place value system of representing numbers, 3 hundreds, 5 tens, and 8 ones is 328. Figure 1.7 shows a simple math drawing that could be drawn.

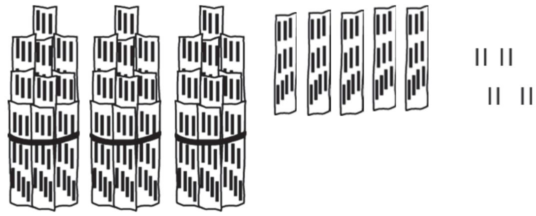


Figure 1.7: Representation of 358

b. The 5 in the number 358 could represent 5 tens, as shown in Figure 1.7. If we took the bundles apart, we would have 50 ones so 5 can stand for that as well. In summary the 5 stands for 5 tens or 50 ones.

c. The 3 in the number 358 could represent 3 hundreds, as shown in Figure 1.7. If we took the bundles apart, we would have 30 tens so 3 can stand for that as well. If we took apart the ten bundles, we would have 300 single toothpicks, so 3 could stand for that as well. In summary, the 3 stands for 3 hundreds, 30 tens or 300 ones.

6.

For instance, if you we are using Popsicle sticks, you could get a large collection (say three-hundred fifty-six sticks) in an unorganized pile. Then you could ask the students that are learning about place value to start counting them. After a bit, someone (you or some of the students) will likely start to group the Popsicle sticks into piles of equal size. You could then count the Popsicle sticks faster by bundling groups of 10 together. Perhaps you might bundle these groups together physically by tying a twist tie around each set of 10 Popsicle sticks.

After a while of doing this, you will have lots of bundled sets of 10 Popsicle sticks. At this stage, you could take 10 sets of twist-tied sets of 10 Popsicle sticks and put them in to a gallon sized plastic bag to make groups of 100 Popsicle sticks (consisting of 10 sets of 10 bundled Popsicle sticks). As you continue making twist-tied bundles of 10 and baggies of 100, you eventually will use up all of the Popsicle sticks. When this is done, you would end up with 3 baggies (or 3 hundreds) and 5 twist-tied bundles (or 5 tens) and 6 left over Popsicle sticks. Now you could point out that your baggies, bundles, and individual Popsicle sticks correspond directly with the base-ten representation of three hundred fifty-six Popsicle sticks (or 356). Since the digit in each place value is representing a count of units that consist of 10 bundles of units that are represented in the digit to the right, we see that each place value represents a number that is 10 times greater than the place value to its immediate right. For example, when we count the farthest left place value in this number (365), we count groups of hundred (3 baggies). Since each baggie is made up of 10 bundles, we note that the place value immediately to right of the hundreds place is the place value where we are counting the bundles (or tens).

7. Young children must learn several key ideas about place value and overcome some linguistic hurdles to learn how to count in the base-ten structure. They must understand

the key role that 10 plays in our base-ten structure or in representing numbers. They must understand how the location affects the value of the number or the unit of the digit in that particular location. They must overcome linguistic difficulties inherent to how we say numbers in English, especially the anomalies like eleven or the inconsistent order of twenty-two (2 tens and two ones) and nineteen (one nine and one ten).

8. The number 1001 looks like 100 and 1 put together. Calling it "one hundred one" makes sense w/o understanding the structure of our number system. See Figure 1.8. Each small block represents 1.

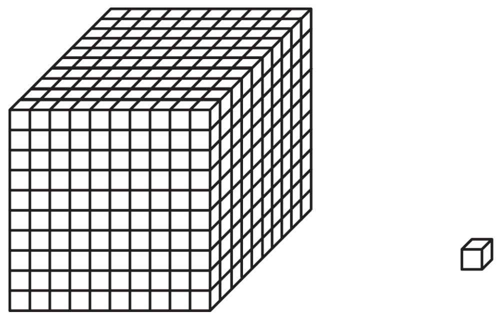


Figure 1.8: Base-Ten representation of 1001

9. If we count what we've got in the math drawing, we see 17 individual toothpicks and 15 bags of 10 toothpicks in each bag. Naively, we might write this as 1517 toothpicks, which would be misleading since as written it would represent one-thousand five-hundred seventeen toothpicks.

Since our place value system can only represent up to 9 of any particular place value, we have to regroup when we have more than 9 of a particular place value (or base-ten unit). In terms of toothpicks, this means that since 17 is greater than 9, we have enough individual toothpicks to regroup into one more baggie of 10. This gives us 7 left over toothpicks but now 16 bags. Similarly, we also have enough bags to regroup (or bundle) them with a rubber band into one group of 100 toothpicks. This gives us 1 bundle of 100 toothpicks (or 1 hundred), 6 bags of ten toothpicks (or 6 tens) and 7 individual toothpicks. See Figure 1.9 for what Figure 1.15 (in the regular text) would look like once you've regrouped the numbers in a way that corresponds to the base-ten structure.

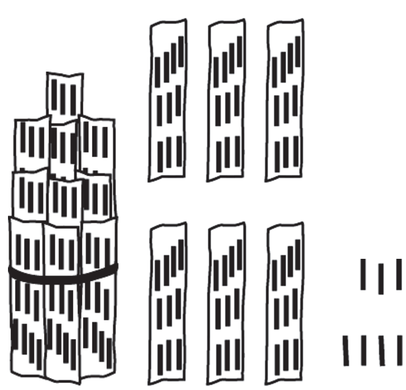


Figure 1.9: Representation of 167

10. Answers will vary. In the base-ten structure, the digits represent different values of objects. The place value is integrally related to the value that any particular digit represents. The number ten plays a vital role in the system and is the basis of the value of each place. The base-ten structure is much easier to represent large numbers than more primitive ways of representing numbers. However, the base-ten structure is not as intuitive and is harder to learn than more primitive systems, such as a simple tally mark system.

11.

a. You cannot tell where to plot 5 on the given number line. Without another point for reference, you cannot determine how much space on the given number line that a unit represents or that some other quantity of units represent.

b. You cannot plot 100 on the given number line. The reasoning is the same as the answer in part a above.

c. You cannot plot N+1 on the given number line. Without knowing what N represents, you don't know how many partitions to make between 0 and N. Those partition sizes would be each 1 unit, so you cannot determine where one more unit past N would be.

12.

a. See Figure 1.10. In the first number line I first used the larger tick marks to represent 400 each. Then I counted up to 800 and then 1200 using these sized tick marks. Realizing that 900 wasn't going to fall perfectly on a 400 tick mark, I partitioned the 400 tick marks spaces into 4 smaller spaces and then each shorter tick mark represented 100. I then counted one more shorter tick mark past 800 to get to 900.

b. See Figure 1.10. For the second number line, we partitioned the space between 0 and 300 into 3 equal spaces so between taller tick marks represents 100. Then we partitioned each of these spaces representing 100 into 2 small spaces representing 50. I counted up to 200 with large tick marks and then over one more smaller space to 250.

c. See Figure 1.10. For the last number line, I let the spaces between taller tick marks represent 2000. I went up to 6,000. Next, since it takes ten 200s to make 2,000 and the gap between 6,000 and 8,000 is 2,000, then I made 10 spaces between 6,000 and 8,000 and plotted 6,200 one of these spaces above 6,000.

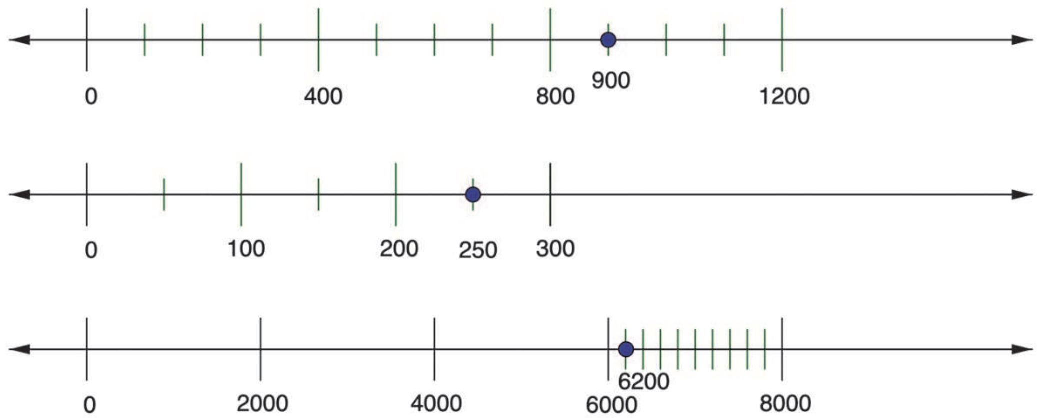


Figure 1.10: Number lines

13.

a. The next two units in base-two after 16 are 32 and 64.

b. See Table 1.1

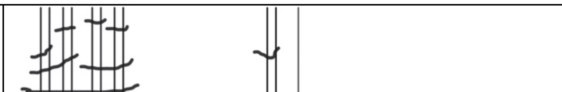
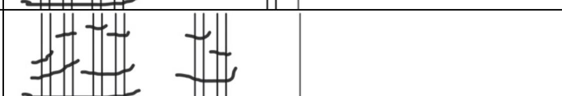
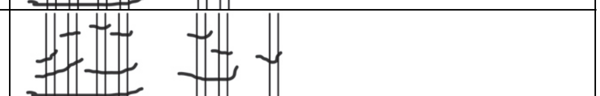
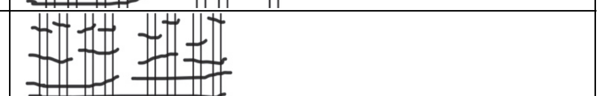
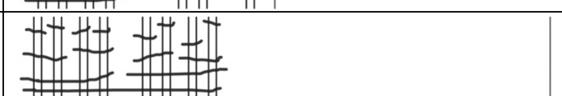
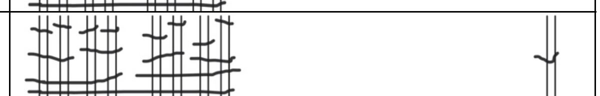
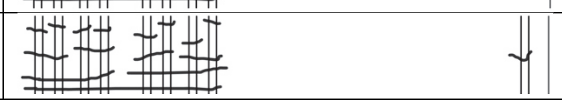
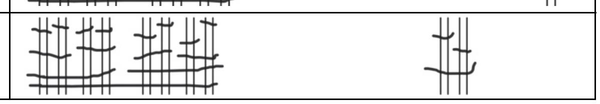
11		12	
13		14	
15		16	
17		18	
19		20	

Table 1.1: 11 through 20 in base-two representation

c. $11 = 1011_2$
$12 = 1100_2$
$13 = 1101_2$
$14 = 1110_2$
$15 = 1111_2$
$16 = 10000_2$
$17 = 10001_2$
$18 = 10010_2$
$19 = 10011_2$
$20 = 10100_2$

d. See Figure 1.11. $35 = 100011_2$.

Figure 1.11: 35 written in base-two.

e. See Figure 1.12. $45 = 101101_2$.

