

# Solutions Manual for Digital Signal Processing 5th Proakis

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# SOLUTIONS MANUAL

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## DIGITAL SIGNAL PROCESSING

Principles, Algorithms, and Applications  
5th Edition

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# Chapter 1

## 1.1

- (a) One dimensional, multichannel, discrete time, and digital.
- (b) Multi dimensional, single channel, continuous-time, analog.
- (c) One dimensional, single channel, continuous-time, analog.
- (d) One dimensional, single channel, continuous-time, analog.
- (e) One dimensional, multichannel, discrete-time, digital.



# Chapter 2

## 2.1

(a)

$$x(n) = \left\{ \dots 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \dots \right\}$$

. Refer to fig 2.1-1.

(b) After folding  $s(n)$  we have

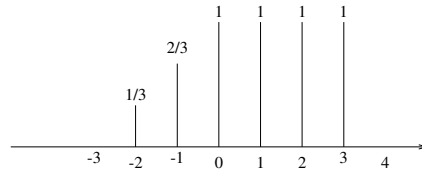


Figure 2.1-1:

$$x(-n) = \left\{ \dots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{2}{3}, \frac{1}{3}, 0, \dots \right\}.$$

After delaying the folded signal by 4 samples, we have

$$x(-n+4) = \left\{ \dots 0, 0, \underset{\uparrow}{1}, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \dots \right\}.$$

On the other hand, if we delay  $x(n)$  by 4 samples we have

$$x(n-4) = \left\{ \dots 0, 0, \underset{\uparrow}{\frac{1}{3}}, \frac{2}{3}, 1, 1, 1, 1, 0, \dots \right\}.$$

Now, if we fold  $x(n-4)$  we have

$$x(-n-4) = \left\{ \dots 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \underset{\uparrow}{0}, \dots \right\}$$

(c)

$$x(-n+4) = \left\{ \dots 0, \underset{\uparrow}{1}, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \dots \right\}$$

(d) To obtain  $x(-n+k)$ , first we fold  $x(n)$ . This yields  $x(-n)$ . Then, we shift  $x(-n)$  by  $k$  samples to the right if  $k > 0$ , or  $k$  samples to the left if  $k < 0$ .

(e) Yes.

$$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n+1) + u(n) - u(n-4)$$

## 2.2

$$x(n) = \left\{ \dots 0, 1, \underset{\uparrow}{1}, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \dots \right\}$$

(a)

$$x(n-2) = \left\{ \dots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \frac{1}{2}, \frac{1}{2}, 0, \dots \right\}$$

(b)

$$x(4-n) = \left\{ \dots 0, \frac{1}{2}, \underset{\uparrow}{\frac{1}{2}}, 1, 1, 1, 1, 0, \dots \right\}$$

(see 2.1(d))

(c)

$$x(n+2) = \left\{ \dots 0, 1, 1, 1, \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{2}, 0, \dots \right\}$$

(d)

$$x(n)u(2-n) = \left\{ \dots 0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \dots \right\}$$

(e)

$$x(n-1)\delta(n-3) = \left\{ \dots 0, \underset{\uparrow}{0}, 0, 1, 0, \dots \right\}$$

(f)

$$\begin{aligned} x(n^2) &= \{ \dots 0, x(4), x(1), x(0), x(1), x(4), 0, \dots \} \\ &= \left\{ \dots 0, \frac{1}{2}, 1, \underset{\uparrow}{1}, 1, \frac{1}{2}, 0, \dots \right\} \end{aligned}$$

(g)

$$\begin{aligned} x_e(n) &= \frac{x(n) + x(-n)}{2}, \\ x(-n) &= \left\{ \dots 0, \frac{1}{2}, \frac{1}{2}, 1, 1, \underset{\uparrow}{1}, 1, 0, \dots \right\} \\ &= \left\{ \dots 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \dots \right\} \end{aligned}$$

(h)

$$\begin{aligned}x_o(n) &= \frac{x(n) - x(-n)}{2} \\&= \left\{ \dots, 0, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0, \dots \right\}\end{aligned}$$

## 2.3

(a)

$$u(n) - u(n-1) = \delta(n) = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 0, & n > 0 \end{cases}$$

(b)

$$\begin{aligned}\sum_{k=-\infty}^n \delta(k) &= u(n) = \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases} \\ \sum_{k=0}^{\infty} \delta(n-k) &= \begin{cases} 0, & n < 0 \\ 1, & n \geq 0 \end{cases}\end{aligned}$$

## 2.4

Let

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)],$$

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)].$$

Since

$$x_e(-n) = x_e(n)$$

and

$$x_o(-n) = -x_o(n),$$

it follows that

$$x(n) = x_e(n) + x_o(n).$$

The decomposition is unique. For

$$x(n) = \left\{ 2, 3, 4, \underset{\uparrow}{4}, 5, 6 \right\},$$

we have

$$x_e(n) = \left\{ 4, 4, 4, \underset{\uparrow}{4}, 4, 4 \right\}$$

and

$$x_o(n) = \left\{ -2, -1, 0, \underset{\uparrow}{1}, 1, 2 \right\}.$$



## 2.5

First, we prove that

$$\begin{aligned}
 \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) &= 0 \\
 \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) &= \sum_{m=-\infty}^{\infty} x_e(-m)x_o(-m) \\
 &= - \sum_{m=-\infty}^{\infty} x_e(m)x_o(m) \\
 &= - \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
 &= \sum_{n=-\infty}^{\infty} x_e(n)x_o(n) \\
 &= 0
 \end{aligned}$$

Then,

$$\begin{aligned}
 \sum_{n=-\infty}^{\infty} x^2(n) &= \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2 \\
 &= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + \sum_{n=-\infty}^{\infty} 2x_e(n)x_o(n) \\
 &= E_e + E_o
 \end{aligned}$$

## 2.6

(a) No, the system is time variant. Proof: If

$$\begin{aligned}
 x(n) \rightarrow y(n) &= x(n^2) \\
 x(n-k) \rightarrow y_1(n) &= x[(n-k)^2] \\
 &= x(n^2 + k^2 - 2nk) \\
 &\neq y(n-k)
 \end{aligned}$$

(b) (1)

$$x(n) = \left\{ 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \dots \right\}$$

(2)

$$y(n) = x(n^2) = \left\{ \dots, 0, 1, \underset{\uparrow}{1}, 1, 0, \dots \right\}$$

(3)

$$y(n-2) = \left\{ \dots, 0, 0, \underset{\uparrow}{1}, 1, 1, 0, \dots \right\}$$

(4)

$$x(n-2) = \left\{ \dots, 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, \dots \right\}$$

(5)

$$y_2(n) = \mathcal{T}[x(n-2)] = \left\{ \dots, 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0, \dots \right\}$$

(6)

$$y_2(n) \neq y(n-2) \Rightarrow \text{system is time variant.}$$

(c) (1)

$$x(n) = \left\{ \underset{\uparrow}{1}, 1, 1, 1 \right\}$$

(2)

$$y(n) = \left\{ \underset{\uparrow}{1}, 0, 0, 0, 0, -1 \right\}$$

(3)

$$y(n-2) = \left\{ \underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1 \right\}$$

(4)

$$x(n-2) = \left\{ \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1 \right\}$$

(5)

$$y_2(n) = \left\{ \underset{\uparrow}{0}, 0, 1, 0, 0, 0, -1 \right\}$$

(6)

$$y_2(n) = y(n-2).$$

The system is time invariant, but this example alone does not constitute a proof.

