TT TEXTBOOK TESTBANKS

Test Bank for Stats Modeling the World 6th Veaux

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TEST BANK AND RESOURCE GUIDE

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STATS
MODELING THE WORLD
SIXTH EDITION

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Contents

About This Guide

Whether you are teaching Statistics for the first time, or just adapting to the new approach we take in this book, we hope this Teacher's Guide will help you optimize your students' experience. Here we explain the reasoning behind our approach to teaching Statistics. We summarize each chapter, highlighting the important concepts and pointing out where they'll show up later in the course. We offer some pedagogical suggestions—do's and don'ts—and include examples and activities you might use in teaching your class. We offer suggested quizzes, tests, and investigative tasks. And we provide references to other resources that you may find helpful.

Each chapter of this Teacher's Guide contains some or all of the following features.

What's It About?

This section summarizes the major topics included in the chapter. More important, we tell the *story* of the chapter. Each chapter introduces new concepts and methods, and each one fits with what students have learned in previous chapters and will learn in subsequent ones. We give you the overview to help you show your students how it all fits together.

Comments

The Comments section explains the statistical and pedagogical reasons for the choices we've made in what to teach, in how to present it, and in what order to discuss it. Some of these choices may differ from those made by other textbooks. We try to point out these differences and explain our approach.

Looking Ahead

The Looking Ahead sections point out ways that many of the ideas we introduce in early chapters foreshadow or pave the way for important features of later chapters. These are often good points to make in class to motivate students and to help them fit all these new concepts together into a coherent whole.

Class Do's

We offer pedagogical advice about approaches that have worked for us, ideas to stress, and other ways to highlight important concepts or take advantage of important features of this text.

The Importance of What You Don't Say

One of the challenges of teaching AP* Stats is that there's so much to say. But too much information at the wrong time can be confusing to the beginning student. Because deciding how much to say and when to say it can be tricky, we offer some suggestions about what *not* to say and what not to say quite *yet*.

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What If…?

You will probably want to expand on the "What If…?" section in the text. Starting in Chapter 2, we provide several questions that will stimulate class discussions, and get students thinking about the important concepts that will lead to deeper understanding of the major topics of the course, such as independence, sampling variability, the Central Limit Theorem, or statistical significance.

Class Examples

It's always good to have another example for class. Students seem to always want one more example. So we provide new examples different from those in the book. These can include actual classroom materials in the form of worksheets or guided explorations.

Resources

We offer a list of resources for background information, data sets, and classroom activities. These may include other books, videos, software, or Web sites.

Assignments

We make general suggestions about pace and timing of your work in the chapter, including the amount of reading and the number of exercises you might assign each night.

Chapter Quiz

We offer four versions of a quiz you might choose to give after completing the chapter.

Investigative Task

Instead of a quiz, you may choose to have students complete a written assignment that asks them to apply the major concepts of the chapter. Along with each classroom-tested task we include a scoring rubric you can use as you grade each student's work and return to the student to provide them with guidance about writing clear, complete, concise statistical analyses. (We prefer these to quizzes, but that's us.)

Unit Test

We offer four versions of a sample exam at the end of each of the text's seven Parts (and occasionally more often). These exams, also classroom-tested, include multiple choice questions, short questions requiring some calculations or written explanations, and longer questions requiring more in-depth analysis. They are not easy. When students take the AP* Exam they will be asked to work on difficult problems asking for clear understanding of important concepts, accurate application of statistical techniques, and proper interpretation of the results – all under pressure of time. Many will not finish, but no one is expected to come close to getting everything right. Do not imagine that we think students should succeed on 90% of these questions to earn an A! After all, that's not the expectation on the AP* Exam either.

Also In This Guide

Suggested time lines appear right after this introduction. The full year timeline is based on Dave Bock's experiences at Ithaca High School, where the school year begins right after Labor Day and classes meet 5 days a week for 44 minutes. The alternating block timeline comes from Greg Timm of Roland Park Country School in Baltimore. That school opens right after Labor Day, and classes meet on alternating days for 70 minutes with an additional 40-minute class each week.

A ***Correlation to the AP* Exam*** is included at the end of this guide.

And a word about the Texas Instruments calculators

You'll find the textbook's TI Tips allow students to learn to use the statistics functions for their TI-83+/84+ family of calculators, freeing you from spending valuable classroom time on button-pushing. We prepared those instructions based on the latest TI operating system at the time the text was published, which we urge you to download and install. Occasionally in this Guide we'll provide additional TI Tips supporting calculators with older operating systems. And if your students use TI-89s or TI-Nspire™ handhelds, they'll find calculator instructions in Appendix B's guide to statistical software. Additionally, this Guide directs you to TI-Nspire activities available on the book's website, www.pearsonhighered.com/bock.

A Few More Words about the Text

The Preface

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We know students won't read the preface, but you should. We think this text is different. We hope to entice students to read the book with our easy-to-understand conversational style—and to entertain them with occasional humor. We have created several features that provide consistent themes and helpful resources for doing Statistics. Take advantage of them!

New to the Sixth Edition

This new edition of *Stats: Modeling the World* extends the series of innovations pioneered in our books, teaching Statistics and statistical thinking as it is practiced today. We've made some important revisions and additions, each with the goal of making it even easier for students to put the concepts of Statistics together into a coherent whole.

- ***Full inclusion of all new topics in the AP Statistics syllabus.*** In the Fall of 2020 the College Board updated the AP Statistics syllabus to better reflect the way the course is taught in colleges around the United States. With this edition of *Stats: Modeling the World* we incorporate all the changes so that teachers and students can prepare for the exam without needing to supplement the text. The new topics range from new graphical tools like the cumulative distribution plot to stricter guidelines on how to carry out some hypothesis tests. Every change is reflected in the exercises as well as the text, to give students opportunities to practice what they've learned.
- ***Updated examples, exercises, and data.*** We've updated our innovative *Think/Show/Tell Step-by-Step* examples with new contexts and data. We've added hundreds of new exercises and updated continuing exercises with the most recent data. Whenever possible, we've provided those data, available through MyMathLab for School or at www.pearsonhighered.com/mathstatsresources. Many of the examples and exercises are

based on recent news stories, research articles, and other real-world sources. We've listed many of those sources so students can explore them further.

Continued Features

- **Chapter 1 (and beyond).** Chapter 1 gets down to business immediately, looking at data. And throughout the book chapters lead with new up-to-the-minute motivating examples and follow through with analyses of the data, and real-world examples provide a basis for sample problems and exercises.
- **Think, Show, Tell.** The worked examples repeat the mantra of Think, Show, and Tell in every chapter. They emphasize the importance of thinking about a Statistics question (What do we know? What do we hope to learn? Are the assumptions and conditions satisfied?) and reporting our findings (the Tell step). The Show step contains the mechanics of calculating results and conveys our belief that it is only one part of the process.
- **Step-by-Step** examples guide students through the process of analyzing a problem by showing the general explanation on the left and the worked-out solution on the right. The result: better understanding of the concept, not just number crunching.
- **For Example**. In every chapter, an interconnected series of For Example elements present a continuing discussion, recapping a story and moving it forward to illustrate how to apply each new concept or skill.
- **Just Checking**. At key points in each chapter, we ask students to pause and think with questions designed to be a quick check that they understand the material they've just read. Answers are at the end of the exercise sets in each chapter so students can easily check themselves.
- **Updated TI Tips.** Each chapter's easy-to-read "TI Tips" now show students how to use TI-84 Plus CE Statistics functions with the StatWizard operating system. (Help using a TI-Nspire appears in Appendix B, and help with a TI-89 is on the book's resource site (www.pearsonhighered.com/bock.) As we strive for a sound understanding of formulas and methods, we want students to use technology for actual calculations. We do emphasize that calculators are just for "Show"—they cannot Think about what to do or Tell what it all means.
- **Math Boxes**. In many chapters we present the mathematical underpinnings of the statistical methods and concepts. By setting these proofs, derivations, and justifications apart from the narrative, we allow students to continue to follow the logical development of the topic at hand, yet also explore the underlying mathematics for greater depth.
- **TI-Nspire Activities**. Margin pointers identify demonstrations and investigations for TI-Nspire handhelds to enhance each chapter. They're found at the book's resource site (www.pearsonhighered.com/bock).
- **What Can Go Wrong?** Each chapter still contains our innovative What Can Go Wrong? sections that highlight the most common errors people make and the misconceptions they have about Statistics. Our goals are to help students avoid these pitfalls and to arm them with the tools to detect statistical errors and to debunk misuses of statistics, whether intentional or not.
- **What Have We Learned?** Chapter-ending study guides help students review key concepts and terms.

- **Exercises**. We've maintained the pairing of examples so that each odd-numbered exercise (with an answer in the back of the book) is followed by an even-numbered exercise illustrating the same concept. Exercises are ordered by approximate level of complexity.
- **Practice Exams.** At the end of each of the book's seven parts you'll find a practice exam, consisting of both multiple choice and free response questions. These cumulative exams encourage students to keep important concepts and skills in mind throughout the course while helping them synthesize their understanding as they build connections among the various topics.
- **Reality Check**. We regularly remind students that Statistics is about understanding the world with data. Results that make no sense are probably wrong, no matter how carefully we think we did the calculations. Mistakes are often easy to spot with a little thought, so we ask students to stop for a reality check before interpreting their result.
- **Notation Alerts.** Clear communication is essential in Statistics, and proper notation is part of the vocabulary students need to learn. We've found that it helps to call attention to the letters and symbols statisticians use to mean very specific things.
- **On the Computer**. Because real-world data analysis is done on computers, at the end of each chapter we summarize what students can find in most Statistics software, usually with an annotated example.

Review of Part …

The 26 chapters of this book are divided into seven units. The end of each Part includes a one page **Quick Review** of the major concepts followed by a large set of **Exercises**. These exercises are comprehensive, often integrate several concepts, and appear in random order. You should find everything you need to prepare your students for tests. We have also provided an opportunity for your students to prepare *themselves* for tests, with an AP*-style Practice Test following the Review Exercises in each Part. Each Practice Test contains both multiple choice and free response questions, just like the AP* Exam.

Some Important Resources

We offer advice on resources we think you'll find helpful.

TI-Nspire Demonstrations and Activities

Looking for a great way to illustrate an important statistical concept? We've created 29 classroom demonstrations that run on your computer's TI-Nspire software, and included them on the resource on the book's website. These demonstrations were written using Version 1.4 of the TI-Nspire Computer Software – Teacher Edition and will run on the CAS Computer Software. Any updates for these activities will be posted, www.pearsonhighered.com/bock.

Note that the TI-Nspire demos are best shown using the software's Normal, Presentation View. (They are not formatted for the handheld units.) Many use the command Control-R on a PC to re-randomize data. The equivalent command on a Mac is Command-R. To rerun the simulations, you can either delete the contents of some of the columns in the spreadsheet or reload the file.

AP Review Book*

There are several review books on the market, from the usual sources. Our favorite is *Pearson Education AP* Test Prep: Statistics for Stats: Modeling the World, Fifth Edition*, 2019. On one level, that's no surprise, as it was written to accompany this text. However, before this review book came out we used three of the others in our own classrooms. Speaking now as teachers rather than authors, we found our students were best prepared after using this *AP* Test Prep* workbook. It was written by four veteran AP* teachers who have long served as AP* exam Readers. For many years they have seen the mistakes students make on the test, they know the AP* grading criteria well, and all have personal experience structuring review with their own students. It works. The review book can be bundled with the textbooks or purchased separately. For more information, contact your Pearson sales representative.

StatCrunch®

StatCrunch is a powerful online tool that allows you to:

- Upload data files from your computer or the Web to you own datasets library.
- Analyze data using the extensive list of numerical and graphical procedures StatCrunch offers.
- Report your insights along with attached data sets and analysis results.
- Share your data, results and reports with the rest of the world or keep them private.
- Comment on your items or those being shared by other subscribers.