Figure 1.12: 45 written in base-two.

14.

a. The next two units in base-three after 3 are 9 and 27.

b. See Table 1.2.

1		2		3	
4		5		6	
7		8		9	
10		11		12	
13		14		15	
16		17		18	
19		20		21	
22		23		24	
25		26		27	
28					
29					
30					

Table 1.2: Numbers 1 to 30 in base-three math drawings

c. See Table 1.3.

$1 = 1_3$	$11 = 102_3$	$21 = 210_3$
$2 = 2_3$	$12 = 110_3$	$22= 211_3$
$3 = 10_3$	$13 = 111_3$	$23 = 212_3$
$4 = 11_3$	$14 = 112_3$	$24 = 220_3$
$5 = 12_3$	$15 = 120_3$	$25 = 221_3$
$6 = 20_3$	$16 = 121_3$	$26 = 222_3$
$7 = 21_3$	$17 = 122_3$	$27 = 1000_3$
$8 = 22_3$	$18 = 200_3$	$28 = 1001_3$
$9 = 100_3$	$19 = 201_3$	$29 = 1002_3$
$10 = 101_3$	$20 = 202_3$	$30 = 1010_3$

Table 1.3: Numbers 1 to 30 in base-three

d. See Figure 1.13. $45 = 120_3$.

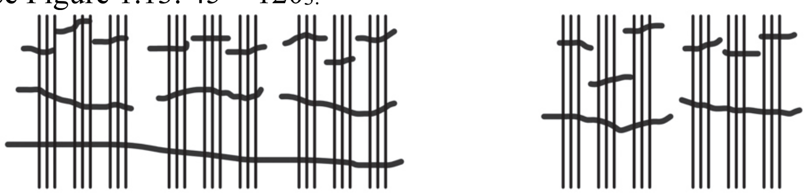


Figure 1.13: 45 written in base-three.

15.

a. The next two units in base-eight after 8 are 64 and 256.

b. See Table 1.4.

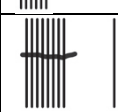
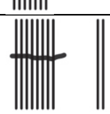
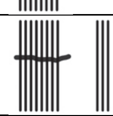
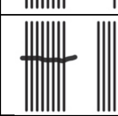
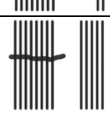
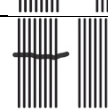
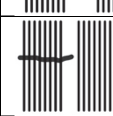
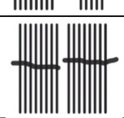
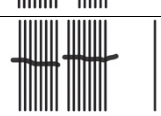
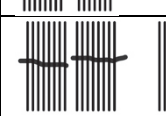
1		2		3	
4		5		6	
7		8		9	
10		11		12	
13		14		15	
16		17		18	
19		20			

Table 1.4: Numbers 1 to 20 in base-eight math drawings

c. See Table 1.5.

$1 = 1_8$	$5 = 5_8$	$9 = 11_8$	$13 = 15_8$	$17 = 21_8$
$2 = 2_8$	$6 = 6_8$	$10 = 12_8$	$14 = 16_8$	$18 = 22_8$
$3 = 3_8$	$7 = 7_8$	$11 = 13_8$	$15 = 17_8$	$19 = 23_8$
$4 = 4_8$	$8 = 10_8$	$12 = 14_8$	$16 = 20_8$	$20 = 24_8$

Table 1.5: Numbers 1 to 20 in base-eight

d. See Figure 1.14. $100 = 144_8$.

Figure 1.14: 100 written in base-eight.

16. Answers will vary. For example, they could use a photocopier and shrink the poster to a small size (but where the dots are still distinguishable hopefully), then they could make 999 copies of this small sized poster. This would work since each of the 1000 shrunk down posters contain 1 million dots and 1,000,000 x 1,000 is 1 billion. It would probably be doable; however, one might need to find a special photocopier (like one that photocopies large maps) if the poster is large enough. Or for a more green approach, one could try to have each student try to draw a portion of the dots themselves. For example, if there were 25 students in the class then each student would need to represent 40 million dots. This would likely be not very feasible because if you worked every day on this for 4 weeks (for a total of 5x4 or 20 days) each student would still need to draw $40 \div 20$ million or 2 million dots a day. If they worked for 50 minutes on it, that would be $2{,}000{,}000 \div 50$ or 40,000 dots every minute. Even if they could do this, the class would likely revolt after two or three days of drawing dots day after day.

1.2 Decimals and Negative Numbers

1.

a. It represents one tenth. Since it takes ten of those to make one, then each long strip is one tenth of a unit or one tenth of one.

b. It represents one hundredth. Since it takes one hundred of those to make one, then each small square is one hundredth of a unit or one hundredth of one.

c. The figure represents 1.74. As described above and in the directions, each large square represents one. In the figure, there is one large square so there is 1 one. Similarly, there are 7 tenths (or 7 long strips of small squares) and 4 hundredths (or 4 small squares).

d. See Figure 1.15. As described above and in the directions, each large square represents one. In the figure, there are 3 large squares, so we have 3 ones. Similarly, there are 2 tenths (or 2 long strips of small squares) and 5 hundredths (or 5 small squares).

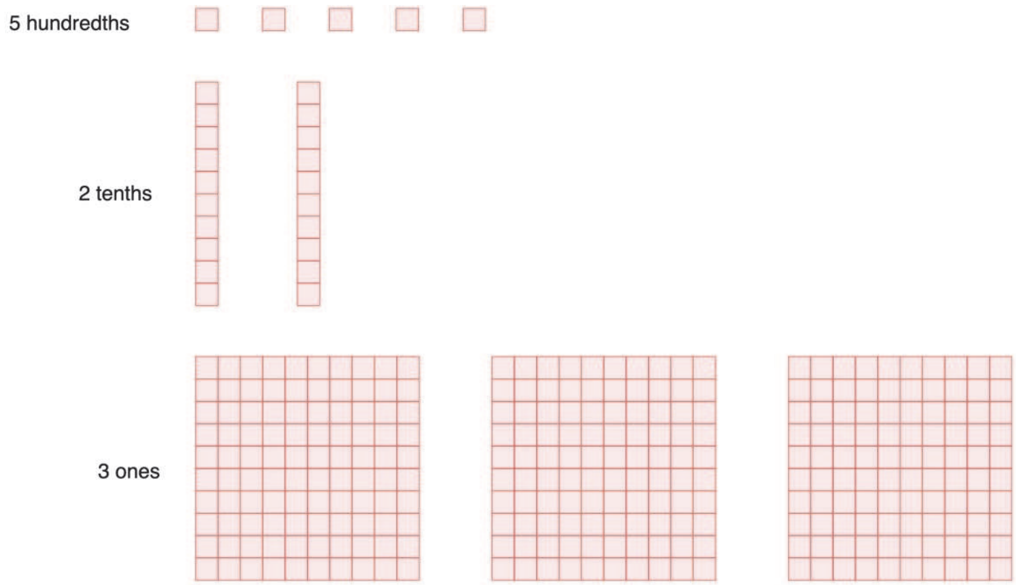


Figure 1.15: 3.25 represented with base-ten blocks

e. See Figure 1.16. As described above and in the directions, each large square represents one. In the figure, there is 1 large square so there is 1 one. Similarly, there are 8 tenths (or 8 long strips of small squares).

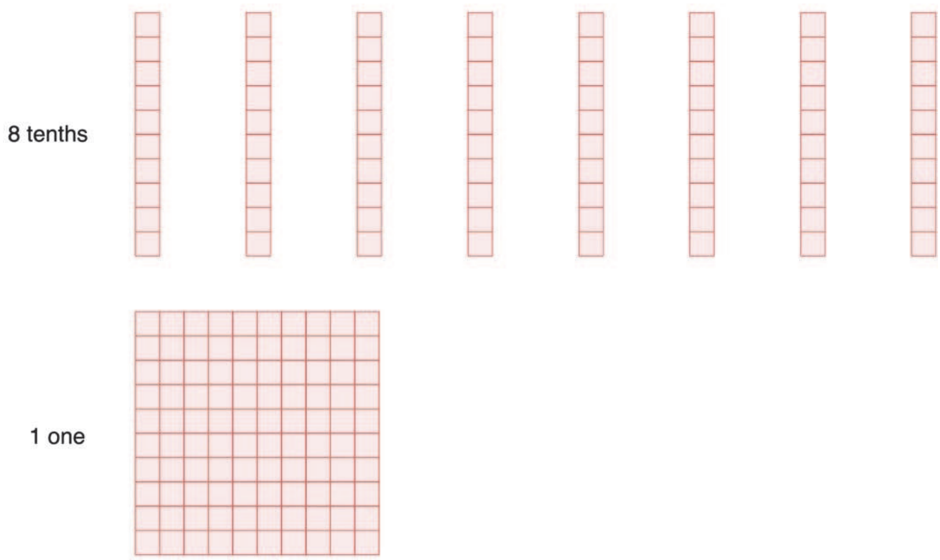


Figure 1.16: 1.8 represented with base-ten blocks

f. See Figure 1.17. As described above and in the directions, each large square represents one. In the figure, there is 1 large square so there is 1 one. Similarly, there are 8 hundredths (or 8 small squares).

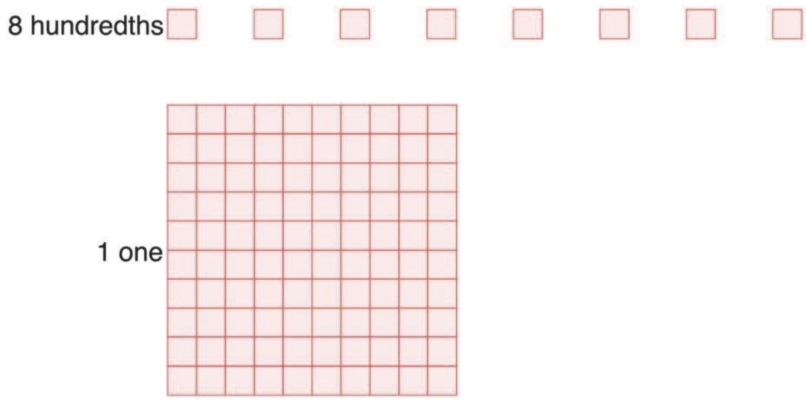


Figure 1.17: 1.08 represented with base-ten blocks

g. See Figure 1.18. As described above and in the directions, each large square represents one. In the figure, there are 4 tenths (or 4 long strips of small squares and 3 hundredths (or 3 small squares).