(d) (1)

$$y(n) = nx(n),$$

$$x(n) = \left\{ \dots, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \dots \right\}$$

(2)

$$y(n) = \left\{ \dots, \underset{\uparrow}{0}, 1, 2, 3, \dots \right\}$$

(3)

$$y(n-2) = \left\{ \dots, \underset{\uparrow}{0}, 0, 0, 1, 2, 3, \dots \right\}$$

(4)

$$x(n-2) = \left\{ \dots, 0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, \dots \right\}$$

(5)

$$y_2(n) = \mathcal{T}[x(n-2)] = \{\dots, 0, 0, 2, 3, 4, 5, \dots\}$$

(6)

$$y_2(n) \neq y(n-2) \Rightarrow \text{the system is time variant.}$$

## 2.7

- (a) Static, nonlinear, time invariant, causal, stable.  
(b) Dynamic, linear, time invariant, noncausal and unstable. The latter is easily proved.  
For the bounded input  $x(k) = u(k)$ , the output becomes

$$y(n) = \sum_{k=-\infty}^{n+1} u(k) = \begin{cases} 0, & n < -1 \\ n+2, & n \geq -1 \end{cases}$$

since  $y(n) \rightarrow \infty$  as  $n \rightarrow \infty$ , the system is unstable.

- (c) Static, linear, timevariant, causal, stable.  
(d) Dynamic, linear, time invariant, noncausal, stable.  
(e) Static, nonlinear, time invariant, causal, stable.  
(f) Static, nonlinear, time invariant, causal, stable.

## 2.8

(a) True. If

$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$

$$v_2(n) = \mathcal{T}_1[x_2(n)],$$

then

$$\alpha_1 x_1(n) + \alpha_2 x_2(n)$$

yields

$$\alpha_1 v_1(n) + \alpha_2 v_2(n)$$

by the linearity property of  $\mathcal{T}_1$ . Similarly, if

$$y_1(n) = \mathcal{T}_2[v_1(n)] \text{ and}$$

$$y_2(n) = \mathcal{T}_2[v_2(n)],$$

then

$$\beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$$

by the linearity property of  $\mathcal{T}_2$ . Since

$$v_1(n) = \mathcal{T}_1[x_1(n)] \text{ and}$$

$$v_2(n) = \mathcal{T}_1[x_2(n)],$$

it follows that

$$A_1 x_1(n) + A_2 x_2(n)$$

yields the output

$$A_1 \mathcal{T}[x_1(n)] + A_2 \mathcal{T}[x_2(n)],$$

where  $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$ . Hence  $\mathcal{T}$  is linear.

(b) True. For  $\mathcal{T}_1$ , if

$$\begin{aligned}x(n) &\rightarrow v(n) \text{ and} \\x(n-k) &\rightarrow v(n-k),\end{aligned}$$

For  $\mathcal{T}_2$ , if

$$\begin{aligned}v(n) &\rightarrow y(n) \\ \text{and } v(n-k) &\rightarrow y(n-k).\end{aligned}$$

Hence, For  $\mathcal{T}_1\mathcal{T}_2$ , if

$$\begin{aligned}x(n) &\rightarrow y(n) \text{ and} \\x(n-k) &\rightarrow y(n-k)\end{aligned}$$

Therefore,  $\mathcal{T} = \mathcal{T}_1\mathcal{T}_2$  is time invariant.

(c) True.  $\mathcal{T}_1$  is causal  $\Rightarrow v(n)$  depends only on  $x(k)$  for  $k \leq n$ .  $\mathcal{T}_2$  is causal  $\Rightarrow y(n)$  depends only on  $v(k)$  for  $k \leq n$ . Therefore,  $y(n)$  depends only on  $x(k)$  for  $k \leq n$ . Hence,  $\mathcal{T}$  is causal.

(d) True. Combine (a) and (b).

(e) True. This follows from  $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

(f) False. For example, consider

$$\begin{aligned}\mathcal{T}_1 : y(n) &= nx(n) \text{ and} \\ \mathcal{T}_2 : y(n) &= nx(n+1).\end{aligned}$$

Then,

$$\begin{aligned}\mathcal{T}_2[\mathcal{T}_1[\delta(n)]] &= \mathcal{T}_2(0) = 0. \\ \mathcal{T}_1[\mathcal{T}_2[\delta(n)]] &= \mathcal{T}_1[\delta(n+1)] \\ &= -\delta(n+1) \\ &\neq 0.\end{aligned}$$

(g) False. For example, consider

$$\begin{aligned}\mathcal{T}_1 : y(n) &= x(n) + b \text{ and} \\ \mathcal{T}_2 : y(n) &= x(n) - b, \text{ where } b \neq 0.\end{aligned}$$

Then,

$$\mathcal{T}[x(n)] = \mathcal{T}_2[\mathcal{T}_1[x(n)]] = \mathcal{T}_2[x(n) + b] = x(n).$$

Hence  $\mathcal{T}$  is linear.

(h) True.

$\mathcal{T}_1$  is stable  $\Rightarrow v(n)$  is bounded if  $x(n)$  is bounded.

$\mathcal{T}_2$  is stable  $\Rightarrow y(n)$  is bounded if  $v(n)$  is bounded .

Hence,  $y(n)$  is bounded if  $x(n)$  is bounded  $\Rightarrow \mathcal{T} = \mathcal{T}_1\mathcal{T}_2$  is stable.

(i) Inverse of (c).  $\mathcal{T}_1$  and for  $\mathcal{T}_2$  are noncausal  $\Rightarrow \mathcal{T}$  is noncausal. Example:

$$\begin{aligned}\mathcal{T}_1 : y(n) &= x(n+1) \text{ and} \\ \mathcal{T}_2 : y(n) &= x(n-2) \\ \Rightarrow \mathcal{T} : y(n) &= x(n-1),\end{aligned}$$

which is causal. Hence, the inverse of (c) is false.