Explore and learn more at www.statcrunch.com

Classroom Activities

- *Activity-Based Statistics*, 2nd Ed., Schaeffer, Gnanadesekan, Watkins, and Witmer; Key College Press, 2004. Here's a great source of excellent classroom activities. We'll cite many in the chapters ahead, and you'll find others that you will want to use.
- *Workshop Statistics: Discovery with Data and the Graphing Calculator*, 3rd Ed., Rossman, Chance, and Von Oehsen; Key Curriculum Press, 2008. Some adopt this workbook style approach as a primary text. We think it's a valuable source of classroom explorations, and will recommend some in particular. Again you are likely to find others you will want to use or adapt.
- *Fifty Fathoms: Statistics Demonstrations for Deeper Understanding,* Erickson; eepsmedia, 2002. This is a book of computer-based demonstrations of concepts. Teachers can perform these demos for the class – using a projection system – either as an introduction to a topic, as review to clarify some idea that the students found challenging, or simply embedded in a lesson as another way to illustrate a concept. We'll indicate appropriate Demos in many chapters.

Videos

- *Decisions Through Data*; COMAP, 1992. This set of 5 DVDs contains 21 lessons to show in class. Each looks at real-life situations and demonstrates the use of statistics to answer important questions. The units are typically 10-15 minutes in length, allowing you to show the video segment and have time to discuss the statistical concepts and techniques introduced. We'll indicate appropriate units in many chapters. Video Guides, Exercises, and Quizzes are available or each unit.
- *The Joy of Stats* – This hour-long video originally aired on BBC in 2010. Host Hans Rosling gives an overview of how statistics can be used to make sense of the world and "is now the sexiest subject around." Available for free on the internet through gap minder, vimeo or youtube. Free worksheets are available on the internet.
- *Why You Need to Study Statistics* – available from the 'This is Statistics' page on the American Statistical Association website or https://www.youtube.com/watch?time_continue=106&v=wV0Ks7aS7YI
- *Teach Statistics Before Calculus* – TED Talk with Arthur Benjamin. https://www.youtube.com/watch?v=BhMKmovNjvc
- *Meaning from Data: Statistics Made Clear*, M. Starbird, The Great Courses, The Teaching Company, 2006. This set of 4 DVDs contains 24 30-minute lectures. Part I contains 12 lectures which explore topics in all 4 AP Statistics topic threads [Exploring Data; Sampling and Experimentation; Anticipating Patterns; Statistical Inference] while Part II contains 12 lectures which apply the concepts to real life scenarios including real estate, sports, insurance and war, economics, social science, biology, and elections and juries.

Other Books and Magazines

- *Statistics*, 3rd Ed., Freedman, Pisani, and Purvis; Norton, 2001. This book contains interesting stories and great explanations of statistical concepts; it's a valuable resource to have on your shelf.
- *Statistics: Concepts and Controversies,* 6th Ed., Moore; Freeman, 2006. This collection of great stories about the uses and misuses of statistics is a valuable resource when you are looking for examples to talk about in class.
- *Chance*, American Statistical Association. This magazine, published quarterly, provides articles about statistics as well as excellent examples and data sets to use in class.
- *Significance*, Royal Statistical Society and American Statistical Association. This bimonthly magazine challenges myths, provide a unique perspective on the stories of the day, and use statistics to answer society's most difficult questions. Excellent class examples.
- *Innumeracy,* Paulos; Holt McDougal, 2001. This book introduces most of the concepts covered in this course in an interesting manner with plenty of real life context to intrigue the high school student. This book can be used as a summer assignment to introduce the course; it reads like a marketing brochure for the need for this course.
- *The Lady Tasting Tea: How Statistics Revolutionized Science in the Twentieth Century,* Salsburg, Holt, 2001. This book examines the development of ever-more-powerful statistical methods for determining scientific truth. A series of historical and biographical sketches illuminate the reader and "are unified in a single theme: the conversion of science from observational natural history into rigorously defined statistical models of data collection and analysis." Excellent for class discussions, particularly for Part III on this textbook.
- *The Cartoon Guide to Statistics*, Gonick, HarperCollins, 1993. This book dovetails nicely with the AP* Statistics curriculum and can be used as a fun way to review topics in a comic strip format.
- BARRON'S AP STATISTICS FLASH CARDS, M. Sternstein, ISBN-13: 9780764194108. Excellent review resource (multiple choice and free response) for AP* Statistics students on the go. Each card is the size of a credit card and includes a punched hole; a key ring is included for easy transport of selected cards

AP Central – AP* Teacher Community*

The AP* Statistics website maintained by the College Board at AP* Central (https://apcentral.collegeboard.org and https://apcommunity.collegeboard.org) is a treasure trove of valuable material. Among other things, after registering you'll be able to:

- *get the latest AP* Statistics course description and outline;*
- *download free response questions, solutions, and scoring rubrics for all prior AP* Exams dating back to 1997;*
- *download teaching materials and insightful articles written by other AP* Statistics teachers from the Resource Library;*
- *read reviews of texts, review books, and other teaching materials;*
- *download the full released 2012 AP* Statistics exam;*
- *download several unreleased Practice Exams, after passing your course audit;*
- *search for professional development workshops.*

The AP Statistics Teacher Community Discussion Board*

The Teacher Community Discussion Board is a gathering place for people teaching AP* Statistics, and perhaps the gentlest discussion group on the web. Whether you are looking for teaching ideas or have questions about statistical concepts or theory, the response will be quick, varied, and supportive. You'll hear from teachers all over the country with fantastic ideas to share and gain statistical knowledge generously offered by some of the biggest names in the field. This is a must.

Sign up at: https://apcommunity.collegeboard.org/getting-started

AP Statistics Course Audit Syllabus*

The College Board requires each teacher submit a syllabus for approval before using the AP* designation in course descriptions and transcripts. Since such a document is quite lengthy, we have provided a sample audit syllabus on the text website, www.pearsonhighered.com/bock. Please be sure that your audit syllabus outlines what will happen in your classroom. The College Board Course Audit page for AP Statistics can be found here: https://apcentral.collegeboard.org/courses/ap-statistics/course-audit?course=ap-statistics

Net Links

The Internet is a valuable source of data sets, examples, tables, random numbers, and current events. The good news is that you can probably find almost everything you need or want to know there. The bad news is that the materials will not be consistent or integrated. Be especially wary of introducing students to a variety of online applets, each with its own interface, notation, terminology, and assumptions.

Many of the data sets and examples of the book are sourced from Internet sites. Where appropriate, we provide URL references to the top level, and key search terms to help locate the particular data or discussion. These references may lead to even more up-to-date data than were available when we found them for the book. The data used in the book are available on our website www.pearsonhighered.com/bock.

We provide below some useful jumping off points, with the obvious caveat that many of them may move, change, or disappear altogether between the time we compile this list and you try to use it. With our apologies in advance when a link fails, we hope you find this effort of value. You'll find information on many other useful links on our website www.pearsonhighered.com/bock.

- Materials posted by other teachers

  courses.ncssm.edu/math/Stat_Inst/Notes.htm

  www.bbn-school.org/us/math/ap_stats/

  exploringdata.net/intro.htm

- Sources of Data

  dartmouth.edu/~chance/teaching_aids/data.html

  exploringdata.net/intro.htm

  www.census.gov

  www.fedstats.sites.usa.gov

  lib.stat.cmu.edu/DASL/ (data sets indexed by topic)

  ucr.fbi.gov/ucr (crime statistics)

  www.amstat.org/publications/jse

  www.wri.org/resources

- Applets you may find useful for classroom demonstration:

  media.pearsoncmg.com/cmg/pmmg_mml_shared/animations/statistics/west_applets_HTML5/index.html (StatCrunch Applets—no log-in required)

  students.brown.edu/seeing-theory/index.html

  www.amstat.org/asa/education/Useful-Websites-for-Teachers.aspx

  www2.stat.duke.edu/sites/java.html

  davidmlane.com/hyperstat/index.html

  wise.cgu.edu/portfolio/

  www.rossmanchance.com/applets/

  onlinestatbook.com/stat_sim/

  sites.berry.edu/vbissonnette/index/stats-homework/documentation/generating-sampling-distributions/

- Software – calculators, commercial software

  education.ti.com

  www.casioeducation.com/home

  edu.casio.com/software_app/app/index.php

  datadesk.com/products/activstats

  www.minitab.com

  www.jmp.com

  www.spss.com

- Statistics Background

  dartmouth.edu/~chance/teaching_aids/data.html

  courses.ncssm.edu/math/Stat_Inst/Notes.htm

AP* Statistics Full Year TimeLine for *Stats: Modeling the World*

Chapter		**Class days**
1	Stats Starts Here	2
2	Displaying & Describing Categorical Data	3
3	Displaying and Summarizing Quantitative Data	6
4	Understanding and Comparing Distributions	3
5	The Standard Deviation as Ruler and the Normal Model	5
	Part I – Review and Testing	2
6	Scatterplots, Association, and Correlation	3
7	Linear Regression (including a test)	8
8	Regression Wisdom	4
9	Re-Expressing Data: Get It Straight!	6
	Part II – Review and Testing	2
10	Understanding Randomness	4
11	Sample Surveys	5
12	Experiments and Observational Studies	4
	Part III – Review and Testing	2
	Project (Data Collection and Analysis)	4
13	From Randomness to Probability	3
14	Probability Rules!	4
15	Random Variables	4
16	Probability Models	4
	Part IV – Review and Testing	2
17	Sampling Distribution Models	4
18	Confidence Intervals for Proportions	4
19	Testing Hypotheses about Proportions	2
20	More About Tests	3
21	Comparing Two Proportions	2
	Part V – Review and Testing	2
22	Inferences about Means	3
23	Comparing Means	2
24	Paired Samples and Blocks	2
	Part VI – Group Project, Review, and Testing	4
25	Comparing Counts	6
26	Inferences for Regression	4
	Part VII – Review and Testing	2
	Review for the AP* Exam, with a practice exam	20

AP* Statistics Block TimeLine for *Stats: Modeling the World*

Chapter		Class days
1	Stats Starts Here	1
2	Displaying & Describing Categorical Data	1.5
3	Displaying and Summarizing Quantitative Data	2.5
4	Understanding and Comparing Distributions	2.5
5	The Standard Deviation as Ruler and the Normal Model	2.5
	Part I – Review and Testing	2
6	Scatterplots, Association, and Correlation	1.5
7	Linear Regression (including a test)	5
8	Regression Wisdom	2.5
9	Re-Expressing Data: Get It Straight!	3.5
	Part II – Review and Testing	2
10	Understanding Randomness	2.5
11	Sample Surveys	2.5
12	Experiments and Observational Studies	2.5
	Part III – Review and Testing	2
	Project (Data Collection and Analysis)	2.5
13	From Randomness to Probability	2
14	Probability Rules!	2.5
15	Random Variables	3
16	Probability Models	2
	Part IV – Review and Testing	2
17	Sampling Distribution Models	2
18	Confidence Intervals for Proportions	2.5
19	Testing Hypotheses about Proportions	2
20	More About Tests	2
21	Comparing Two Proportions	1.5
	Part V – Review and Testing	2
22	Inferences about Means	2
23	Comparing Means	1.5
24	Paired Samples and Blocks	2
	Part VI – Group Project, Review, and Testing	3
25	Comparing Counts	2.5
26	Inferences for Regression	2
	Part VII – Review and Testing	2
	Review for the AP* Exam, with a practice exam	6

Part I: Exploring and Understanding Data: Chapters 1 – 6

This Part of the book covers data displays and summaries. Many students will recognize some of the material from middle and high school, so our emphasis is on statistical thinking. Of course, we define terms and provide examples. But we also discuss *why* methods presented are used, and what we hope to learn from them. These are concepts that appear throughout the course. Even more important than what to look for in a histogram or how to summarize the spread of a distribution is the underlying lesson that there *are* reasons for displaying and summarizing data. These reasons inform and motivate the entire course.

Chapter 1
Stats Starts Here

What's it about?

This chapter is about Statistics – what it is and why we care. It also describes the important features of the text. We've given the chapter an unusual title and tried to grab students' attention with a humorous footnote. (Some have e-mailed us to assure us that they *do* read the footnotes.) If we can get them to read three words and the footnote, maybe we can get them to read on.

We then introduce students to *data*. We talk about the importance of context (the W's), about variables, and make the distinction between categorical and quantitative data. We begin to introduce the vocabulary of Statistics. And the TI Tips section shows students how to enter data into the calculator's lists.

Comments

Do expect your students to read the book. Give them specific reading assignments, starting with this first chapter. We've tried to make the book engaging. You can assign a few pages of reading along with some problems each night.