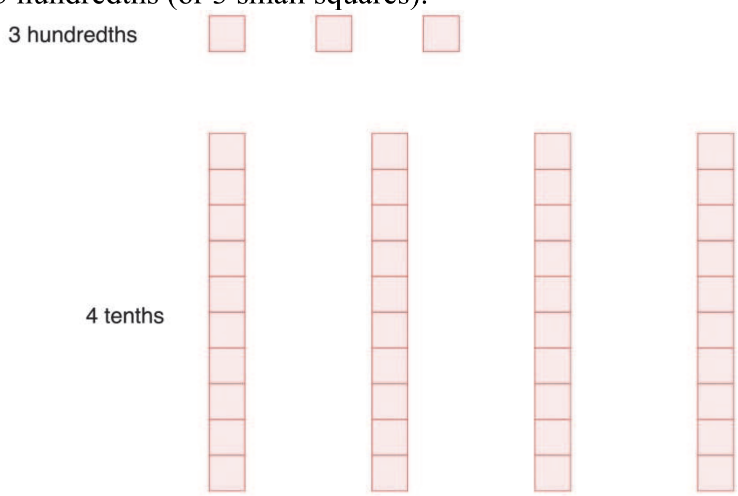


Figure 1.18: 0.43 represented with base-ten blocks

h. To represent a thousandth, we would need a very thin vertical strip such that ten of those strips would make up the small square that represents a hundredth.

2. Answers will vary. For example, to create tenths, you must first establish what shape represents one. Once you have done that, you can partition that shape into ten equal parts. Each of these parts will represent one tenth. Similarly, you can partition one tenth into ten equal parts and each of these parts will represent one hundredth. Finally, you can partition one hundredth into ten equal parts, which will each represent one thousandth. If you want to represent one thousandth and one unit using the same model, then you must pick a one that can be split easily into 1000 pieces so that the thousandths are visible. Base-ten blocks are often used for this representation.

3.

a. See Figure 1.19. The large square is one. The lightly shaded column represents one tenth. If you count the columns you can see that there are a total of 10 columns, indicating that each one is made up of 10 tenths.

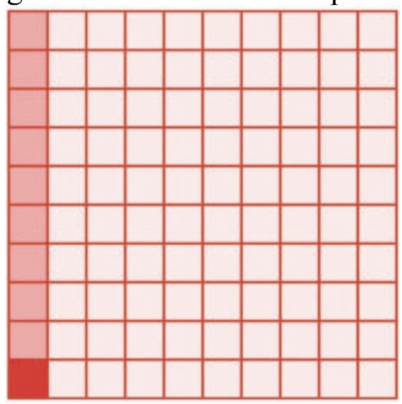


Figure 1.19: Representation of ones, tenths, and hundredths.

b. See Figure 1.19. The large square is one. The darkly shaded small square represents one hundredth. The columns each represent a tenth. If you count the number of small squares per column, you can see the answer is that 10 hundredths make a tenth.

c. See Figure 1.19. The large square is one. The darkly shaded small square represents one hundredth. The columns each represent a tenth. From part a, we know there are 10 tenths in a one. From part b, we know that each of these tenths have 10 hundredths. To find how many hundredths make a one, we could just add 10 hundredths, 10 times or we could multiply and see that $10 \times 10 = 100$ total hundredths.

4.

a. See Figure 1.20. If each paperclip is a thousandth, then the darkly shaded red column in the upper left-hand corner represents a hundredth. This is true because we know that 10 thousandths make a hundredth and there are ten paperclips in that column.

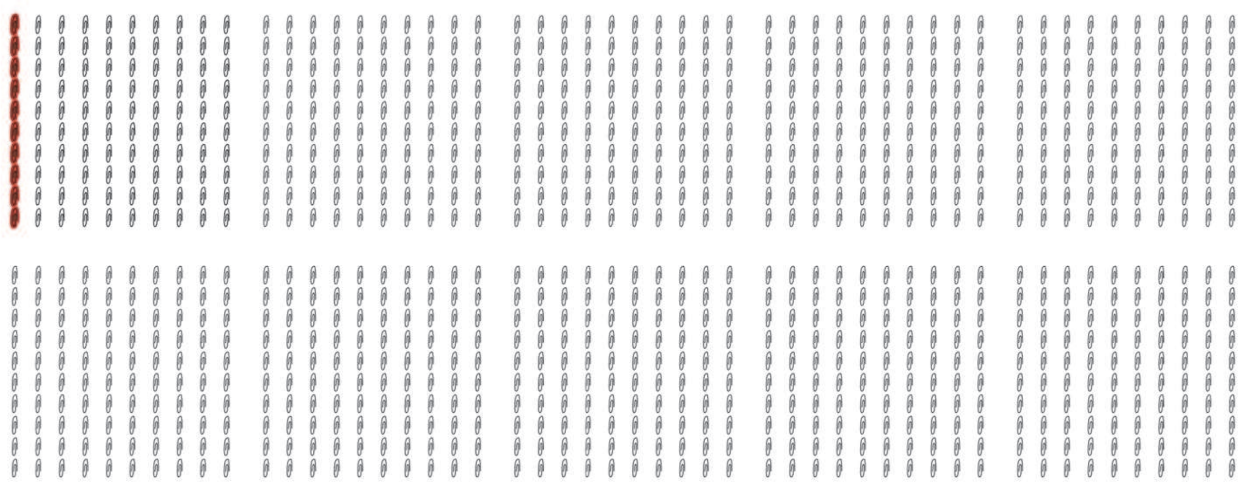


Figure 1.20: Base-Ten representations with paperclips

b. See Figure 1.19. If each paperclip is a thousandth, then the squares represent a tenth. This is true because we know that there are 10 thousandths make a hundredth (or column). Also, we know that 10 hundredths make a tenth. We can see that there are ten columns in each square, so the squares are tenths.

c. See Figure 1.19. We know that ten tenths (squares) make a one. Since we can see ten squares in the entire collection, then the entire collection is one.

5.

a.

i.

See Figure 1.21. If each paperclip is a thousandth, then each red column represents a hundredth because there are 10 paperclips in each column and each black squares of toothpicks represents a tenth because there are 10 columns in each black square. In the figure, there are 3 black squares or 3 tenths, and 2 red columns or 2 hundredths and finally 9 additional paperclips or thousandths in blue. So, 0.329 has been represented.

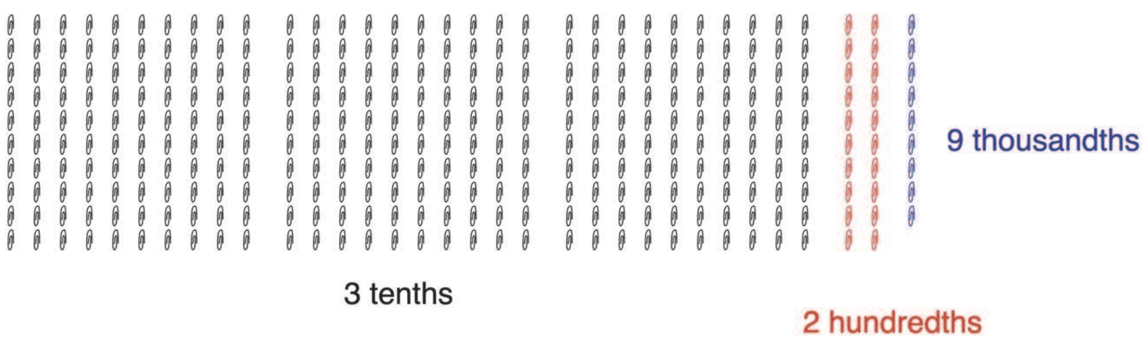


Figure 1.21: 0.329 representation

ii. See Figure 1.22. In the figure, there are 2 red columns or 2 hundredths and 7 additional paperclips or thousandths in blue. So, 0.027 has been represented.

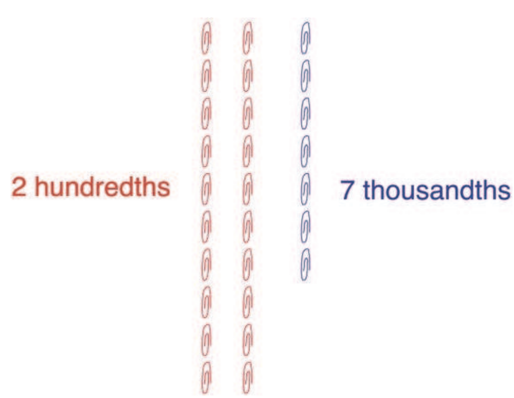


Figure 1.22: 0.027 representation

iii. See Figure 1.23. In the figure, there is 1 black square or 1 tenth and 4 red columns or 4 hundredths. So, 0.14 has been represented.

Figure 1.23: 0.14 representation

b. The 2 means 2 hundredths. However, since there are 10 thousandths in each hundredth, you could think of it as 20 thousandths. You can see this in the Figure 1.21 above as 20 toothpicks in the 2 red columns.

c. The 3 means 3 tenths. However, since there are 10 hundredths in each tenth, then you could think of it as 30 hundredths. You can see this in Figure 1.21 above as 30 columns in the 3 black squares. Moreover, because each hundredth has 10 thousandths in it, the 3 tenths could also be thought of as 300 thousandths, which would be the total number of paperclips in the 3 black squares.

6. If one toothpick represents one thousandth. then the tubes of ten toothpicks would represent one hundredth because 10 thousandths make up 1 hundredth and there are 10 toothpicks in each tube. And the bundles of tubes would represent one tenth because 10 hundredths make up 1 tenth and there are 10 tubes in each bundle. In that case, the figure would represent the number 0.346 because there are 3 tenths (bundles) shown and 4 hundredths(tubes) shown, and 6 thousandths (individual toothpicks) shown.

However, if one toothpick represents one tenth. then the tubes of ten toothpicks would represent one because 10 tenths make up 1 one and there are 10 toothpicks in each tube. And the bundles of tubes would represent one ten because 10 ones make up 1 one and there are 10 tubes in each bundle. In that case, the figure would represent the number 34.6 because there are 3 tens (bundles) shown and 4 ones(tubes) shown and 6 tenths (individual toothpicks) shown.

Finally, if one toothpick represents one ten. then the tubes of ten toothpicks would represent one hundred because 10 tens make up 1 hundred and there are 10 toothpicks in each tube. And the bundles of tubes would represent one thousand because 10 hundreds make up 1 thousand and there are 10 tubes in each bundle. In that case, the figure would represent the number 3,460 because there are 3 thousands (bundles) shown and 4 hundreds(tubes) shown and 6 tens (individual toothpicks) shown.

As in Practice Exercise 3, one toothpick could represent any number of quantities. If one toothpick represents 1, then the given collection represents 346. Other possibilities are shown in the table. See Table 1.6.

If 1 toothpick represents:	then the collection represents:
100	34,600
10	3,460
1	346
$\frac{1}{10}$	34.6
$\frac{1}{100}$	3.46
$\frac{1}{1000}$	0.346

Table 1.6: Base-Ten representations of a math drawing

7. Base-ten block sketches will be shown here. Similar sketches can be used for the bundled units model.

   a. Figure 1.24 represents 0.26 if we consider each block to represent a hundredth. Then by counting we see that we have 6 blocks (or 6 hundredths) and 2 bundles of 10 blocks (or 2 tenths). If we decided to have each block represent a tenth instead, then Figure 1.24 would represent 2.6; whereas if each block represented a hundred then Figure 1.24 would represent 2,600.