Inverse of (h):  $\mathcal{T}_1$  and/or  $\mathcal{T}_2$  is unstable, implies  $\mathcal{T}$  is unstable. Example:

$$\mathcal{T}_1 : y(n) = e^{x(n)}, \text{ stable and } \mathcal{T}_2 : y(n) = \ln[x(n)], \text{ which is unstable.}$$

But  $\mathcal{T} : y(n) = x(n)$ , which is stable. Hence, the inverse of (h) is false.

## 2.9

(a)

$$\begin{aligned}
 y(n) &= \sum_{k=-\infty}^n h(k)x(n-k), x(n) = 0, n < 0 \\
 y(n+N) &= \sum_{k=-\infty}^{n+N} h(k)x(n+N-k) = \sum_{k=-\infty}^{n+N} h(k)x(n-k) \\
 &= \sum_{k=-\infty}^n h(k)x(n-k) + \sum_{k=n+1}^{n+N} h(k)x(n-k) \\
 &= y(n) + \sum_{k=n+1}^{n+N} h(k)x(n-k)
 \end{aligned}$$

For a BIBO system,  $\lim_{n \rightarrow \infty} |h(n)| = 0$ . Therefore,

$$\begin{aligned}
 \lim_{n \rightarrow \infty} \sum_{k=n+1}^{n+N} h(k)x(n-k) &= 0 \text{ and} \\
 \lim_{n \rightarrow \infty} y(n+N) &= y(N).
 \end{aligned}$$

(b) Let  $x(n) = x_o(n) + au(n)$ , where  $a$  is a constant and

$x_o(n)$  is a bounded signal with  $\lim_{n \rightarrow \infty} x_o(n) = 0$ .

Then,

$$\begin{aligned}
 y(n) &= a \sum_{k=0}^{\infty} h(k)u(n-k) + \sum_{k=0}^{\infty} h(k)x_o(n-k) \\
 &= a \sum_{k=0}^n h(k) + y_o(n)
 \end{aligned}$$

clearly,  $\sum_n x_o^2(n) < \infty \Rightarrow \sum_n y_o^2(n) < \infty$  (from (c) below) Hence,

$$\lim_{n \rightarrow \infty} |y_o(n)| = 0.$$

and, thus,  $\lim_{n \rightarrow \infty} y(n) = a \sum_{k=0}^{\infty} h(k) = \text{constant}$ .

(c)

$$\begin{aligned}
 y(n) &= \sum_k h(k)x(n-k) \\
 \sum_{-\infty}^{\infty} y^2(n) &= \sum_{-\infty}^{\infty} \left[ \sum_k h(k)x(n-k) \right]^2 \\
 &= \sum_k \sum_l h(k)h(l) \sum_n x(n-k)x(n-l)
 \end{aligned}$$

But

$$\sum_n x(n-k)x(n-l) \leq \sum_n x^2(n) = E_x.$$

Therefore,

$$\sum_n y^2(n) \leq E_x \sum_k |h(k)| \sum_l |h(l)|.$$

For a BIBO stable system,

$$\sum_k |h(k)| < M.$$

Hence,

$$E_y \leq M^2 E_x, \text{ so that} \\ E_y < 0 \text{ if } E_x < 0.$$

## 2.10

The system is nonlinear. This is evident from observation of the pairs

$$x_3(n) \leftrightarrow y_3(n) \text{ and } x_2(n) \leftrightarrow y_2(n).$$

If the system were linear,  $y_2(n)$  would be of the form

$$y_2(n) = \{3, 6, 3\}$$

because the system is time-invariant. However, this is not the case.

## 2.11

since

$$x_1(n) + x_2(n) = \delta(n)$$

and the system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \left\{ 0, \underset{\uparrow}{3}, -1, 2, 1 \right\}.$$

If the system were time invariant, the response to  $x_3(n)$  would be

$$\left\{ \underset{\uparrow}{3}, 2, 1, 3, 1 \right\}.$$

But this is not the case.

## 2.12

- (a) Any weighted linear combination of the signals  $x_i(n), i = 1, 2, \dots, N$ .
- (b) Any  $x_i(n - k)$ , where  $k$  is any integer and  $i = 1, 2, \dots, N$ .

## 2.13

A system is BIBO stable if and only if a bounded input produces a bounded output.

$$\begin{aligned} y(n) &= \sum_k h(k)x(n-k) \\ |y(n)| &\leq \sum_k |h(k)||x(n-k)| \\ &\leq M_x \sum_k |h(k)| \end{aligned}$$

where  $|x(n-k)| \leq M_x$ . Therefore,  $|y(n)| < \infty$  for all  $n$ , if and only if

$$\sum_k |h(k)| < \infty.$$

## 2.14

(a) A system is causal  $\Leftrightarrow$  the output becomes nonzero after the input becomes non-zero. Hence,

$$x(n) = 0 \text{ for } n < n_o \Rightarrow y(n) = 0 \text{ for } n < n_o.$$

(b)

$$y(n) = \sum_{-\infty}^n h(k)x(n-k), \text{ where } x(n) = 0 \text{ for } n < 0.$$

If  $h(k) = 0$  for  $k < 0$ , then

$$y(n) = \sum_0^n h(k)x(n-k), \text{ and hence, } y(n) = 0 \text{ for } n < 0.$$

On the other hand, if  $y(n) = 0$  for  $n < 0$ , then

$$\sum_{-\infty}^n h(k)x(n-k) \Rightarrow h(k) = 0, k < 0.$$

## 2.15

(a)

$$\begin{aligned} \text{For } a = 1, \sum_{n=M}^N a^n &= N - M + 1 \\ \text{for } a \neq 1, \sum_{n=M}^N a^n &= a^M + a^{M+1} + \dots + a^N \\ (1-a) \sum_{n=M}^N a^n &= a^M + a^{M+1} - a^{M+1} + \dots + a^N - a^N - a^{N+1} \\ &= a^M - a^{N+1} \end{aligned}$$

(b) For  $M = 0$ ,  $|a| < 1$ , and  $N \rightarrow \infty$ ,

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, |a| < 1.$$

## 2.16

(a)

$$\begin{aligned}
 y(n) &= \sum_k h(k)x(n-k) \\
 \sum_n y(n) &= \sum_n \sum_k h(k)x(n-k) = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k) \\
 &= \left( \sum_k h(k) \right) \left( \sum_n x(n) \right)
 \end{aligned}$$