This is the students' first look at the style of the book, and we do lay it on more heavily here than we will when discussing, say, confidence intervals. We want to shake things up. We want them to notice that this is not the same old math or science textbook they've seen before. And we'd like to get them on our side. That's the reason for the humor and self-deprecating remarks.

Every Statistics text starts with a definition of *Statistics.* We do too, but ours is different. And the difference matters. We say that Statistics is *a way of reasoning* and that the goal is *to help us understand the world.* We've found it helpful to reinforce this idea throughout the semester, especially when we get into the methods sections of the course. This book is first and primarily about statistical thinking. Methods, definitions, and skills are all here, but each is presented with the purpose of understanding the world. That's why every example follows the *Think, Show, Tell* pattern, starting with careful reasoning and concluding with a sentence or two telling what we've learned about the world.

It is easy to be drawn into a focus on definitions, on algorithms, and on getting the "right answer." Those are easier to teach and certainly easier to grade. Please resist the temptation. If you can help your students to stay focused on statistical thinking and understanding, this course can change the way they view the world.

Get students involved with data from the start. We don't take a "big picture" approach at this time. There will be plenty of time to build models and draw inferences later. For now, let's just get our hands dirty playing with the data. When students have a good sense of what kinds of things data can say to us, they learn to expect to listen to the data. Throughout the course, we insist that no analysis of data is complete without telling what it means. This is where that understanding starts.

Rather than head directly for the "real purpose" of the course in the inference chapters, we prefer to emphasize the connection between our work with data and what they tell us about the world. No analysis (and no AP* Part II answer) is complete without a connection back to the real-world circumstances. Setting that stage is the underlying motivation for this chapter. We'll spend the next 10 chapters or so looking at and exploring data without making formal inferences.

Looking Ahead

Most of our own students actually do the reading. (Yes, we were surprised at first.) If you can hook students on reading the text, there are big benefits later on. As they read they learn that the book is a valuable resource to help them understand sticky concepts, to help lead them through writing complete and clear solutions, and to help them avoid common mistakes. Many find that they can easily catch up after absences by reading the relevant chapters on their own.

Technology plays an important role in this book. We expect students to use a calculator or computer statistics package for finding the numerical "answers." So we won't spend much time worrying about the calculation details, although we do expect them to understand what's happening. Instead, we focus on understanding and meaning. But the book is "technology neutral." The "computer output" in the book is designed to look like the results of many statistical packages, but exactly match none of them. Students should feel comfortable using output from almost any statistics program or graphing calculator.

You might have the students thumb through the book and read the opening of some chapters. Each one starts with a story and data, and most have additional stories and more data inside. Statistics is about the real world. Among other topics, we'll be discussing how well we can predict hurricane tracks, whether generic batteries perform as well as their brand name competitors, the debate about the safety of a widely used drug, and whether or not people over 30 snore more. We need to get students thinking about the context of data and able to make the distinction between categorical and quantitative data. These are fundamental skills for everything that follows, and they'll be used throughout the course.

We also want students to discover how helpful the TI Tips are. If you can get them to follow these instructions in each chapter you won't need to spend much class time helping with button-pushing, and students will be up to speed for the tests. If you use computer software in addition to calculators, you may want to give students a customized handout to get them started in your school's particular computing environment.

Class Do's

One of our favorite definitions says "Statistics is the art of distilling meaning from data." Data have a story to tell. Our objective is to uncover that story. Collect some data in class, and ask students to look for interesting facts hiding there.

Encourage students to think about the concepts and definitions in this chapter. For example, why do we talk about "a statistic" when we don't discuss "a mathematic" or "a physic"? Statistics is a whole that is made up of many parts, and each of those parts has its own meaning and its own story to tell.

Get the class thinking about what the term "data" means. Students need to understand that data are not just numbers and that they must have a context (the W's). When data are quantitative, they should also have units. There are two ways we treat data: *categorical* and *quantitative*. Don't get distracted by worrying about ratio, interval, and other distinctions. These are problematic and don't matter for the concepts and methods discussed in this book. Emphasize that the distinction between treating data as categorical or quantitative may be more about how *we* display and analyze data than it is about the variable itself. The variable "sex" is data, but just because we might label the males as 1 and the females as 0 doesn't mean that it's quantitative. On the other hand, taking the average of those 0's and 1's does give us the percentage of males. How about *age*? It is often quantitative, but could be categorical if broken down only into *child*, *adult*, and *senior*. Zip code is usually categorical, but if one business had an "average" zip code for their customers of 10000 while another had 90000, we'd know the latter had more customers in the western United States. Emphasize the importance of the context and the W's in summarizing these data.

Students should recognize that every discipline has its own vocabulary, and Statistics is no exception. They'll need to understand and use that vocabulary properly. Unfortunately, many Statistics words have a common everyday usage that's not quite the same. We'll be pointing those out as we go along.

Write vocabulary words on the board as they come up. One of the first should be *variable*. Make the point that it does not mean exactly the same thing as it did in Algebra. There, we call "*x*" a variable, but often that means that we just don't currently know its value. In Statistics a variable is an attribute or characteristic of an individual or object whose value varies from case to case.

A *statistic* is a numerical summary of data; one of the first you'll likely hear is that the class is x% male. Point out the difference between statistics and data. The book wisecracks that, contrary to an advertised saying, you can't be a statistic, only a datum.

Some students will suggest pie charts or histograms. It's sufficient for now to point out that graphical displays are useful visual summaries of data.

Point out that summaries of data can be verbal, visual, and numerical. All are important. In fact, any complete analysis of data almost always includes all three of these.

After looking at the data from your class survey, some students may say things like, "The males are more conservative." Point out the difference between *univariate* and *bivariate* analysis. Note that bivariate is a lot more interesting.

Hope that someone objects to finding an overall average shoe size or to comparing men's and women's sizes—shoe sizes are inconsistent in terms of units. This adds emphasis to the importance of units and the W's.

The Importance of What You Don't Say

One of the reasons Statistics can be difficult to teach is that we often deal with vague concepts. Students and teachers both like clear definitions; they're easier to teach, learn, and grade. But reasonable people can disagree about whether a histogram is symmetric or skewed, whether a straggling point is really an outlier or just the largest value, or whether the spread really is the same in two groups we want to compare. It is important to allow students their own opinions and insights into data.

This raises the issue of ethical practice in Statistics. We are engaged in an honest search for truth and understanding, and that's what should guide our (and our students') judgments. Emphasize this point now to alert students that this isn't a course about calculating the right answer, but about understanding the world.

Avoid the temptation to lead students in any particular direction or give them hints about how to examine the data you collected. They know some things to do—they'll find percents and averages, maybe draw some simple graphs, and write a few sentences about what they see. Just let it happen. You'll have plenty of chances to suggest, lead, and modify later on.

The motivation of hoping to understand the world is quite sufficient to justify thinking beyond the numbers at hand. This is not the time to introduce technical concepts such as *population* and *sample*. We'd rather not even see these terms yet (and you won't find them this early in the text). We all know that we'll be heading that way eventually, but if we make reasoning about patterns in the data too formal too soon, we'll stifle students' interest and enthusiasm and burden them with a lot of terms they don't have a context for just yet.

There was a time, not long ago, when our students' first question, "Why am I taking this course?" was typically answered by "It's required. Sit down and be quiet." We propose that a better answer is "So you can learn how data can tell us about the world. Stand up and tell us what you see."

We are laying a foundation here. Stretching up to the attic at this point just makes everyone feel unsafe. Many fundamental Statistics terms are left unmentioned in this chapter. You'll find it best to leave it that way. We'll get to them when the students have a safe place to file them along with their other knowledge. So we have an unusually long list of terms we recommend leaving for later in the course. In particular, avoid saying the following:

Hypothesis, Inference. These are certainly important in this course, but we have no background for discussing them honestly now, so they would just be confusing and intimidating.

Nominal, Ordinal, Interval, Ratio. "Nominal" is used by some software packages as a synonym for "categorical" as "continuous" is used for "quantitative." These distinctions arise from studies of measurement scales. But it isn't correct to claim that each variable falls into one of these categories. It is the use to which the data are put that determines what properties the variable must have. Ordinal categorical data may come up, but there are no special techniques for dealing with ordered categories in this course. And any differences between interval- and ratio-scaled data are commonly ignored in statistical analyses. If any of these terms were mentioned now, they'd never come up again anyway.

Random, Probability, Correlation. Everyone has some intuitive sense of these terms, and we'll deal with them formally—but not for a while. Students may want to use these terms, but at this early stage in the course, we don't need them. Without background and careful definition, they are likely to be misused and can simply be frightening.

Class Examples

Take a quick class survey. We suggest asking for things like gender, political leaning (Liberal, Moderate, Conservative), number of siblings, number of states visited, number of countries visited, whether they play varsity sports, GPA, height, handedness (left or right), and shoe size. Be sure to include both categorical and quantitative values. Recall what you were interested in at that age and try some carefully worded questions on those subjects. Try this question after getting everyone's attention: Ask your students to pick a number between 1 and 10 and write it down quickly. (Later you can look to see how "random" these numbers really are.) Start passing the survey sheet (a sample is provided in the resources section of Chapter 1) around at the very beginning of class; it can circulate quickly while you are introducing yourself and talking about the course, expectations, and so on. If you choose to use the number of states visited, passing out slips of paper with a list of states for each student to check off will speed things up, so we provide that, too. You should be able to have the data collected and duplicated by the end of the class, or put up on a course Web site soon. For their first assignment, ask students what story the data tell about their class. You'll find that a lot of important issues will surface during discussions over the next few days.

Hints: Data are rarely as simple as they seem. Suggest the variable above, then pause for some discussion. Does touching down at an airport qualify as "visiting" a state? Does an only child count herself when counting siblings? Should shoe sizes be adjusted because men's and women's size 7 are different sizes? If you write with your left hand , but throw with your right, are you left handed? Give them a chance and they'll find other issues—and they'll be developing a healthy skepticism for data. That's just what we want, so they'll value the tools that help them look at data more carefully.

If you don't specify units for height, you may get some values in centimeters. Alternatively, if you specify inches, you may get a "55" from someone who meant 5'5". Those outliers make for good class discussion.

If you teach several sections, consider collecting data online. There are a number of services that will let you design an online survey and will host it for a modest price, letting students respond online at their convenience and providing you with anonymous and machine-readable responses. One we have used successfully is at www.surveymonkey.com. If you use Fathom software, Fathom Surveys is another nice option.

Some follow-up ideas:

1. Ask students to tell some things they learned about the class from inspecting the data collected in the survey. You can use that discussion to develop several of the important points later in the chapter.
2. Consider 17, 21, 44, and 76. Are those data? Context is critical—they could be test scores, ages in a golf foursome, or uniform numbers of the starting backfield on the football team. In each case, our reaction changes.
3. Run through some other examples of data, asking about the W's, the variables (what are they, what type is each used as, and what are the units), and so on.

   •A Consumer Reports article on energy bars gave the brand name, flavor, price, number of calories, and grams of protein and fat.

   •A report on the Boston Marathon listed each runner's gender, country, age, and time.

Solution:

Consumer Reports
Who: energy bars
What: brand name, flavor, price, calories, protein, fat
When: not specified
Where: not specified
How: not specified. Are data collected from the label? Are independent tests performed?
Why: information for potential consumers
Categorical variables: brand name, flavor
Quantitative variables: price (US$), number of calories (calories), protein (grams), fat(grams)

Boston Marathon
Who: Boston Marathon runners
What: gender, country, age, time
When: not specified
Where: Boston
How: not specified. Presumably, the data were collected from registration information.
Why: race result reporting
Categorical variables: gender, country
Quantitative variables: age (years), time (hours, minutes, seconds)

Resources

Decisions Through Data

- Video Unit 1: What Is Statistics?
- *Why You Need to Study Statistics* – available from the 'This is Statistics' page on the American Statistical Association website or https://www.youtube.com/watch?time_continue=106&v=wV0Ks7aS7YI
- *Teach Statistics Before Calculus* – TED Talk with Arthur Benjamin. https://www.youtube.com/watch?v=BhMKmovNjvc

Web Links

- The Data and Story Library (DASL) is a source of data for student projects and classroom examples. dasl.datadesk.com
- The U.S. Census Bureau, www.census.gov
- See many sources of data in the "About this Guide" section (pages 0-10 and 0-11)

Other

- Read polls, studies, or other reports in newspaper and magazine articles. It's always interesting to see how well (or poorly) they provide information about the W's.