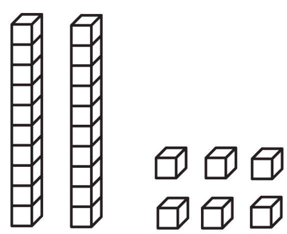


Figure 1.24: A representation of 0.26

   b. Figure 1.25 represents 13.4 if we consider each block to represent a tenth. Then by counting we see that we have 4 blocks (or 4 tenths) and 3 bundles of 10 blocks (or 3 ones) and 1 bundle of 100 blocks (or 1 ten). If we decided to have each block represent a hundredth instead, then Figure 1.25 would represent 0.134; whereas if each block stood for 1 unit then Figure 1.24 would represent 134.

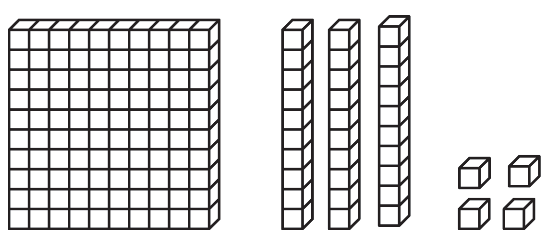


Figure 1.25: A representation of 13.4

c. Figure 1.26 represents 1.28 if we consider each block to represent a tenth. Then by counting we see that we have 8 blocks (or 8 hundredths) and 2 bundles of 10 blocks (or 2 tenths) and 1 bundle of 100 blocks (1 one). If we decided to have each block represent a hundred thousandth of a unit instead, then Figure 1.26 would represent 0.00128; whereas if each block stood for 10 then Figure 1.26 would represent 1,280.

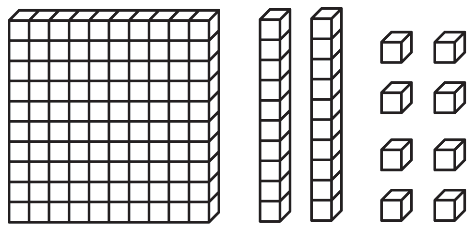


Figure 1.26: A representation of 1.28

d. Figure 1.27 represents 0.000032 if we consider each block to represent a millionth. Then by counting we see that we have 2 blocks (or 2 millionths) and 3 bundles of 10 blocks (or 3 hundred-thousandths). If we decided to have each block represent a tenth instead, then Figure 1.27 would represent 3.2; whereas if each block stood for a thousandth, then Figure 1.27 would represent 0.032.

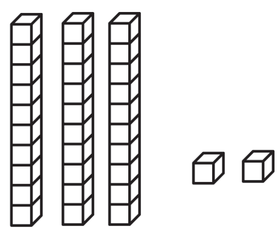


Figure 1.27: A representation of 0.000032

8. See Figure 1.28. In the figure below the darkened strip next to the phrase “each 0.001”is meant to be 1 hundredth strip partitioned into 10 smaller strips. To represent 1.438 as a length, make a long strip by laying a 1 unit long strip next to 4 tenth unit long strips, 3 hundredth unit long strips and 8 thousandth unit long strips. To represent 0.804, lay 8 tenth unit long strips next to 4 thousandth unit long strips.

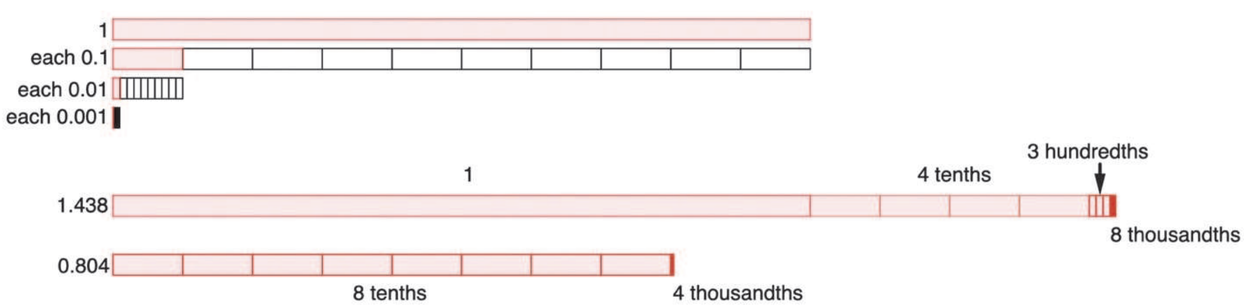


Figure 1.28: Representing base-ten numbers as lengths

9.

a. See Figure 1.29. If each paper clip is worth a hundredth, then each column must represent a tenth because 10 hundredths make up 1 tenth and there are 10 paper clips in each column. Similarly, the entire square represents one because 10 tenths make up 1 one and there are 10 columns in the square. The entire square has 10 columns with 10 paper clips in each column so there are a total of $10 \times 10 = 100$ paper clips. So, 100 hundredths make up a one.

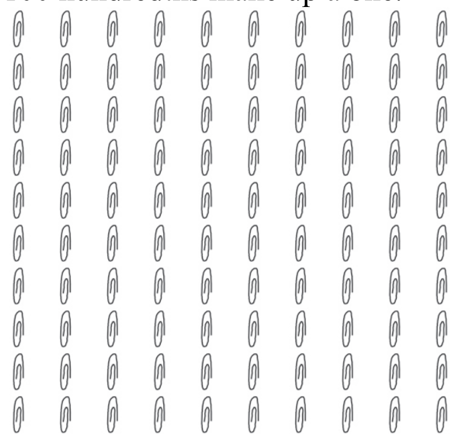


Figure 1.29: Base-ten representation with paperclips

b. See Figure 1.29. If each paper clip is worth a tenth, then each column must represent one because 10 tenths make up 1 one and there are 10 paper clips in each column. Similarly, the entire square represents ten because 10 ones make up 1 ten and there are 10 columns in the square. The entire square has 10 columns with 10 paper clips in each column so there are a total of $10 \times 10 = 100$ paper clips. So, 100 tenths make up a ten.

c. See Figure 1.29. If each paper clip is worth a thousandth, then each column must represent a hundredth because 10 thousandths make up 1 hundredth and there are 10 paper clips in each column. Similarly, the entire square represents a tenth because 10 hundredths make up 1 tenth and there are 10 columns in the square. The entire square has 10 columns with 10 paper clips in each column so there are a total of $10 \times 10 = 100$ paper clips. So, 100 thousandths make up a tenth.

d. See Figure 1.29. If each paper clip is worth a ten, then each column must represent a hundred because 10 tens make up 1 hundred and there are 10 paper clips in each column. Similarly, the entire square represents a thousand because 10 hundreds make up 1 thousand and there are 10 columns in the square. The entire square has 10 columns with 10 paper clips in each column so there are a total of $10 \times 10 = 100$ paper clips. So, 100 tens make up a thousand.

e. The arguments were essentially identical. When you are looking at base-ten place values that are two digits apart, 100 of the smaller units make up 1 of the larger unit.

10. We extend the base-ten structure from whole numbers to decimals by allowing the pattern established with whole numbers to continue. Specifically, as I move to the left from one place value to another, it takes 10 base-ten units from the right-hand spot to make 1 base-ten unit in the left-hand spot. For example, in the number 4,563, if we consider the digits 4 and 5, we note that the base-ten unit for the 5 digit is a hundred (5 is in the hundreds spot) and that the base-ten unit for the 4 digit is a thousand (4 is in the thousands spot). This works because as state above, it takes 10 hundreds (right hand spot) to make 1 thousand (left hand spot). This applies to decimals as well. If I consider the number 0.4563 and the same digits. Now the base-ten unit for 5 is hundredths and the base-ten unit for 4 is tenths. The reason those are the base-ten values is because it is true that 10 hundredths will make 1 tenth so the value of the digits follows the same pattern as before.

11. Jerome is trying to follow the pattern shown on the number line by thinking that the number seven and ten tenths comes after seven and nine tenths. In fact, that thinking is correct. What is incorrect is to write seven and ten tenths as 7.10. Pointing out to Jerome that 7.10 is already on the number line (7.1) might help him understand his error. Base-ten blocks could also help by allowing him to see that seven units and ten tenths of a unit is properly rearranged as eight units.

12. Answers will vary. Examples in which zeros can be dropped include such numbers as 01, 1.0, and 00.20100. Examples in which zeros cannot be dropped include such numbers as 1.01, 2.00301, and 0.00002. The issue of whether one can drop the zero directly in front of the decimal point (e.g., .1 vs. 0.1) is not a mathematical issue but one of style. Sometimes zeros which could be dropped mathematically are kept because of tolerance levels. This is discussed in the "What Is the Significance of Rounding When Working with Numbers That Represent Actual Quantities", Section 1.4.

13. Answers will vary. For example, see Figure 1.30.

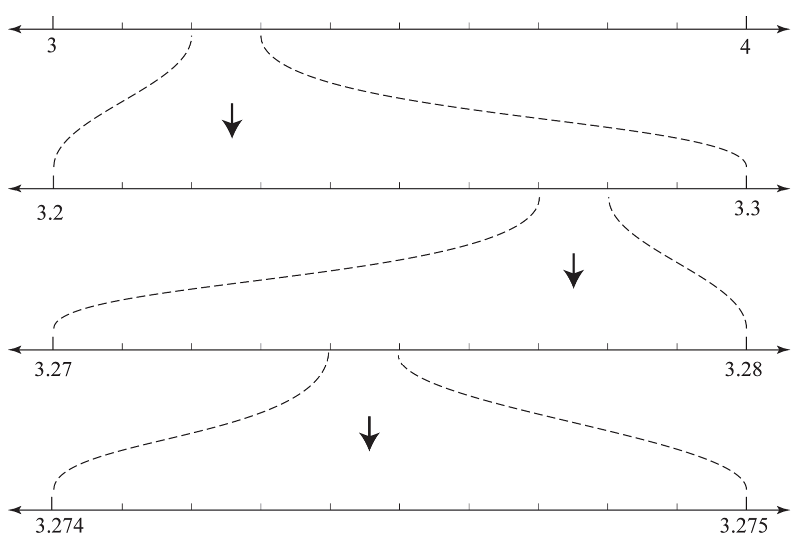


Figure 1.30: Zooming in on a number line.