(b) (1)

$$y(n) = h(n) * x(n) = \{1, 3, 7, 7, 7, 6, 4\}$$

$$\sum_n y(n) = 35, \quad \sum_k h(k) = 5, \quad \sum_k x(k) = 7$$

(2)

$$y(n) = \{1, 4, 2, -4, 1\}$$

$$\sum_n y(n) = 4, \quad \sum_k h(k) = 2, \quad \sum_k x(k) = 2$$

(3)

$$y(n) = \left\{ 0, \frac{1}{2}, -\frac{1}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, -2 \right\}$$

$$\sum_n y(n) = -5, \quad \sum_n h(n) = 2.5, \quad \sum_n x(n) = -2$$

(4)

$$y(n) = \{1, 2, 3, 4, 5\}$$

$$\sum_n y(n) = 15, \quad \sum_n h(n) = 1, \quad \sum_n x(n) = 15$$

(5)

$$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y(n) = 8, \quad \sum_n h(n) = 4, \quad \sum_n x(n) = 2$$



## 2.17

(a)

$$\begin{aligned}
 x(n) &= \left\{ \underset{\uparrow}{1}, 1, 1, 1 \right\} \\
 h(n) &= \left\{ \underset{\uparrow}{6}, 5, 4, 3, 2, 1 \right\} \\
 y(n) &= \sum_{k=0}^n x(k)h(n-k) \\
 y(0) &= x(0)h(0) = 6, \\
 y(1) &= x(0)h(1) + x(1)h(0) = 11 \\
 y(2) &= x(0)h(2) + x(1)h(1) + x(2)h(0) = 15 \\
 y(3) &= x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0) = 18 \\
 y(4) &= x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1) + x(4)h(0) = 14 \\
 y(5) &= x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(2) + x(4)h(1) + x(5)h(0) = 10 \\
 y(6) &= x(1)h(5) + x(2)h(4) + x(3)h(3) = 6 \\
 y(7) &= x(2)h(5) + x(3)h(4) = 3 \\
 y(8) &= x(3)h(5) = 1 \\
 y(n) &= 0, n \geq 9 \\
 y(n) &= \left\{ \underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1 \right\}
 \end{aligned}$$

(b) By following the same procedure as in (a), we obtain

$$y(n) = \left\{ 6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1 \right\}$$

(c) By following the same procedure as in (a), we obtain

$$y(n) = \left\{ 1, \underset{\uparrow}{2}, 2, 2, 1 \right\}$$

(d) By following the same procedure as in (a), we obtain

$$y(n) = \left\{ \underset{\uparrow}{1}, 2, 2, 2, 1 \right\}$$

## 2.18

(a)

$$\begin{aligned}
 x(n) &= \left\{ \underset{\uparrow}{0}, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2 \right\} \\
 h(n) &= \left\{ 1, 1, \underset{\uparrow}{1}, 1, 1 \right\} \\
 y(n) &= x(n) * h(n) \\
 &= \left\{ \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2 \right\}
 \end{aligned}$$

(b)

$$\begin{aligned}x(n) &= \frac{1}{3}n[u(n) - u(n-7)], \\h(n) &= u(n+2) - u(n-3) \\y(n) &= x(n) * h(n) \\&= \frac{1}{3}n[u(n) - u(n-7)] * [u(n+2) - u(n-3)] \\&= \frac{1}{3}n[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2) + u(n-7) * u(n-3)] \\y(n) &= \frac{1}{3}\delta(n+1) + \delta(n) + 2\delta(n-1) + \frac{10}{3}\delta(n-2) + 5\delta(n-3) + \frac{20}{3}\delta(n-4) + 6\delta(n-5) \\&\quad + 5\delta(n-6) + 5\delta(n-6) + \frac{11}{3}\delta(n-7) + \delta(n-8)\end{aligned}$$

## 2.19

$$\begin{aligned}y(n) &= \sum_{k=0}^4 h(k)x(n-k), \\x(n) &= \left\{ \alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \dots, \alpha^5 \right\} \\h(n) &= \left\{ \underset{\uparrow}{1}, 1, 1, 1, 1 \right\} \\y(n) &= \sum_{k=0}^4 x(n-k), -3 \leq n \leq 9 \\&= 0, \text{ otherwise.}\end{aligned}$$

Therefore,

$$\begin{aligned}y(-3) &= \alpha^{-3}, \\y(-2) &= x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}, \\y(-1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \\y(0) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 \\y(1) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \\y(2) &= \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^2 \\y(3) &= \alpha^{-1} + 1 + \alpha + \alpha^2 + \alpha^3, \\y(4) &= \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 \\y(5) &= \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \\y(6) &= \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5 \\y(7) &= \alpha^3 + \alpha^4 + \alpha^5, \\y(8) &= \alpha^4 + \alpha^5, \\y(9) &= \alpha^5\end{aligned}$$

## 2.20

(a)  $131 \times 122 = 15982$

(b)  $\{1_{\uparrow}, 3, 1\} * \{1_{\uparrow}, 2, 2\} = \{1, 5, 9, 8, 2\}$

- (c)  $(1 + 3z + z^2)(1 + 2z + 2z^2) = 1 + 5z + 9z^2 + 8z^3 + 2z^4$   
 (d)  $1.31 \times 12.2 = 15.982$ .  
 (e) These are different ways to perform convolution.

## 2.21

(a)

$$y(n) = \sum_{k=0}^n a^k u(k) b^{n-k} u(n-k) = b^n \sum_{k=0}^n (ab^{-1})^k$$

$$y(n) = \begin{cases} \frac{b^{n+1} - a^{n+1}}{b-a} u(n), & a \neq b \\ b^n (n+1) u(n), & a = b \end{cases}$$

(b)

$$\begin{aligned} x(n) &= \left\{ 1, 2, \underset{\uparrow}{1}, 1 \right\} \\ h(n) &= \left\{ \underset{\uparrow}{1}, -1, 0, 0, 1, 1 \right\} \\ y(n) &= \left\{ 1, 1, -\underset{\uparrow}{1}, 0, 0, 3, 3, 2, 1 \right\} \end{aligned}$$

(c)

$$\begin{aligned} x(n) &= \left\{ 1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1 \right\}, \\ h(n) &= \left\{ 1, 2, \underset{\uparrow}{3}, 2, 1 \right\} \\ y(n) &= \left\{ 1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, -1 \right\} \end{aligned}$$

(d)

$$\begin{aligned} x(n) &= \left\{ \underset{\uparrow}{1}, 1, 1, 1, 1 \right\}, \\ h'(n) &= \left\{ \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1 \right\} \\ h(n) &= h'(n) + h'(n-9), \\ y(n) &= y'(n) + y'(n-9), \text{ where} \\ y'(n) &= \left\{ \underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1 \right\} \end{aligned}$$