  If you have a computer and projection capabilities in class, you can find daily surveys at Gallup and other polling organizations. Current data are often particularly interesting to students. But don't use results of voluntary-response online surveys. We'll be making the point that these are fatally flawed—but we can't say that clearly without concepts and terms that we haven't developed yet.

Assignment:

Have students read Chapter 1. Explore the data collected, and make some observations about the class. What could you report to someone who asked you to describe your class? What did you find interesting (or perhaps surprising) about the class?

Be sure that they work through the TI Tips. We want them to learn that the book will show them how to use their calculators. Students should become able to enter and edit data in the lists without help from you.

Assign 5 or 6 of the exercises. After discussion tomorrow, assign another one or two of these exercises each day for a few days (as you are working along into Chapter 2) until everyone seems to have all the issues figured out.

Four chapter quizzes are provided.

States – Count the number you have visited

Alabama
Alaska
Arizona
Arkansas
California
Colorado
Connecticut
Delaware
Florida
Georgia
Hawaii
Idaho
Illinois
Indiana
Iowa
Kansas
Kentucky
Louisiana
Maine
Maryland
Massachusetts
Michigan
Minnesota
Mississippi
Missouri
Montana
Nebraska
Nevada
New Hampshire
New Jersey
New Mexico
New York
North Carolina
North Dakota
Ohio
Oklahoma
Oregon
Pennsylvania
Rhode Island
South Carolina
South Dakota
Tennessee
Texas
Utah
Vermont
Virginia
Washington
West Virginia
Wisconsin
Wyoming

States – Count the number you have visited

Alabama
Alaska
Arizona
Arkansas
California
Colorado
Connecticut
Delaware
Florida
Georgia
Hawaii
Idaho
Illinois
Indiana
Iowa
Kansas
Kentucky
Louisiana
Maine
Maryland
Massachusetts
Michigan
Minnesota
Mississippi
Missouri
Montana
Nebraska
Nevada
New Hampshire
New Jersey
New Mexico
New York
North Carolina
North Dakota
Ohio
Oklahoma
Oregon
Pennsylvania
Rhode Island
South Carolina
South Dakota
Tennessee
Texas
Utah
Vermont
Virginia
Washington
West Virginia
Wisconsin
Wyoming

States – Count the number you have visited

Alabama
Alaska
Arizona
Arkansas
California
Colorado
Connecticut
Delaware
Florida
Georgia
Hawaii
Idaho
Illinois
Indiana
Iowa
Kansas
Kentucky
Louisiana
Maine
Maryland
Massachusetts
Michigan
Minnesota
Mississippi
Missouri
Montana
Nebraska
Nevada
New Hampshire
New Jersey
New Mexico
New York
North Carolina
North Dakota
Ohio
Oklahoma
Oregon
Pennsylvania
Rhode Island
South Carolina
South Dakota
Tennessee
Texas
Utah
Vermont
Virginia
Washington
West Virginia
Wisconsin
Wyoming

AP Stats – Class Survey

Gender (M/F)	Politics (L, M, C)	Number of Siblings	States Visited	Shoe Size

AP* Statistics Quiz A – Chapter 1 ***Name*** ________________________

1. One of the reasons that the Monitoring the Future (MTF) project was started was "to study changes in the beliefs, attitudes, and behavior of young people in the United States." Data are collected from 8th, 10th, and 12th graders each year. To get a representative nationwide sample, surveys are given to a randomly selected group of students. In Spring 2004, students were asked about alcohol, illegal drug, and cigarette use. Describe the W's, if the information is given. If the information is not given, state that it is not specified.

   - Who:
   - What:
   - When:
   - Where:
   - How:
   - Why:

2. Consider the following part of a data set:

Age (years)	Sex	Only child?	Height (inches)	Weight (pounds)	Credit Hours	GPA	Major
21	Female	Yes	67.00	140.0	16	3.60	animal science
20	Female	No	62.00	130.0	18	3.86	biology
28	Female	No	64.00	188.0	21	3.25	psychology
21	Male	No	65.00	140.0	15	2.95	psychology
24	Female	No	67.00	130.0	20	3.00	anthropology
22	Male	Yes	68.00	135.0	15	2.94	journalism

   List the variables in the data set. Indicate whether each variable is treated as categorical or quantitative in this data set. If the variable is quantitative, state the units.

AP Statistics Quiz A – Chapter 1 – Key*

1. One of the reasons that the Monitoring the Future (MTF) project was started was "to study changes in the beliefs, attitudes, and behavior of young people in the United States." Data are collected from 8th, 10th, and 12th graders each year. To get a representative nationwide sample, surveys are given to a randomly selected group of students. In Spring 2004, students were asked about alcohol, illegal drug, and cigarette use. Describe the W's, if the information is given. If the information is not given, state that it is not specified.
   - Who: 8th, 10th, and 12th graders
   - What: alcohol, illegal drug, and cigarette use
   - When: Spring 2004
   - Where: United States
   - How: survey
   - Why: "to study changes in the beliefs, attitudes, and behavior of young people in the United States"

2. Consider the following part of a data set:

Age (years)	Sex	Only child?	Height (inches)	Weight (pounds)	Credit Hours	GPA	Major
21	Female	Yes	67.00	140.0	16	3.60	animal science
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24	Female	No	67.00	130.0	20	3.00	anthropology
22	Male	Yes	68.00	135.0	15	2.94	journalism

List the variables in the data set. Indicate whether each variable is treated as categorical or quantitative in this data set. If the variable is quantitative, state the units.

Categorical: sex, only child?, major
Quantitative: age (years), height (inches), weight (pounds), credit hours, GPA

AP* Statistics Quiz B – Chapter 1 ***Name*** ________________________

In November 2003 *Discover* published an article on the colonies of ants. They reported some basic information about many species of ants and the results of some discoveries found by myrmecologist Walter Tschinkel of the University of Florida. Information included the scientific name of the ant species, the geographic location, the depth of the nest (in feet), the number of chambers in the nest, and the number of ants in the colony. The article documented how new ant colonies begin, the ant-nest design, and how nests differ in shape, number, size of chambers, and how they are connected, depending on the species. It reported that nest designs include vertical, horizontal, or inclined tunnels for movement and transport of food and ants.

1. Describe the W's, if the information is given:
   - Who:
   - What:
   - When:
   - Where:
   - How:
   - Why:

2. List the variables. Indicate whether each variable is categorical or quantitative. If the variable is quantitative, tell the units.

AP Statistics Quiz B – Chapter 1 – Key*

In November 2003 *Discover* published an article on the colonies of ants. They reported some basic information about many species of ants and the results of some discoveries found by myrmecologist Walter Tschinkel of the University of Florida. Information included the scientific name of the ant species, the geographic location, the depth of the nest (in feet), the number of chambers in the nest, and the number of ants in the colony. The article documented how new ant colonies begin, the ant-nest design, and how nests differ in shape, number, size of chambers, and how they are connected, depending on the species. It reported that nest designs include vertical, horizontal, or inclined tunnels for movement and transport of food and ants.

1. Describe the W's, if the information is given:
    - Who: Colonies of ants. "Many species of ants," but no indication of exactly how many.
    - What: scientific name, geographic location, average nest depth, average number of chambers, average colony size, how new ant colonies begin, the ant-nest design, and how nests differ in architecture.
    - When: November 2003
    - Where: not specified
    - How: The results of some discoveries found by myrmecologist Walter Tschinkel of the University of Florida
    - Why: Information of interest to readers of the magazine

2. List the variables. Indicate whether each variable is categorical or quantitative. If the variable is quantitative, tell the units.

    Categorical: species, geographic location, how new ant colonies begin, and nest design.
    Quantitative: nest depth (feet), number of chambers (units), and colony size (units).

AP* Statistics Quiz C – Chapter 1 **Name** ______________________

In June 2003 *Consumer Reports* published an article on some sport-utility vehicles they had tested recently. They reported some basic information about each of the vehicles and the results of some tests conducted by their staff. Among other things, the article told the brand of each vehicle, its price, and whether it had a standard or automatic transmission. They reported the vehicle's fuel economy, its acceleration (number of seconds to go from zero to 60 mph), and its braking distance to stop from 60 mph. The article also rated each vehicle's reliability as much better than average, better than average, average, worse, or much worse than average.

1. Describe the W's, if the information is given:
- Who:
- What:
- When:
- Where:
- How:
- Why:

2. List the variables. Indicate whether each variable is categorical or quantitative. If the variable is quantitative, tell the units.

AP Statistics Quiz C – Chapter 1 – Key*

In June 2003 *Consumer Reports* published an article on some sport-utility vehicles they had tested recently. They reported some basic information about each of the vehicles and the results of some tests conducted by their staff. Among other things, the article told the brand of each vehicle, its price, and whether it had a standard or automatic transmission. They reported the vehicle's fuel economy, its acceleration (number of seconds to go from zero to 60 mph), and its braking distance to stop from 60 mph. The article also rated each vehicle's reliability as much better than average, better than average, average, worse, or much worse than average.

1. Describe the W's, if the information is given:

- Who: SUV's currently on the market. We don't know how many models.
- What: brand of vehicle, price, type of transmission, fuel economy, acceleration, braking distance, and reliability.
- When: prior to June 2003
- Where: not specified, probably the United States
- How: testing the vehicles by driving each
- Why: information for potential consumers

2. List the variables. Indicate whether each variable is categorical or quantitative. If the variable is quantitative, tell the units.

   Categorical: brand, transmission type, reliability
   Quantitative: price (US$), fuel economy (mpg), acceleration (seconds), braking distance (probably feet?)

AP Statistics Quiz D – Chapter 1* **Name ________________________**

In July 2013, the Federal Drug Administration approved a new version of a drug used to treat opium dependence. The old version of the drug had received complaints from patients about a bitter taste, an aftertaste, and that it took a long time to dissolve. Clinical studies were conducted in the United States to see if the new drug met the goal of getting more patients to take the drug as prescribed by addressing these issues. In addition to these improvements, experimenters monitored the existence and types of side effects of the drug.
.

1. Describe the W's, if the information is given:

- Who:
- What:
- When:
- Where:
- How:
- Why:

2. List the variables. Indicate whether each variable is categorical or quantitative. If the variable is quantitative, tell the units.

AP Statistics Quiz D – Chapter 1* **Name** ________________________

In July 2013, the Federal Drug Administration approved a new version of a drug used to treat opium dependence. The old version of the drug had received complaints from patients about a bitter taste, an aftertaste, and that it took a long time to dissolve. Clinical studies were conducted in the United States to see if the new drug met the goal of getting more patients to take the drug as prescribed by addressing these issues. In addition to these improvements, experimenters monitored the existence and types of side effects of the drug.

1. Describe the W's, if the information is given:

- Who: Patients being treated for opium addiction
- What: Taste, aftertaste, time to dissolve, patient compliance, types of side effects
- When: Prior to July 2013
- Where: United States
- How: Clinical trials
- Why: To determine whether patients would comply with treatment better with the new version of the drug.

2. List the variables. Indicate whether each variable is categorical or quantitative. If the variable is quantitative, tell the units.

   Categorical: taste, aftertaste, compliance, side effects
   Quantitative: time to dissolve (minutes?)

Chapter 2
Displaying and Describing Categorical Data

What's It About?

We introduce students to distributions of categorical variables. The mathematics is easy (summaries are just percentages) and the graphs are straightforward (pie charts and bar graphs). We challenge students to uncover the story the data tell, and to write about it in complete sentences in context.

Then we up the ante, asking them to compare distributions in two-way tables. Constructing comparative graphs, discussing conditional distributions, and considering (informally) the idea of independence give students a look at issues that require deeper thought, careful analysis, and clear writing.

Comments

By Chapter 2, it will begin to dawn on your students that this isn't a math class. At the very least, they are going to be expected to write often and clearly. For those who have not yet developed the skill of writing clearly, this may be one of the most valuable things learned in this course. This chapter provides an early challenge to students to write conclusions that are clear, concise, complete, and in context – The Four C's.