14. See Figure 1.31.

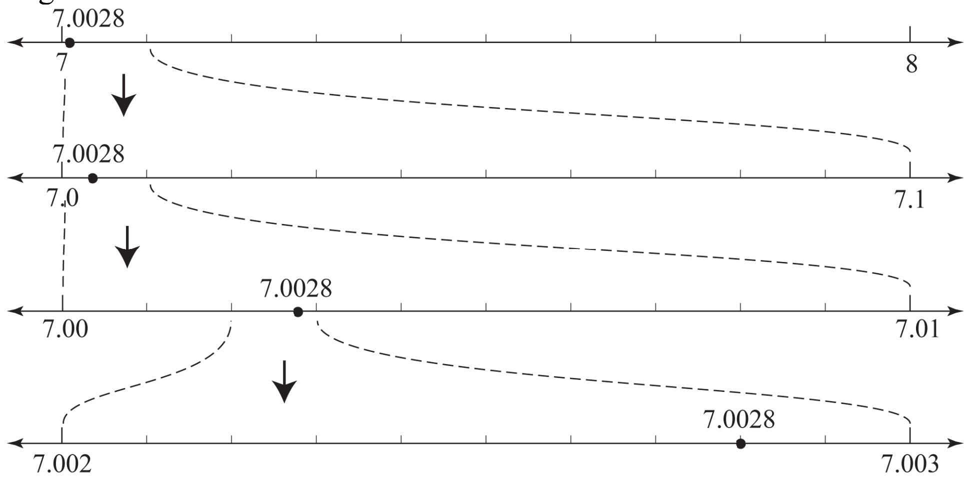


Figure 1.31: Zooming in on 7.0028

15. See Figure 1.32.

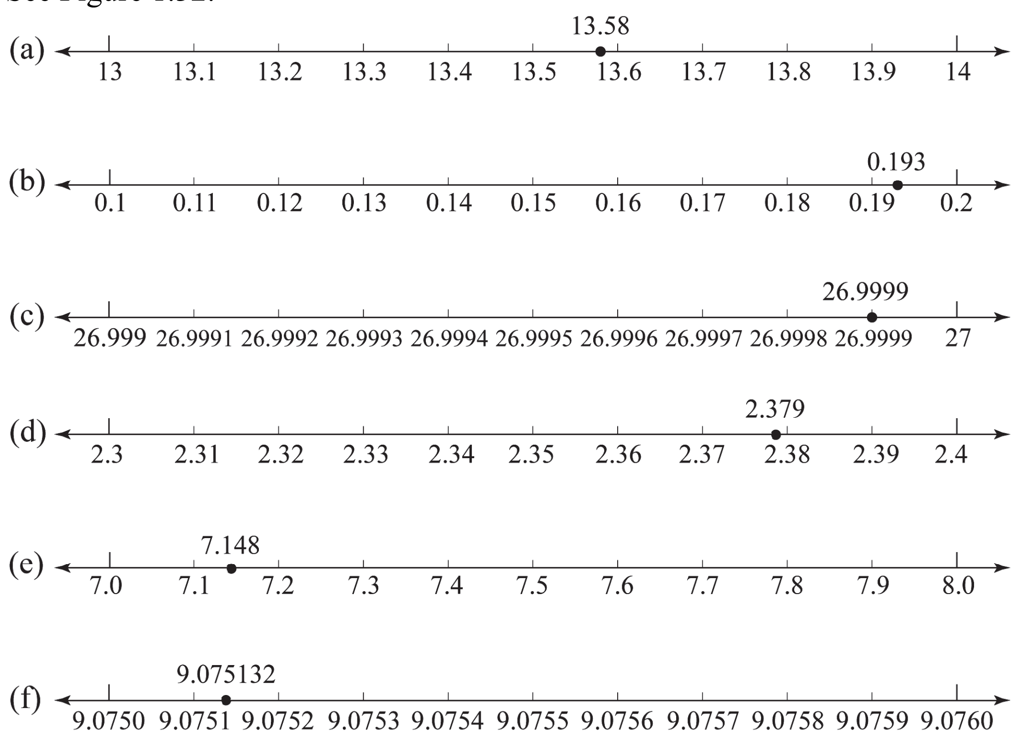


Figure 1.32: Locating numbers on various scales

16. Yes, Cierral may label the tick mark that way. Starting from the left, the tick marks should then be labeled 7.0001, 7.0002, 7.0003, …, 7.0009.

17. Yes, Juan may plot 9.999 where he did. Starting from the left, the intervening tick marks should then be labeled 9.9991, 9.9992, …, 9.9999.

18. a. This number line can be labeled in many different ways. Three examples are shown in Figure 1.33.

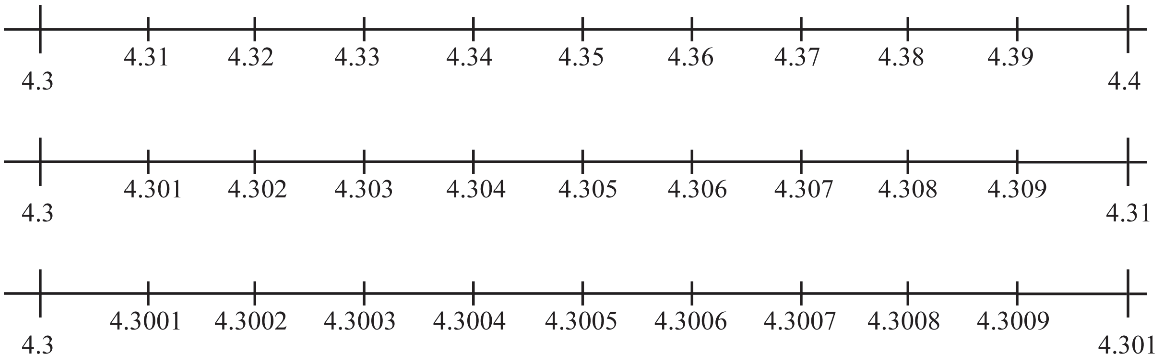


Figure 1.33: Three ways to label the tick marks

b. This number line can be labeled in many different ways. Three examples are shown in Figure 1.34.

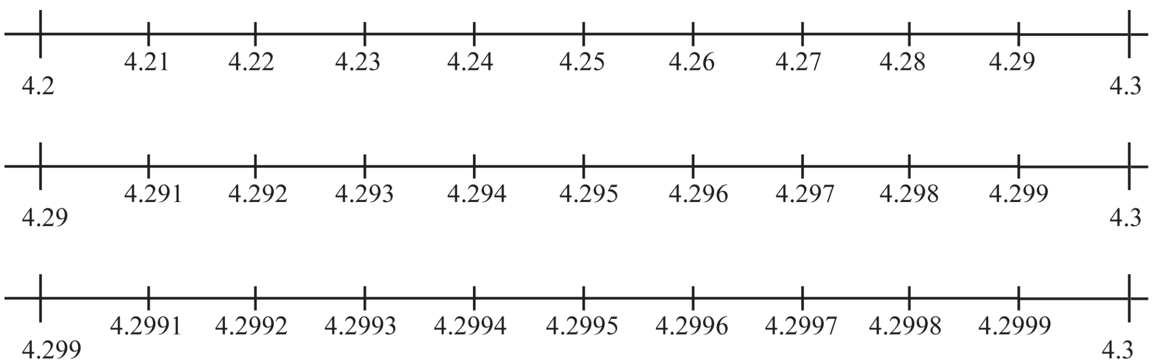


Figure 1.34: Three ways to label the tick marks

c. This number line can be labeled in many different ways. Three examples are shown in Figure 1.35.

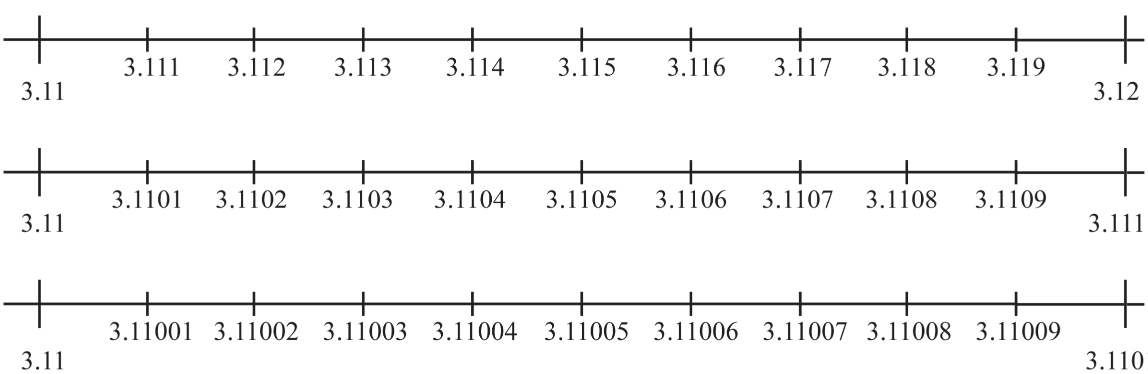


Figure 1.35: Three ways to label the tick marks

d. This number line can be labeled in many different ways. Three examples are shown in Figure 1.36.

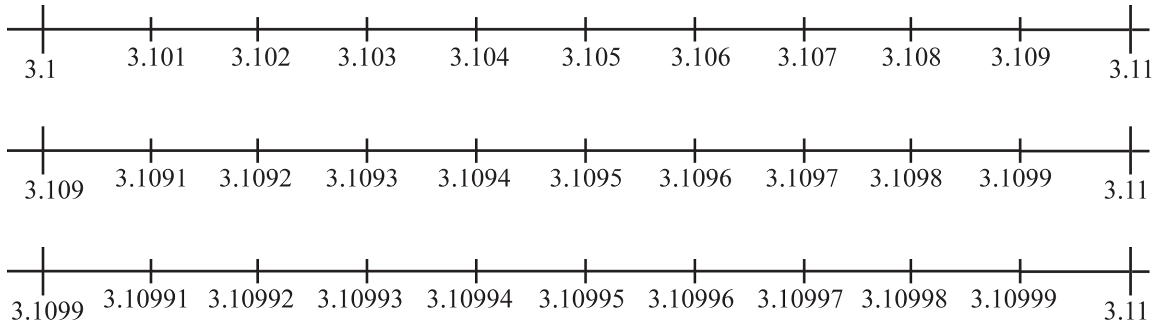


Figure 1.36: Three ways to label the tick marks

19.

a. First, note that there are ten tick marks between A and B indicating that each of the spaces between tick marks represent a base-ten unit. The red dot is between the third and fourth tick mark. If we consider some options for A and B that work for the dot falling between the 3rd and 4th tick mark, we see that A could be 5 and B could be 6. This would make the tick marks tenths and the tick mark immediately to the left of the red mark would be 5.3 and the one immediately to the right of the red mark would be 5.4. Alternatively, A could be 5.34 and B could be 5.35. This would make the tick marks thousandths and the tick mark immediately to the left of the red dot would be 5.343 and the one immediately to the right of the red dot would be 5.344. The red dot is very close to the tick mark on the right. So, it must be a number that is very close to either 5.344 or 5.4. 5.3438 is only 2 ten thousandths away from 5.344 so that is the correct choice. Therefore A = 5.34 and B = 5.35.

b. Unlike part a, the red dot is slightly less than halfway between the two tick marks. If we consider our options for the A and B, we can see that 5.34 is about halfway between 5.3 and 5.4. So that is our choice and A = 5 and B = 6.

c. The red dot is two tick marks away from A. This could work if A was 1.2 and each tick mark was a hundredth (0.01). Or this could work if A was 1 and each tick mark was a tenth (0.1). If each tick mark were 0.01, then the 7 thousandths would indicate that the red dot should be closer to the third tick mark than the second (as shown in the number line shown in the text for part d below). So that leaves the other option as the right option. A = 1. Each tick mark is a tenth so two tick marks over is 1.2 and then you are closer to the second tick mark than the third tick mark so 1.227 makes sense. Ten tenths = 1 so B is 1 more so B = 2.

d. A is 1.2 as indicated above in part c. The tick marks are hundredths so the 2nd tick mark represents 1.22 and the third tick mark 1.23. Since 1.227 is closer to 1.23 than 1.22 the tick mark is in the correct location. Ten hundredths = 1 more tenth so B = 1.3.