## 2.22

(a)

$$\begin{aligned}
 y_i(n) &= x(n) * h_i(n) \\
 y_1(n) &= x(n) + x(n-1) \\
 &= \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}, \text{ similarly} \\
 y_2(n) &= \{1, 6, 11, 11, 13, 16, 14, 13, 15, 21, 25, 28, 24, 9\} \\
 y_3(n) &= \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\} \\
 y_4(n) &= \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\} \\
 y_5(n) &= \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}
 \end{aligned}$$

(b)

$$\begin{aligned}
 y_3(n) &= \frac{1}{2}y_1(n), \text{ because} \\
 h_3(n) &= \frac{1}{2}h_1(n) \\
 y_4(n) &= \frac{1}{4}y_2(n), \text{ because} \\
 h_4(n) &= \frac{1}{4}h_2(n)
 \end{aligned}$$

(c)  $y_2(n)$  and  $y_4(n)$  are smoother than  $y_1(n)$ , but  $y_4(n)$  will appear even smoother because of the smaller scale factor.

(d) System 4 results in a smoother output. The negative value of  $h_5(0)$  is responsible for the non-smooth characteristics of  $y_5(n)$

(e)

$$y_6(n) = \left\{ \frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2} \right\}$$

$y_2(n)$  is smoother than  $y_6(n)$ .

## 2.23

We can express the unit sample in terms of the unit step function as  $\delta(n) = u(n) - u(n-1)$ . Then,

$$\begin{aligned}
 h(n) &= h(n) * \delta(n) \\
 &= h(n) * (u(n) - u(n-1)) \\
 &= h(n) * u(n) - h(n) * u(n-1) \\
 &= s(n) - s(n-1)
 \end{aligned}$$

Using this definition of  $h(n)$

$$\begin{aligned}
 y(n) &= h(n) * x(n) \\
 &= (s(n) - s(n-1)) * x(n) \\
 &= s(n) * x(n) - s(n-1) * x(n)
 \end{aligned}$$

## 2.24

If

$$\begin{aligned}y_1(n) &= ny_1(n-1) + x_1(n) \text{ and} \\y_2(n) &= ny_2(n-1) + x_2(n) \text{ then} \\x(n) &= ax_1(n) + bx_2(n)\end{aligned}$$

produces the output

$$\begin{aligned}y(n) &= ny(n-1) + x(n), \text{ where} \\y(n) &= ay_1(n) + by_2(n).\end{aligned}$$

Hence, the system is linear. If the input is  $x(n-1)$ , we have

$$\begin{aligned}y(n-1) &= (n-1)y(n-2) + x(n-1). \text{ But} \\y(n-1) &= ny(n-2) + x(n-1).\end{aligned}$$

Hence, the system is time variant. If  $x(n) = u(n)$ , then  $|x(n)| \leq 1$ . But for this bounded input, the output is

$$y(0) = 1, \quad y(1) = 1 + 1 = 2, \quad y(2) = 2 \times 2 + 1 = 5, \dots$$

which is unbounded. Hence, the system is unstable.

## 2.25

(a)

$$\begin{aligned}\delta(n) &= \gamma(n) - a\gamma(n-1) \text{ and,} \\ \delta(n-k) &= \gamma(n-k) - a\gamma(n-k-1). \text{ Then,} \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \\ &= \sum_{k=-\infty}^{\infty} x(k)[\gamma(n-k) - a\gamma(n-k-1)] \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1) \\ x(n) &= \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a \sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k) \\ &= \sum_{k=-\infty}^{\infty} [x(k) - ax(k-1)]\gamma(n-k)\end{aligned}$$

$$\text{Thus, } c_k = x(k) - ax(k-1)$$

(b)

$$\begin{aligned}y(n) &= \mathcal{T}[x(n)] \\ &= \mathcal{T}\left[\sum_{k=-\infty}^{\infty} c_k\gamma(n-k)\right] \\ &= \sum_{k=-\infty}^{\infty} c_k\mathcal{T}[\gamma(n-k)] \\ &= \sum_{k=-\infty}^{\infty} c_kg(n-k)\end{aligned}$$

(c)

$$\begin{aligned}h(n) &= \mathcal{T}[\delta(n)] \\&= T[\gamma(n) - a\gamma(n-1)] \\&= g(n) - ag(n-1)\end{aligned}$$

## 2.26

With  $x(n) = 0$ , we have

$$\begin{aligned}y(n-1) + \frac{4}{3}y(n-1) &= 0 \\y(-1) &= -\frac{4}{3}y(-2) \\y(0) &= \left(-\frac{4}{3}\right)^2 y(-2) \\y(1) &= \left(-\frac{4}{3}\right)^3 y(-2) \\&\vdots \\y(k) &= \left(-\frac{4}{3}\right)^{k+2} y(-2) \leftarrow \text{zero-input response.}\end{aligned}$$

## 2.27

$$\begin{aligned}h(n) &= h_1(n) * h_2(n) \\&= \sum_{k=-\infty}^{\infty} a^k [u(k) - u(k-N)][u(n-k) - u(n-k-M)] \\&= \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k)u(n-k-M) \\&\quad - \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N)u(n-k-M) \\&= \left( \sum_{k=0}^n a^k - \sum_{k=0}^{n-M} a^k \right) - \left( \sum_{k=N}^n a^k - \sum_{k=N}^{n-M} a^k \right) \\&= 0\end{aligned}$$

## 2.28

(a)  $L_1 = N_1 + M_1$  and  $L_2 = N_2 + M_2$

(b) Partial overlap from left:

$$\text{low } N_1 + M_1 \quad \text{high } N_1 + M_2 - 1$$

$$\text{Full overlap: low } N_1 + M_2 \quad \text{high } N_2 + M_1$$

Partial overlap from right:

$$\text{low } N_2 + M_1 + 1 \quad \text{high } N_2 + M_2$$

(c)

$$\begin{aligned}x(n) &= \left\{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\right\} \\h(n) &= \left\{2, \underset{\uparrow}{2}, 2, 2\right\} \\N_1 &= -2, \\N_2 &= 4, \\M_1 &= -1, \\M_2 &= 2,\end{aligned}$$