1. Continue to emphasize precision of vocabulary (and notation). These are an important part of clear communication, critical to success.

2. Emphasize *Think-Show-Tell* right from the start. The key to doing well in Statistics (and on the AP* Exam) is to think carefully about what each question is asking and what statistical techniques can address those issues before starting to write an answer. And then, after showing some calculations or other work, to write clear and concise explanations of what it all means. Your students may rebel at first at having to write sentences, much less paragraphs, in a course they may have thought was a math class. They are used to just doing the *Show*. *Tell* is at least 50% of each solution. If you make that point consistently right from the start of the course it becomes second nature soon, and puts each student in the right mindset for writing solid AP* answers. Continually remind them: ***Answers are sentences, not numbers.*** (Indeed, on the AP* Exam, clear communication usually accounts for at least 50% of the credit for a problem.)

3. This is the first substantial chapter, and it gets pretty involved right away. We deal with conditional distributions, independence, and confounding (Simpson's paradox). It may seems early to bring up such sophisticated concepts, but our experience is that students can get lulled into a false sense of security in the early part of this course, if all they see is things like means and histograms that they have dealt with since middle school. They think the course is going to be pretty easy, and they may not recognize the level of sophistication that is required until it's too late. The ideas in Chapter 2 are not hard and are introduced only informally, but they do require some thought. Students will find it difficult to make clear explanations. We want these ideas to be interesting, to engage imaginations, and to challenge students. We hope the level of thought required will get their attention and arouse their interest.

Looking Ahead

There are many important skills and ideas here that prepare students for later topics. They need to think about the type of data, checking a condition before plunging ahead. They need to think about what comparisons will answer the questions posed, and write clear explanations in context. They begin to think about independence, one of the most important issues in Statistics. And, in Simpson's paradox, they see the need to think more deeply to avoid being misled by lurking or confounding variables.

Class Do's

Weave the key step of checking the assumptions and conditions into the fabric of doing Statistics. It's easy: have students check that the data are being treated as categorical before they proceed with pie charts, conditional distributions, and the like. As the course goes on, *Thinking* about assumptions and conditions will help students select appropriate statistical procedures – and it's a requirement for a complete solution on the AP* Exam. Start now.

Discuss categorical data and appropriate summaries: numerical (counts/percentages), graphical (pie charts, bar graphs). Discuss *distribution, frequency, relative frequency*.

It gets more interesting when we make comparisons (using bivariate data): e.g., political leanings by gender? Discuss two-way tables, *marginal* and *conditional* distributions. Political views may be interesting, but looking at the differences in political view by gender adds much more to the discussion. You can emphasize the vocabulary by asking things like "What is the marginal frequency distribution of gender?" vs. "What is the conditional relative frequency distribution of gender among Conservatives?"

Make sure students can correctly sort out (*Think-Show-Tell*) answers to similar sounding questions:

1. What percent of the class are women with liberal political views?
2. What percent of the liberals are women?
3. What percent of the women are liberals?

Raise the issue of independence. It's not formal independence yet, just the general idea that if gender and political view were independent, the percentages for either gender would mirror the class as a whole, or the percentages of Liberal, Moderate, and Conservative would be the same for both genders. If they are not, we encounter what the politicians refer to as the "gender gap". Statisticians would say this indicates that voting preference is not independent of gender.

Pay attention in each chapter to the What Can Go Wrong? (WCGW) sections. Helping students avoid common pitfalls is one of the keys to success in this course.

Simpson's Paradox is fun, but don't overemphasize it. It's not a critical issue, but it's a good discussion point about making valid comparisons, and not overlooking lurking or confounding variables.

The Importance of What You Don't Say

Probability. You can see that we are patrolling the perimeter of probability. Concepts like relative frequency, conditional relative frequency, and independence cry out for a formal discussion in probabilistic terms. Don't heed the cry. You and we know that we are setting up the habits of thought that students will need for learning about probability. But this isn't the time to discuss the formalities. Or even to say the word "probability" out loud. (Notice that the book doesn't use the term in this chapter at all – it'll still be a while before we get to it.) Talk about "relative frequency" instead. In this class probability is a relative frequency, so we are encouraging students to think about the concepts correctly. By the time we introduce formal probability, they will have a sound intuitive foundation.

Class Examples

1. Use the class data about gender and political view – liberal, moderate, conservative. Help students develop their *Think-Show-Tell* skills with questions like:
   - What percent of the class are girls with liberal political views?
   - What percent of the liberals are girls?
   - What percent of the girls are liberals?
   - What is the marginal frequency distribution of political views?
   - What is the conditional relative frequency distribution of gender among conservatives?
   - Are gender and political view independent?
2. Use the worksheet about smoking and education level. Students should conclude that smokers tend to have higher education levels than non-smokers. 64% of the smokers had only a high school education compared to 47% overall. And non-smokers were almost twice as likely as smokers (48% to 26%) to have completed at least 4 years of college.

   As a "What Can Go Wrong" exercise, it's fun to see how many incorrect "conclusions" students can think up beyond the one suggested on the sheet. Of course, this does not indicate that kids who smoke in high school will quit when they go off to college. Perhaps smokers can't afford college because of all the money they waste on cigarettes. We simply don't have data to support any such conclusions, and, being a cautious bunch, statisticians should avoid such speculation.
3. Is the color distribution of M&Ms independent of the type of candy? Break open bags of plain and peanut M&Ms and count the colors. Is the color distribution of M&Ms independent of type of candy? (Then eat the data…)
4. Simpson's paradox example:
   It's the last inning of important game. Your team is a run down with the bases loaded and two outs. The pitcher is due up, so you'll be sending in a pinch-hitter. There are 2 batters available on the bench. Whom should you send in to bat? First show the students the overall success history of the two players.

Player	Overall	vs LHP	vs RHP
A	33 for 103	28 for 81	5 for 22
B	45 for 151	12 for 32	33 for 119

A's batting average is higher than B's (.320 vs. .298), so he looks like the better choice. Someone, though, will raise the issue that it matters whether the pitcher throws right- or left-handed. Now add the rest of the table. It turns out that B has a higher batting average against both right- and left-handed pitching, even though his overall average is lower. Students are stunned.

Here's an explanation. B hits better against both right- and left-handed pitchers. So no matter the pitcher, B is a better choice. So why is his batting "average" lower? Because B sees a lot more right-handed pitchers than A, and (at least for these guys) right-handed pitchers are harder to hit. For some reason, A is used mostly against left-handed pitchers, so A has a higher average.
Suppose all you know is that A bats .227 against righties and .346 against lefties. Ask the students to guess his overall batting average. It could be anywhere between .227 and .346, depending on how many righties and lefties he sees. And B's batting average may slide between .277 and .375. These intervals overlap, so it's quite possible that A's batting average is either higher or lower than B's, depending on the mix of pitchers they see.

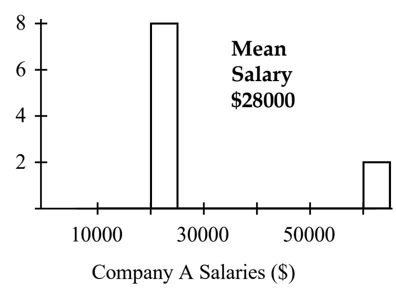
Pooling (nice word to use . . .) the data together loses important information and leads to the wrong conclusion. We always should take into account any factor that might matter.

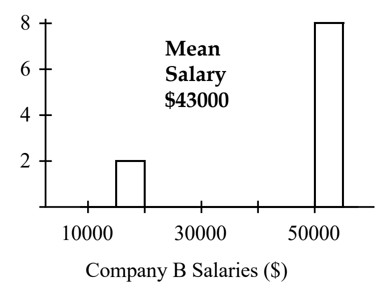
5. Refer to Simpson, again. Here's a nice thought problem to pose to the class; give them a few minutes to work it out. Two companies have labor and management classifications of employees. Company A's laborers have a higher average salary than company B's, as do Company A's managers. But overall company B pays a higher average salary. How can that be? And which is the better way to compare earning potential at the two companies?

Solution:

First of all, make sure you point out that this example deals with quantitative variables, not categorical. The paradox can be explained when you realize that Company A must employ a greater percentage of laborers than Company B. Also, Company A must employ a smaller percentage of managers than Company B. If laborers earn salaries that are considerably lower than managers, the salaries of Company A's laborers will pull the company average down, and the salaries of Company B's managers will pull the company average up.

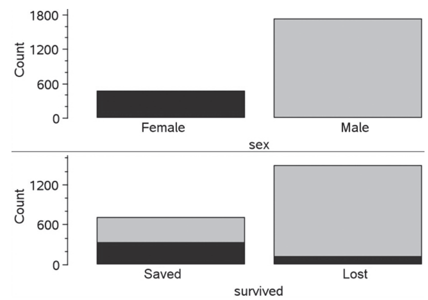
The proper way to compare the companies is to use the salaries that are broken down by job type. Using the overall average salary leads to a misleading conclusion.





Resources

TI-Nspire™ Demonstration



- Conditional distributions and association
  - Explore the Titanic data to see which passengers were most likely to survive.
  - Demonstrate conditional distributions through interactive graphical displays.
  - Look for associations between variables.
  - Introduce the concept of independence.

  For more information, see page 0-6 of this guide or the online instructor resources.

Workshop Statistics

- Topic 7 – "Comparing Distributions: Categorical Variables" has many investigations for class room use.

Web Links

- The AP* Statistics Teacher Community archives contain a discussion about a private school where the administration announced that their school had the highest average faculty salary among eight similar schools surveyed. Further analysis revealed that the school ranked no higher than third for the average salary in every age group (20 – 29, 30 – 39, and so on).

- Gallery of Data Visualization: The Best and Worst of Statistical Graphs. Available at AP* Statistics Teacher Community Resources Library. An homage to John Tukey, this site collects current and historical graphical displays of data. It claims to have the "best statistical graphic ever drawn" as well as the "current record holder for the worst." The site is a large collection of such images, and encourages contribution. A really fun yet sophisticated collection to share with students when discussing graphical displays of data. Attribution: Michael Friendly, York University. http://www.datavis.ca/gallery/index.php

Assignments

This chapter is about 3 days work. Have students read the whole chapter in three segments. About 5 or 6 exercises a night seems to be plenty. (Remember to add one or two from Chapter 1 until those issues seem clear.)

Emphasize that the "*Tell*" is important. Be sure to ask students to read sentences they have written about the data. Take time to revise those sentences until they can be described by the 4 C's: clear, complete, concise, and in context.

Four chapter quizzes are provided.

Investigative Task

We recommend assigning written analyses from time to time, called Investigative Tasks. The first one appears here. The intent is to get students to examine data, to reach some conclusions, to then create graphical and numerical analyses, and interpret in writing what it all means (*Think, Show, Tell*). This would be the only assignment one night, due the next day. In the best of all possible worlds it works like this:

- Wednesday: hand out the task. Along with the usual reading and handful of problems for homework, ask students to read the Task to see if they understand it.
- Thursday in class: answer their questions about the Task (without giving it all away, of course).
- Thursday night: they do the Task (no other homework).
- Friday: collect the written work.
- Weekend: read and grade the Tasks. Use the rubric leading to a score on a 4-point scale, just like the AP* exam. Set the bar high - the AP* exam certainly does! Scores on this first Task will probably be pretty low.
- Monday: return the Tasks, with rubrics attached.
- Tuesday (or later): discuss the Task and the rubric.

The grading rubric we give for the Task is modeled on the AP* grading rubrics. This is typical of the way students will be graded on the exam. By showing them the scoring guidelines we help them understand the expectations and provide them with valuable feedback about how to improve their performance.

We also provide a model solution. Please keep in mind that these solutions are <u>*not*</u> scoring keys. In other words, students should not be graded on how closely their solutions parallel ours, rather on how well their solutions reflect the requirements set forth in the rubric. Our solution is only one example of what a student might write.

Smoking and Education

200 adults shopping at a supermarket were asked about the highest level of education they had completed and whether or not they smoke cigarettes. Results are summarized in the table.