20. The distance between 0 and 1 is the unit. We place -1 one unit to the left of the 0. We place -2 two units to the left of the 0. For -1.68, we start at 0 and move 1 unit to the left.

Then from this spot, we move 6 tenths of a unit to the left from this spot. Finally, we move 8 hundredths of a unit to the left from this spot to arrive at -1.68.

21. Figure 1.37 shows the decimal numbers: -1, -0.92, -0.3, -0.03, 0, 0.07, 0.1, 0.3, 0.9, and 1. These choices allow students to distinguish between numbers such as 0.07 and 0.1, which allows students to consider place value when plotting each number. These choices also allow a comparison of the decimals -0.03, -0.3, and 0.3, which helps students consider the place value meaning of -0.03 compared to -0.3 and also to consider that, like the numbers on the positive part of the number line, smaller decimals (-0.3 for instance) are farther to the left from larger decimals like -0.03.

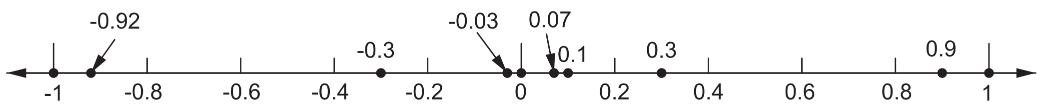


Figure 1.37: Comparing decimals on a number line

22. a. See Figure 1.38.

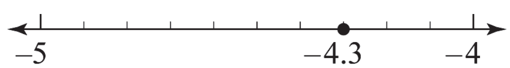


Figure 1.38: -4.3 on a number line

b. See Figure 1.39.

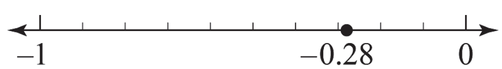


Figure 1.39: -0.28 on a number line

c. See Figure 1.40.

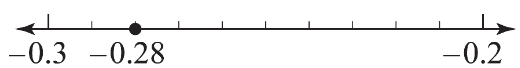


Figure 1.40: -0.28 on a number line, zoomed in

d. See Figure 1.41.

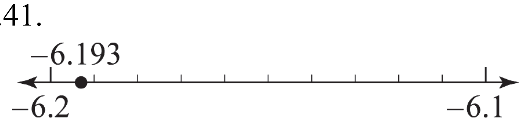


Figure 1.41: -6.193 on a number line

e. See Figure 1.42.

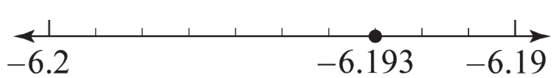


Figure 1.42: -6.193 on a number line, zoomed in

23. Thinking of –N as the opposite of N, then if N were a negative number, then –N would be the opposite of a negative number. In other words, -N would be a positive number. $|N|$ would also be a positive number. So –N and $|N|$ would be the same number.

1.3 Reasoning to Compare Numbers in Base Ten

1. a. 3 hundreds and 19 tens and 2 ones is greater (even though 3 hundreds are fewer than 4 hundreds). This is because 10 tens make 1 hundred so 19 tens make 1 hundred and 9 tens so we can describe 3 hundreds and 19 tens and 2 ones as 4 hundreds and 9 tens and 2 ones and that is clearly bigger than 4 hundreds and 8 tens and 7 ones.

   b. When you write a number in base-ten notation like 198 and 213, you automatically have regrouped any number of lower place value units into their higher place value equivalents. For example, how would you even write 3 hundreds 19 tens and 2 ones? You couldn't write 3192 because that makes it look like 3 thousand, one hundred and ninety-two. Instead, you have to regroup the 19 tens into 1 additional hundred and you are left with 9 tens. Then the number is written in a more traditional form, 492. It just so happens that 492 is the same as 3 hundreds 19 tens and 2 ones. Because 492 and 487 are now both in a form where everything that could be made into a higher value base-ten unit has been so changed, you can go ahead and just compare the hundreds and then the tens and if necessary then the ones to see which number is bigger

2. a. 4 ones is greater. This is because 10 tenths make 1 one so 37 tenths make 3 ones. and 9 tenths. 4 ones is greater than 3 ones and you don't have enough tenths to make another one so 4 ones is greater than 37 tenths.

   b. 61 tenths is greater. This is because 10 tenths make 1 one so 61 tenths make 6 ones and 1 tenths. 6 ones is greater than 4 so 61 tenths is greater than 4 ones.

3. Since 10 hundredths make up each tenth, then 2 tenths can be broken down into 20 hundredths. This shows us that 2 tenths is indeed bigger than 13 hundredths, but that is because 20 hundredths is bigger than 13 hundredths. 2 is also less than 24 but 2 tenths is less than 24 hundredths because 20 hundredths is less than 24 hundredths. 2 tenths is always less than anything more than 20 hundredths, but is always greater than anything less than 20 hundredths because 2 tenths is exactly 20 hundredths.

4. We compare numbers in the base-ten structure in the way that we do because base-ten places of larger value count more than the largest combined value made with lower places. For instance, when comparing 234 to 219, we see that both numbers have the same value in the hundreds place but that the 234 has a greater value in the tens place. Since the tens place counts more than the largest amount you could have in the ones place, we can essentially ignore the values in the ones place.

5. See Figure 1.43.

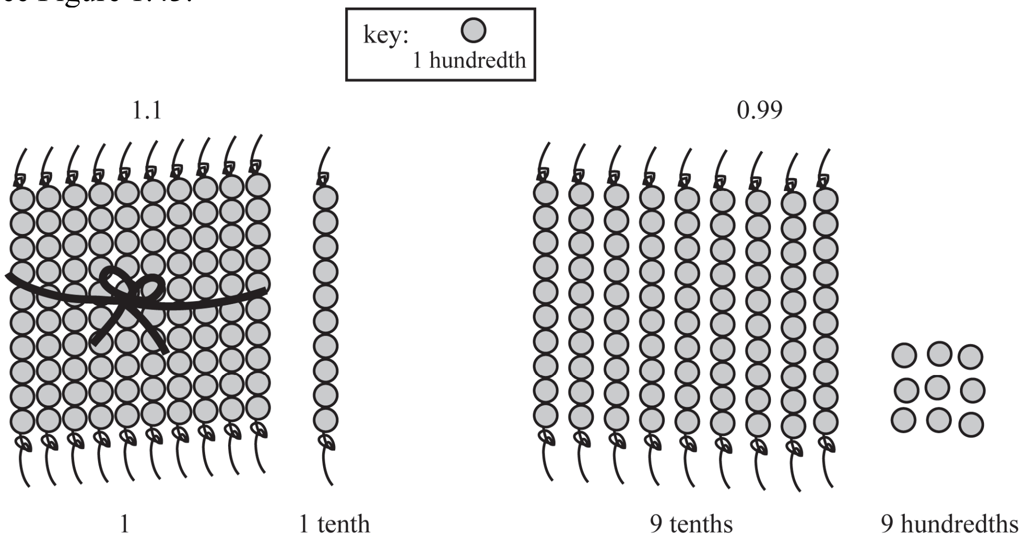


Figure 1.43: Using bundled objects to compare decimals

6.

a. See the solution to practice exercise #4.

b. See Figure 1.44 Since 1.1 is farther to the right of 0.999 so 1.1 is greater than 0.999.

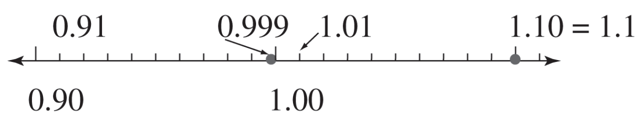


Figure 1.44: Comparing 1.1 and 0.999 with a number line.

c. See Figures 1.45 and 1.46. 1.1 is greater because it has more overall toothpicks.

Key
– = 1 toothpick = one hundredth

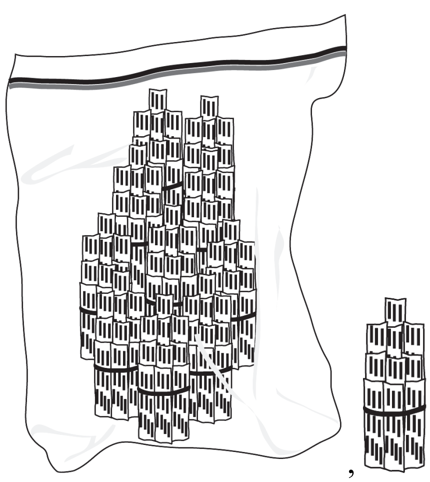


Figure 1.45: Math drawing for 1.1

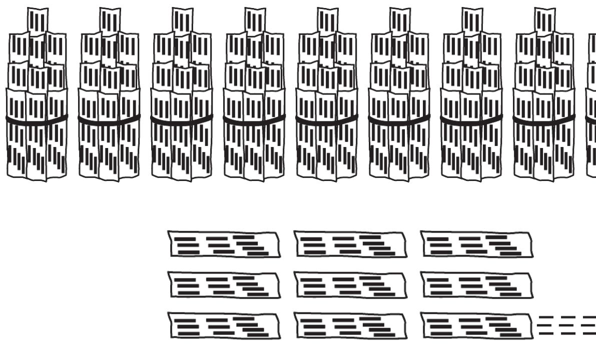


Figure 1.46: Math drawing for 0.999.

7. See Figure 1.47.
   a. 0 is to the left of 0.6 in the number line so 0 < 0.6.
   b. 0.00 = 0, which is to the left of 0.7, so 0.00 < 0.7.
   c. 3.00, 3.0, and 3 are all the same point.
   d. 3.7777 is to the right of 3.77 so 3.7777 > 3.77.

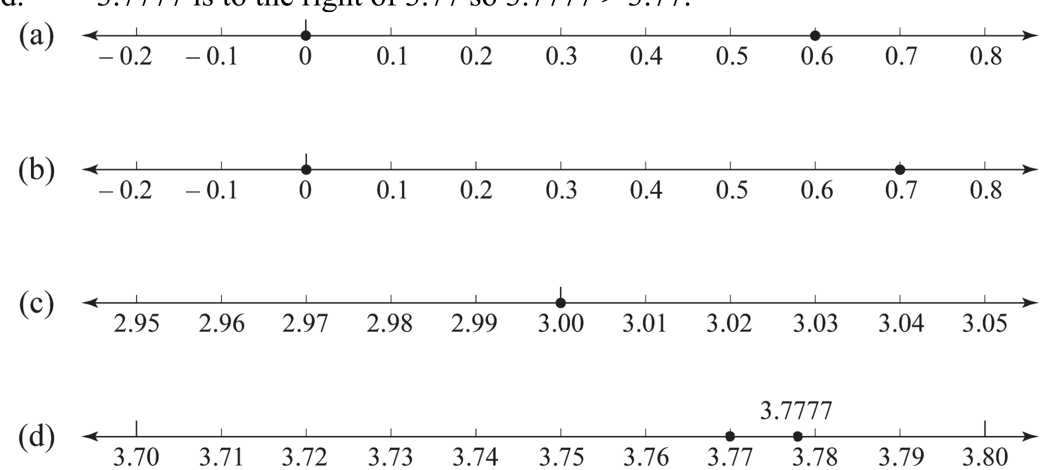


Figure 1.47: A number line to compare 0 and 0.6.