Partial overlap from left:  $n = -3$      $n = -1$      $L_1 = -3$

Full overlap:  $n = 0$      $n = 3$

Partial overlap from right:  $n = 4$      $n = 6$      $L_2 = 6$

## 2.29

(a)

$$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n).$$

The characteristic equation is

$$\lambda^2 - 0.6\lambda + 0.08 = 0.$$

$\lambda = 0.2, 0.4$  Hence,

$$y_h(n) = c_1 \frac{1}{5}^n + c_2 \frac{2}{5}^n.$$

With  $x(n) = \delta(n)$ , the initial conditions are

$$y(0) = 1,$$

$$y(1) - 0.6y(0) = 0 \Rightarrow y(1) = 0.6.$$

$$\text{Hence, } c_1 + c_2 = 1 \text{ and}$$

$$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6 \Rightarrow c_1 = -1, c_2 = 3.$$

$$\text{Therefore } h(n) = \left[ -\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n \right] u(n)$$

The step response is

$$\begin{aligned}s(n) &= \sum_{k=0}^n h(n-k), n \geq 0 \\&= \sum_{k=0}^n \left[ 2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k} \right] \\&= \left\{ \frac{1}{0.12} \left[ \left(\frac{2}{5}\right)^{n+1} - 1 \right] - \frac{1}{0.16} \left[ \left(\frac{1}{5}\right)^{n+1} - 1 \right] \right\} u(n)\end{aligned}$$

(b)

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2).$$

The characteristic equation is

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$\lambda = \frac{1}{2}, \frac{1}{5}$  Hence,

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n.$$

With  $x(n) = \delta(n)$ , we have

$$\begin{aligned} y(0) &= 2, \\ y(1) - 0.7y(0) &= 0 \Rightarrow y(1) = 1.4. \\ \text{Hence, } c_1 + c_2 &= 2 \text{ and} \\ \frac{1}{2}c_1 + \frac{1}{5} &= 1.4 = \frac{7}{5} \\ \Rightarrow c_1 + \frac{2}{5}c_2 &= \frac{14}{5}. \end{aligned}$$

These equations yield

$$c_1 = \frac{10}{3}, c_2 = -\frac{4}{3}.$$

$$h(n) = \left[ \frac{10}{3} \left(\frac{1}{2}\right)^n - \frac{4}{3} \left(\frac{1}{5}\right)^n \right] u(n)$$

The step response is

$$\begin{aligned} s(n) &= \sum_{k=0}^n h(n-k), \\ &= \frac{10}{3} \sum_{k=0}^n \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^n \left(\frac{1}{5}\right)^{n-k} \\ &= \frac{10}{3} \left(\frac{1}{2}\right)^n \sum_{k=0}^n 2^k - \frac{4}{3} \left(\frac{1}{5}\right)^n \sum_{k=0}^n 5^k \\ &= \frac{10}{3} \left(\frac{1}{2}\right)^n (2^{n+1} - 1)u(n) - \frac{1}{3} \left(\frac{1}{5}\right)^n (5^{n+1} - 1)u(n) \end{aligned}$$

## 2.30

$$\begin{aligned} h(n) &= \left\{ \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \right\} \\ y(n) &= \left\{ \underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0 \right\} \\ x(0)h(0) &= y(0) \Rightarrow x(0) = 1 \\ \frac{1}{2}x(0) + x(1) &= y(1) \Rightarrow x(1) = \frac{3}{2} \end{aligned}$$

By continuing this process, we obtain

$$x(n) = \left\{ 1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \dots \right\}$$



## 2.31

- (a)  $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$   
 (b)

$$\begin{aligned}
 h_3(n) * h_4(n) &= (n-1)u(n-2) \\
 h_2(n) - h_3(n) * h_4(n) &= 2u(n) - \delta(n) \\
 h_1(n) &= \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) \\
 \text{Hence } h(n) &= \left[ \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) \right] * [2u(n) - \delta(n)] \\
 &= \frac{1}{2}\delta(n) + \frac{5}{4}\delta(n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)
 \end{aligned}$$

(c)

$$\begin{aligned}
 x(n) &= \left\{ 1, 0, 0, 3, 0, -4 \right\} \\
 &\quad \quad \quad \uparrow \\
 y(n) &= \left\{ \frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \dots \right\}
 \end{aligned}$$

## 2.32

First, we determine

$$\begin{aligned}
 s(n) &= u(n) * h(n) \\
 s(n) &= \sum_{k=0}^{\infty} u(k)h(n-k) \\
 &= \sum_{k=0}^n h(n-k) \\
 &= \sum_{k=0}^{\infty} a^{n-k} \\
 &= \frac{a^{n+1} - 1}{a - 1}, n \geq 0
 \end{aligned}$$

For  $x(n) = u(n+5) - u(n-10)$ , we have the response

$$s(n+5) - s(n-10) = \frac{a^{n+6} - 1}{a - 1}u(n+5) - \frac{a^{n-9} - 1}{a - 1}u(n-10)$$

From figure P2.33,

$$\begin{aligned}
 y(n) &= x(n) * h(n) - x(n) * h(n-2) \\
 \text{Hence, } y(n) &= \frac{a^{n+6} - 1}{a - 1}u(n+5) - \frac{a^{n-9} - 1}{a - 1}u(n-10) \\
 &\quad - \frac{a^{n+4} - 1}{a - 1}u(n+3) + \frac{a^{n-11} - 1}{a - 1}u(n-12)
 \end{aligned}$$

### 2.33

$$\begin{aligned}
 h(n) &= [u(n) - u(n - M)] / M \\
 s(n) &= \sum_{k=-\infty}^{\infty} u(k)h(n - k) \\
 &= \sum_{k=0}^n h(n - k) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}
 \end{aligned}$$

### 2.34

$$\begin{aligned}
 \sum_{n=-\infty}^{\infty} |h(n)| &= \sum_{n=0, \text{even}}^{\infty} |a|^n \\
 &= \sum_{n=0}^{\infty} |a|^{2n} \\
 &= \frac{1}{1 - |a|^2}
 \end{aligned}$$

Stable if  $|a| < 1$

### 2.35

$h(n) = a^n u(n)$ . The response to  $u(n)$  is

$$\begin{aligned}
 y_1(n) &= \sum_{k=0}^{\infty} u(k)h(n - k) \\
 &= \sum_{k=0}^n a^{n-k} \\
 &= a^n \sum_{k=0}^n a^{-k} \\
 &= \frac{1 - a^{n+1}}{1 - a} u(n) \\
 \text{Then, } y(n) &= y_1(n) - y_1(n - 10) \\
 &= \frac{1}{1 - a} [(1 - a^{n+1})u(n) - (1 - a^{n-9})u(n - 10)]
 \end{aligned}$$

### 2.36

We may use the result in problem 2.32 with  $a = \frac{1}{2}$ . Thus,

$$y(n) = 2 \left[ 1 - \left(\frac{1}{2}\right)^{n+1} \right] u(n) - 2 \left[ 1 - \left(\frac{1}{2}\right)^{n-9} \right] u(n - 10)$$