	Smoker	**Non-smoker**	**Total**
High school	32	61	93
2 yr college	5	17	22
4+ yr college	13	72	85
Total	50	150	200

1. Discuss the W's.

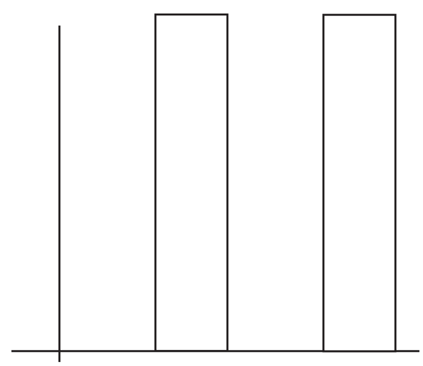
2. Identify the variables.

3. a) What percent of the shoppers were smokers with only high school educations? _______

   b) What percent of the shoppers with only high school educations were smokers? _______

   c) What percent of the smokers had only high school educations? _______

4. Create a segmented bar graph comparing education level among smokers and non-smokers. Label your graph clearly

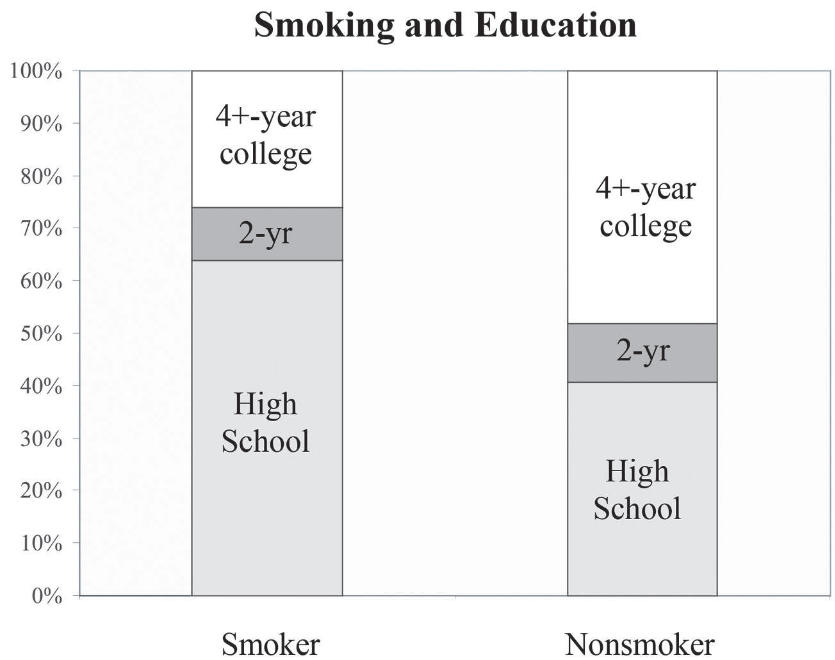


5. Do these data suggest there is an association between smoking and education level? Give statistical evidence to support your conclusion.

6. Follow-up question: Does this indicate that students who start smoking while in high school tend to give up the habit if they complete college? Explain.

AP Statistics Classwork – Smoking and Education Key*

1. Who: 200 adults
   What: education level and smoking habits
   When: not specified
   Where: shopping mall
   How: not specified. Was this a random sample, or were some people simply asked?
   Why: to examine possible links between smoking and education level
2. Categorical variables: Education level, and whether or not the person was a smoker.
3. a) $\frac{32}{200} = 16\%$
   b) $\frac{32}{93} \approx 34.4\%$
   c) $\frac{32}{50} = 64\%$
4. The segmented bar graph comparing education level among smokers and non-smokers is at the right.



5. These data provide evidence of an association between smoking and education level. 64% of smokers had only a high school diploma, while only 40.7% of non-smoker had only high school diplomas. Only 26% of smokers had four or more years of college, compared to 48% of smokers.
6. These data do not indicate that students who start smoking in high school tend to give up the habit if they complete college. These data were gathered at one time, about two different groups, smokers and non-smokers. We have no idea if smoking behavior changes over time.

AP* Statistics Quiz A – Chapter 2 ***Name*** ________________________

Has the percentage of young girls drinking milk changed over time? The following table is consistent with the results from "Beverage Choices of Young Females: Changes and Impact on Nutrient Intakes" (Shanthy A. Bowman, *Journal of the American Dietetic Association*, *102*(9), pp. 1234-1239):

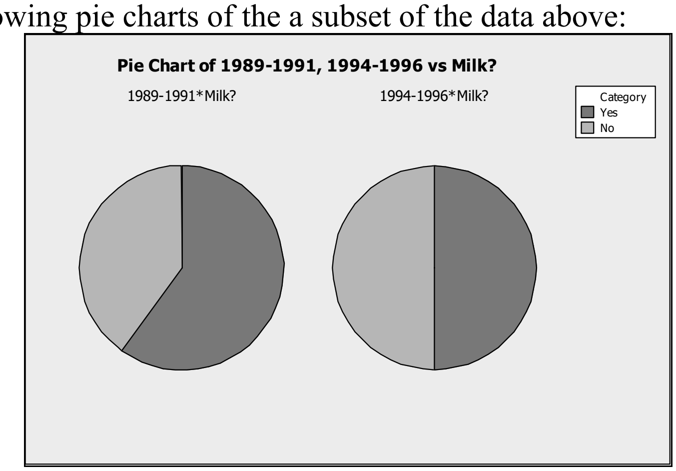
		Nationwide Food Survey Years			
		1987-1988	1989-1991	1994-1996	**Total**
Drinks Fluid Milk	Yes	354	502	366	**1222**
	No	226	335	366	**927**
	Total	**580**	**837**	**732**	**2149**

1. Find the following:
   a. What percent of the young girls reported that they drink milk? _______
   b. What percent of the young girls were in the 1989-1991 survey? _______
   c. What percent of the young girls who reported that they drink milk were in the 1989-1991 survey? _______
   d. What percent of the young girls in 1989-1991 reported that they drink milk? _______

2. What is the marginal distribution of milk consumption?

3. Do you think that milk consumption by young girls is independent of the nationwide survey year? Use statistics to justify your reasoning.

4. Consider the following pie charts of the a subset of the data above:



Do the pie charts above indicate that milk consumption by young girls is independent of the nationwide survey year? Explain.

AP* Statistics Quiz A – Chapter 2 - Key

Has the percentage of young girls drinking milk changed over time? The following table is consistent with the results from "Beverage Choices of Young Females: Changes and Impact on Nutrient Intakes" (Shanthy A. Bowman, *Journal of the American Dietetic Association*, *102*(9), pp. 1234-1239):

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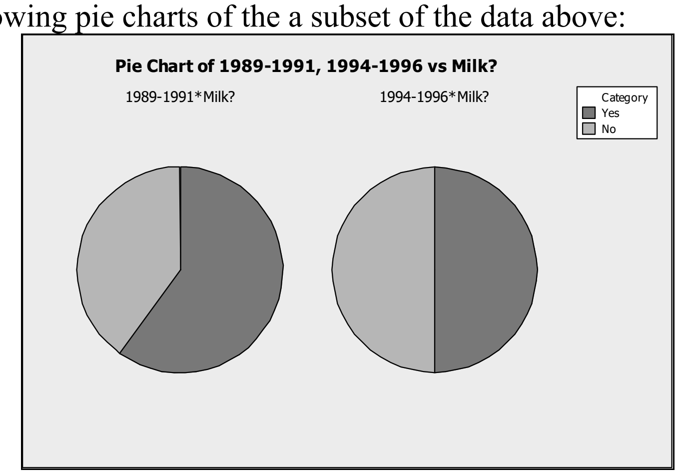
Drinks Fluid Milk	Yes	354	502	366	**1222**
	No	226	335	366	**927**
	Total	**580**	**837**	**732**	**2149**

1. Find the following:
   a. What percent of the young girls reported that they drink milk? <u>56.9%</u>
   b. What percent of the young girls were in the 1989-1991 survey? <u>38.9%</u>
   c. What percent of the young girls who reported that they drink milk were in the 1989-1991 survey? <u>41.1%</u>
   d. What percent of the young girls in 1989-1991 reported that they drink milk? <u>60.0%</u>

2. What is the marginal distribution of milk consumption?
   Yes: 1,222; No: 927

3. Do you think that milk consumption by young girls is independent of the nationwide survey year? Use statistics to justify your reasoning.
   No. 56.9% of all young girls surveyed reported drinking milk, but 60% of the young girls reported drinking milk in the 1989-1991 survey. Since these percentages differ, milk consumption and year are not independent.

4. Consider the following pie charts of the a subset of the data above:



   Do the pie charts above indicate that milk consumption by young girls is independent of the nationwide survey year? Explain.
   No. It looks like there is some sort of relationship between milk consumption and nationwide survey year, since the percentage of young girls who reported drinking milk is a larger slice of the pie chart for the 1989-1991 survey than the same response for the 1994-1996 survey.

AP Statistics Quiz B – Chapter 2* *Name* ________________________

To determine if people's preference in dogs had changed in the recent years, organizers of a local dog show asked people who attended the show to indicate which breed was their favorite. This information was compiled by dog breed and gender of the people who responded. The table summarizes the responses.

	Female	**Male**	**Total**
Yorkshire Terrier	73	59	**132**
Dachshund	49	47	**96**
Golden Retriever	58	33	**91**
Labrador	37	41	**78**
Dalmatian	45	28	**73**
Other breeds	86	67	**153**
Total	**348**	**275**	**623**

1. Identify the variables and tell whether each is categorical or quantitative.

2. Which of the W's are unknown for these data?

3. Find each percent.
   a. What percent of the responses were from males who favor Labradors? ________
   b. What percent of the male responses favor Labradors? ________
   c. What percent of the people who choose Labradors were males? ________

4. What is the marginal distribution of breeds?

5. Write a sentence or two about the conditional relative frequency distribution of the breeds among female respondents.

6. Do you think the breed selection is independent of gender? Give statistical evidence to support your conclusion.

AP Statistics Quiz B – Chapter 2 – Key*

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	Female	**Male**	**Total**
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Labrador	37	41	**78**
Dalmatian	45	28	**73**
Other breeds	86	67	**153**
Total	**348**	**275**	**623**

1. Identify the variables and tell whether each is categorical or quantitative.

   Gender and Breed; both categorical.

2. Which of the W's are unknown for these data?

   We do not know how or when the people were surveyed, or where the local dog show was located.

3. Find each percent.
   a. What percent of the responses were from males who favor Labradors? 6.6%
   b. What percent of the male responses favor Labradors? 14.9%
   c. What percent of the people who choose Labradors were males? 52.6%

4. What is the marginal distribution of breeds?

   There were 132 Yorkshire terrier responses, 96 Dachshund responses, 91 Golden Retriever responses, 78 Labrador responses, 73 Dalmatian responses, and 153 Other responses.

5. Write a sentence or two about the conditional relative frequency distribution of the breeds among female respondents.

   Among females, 20.9% chose Yorkshire Terriers, 14.2% Dachshunds, 16.7% Golden Retrievers, 10.6% Labs, and 12.9% Dalmatians. The remaining 24.7% of females preferred other breeds.

6. Do you think the breed selection is independent of gender? Give statistical evidence to support your conclusion.

   The breed selection does not appear to be independent of gender. Overall, 56% of the respondents were females, but females were over-represented among those who favored Golden Retrievers (64%) and Dalmatians (62%), yet a much lower percentage (47%) among those who chose Labradors.

AP* Statistics Quiz C – Chapter 2 ***Name*** ______________________

In order to plan transportation and parking needs at a private high school, administrators asked students how they get to school. Some rode a school bus, some rode in with parents or friends, and others used "personal" transportation – bikes, skateboards, or just walked. The table summarizes the responses from boys and girls.

	Male	**Female**	**Total**
Bus	30	34	**64**
Ride	37	45	**82**
Personal	19	23	**42**
Total	**86**	**102**	**188**

1. Identify the variables and tell whether each is categorical or quantitative.

2. Which of the W's are unknown for these data?

3. Find each percent.
   a) What percent of the students are girls who ride the bus? __________
   b) What percent of the girls ride the bus? __________
   c) What percent of the bus riders are girls? __________

4. What is the marginal distribution of gender?

5. Write a sentence or two about the conditional relative frequency distribution of modes of transportation for the boys.