8. a. Mary has labeled the tick mark after 4.99 incorrectly as 4.100 instead of 5.00. She might be making this mistake by thinking of the decimal parts of the numbers like one might read them “four point ninety seven, four point ninety eight, four point ninety nine, and now four point one hundred”, since 100 comes after 99.

   b. You could help Mary see that her answer is wrong by helping her read the answer properly as four and one hundred thousandths. Also, you could show her that, using bundled objects, this number is the same as 4.10 (and 4.1) which she will probably recognize as less than 4.99. Showing this 4.1 on the number line would help reinforce this idea. To help her correct the error, it might be good to use bundled objects showing that each time you go up a tick mark on the number line that you are adding one of the objects representing one hundredth. But then when you have nine objects that each represent tenths and nine additional objects that each represent a hundredth, you get to regroup when you add the next hundredths. Specifically, 10 hundredths regroup into one more tenth, giving us 10 tenths, which regroup into one more unit or one. This helps you see that the next tick mark would have 4+1 or 5 units, zero tenths, and zero hundredths so it would be 5.00 or just 5.

9. Mark may think this because 178 > 25 when you ignore the decimal or perhaps because when you read 0.178 you use the word "hundred" and with 0.25 you only use the words "twenty" and "five" or even thinking 0.178 is longer than 0.25. Using money to model decimals, 0.178 would represent 17¢ + $\frac{8}{10}$ ¢, but 0.25 represents 25¢. Since 25¢ is greater than 17¢ and part of a cent, 0.25 must be the bigger of the two numbers. Using a number line, you could help Mark see that 0.178 lies to the left of 0.25 and so it is the smaller of the two numbers.

10. Answers will vary. For example, 3.2405, 3.2407, 3.24032 are all between 3.24 and 3.241.

11. There are infinitely many such numbers. For example, 8.46, 8.456, 8.4502, and so on.

12. a. Many answers are possible. For example, 3.82 is between 3.8 and 3.9. The number line can be marked in many ways. One way is shown in Figure 1.48.

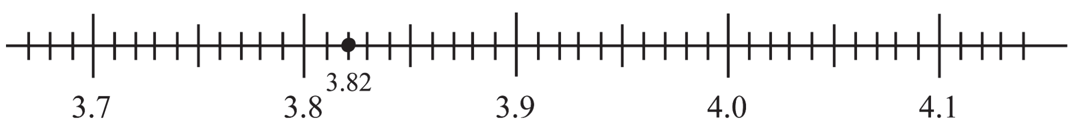


Figure 1.48: A number line for problem 12

b. 3.8 is like \$3.80, and 3.9 is like \$3.90. To find a number between 3.8 and 3.9, think of adding some cents (fewer than ten) to \$3.80.

13. a. The number line can be marked in many ways. One way is shown in Figure 1.49.

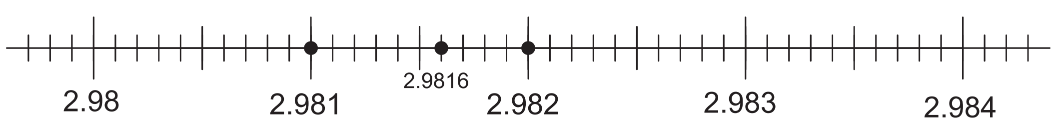


Figure 1.49: A number line for problem 13a.

b. The number line can be marked in many ways. One way is shown in Figure 1.50.

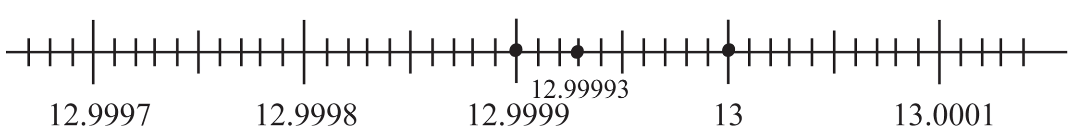


Figure 1.50: A number line for problem 13b.

c. The number line can be marked in many ways. One way is shown in Figure 1.51.

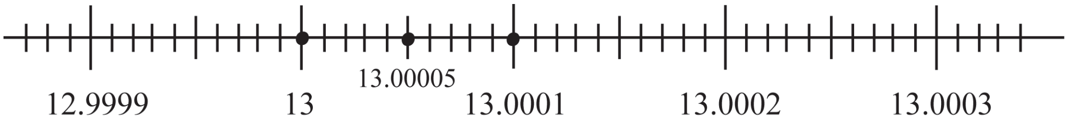


Figure 1.51: A number line for problem 13c.

14. On a number line, –8 is three units to the left of –5. Thinking of debt, to owe someone \$8 is to have less net value than if you owed \$5.

15. On a number line, –3 is to the left of –1, so –3.25 is to the left of –1.4. Thinking of debt, to owe someone \$3.25 is to have less net value overall than if you owed \$1.40.

16. a. See Figure 1.52.

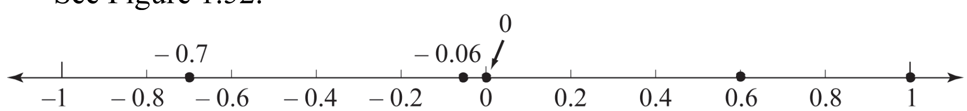


Figure 1.52: Comparing decimals on a number line

b. –0.7 < –0.06 < 0 < 0.6 < 1.

17. a. Answers will vary. -1.52 is between -1.6 and -1.5. The number line can be marked in many ways. One way is shown in Figure 1.53.

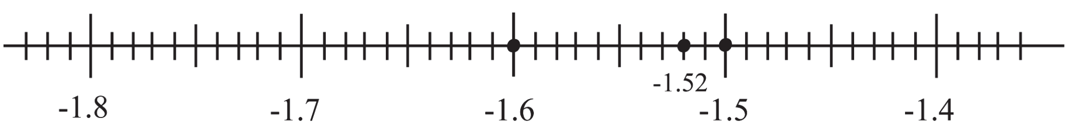


Figure 1.53: A number line for problem 17a.

b. Answers will vary. -34.98 is between -34.9835 and -34.9714. The number line can be marked in many ways. One way is shown in Figure 1.54

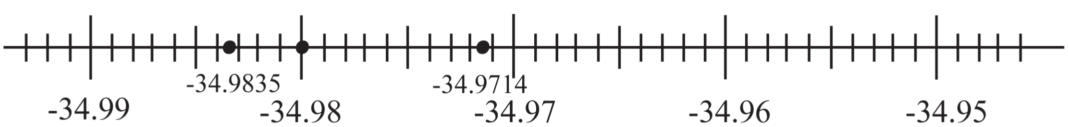


Figure 1.54: A number line for problem 17b.

c. Answers will vary. -7.835 is between -7.834 and -7.83561. The number line can be marked in many ways. One way is shown in Figure 1.55.

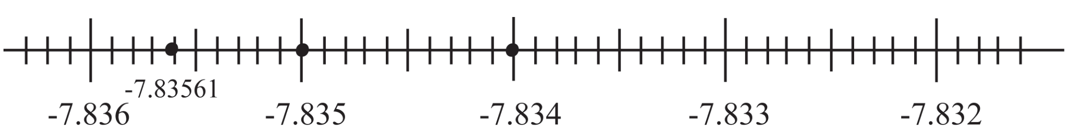


Figure 1.55: A number line for problem 17c

18. One list that fits the requirements of the problem follows. Note that the numbers are getting *smaller:* 3.5141, 3.51401, 3.514001, 3.5140001, 3.51400001 …

19. No. This problem is essentially the same as Practice Exercise 11. We can continue with a similar list of decimal numbers getting closer and closer to 2 without ever reaching 2. This would be 2.1, 2.01, 2.001, 2.0001, 2.00001,

Another solution involving averages (or midpoints) can easily show a list of numbers getting smaller, but with all the numbers that are bigger than 2.
2.1, 2.05, 2.025, 2.0125, 2.00625,… This list is generated by simply dividing the decimal part of the number by 2. In essence, if you considered a number line, you start at 2.1 and then find the midpoint between 2 and 2.1, then the midpoint between 2.05 and 2, then between 2.025 and 2, and so on. Since you will always be able to find a new midpoint, you'll always get closer to 2, but still bigger than 2.

20. Since A < B then the 0 point would be closer to A. As the values of the digits increase, negative numbers get increasingly smaller and farther away from 0. For instance, -8 is farther away from 0 than -6 is. Since A<B, then if we were to plot –B, it would be farther from 0 and thus left of -A. Since 0 must be exactly halfway between B and –B, then the 0 would be closer to –A.

21. Since A>B, then the 0 point would be closer to B. As the values of the digits increase, negative numbers get increasingly smaller and farther away from 0. With A>B, then if we were to plot –B, it would be closer to 0 and thus right of -A. Since 0 must be exactly halfway between B and –B, then the 0 would be closer to B.

1.4 Reasoning to Round Numbers

1. 2.1349 would round to 2.13. This is because of the 4 in the thousandths place, 2.1349 is less than 2.135. Thus, 2.1349 is closer to 2.13 than to 2.14. Figure 1.56 shows the relationship on a number line.

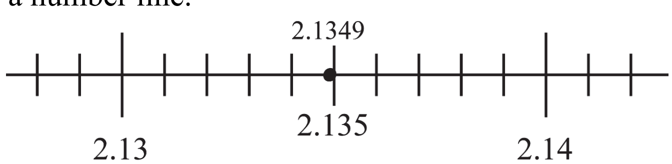


Figure 1.56: Rounding to the nearest hundredth

2. 27,003 would round to 27,000. 27,003 is much less than 27,050, so it is closer to 27,000 than to 27,100. Figure 1.57 shows the relationship on a number line.

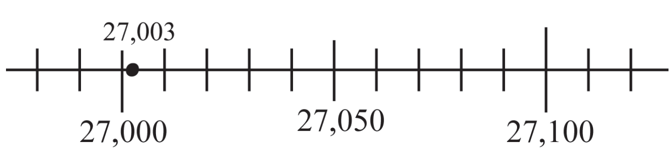


Figure 1.57: Rounding to the nearest hundred

3. 9,995.2 would round to 10,000. The 5 in the ones place puts 9995 exactly halfway between 9990 and 10,000. The conventional rounding rule is to round up in this case. But we do not have rely on just convention, because the 2 in the tenths place actually puts the number *over* halfway to 10,000. Figure 1.58 shows the relationship on a number line.