## 2.37

(a)

$$\begin{aligned}
 y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\
 &= \sum_{k=0}^n \left(\frac{1}{2}\right)^k 2^{n-k} \\
 &= 2^n \sum_{k=0}^n \left(\frac{1}{4}\right)^k \\
 &= 2^n \left[ 1 - \left(\frac{1}{4}\right)^{n+1} \right] \left(\frac{4}{3}\right) \\
 &= \frac{2}{3} \left[ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \right] u(n)
 \end{aligned}$$

(b)

$$\begin{aligned}
 y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n-k) \\
 &= \sum_{k=0}^{\infty} h(k) \\
 &= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, n < 0 \\
 y(n) &= \sum_{k=n}^{\infty} h(k) \\
 &= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k \\
 &= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k \\
 &= 2 - \left(\frac{1 - \left(\frac{1}{2}\right)^n}{\frac{1}{2}}\right) \\
 &= 2\left(\frac{1}{2}\right)^n, n \geq 0.
 \end{aligned}$$

## 2.38

(a)

$$\begin{aligned}
 h_e(n) &= h_1(n) * h_2(n) * h_3(n) \\
 &= [\delta(n) - \delta(n-1)] * u(n) * h(n) \\
 &= [u(n) - u(n-1)] * h(n) \\
 &= \delta(n) * h(n) \\
 &= h(n)
 \end{aligned}$$

(b) No.

## 2.39

- (a)  $x(n)\delta(n - n_0) = x(n_0)$ . Thus, only the value of  $x(n)$  at  $n = n_0$  is of interest.  
 $x(n) * \delta(n - n_0) = x(n - n_0)$ . Thus, we obtain the shifted version of the sequence  $x(n)$ .  
 (b)

$$\begin{aligned}
 y(n) &= \sum_{k=-\infty}^{\infty} h(k)x(n - k) \\
 &= h(n) * x(n) \\
 \text{Linearity: } x_1(n) \rightarrow y_1(n) &= h(n) * x_1(n) \\
 x_2(n) \rightarrow y_2(n) &= h(n) * x_2(n) \\
 \text{Then } x(n) &= \alpha x_1(n) + \beta x_2(n) \rightarrow y(n) = h(n) * x(n) \\
 y(n) &= h(n) * [\alpha x_1(n) + \beta x_2(n)] \\
 &= \alpha h(n) * x_1(n) + \beta h(n) * x_2(n) \\
 &= \alpha y_1(n) + \beta y_2(n) \\
 \text{Time Invariance:} \\
 x(n) \rightarrow y(n) &= h(n) * x(n) \\
 x(n - n_0) \rightarrow y_1(n) &= h(n) * x(n - n_0) \\
 &= \sum_k h(k)x(n - n_0 - k) \\
 &= y(n - n_0)
 \end{aligned}$$

- (c)  $h(n) = \delta(n - n_0)$ .

## 2.40

- (a)  $s(n) = -a_1s(n - 1) - a_2s(n - 2) - \dots - a_Ns(n - N) + b_0v(n)$ . Refer to fig 2.40-1.  
 (b)  $v(n) = \frac{1}{b_0} [s(n) + a_1s(n - 1) + a_2s(n - 2) + \dots + a_Ns(n - N)]$ . Refer to fig 2.40-2

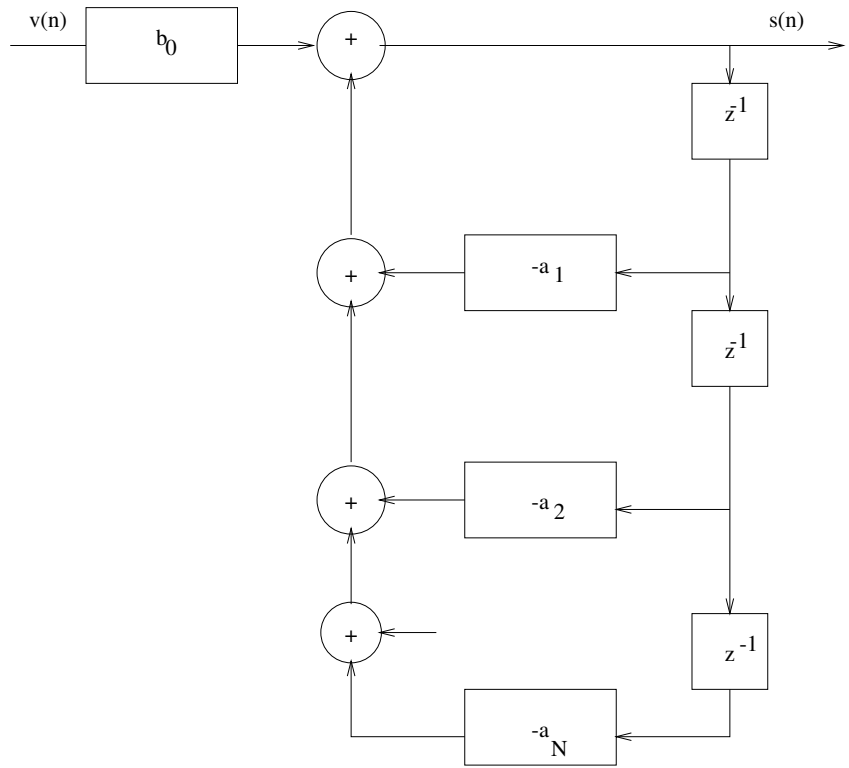


Figure 2.40-1:

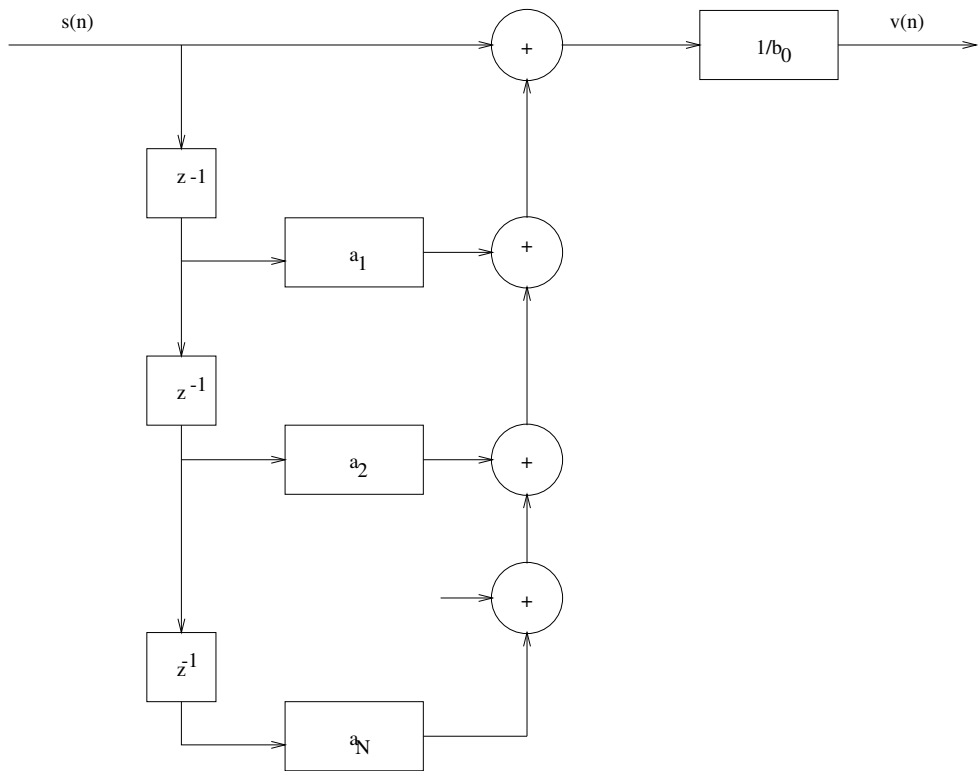


Figure 2.40-2:

## 2.41

$$\begin{aligned}
 y(n) &= -\frac{1}{2}y(n-1) + x(n) + 2x(n-2) \\
 y(-2) &= -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1 \\
 y(-1) &= -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = \frac{3}{2} \\
 y(0) &= -\frac{1}{2}y(-1) + 2x(-2) + x(0) = \frac{17}{4} \\
 y(1) &= -\frac{1}{2}y(0) + x(1) + 2x(-1) = \frac{47}{8}, \text{ etc}
 \end{aligned}$$

## 2.42

(a) Refer to fig 2.42-1

(b) Refer to fig 2.42-2

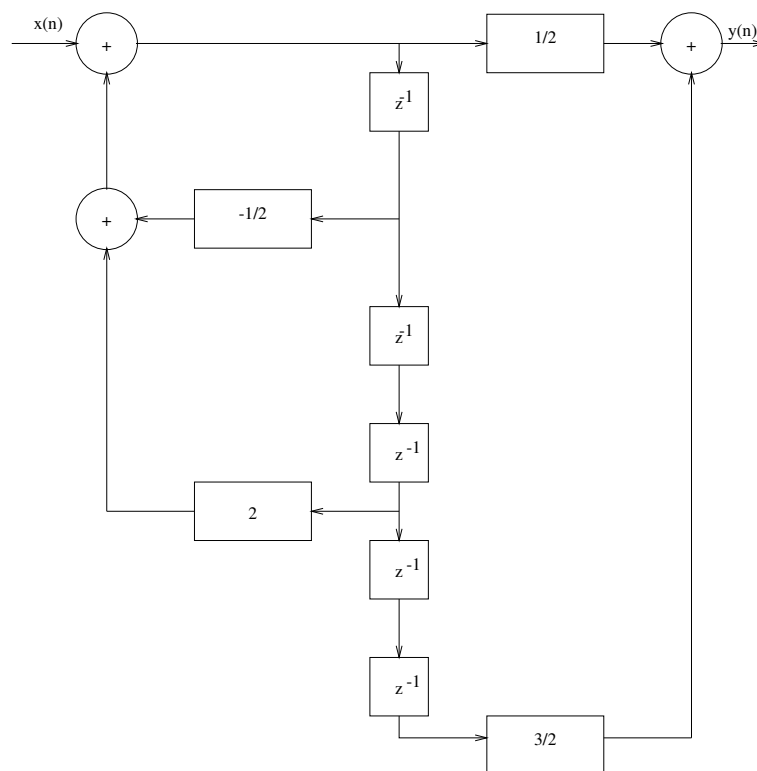


Figure 2.42-1:

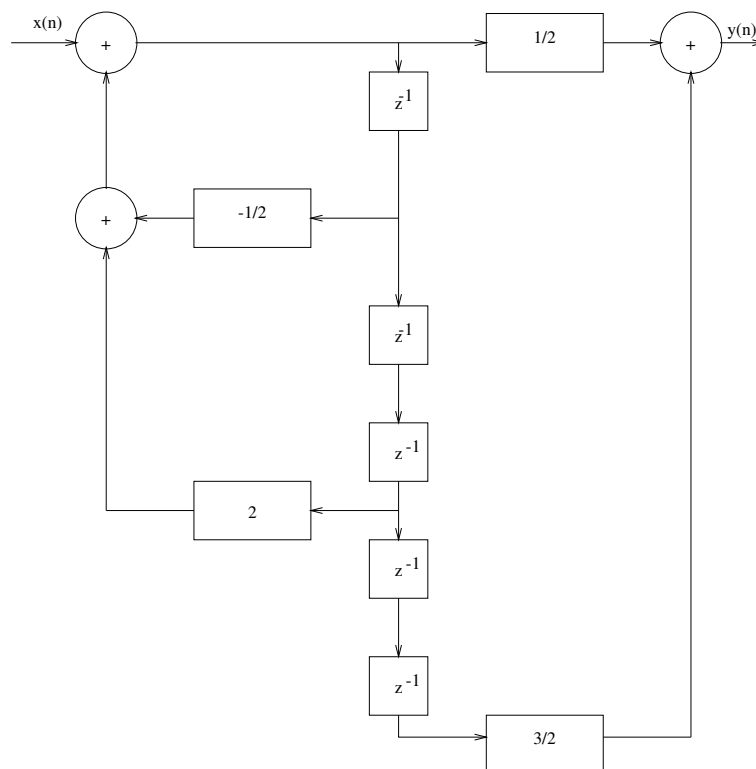


Figure 2.42-2:

## 2.43

(a)

$$\begin{aligned}
 x(n) &= \left\{ \underset{\uparrow}{1}, 0, 0, \dots \right\} \\
 y(n) &= \frac{1}{2}y(n-1) + x(n) + x(n-1) \\
 y(0) &= x(0) = 1, \\
 y(1) &= \frac{1}{2}y(0) + x(1) + x(0) = \frac{3}{2} \\
 y(2) &= \frac{1}{2}y(1) + x(2) + x(1) = \frac{3}{4}. \text{ Thus, we obtain} \\
 y(n) &= \left\{ 1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \dots \right\}
 \end{aligned}$$

(b)  $y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

(c) As in part(a), we obtain

$$y(n) = \left\{ 1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \dots \right\}$$