6. Do you think mode of transportation is independent of gender? Give statistical evidence to support your conclusion.

AP Statistics Quiz C – Chapter 2 – Key*

In order to plan transportation and parking needs at a private high school, administrators asked students how they get to school. Some rode a school bus, some rode in with parents or friends, and others used “personal” transportation – bikes, skateboards, or just walked. The table summarizes the responses from boys and girls.

	Male	**Female**	**Total**
Bus	30	34	**64**
Ride	37	45	**82**
Personal	19	23	**42**
Total	**86**	**102**	**188**

1. Identify the variables and tell whether each is categorical or quantitative.

   Gender and mode of transportation, both categorical.

2. Which of the W’s are unknown for these data?

   We don’t know how or when the students were surveyed, nor where the school is.

3. Find each percent.
   a) What percent of the students are girls who ride the bus? <u>18.1%</u>
   b) What percent of the girls ride the bus? <u>33.3%</u>
   c) What percent of the bus riders are girls? <u>53.1%</u>

4. What is the marginal distribution of gender?

   There are 86 males and 102 females.

5. Write a sentence or two about the conditional relative frequency distribution of modes of transportation for the boys.

   More boys (43%) caught rides to school than any other means of transportation. 35% rode the bus while only 22% used personal transportation like biking, skateboarding, or walking.

6. Do you think mode of transportation is independent of gender? Give statistical evidence to support your conclusion.

   The way students get to school does seem to be independent of gender. Overall, 34% of students ride the bus, compared to 35% of the boys and 33% of the girls. 44% of all students caught rides with someone and 22% used personal transportation, almost the same as the percentages for boys (43% and 22%) or girls (44% and 23%) separately. These data provide little indication of a difference in mode of transportation between boys and girls at this school.

AP* Statistics Quiz D – Chapter2 ***Name*** ________________________

A research company frequently monitors trends in the use of social media by American Adults. The results of one survey of 1846 randomly selected adults looked at social media use versus age group. The table summarizes the survey results.

		Age Group				
		18-29	30-49	50-64	65+	**Total**
Uses Social Media	Yes	328	417	288	114	**1147**
	No	67	125	265	242	**699**
	Total	**395**	**542**	**553**	**356**	**1846**

1. Identify the variables and tell whether each is categorical or quantitative.

2. Which of the W's are unknown for these data?

3. Find each percent.
   a) What percent of adults surveyed are social media users aged 30-49? __________
   b) What percent of the social media users are aged 30-49? __________
   c) What percent of adults aged 30-49 are social media users? __________

4. What is the marginal distribution of age groups?

5. Write a sentence or two about the conditional relative frequency distribution of ages of social media users.

6. Do you think social media use is independent of age? Give statistical evidence to support your conclusion.

AP* Statistics Quiz D – Chapter2 ***Name*** ______________________

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		Age Group				
		18-29	30-49	50-64	65+	**Total**
Uses Social Media	Yes	328	417	288	114	**1147**
	No	67	125	265	242	**699**
	Total	**395**	**542**	**553**	**356**	**1846**

1. Identify the variables and tell whether each is categorical or quantitative.

   Age is numerical, but the grouping treats it as categorical, and social media use is categorical.

2. Which of the W's are unknown for these data?

   We don't know when the adults were surveyed.

3. Find each percent.
   a. What percent of adults surveyed are social media users aged 30-49? 22.6%
   b. What percent of the social media users are aged 30-49? 36.4%
   c. What percent of adults aged 30-49 are social media users? 76.9%

4. What is the marginal distribution of age groups?

   There were 395 adults aged 18-29, 542 aged 30-49, 553 aged 50-64, and 356 that were 65 or older.

5. Write a sentence or two about the conditional relative frequency distribution of ages of social media users.

   More social media users in the survey (36.4%) were aged 30-49 than any other age group. Next was the 18-29 age group at 28.5%, then the 50-64 group at 25.1%, and the smallest group of social media users (9.6%) was the 65 and older group.

6. Do you think social media use is independent of age in the population of American adults? Give statistical evidence to support your conclusion.

Social media use does not appear to be independent of age. Overall, 21.4% of adults surveyed were 18-29 years old, but 28.6% of social media users are in that age group. And 19.2% of adults surveyed were 65 and older, but only 9.9% of social media users were. In general, older groups seem to be underrepresented among social media users.

Race and the Death Penalty

In 1976 the Supreme Court ruled that the death penalty does not violate the U.S. Constitution's ban on "cruel and unusual punishments". Since then many states have passed capital punishment statutes, and over 500 convicted murderers have been executed nationwide.

Capital punishment may be constitutional, but there continues to be a debate about whether or not it is fair. One of the major issues in this debate involves race – the race of both the defendant and the murder victim. The central question: is justice blind?

In 1998 the Death Penalty Information Center published *The Death Penalty in Black and White*, a study examining the sentences following 667 murder convictions in Philadelphia courts between 1983 and 1993. This 3-way table shows how many death sentences were given among all the murder convictions.

DEATH SENTENCES	**Black Victim**	**White Victim**	**Total**
Black Defendant	76 of 422	21 of 99	97 of 521
White Defendant	1 of 25	17 of 121	18 of 146
Total	77 of 447	38 of 220	115 of 667

Is our system of justice colorblind in the administration of the death penalty? Based upon the above information, write a newspaper article discussing the association between race and death sentences in the United States.

(Don't forget: the best analyses of data usually combine visual, numerical, and verbal descriptions.)

AP Statistics Task* *Chapter 3*

	Components	Comments
Think	Identifies useful marginal and conditional distributions (or %'s) to make effective comparisons	
Show	**Numerical** – no major errors **Visual** – includes comparative pie charts or (segmented?) bar graphs ○ well-labeled & fairly accurate	
Tell	**Verbal** - Written article … ○ is clear and concise ○ identifies the W's ○ is in the proper context ○ uses vocabulary correctly ○ avoids speculation States a conclusion about the association between race and the death penalty, explaining at least two examples of statistical support	

<u>Components are scored as **E**ssentially correct, **P**artially correct, or **I**ncorrect</u>

1: **Use of marginal and conditional distributions**

E - Demonstrates understanding of marginal vs conditional for comparison
P - Calculates useful %'s but may not understand why, or has major arithmetic errors
I – Calculations are not %'s, or not useful for comparisons

2: **Graphical display**

E – Uses comparative pie/bar graph, well-labeled and fairly accurate
P – Graph is comparative but poorly constructed or explained
I – Graph is not comparative or is missing

3: News article

E – Article has all 5 listed properties
P – Article has 3 or 4 of the listed properties
I – Article has fewer than 3 of the properties

4: Conclusion

E – Correct general conclusion is supported by 2 appropriate comparisons
P – Conclusion is not clearly stated, or only one supporting comparison is given
I – Conclusion is incorrect, unsupported by statistics, or missing.

Scoring

- E's count 1 point, P's are 1/2
- AP* score = sum of 4 components; rounding based on overall communication quality
- Grade: A = 4, B = 3, etc., with +/- based on rounding (ex: 3.5 rounded to 3 is a B+)

***Name* ___________________________________ *AP* Score* ____ *Grade* ____**

NOTE: We present a model solution with some trepidation. This is not a scoring key, just an example. Many other approaches could fully satisfy the requirements outlined in the scoring rubric. That (not this) is the standard by which student responses should be evaluated.

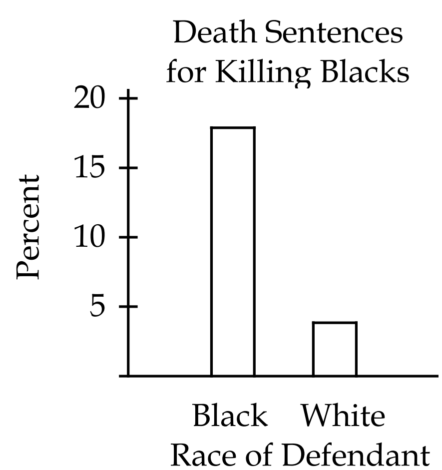
Model Solution – Investigative Task – Race and the Death Penalty

In the 10 years between 1983 and 1993, the city of Philadelphia saw 667 defendants convicted of murder. Of these, 17% were sentenced to death. Was the death penalty administered without regard to the race of the defendant or the victim? Is justice colorblind?

Overall, blacks convicted of murder were sentenced to the death penalty in 18.6% of cases. The death penalty was the sentence for only 12.3% of whites convicted of murder. There may be evidence that black defendants fared worse in regard to death penalty sentences.

At first, there doesn't appear to be an association between the race of the victim and the rate of death sentences. Defendants convicted of murdering whites were sentenced to death about 17% of the time, as were defendants convicted of murdering blacks. But these percentages are misleading. It's not until we look a bit deeper that we see the true picture.

When race of the defendant and race of the victim are both taken into account, blacks are convicted at higher rates across the board than whites. When the victim was black, 18% of black defendants were sentenced to death, compared to only 4% for white defendants. Likewise, when the victim was white, blacks were convicted 21% of the time, while only 14% of white defendants were sentenced to death. Note also that, no matter what the race of the defendant, death penalty rates for killing whites are higher than the rates for killing blacks, 21% to 18% for black defendants, and 14% to 4% for white defendants.



These startling statistics present evidence that justice is not blind to color, at least not in the city of Philadelphia.

Chapter 3
Displaying and Summarizing Quantitative Data

What's It About?

In Chapter 3, we turn our attention to quantitative variables. First we examine visual displays: histograms, stem-and-leaf plots, and dotplots. We want students to learn to describe the important features of a distribution: shape, center, spread, and any unusual features such as outliers, gaps, or clusters. Next, we learn how to summarize the center and spread of a distribution. We start with median and interquartile range because it's always appropriate to describe distributions with these two statistics. Then we look at mean and standard deviation as special statistics for use with symmetric distributions. We discuss the problems that arise when these statistics are used for skewed data or in the presence of outliers.

The chapter introduces the 5-number summaries (extremes, quartiles, and median) and uses them to construct boxplots. We define a rule of thumb for nominating outliers. The rule is based on the quartiles, which have the advantage of not being influenced by outliers. They also allow the nomination rule to be asymmetric around the median, stretching out farther on one side of the center if the data are skewed and the quartile on that side is farther from the center.

This is a very useful rule, which has been validated in simulation studies. Nevertheless, it is a rule of thumb and not a *definition* of "outlier." "Outlierhood" is one of those judgments that we are called on to make, take responsibility for, and defend if necessary.

Again, the TI Tips help students use their calculators, this time to create histograms and calculate summary statistics for a single variable. In the On the Computer section, students learn to read summary statistics in common output format.

Comments

We continue to focus on understanding and interpretation. Many of the concepts in this chapter are vague—something students might not expect if they were taking a math class. For example, there's no fixed criterion for declaring a distribution to be "skewed" or even "bimodal." How far away must a point get before it is an outlier? Is a gap real if it disappears when we rescale the histogram?

It's a good idea to point out to students that these are deliberately vague concepts. That insight can help students who might otherwise think that they aren't understanding them. Don't underestimate the importance of these vague concepts. Although methods we deal with later for inference require precise distributional assumptions for their *theoretical* background, in *practice* the best we can do is look at the *shape* of histograms to judge whether they are roughly unimodal and symmetric. And we will insist that students do just that before using these advanced methods.

A Word About Precision. Some math-oriented students may show signs of frustration with what they see as lack of precision here. The fact that Statistics has a lot of gray areas may frustrate them.

It may help to point out to your students that Statistics deals with the real world, not *MathWorld*. They are used to dwelling in *MathWorld* in all of their other math courses. In *MathWorld* the bicyclist rides in a straight line along a level road at a constant speed without encountering any air resistance past a tree that stands perpendicular to the ground. For this scenario, there is a "right answer." In Statistics we liberate ourselves from these unrealistic constraints, but that liberty comes at a price. Uncertainty, caution, and approximation will accompany everything we do. The real world is more complicated – and more interesting – than *MathWorld*.

It is the demands of dealing with the real world that generate the ambiguities in Statistics. So we deal with them by maintaining our connection to the real world and facing the ambiguities head on. The answer is almost never just a number. Instead, it must come with interpretation in terms of the context of the data and the questions we asked of the data—the things we considered at the *Think* step and now must report in the *Tell* step.