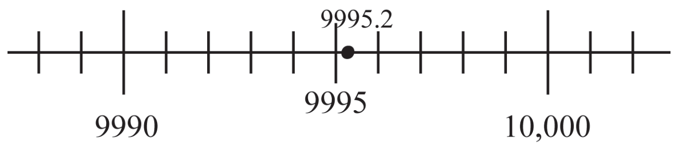


Figure 1.58: Rounding to the nearest ten

4. Adam's rule is not valid. For example, apply his algorithm to round 1.492 to the nearest unit:

$$1.492 \to 1.49 \to 1.5 \to 2\,.$$

But 1.492 is actually closer to 1 than to 2 because it is less than 1.5.

5. If the number has been rounded to the nearest whole number then it is possible that a serving of the snack food might have any amount less than 0.5 grams of trans fat, since these would be rounded to 0. For example, if they food testers found that the product had 0.2512 grams of trans fat, then if they were supposed to round the answer to the nearest whole number, they would round it to 0.

6. No, it is possible and even likely that the weight has been rounded to the nearest thousand pounds. If so, the weight could possibly be as low as 11,500 pounds and up to (but not including) 12,500 pounds.

7. No, the population may have been rounded to the nearest hundred. The exact population is at least 2,650 but less than (but not including) 2,750.

Chapter 1 Review

1. The child needs to be able to make a one-to-one correspondence between the number words 1, 2, 3, 4, 5 and the beads. For example, the child might point twice to one of the beads and count two numbers for that bear, or the child might skip over a bead while counting. The child must also understand that the last number word that is said while counting the beads tells how many beads there are in the collection.

2. The 2 really means 2 tens. For instance, you could group the sticks into sets of 10 sticks in each group. Then you would have 2 groups of 10 sticks in each group and there would be 4 sticks left over. However, if you recognize also that there are 20 single sticks in the two groups, then you could say that the 2 stands for 20 ones as well.

3. a. Bundle every set of ten toothpicks by putting a rubber band about them. These will represent tens. When you do this, you will get 24 tens and have 7 individual

toothpicks left over. Now put every set of 10 bundles (or tens) into a hundred by putting them into piles of 10 ten bundles or 10 tens. These piles will represent a hundred. You will get 2 piles and you will have 4 bundles left and you will still have those 7 individual toothpicks as well. That will be representing 2 hundreds, 4 tens and 7 ones.

b. The 4 represents 4 bundles or 4 tens.

c. The 2 represents 2 piles or 2 hundreds.

4. a. See Figure 1.59. If each paperclip is a thousandth, then each red column represents a hundredth because there are 10 paperclips in each column and each black square of toothpicks represents a tenth because there are 10 columns in each black square. In the figure, there is 3 black square or 1 tenth, and 4 red columns or 4 hundredths and finally 6 additional paperclips or thousandths in blue. So, 0.146 has been represented.

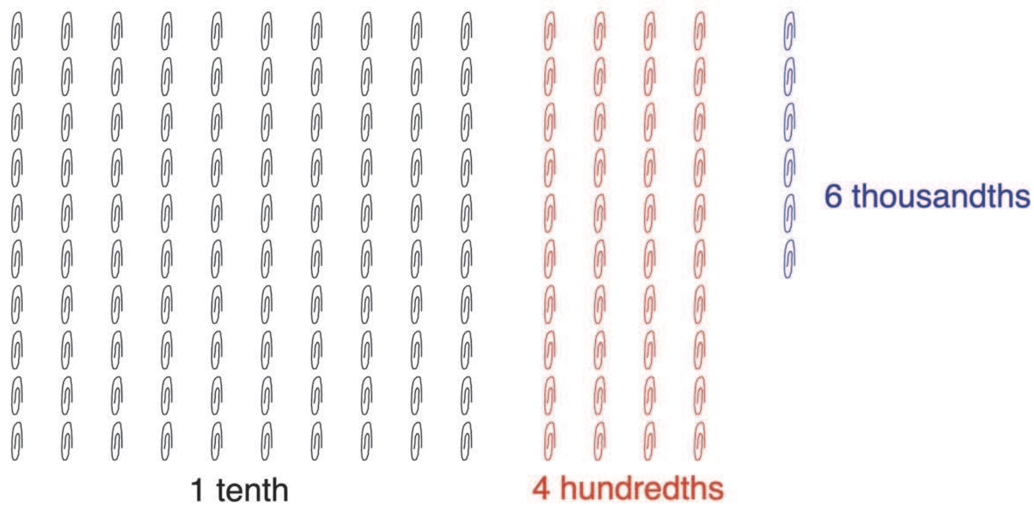


Figure 1.59: Representation of 0.146

b. The 4 stands for 4 hundredths. However, since 10 thousandths make up 1 hundredth, 4 also stands for 40 thousandths. You can see this in the picture by counting the number of paper clips (thousandths) that are found in the 4 red columns.

5. See Figure 1.60.

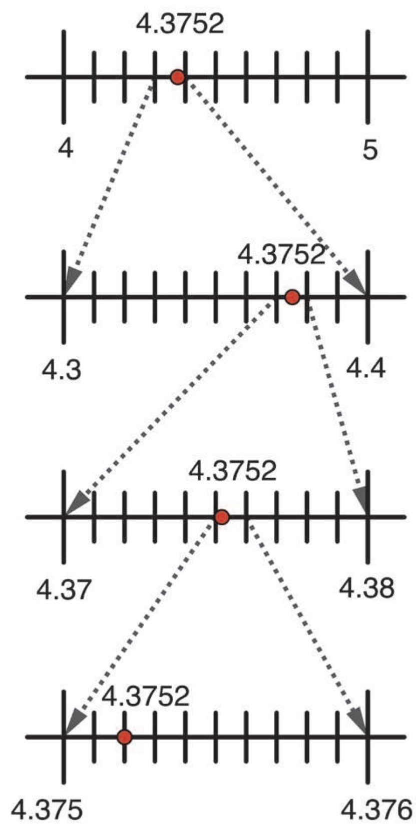


Figure 1.60: 4.3752 on number lines with various levels of precision

6.

a. 8 ones is greater than 73 tenths. Each one can be broken down into 10 tenths. Taking 8 ones we get 80 tenths. Now both quantities are represented with the same base-ten unit, tenths and 80>73 so 80 tenths (or 8 ones) is greater than 73 tenths.

b. 8 ones is smaller than 93 tenths. Each one can be broken down into 10 tenths. Taking 8 ones we get 80 tenths. Now both quantities are represented with the same base-ten unit, tenths and 80<93 so 80 tenths (or 8 ones) is smaller than 93 tenths.

7. 103>80 and 1.03>0.8. If the single bead is a base-ten unit, then as shown in the picture, ten beads can be put together with a string and since 10 base-ten units make 1 base-ten unit of the next place value, then the stringed column represents the adjacent higher place value base-ten unit. And then 100 beads made the square and since 100 base-ten units make 1 base-ten unit of two place values higher, then the square represents the next higher place value base-ten unit after the one represented by the stringed column. In other words, whatever the 3 beads represent, the 1 square must represent a place value two spots higher (or to the left) of the digit 3. Only 103, 1.03, and 0.103 fit that requirement. And the 8 stringed columns must represent a place value one spot higher that than what the 3 beads represent. Of the choices left, only 103>80 and 1.03>0.8 fit.

8. 6.8 is between 6.777 and 6.888. See Figure 1.61.

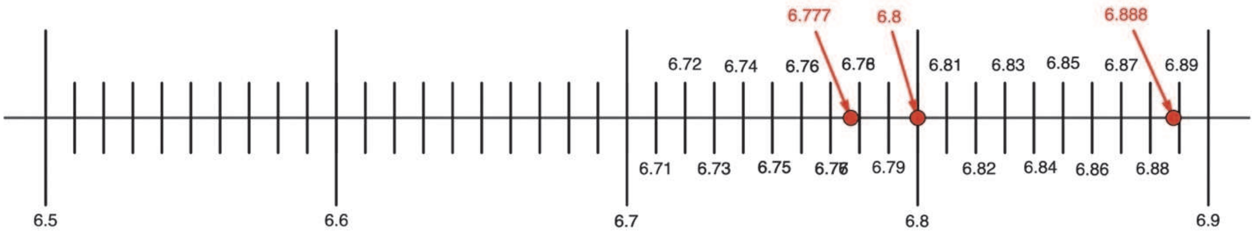


Figure 1.61: Finding a number between 6.777 and 6.888

9. See Figure 1.62.

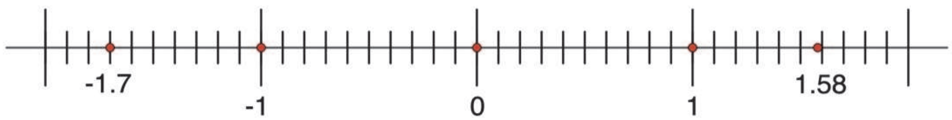


Figure 1.62: Plotting 0,1,-1,-1.7, 1.58 on a number line

10. On a number line, –6 is one unit to the left of –5. The farther left you go on a number line, the smaller the numbers get. Thinking of debt, to owe someone $6 is to have less net value than if you owed $5.

11. a. 274.48 would round to 274.5 if rounded to the nearest tenth. This is because of the 8 in the hundredths place, 274.48 is more than 274.45. Thus, 274.48 is closer to 274.5 than to 274.4. Figure 1.63 shows the relationship on a number line.

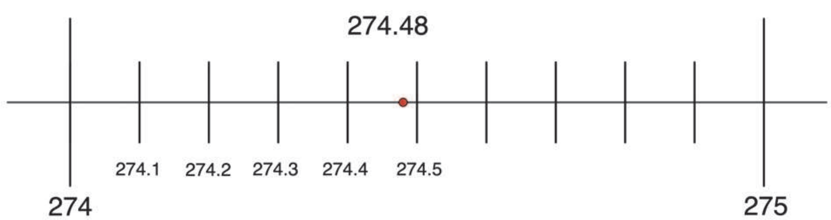


Figure 1.63: Rounding 274.48 using a number line to the nearest tenth

b. 274.48 would round to 270 if rounded to the nearest ten. This is because of the 4 in the ones place, 274 is smaller than 275. Thus, 274.48 is closer to 270 than to 280. Figure 1.64 shows the relationship on a number line.

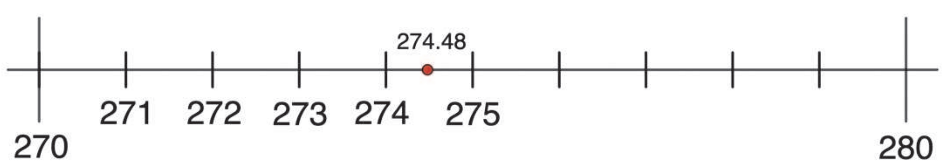


Figure 1.64: Rounding 274.48 using a number line to the nearest ten