Even the numbers that are part of the answer may not – indeed, usually cannot – be known with perfect precision. Sometimes the problem is translating an ideal, model-based concept to the reality of a finite sample. We point in the chapter that many alternative calculations match the ideal definition of a quartile well enough that several are actively in use. They generally converge to the same values for large samples, but the differences for small samples can be striking.

Sometimes, the challenge is the reality of computing with real-world numbers. Digital devices such as computers and calculators cannot represent most numbers without some rounding error. Students who round intermediate results (perhaps as part of dutifully showing their work) exacerbate the problem. Statistics calculations are notorious for magnifying the errors introduced by rounding and truncation. For example, rounding the mean, say, to three decimal places and then using it to find the standard deviation can introduce a substantial change in the calculated standard deviation.

We advise the following:

- Don't round or truncate intermediate results. Keep the full precision that your technology can carry.
- Report statistics to one decimal place more than the precision of the data.
- Focus on the meaning in the Tell section and not on the minor differences in numeric results. Don't sweat the small differences.

You may need to reassure your students that their answers need not match those in the back of the book to the last digit and that their interpretation is more important than their calculation.

Center and Spread

We introduce *center* as the first thing we want to know about a variable. *Spread* is the second thing we want to know. These two numbers, center and spread, should always be reported together.

Emphasize the importance of spread to your students by showing them several histograms with the same center but very different spreads. This gets the point across that you can't describe a variable by using only its center.

In Chapter 6, we'll make it clear that the central concept in all of Statistics is that distances and differences are measured with a ruler based on *spread*. The practical approach we take in this book makes the vague concepts of this chapter central to even the most advanced methods found in the concluding chapters. That may not be what you've seen in older texts where one could fly by these ideas and never look back, but it is the way that practicing statisticians think and work.

Why *y*? This is the first chapter in which it becomes clear that we use the letter *y* to denote the variable we are summarizing, about which we would like to know something, or that we are modeling. This won't bother your students, who are not likely to have any preconceived ideas about what letter to use. But it might take you by surprise if you studied Statistics from a text that used *x* by default or are reading other texts for background. We made this choice with much thought. Most Statistics methods model the world with equations. Once we get beyond one-variable summaries, the Statistics convention is to denote the variable that is being modeled, described, or predicted as *y*, and to plot it on the vertical axis of graphs. We consistently use *x* (and even x_1, x_2,...) to denote the variables that describe, predict, or model *y*. Why should we do otherwise when we consider only one variable? We want to emphasize that we are modeling the world through Statistics. The consistency of naming the variable that we model *y* whenever we need a generic letter helps to convey that message.

As a purely practical matter, if we had used *x* in early chapters, we'd have had to switch to modeling *y* for Chapters 7, 8, and 9, then back to *x* for the early chapters on inference and back again to *y* for regression inference and beyond. That seemed unnecessarily confusing.

Nevertheless, it may be worth pointing out the choice to your students. Other textbooks and statistics packages do use *x,* as does the formula sheet for the AP* exam, some functions on the TI calculator.

Looking Ahead

Almost everything we do in Statistics starts by making a picture of the data. We'll be making histograms and other displays to check out our data in almost every chapter of the book. Visual displays can reveal things that the data itself and numerical summaries may miss or hide. You may want to invite your students to thumb through the book to see how often we show displays of data even in the later chapters, and even in the exercises. They should be encouraged to understand that these skills and concepts are fundamental and will be used throughout the course.

The notion that we need to look at the data – a histogram, for example – before deciding which descriptive statistics are appropriate is important in a broader sense. Every statistical method we encounter will require some underlying assumptions. We should not forge ahead blindly until we *Think* about whether those assumptions are reasonable in our situation. Often we need to check some conditions before proceeding. Making a histogram of the data to rule out skewness, multiple modes, and outliers before finding the mean and standard deviation sets the stage for other such checks that we'll require later on.

Concepts introduced here will be covered more in depth soon. We'll look at a rule for identifying outliers in Chapter 4, and we'll explore more deeply the importance of one measure of spread, the standard deviation, in Chapter 5.

We'll also use re-expressions of data to accomplish a variety of objectives in several chapters, most notably in constructing regression models. Chapter 4 offers the first glimpse of what lies ahead. We'll return to the idea in subsequent chapters, pull the concepts together in Chapter 9, and then use these methods as needed later in the course.

Class Do's

Continue to build students' vocabularies. There are many terms that are critical to sound understanding and clear communication. We can't *Think, Show,* or *Tell* without understanding and using the proper terminology. Write vocabulary words on the board as they come up: *uniform, unimodal, bimodal, symmetric, skewed, cluster, gap, outlier.*

Many students will think that skewness is backwards – that the body rather than the tail should determine whether we say a distribution is skewed left or right. If you liken "skewed" to "strewn" it can help. The bulk of people's incomes are low, but a histogram looks as though a few have been strewn further to the right of the main body – hence "skewed to the right." (Think Bill Gates and Oprah Winfrey.) It might be a good time to talk about the difference between skewed and biased (although bias won't come back until Chapter 11). News media and others sometimes use "skewed" to mean "biased."

Students have better intuition about distributions than they may realize. Exercises 5 and 6 in this chapter demonstrate that fact. Exercises in which students see that they can indeed picture what a distribution ought to look like just from knowing what has been measured and how (and something about the world) help them realize that they can understand and work with these vague concepts.

Ask students when to use a bar graph and when to use a histogram. They have to *Think* about the type of variable before getting on with the *Show.* And be sure they understand that bars represent piles of data. We often see ill-conceived displays in which a bar is used for each datum.

Get students to *Think* about why one might prefer one type of graph to another. In what ways are stem-and-leaf displays, dotplots, and histograms all similar? What information is apparent in some but not others? What are the advantages and disadvantages of using each type of graph?

Present the summary statistics as ways of describing center and spread. Group them that way: first median and IQR, then mean and standard deviation. Median and IQR are appropriate for any data. Mean and standard deviation area bit more powerful, as they have some important mathematical properties we can exploit. But we cannot use mean and standard deviation effectively when the data are strongly skewed or outliers are present.

When reporting measures of center and spread, use statistics that "go together." Median and IQR are the same type of measurement, based on positions in the data. Likewise, mean and standard deviation are based on the size of the data, so they should be paired. We would not talk about, say, median and standard deviation together.

Focus on the meaning of the quartiles rather than on the details of calculation. There are alternative methods for finding the quartiles. Some sources advise including the median in each half of the data; others leave it out entirely. In large data sets this is rarely an issue. Tell students that they should clearly state their choice of method if there can be any doubt about the answer.

Students often misuse the terms "range" and "interquartile range", mistakenly thinking these are intervals rather than numbers. They will say things like "His test score was in the IQR" when they mean the score was between the first and third quartiles (or within the middle 50% of all scores). As always, clear and correct use of vocabulary is critical.

This is the place to discuss the *concept* of an outlier. Point out that whether a point is appears to be an outlier can depend on how we define the group to which we compare it. Outliers seem to come and go in these data as we aggregate it differently. Does this mean they weren't "really" outliers? Or is the concept of "outlier" only relative to some base group?

Be sure students understand the effect of outliers on summary statistics. When there is an outlier in the data, the mean, range, and standard deviation can change a lot. The median and IQR are more stable, and therefore better descriptors in the presence of outliers.

More generally, be sure students understand the relationship between the median and the mean in skewed distributions. Once they see that a high outlier can raise the mean without having much impact on the median, it's easy to talk about skewed distributions; maybe there's no outlier, but there are several unusually large values. When they see that the median divides the area in half but does not provide a balance point for the distribution, it becomes clearer why we use different summaries for skewed distributions.

Students who check assumptions and conditions before proceeding are more likely to deal properly with outliers, skewness, and other issues. Keep requiring this key step.

It may help some students to calculate a few standard deviations by hand—but don't do many. This is not an essential skill. It's more important that they know how to have a calculator or statistics package find SDs, and even more important that they grasp what is being measured.

Remind students that summarizing distributions, or comparing them, still requires discussion of shape, center, spread, and unusual features. As always, answers are sentences, not numbers.

Emphasize and practice verbal descriptions of distributions that observe the four C's: clear, complete, concise, and in context.

The Importance of What You Don't Say

- *Don't teach "measures of center"and then "measures of spread".* Students should think of these statistics grouped in pairs that describe center and spread: median and IQR, or mean and standard deviation.
- *Don't ignore or "throw away" outliers.* Outliers can affect data summaries, but we can't just toss them out. If they are errors, we should correct them (if possible) or set them aside. Outliers are often the most interesting values in a data set, because they don't follow the same pattern as the other data values. We should call attention to them, not conceal them. The best policy when we identify an outlier is to make note of it and gather more information about it if possible. If you can't identify a reason for that point to be different, then do the calculations with and without the outlier to see how much effect it has on your results.

- *Don't say "Measure of central tendency."* At this point, without a Law of Large Numbers, there's no "tending" involved. The mean and median are measures of center; the IQR and standard deviation are measures of spread. Simpler, shorter phrases encourage correct use.
- Many students will have heard about three centers: mean, median, and mode. *Don't make a big deal out of trying to attach a number to the mode.* Especially in a small data set, this can be problematic. A single data value that is repeated can be declared the "mode" even when it is nowhere near the middle hump of a unimodal distribution. Also, the value of the mode can be changed simply by the extent to which the data are rounded. We care more about the overall shape of a histogram, and especially if it tells us there are two commonly occurring clusters of data rather than just one. It's the number of modes that matters, not the numerical value of a mode.

- *Don't fret over the different ways to find quartiles***.** There are many different ways to find the quartiles. Some include the median in both halves; some exclude it (this textbook and the TI, for example). Still others use various methods of interpolation. This means two things: (1) different methods don't always agree with each other about the precise values of the quartiles, and (2) it really doesn't matter. The values will usually differ very little, especially for large data sets. Our goal is to get a feel for the story the data tell. Knowing (say) that the upper quartile of incomes is around $45,000 paints the picture we want. It's really irrelevant that the book's method finds Q3 to be $45,103 and the calculator says it's $45,104.50. Close enough.
- *Don't get hung up over the divisor of (n* – 1*) in the formula for standard deviation.* It's an issue we'll discuss in more detail later in the course. Some students accept it willingly, others grudgingly. You can appeal to the fact that if the mean is used in the calculation, then only $n - 1$ of the n scores are actually free to vary. Or you can discuss (a better explanation) the fact that using n in finding standard deviations of small groups would underestimate the variability actually present in the larger groups they represent—and this is perfectly compensated for by dividing by $n - 1$.
- *Don't get hung up on calculating the standard deviation by hand.* That's what the technology is for. It's far more important that students understand what the standard deviation says about the distribution.
- *Don't worry about always reporting answers to a fixed number of decimal places.* This is the real world, not *MathWorld*. We simply can't find an average height to the nearest thousandth of an inch. Just keep a reasonable number of significant digits when rounding off, and be careful about using rounded values in future calculations.
- *Don't offer rules for deciding when a point is an outlier, when a distribution is bimodal, or what constitutes skewness.* We'll talk about a rule for deciding about outliers in Chapter 5. Until then, encourage students to describe unusual values in a histogram as "possible" outliers. Consider quiz questions for which the "correct" answer isn't clear and alternative interpretations can be considered if they are explained and defended appropriately.

What If…?

Here we see another example of the power of simulation to answer important questions. The theory behind n–1 is complex and beyond the scope of this course. But the simulation allows us to see why using n–1 is better than using n for the sample standard deviation. Here are some questions you might want to use in your discussion.

- Explain the process used in generating one trial in the simulation. (Students should be able to explain that you select a sample of five numbers from the population, calculate the mean, the standard deviation using n, and the standard deviation using n–1.
- What is it that convinces us that n–1 is "better?" (Students should explain that using n–1, on average, gives a value closer to the population standard deviation than using n.)

Class Examples and Activities

1. Use the data you collected the first day. There are many distributions you can compare:

- Make a dotplot of the number of siblings. It will be skewed to the right.
- Make a histogram of the number of states visited.
- Compare heights for males and females. (Or use shoe sizes.) Point out the importance of having the same scale for both graphs.
- Describe the center, spread, and unusual features in each of the variables.

2. Spend a while discussing stem-and-leaf displays, especially how to create stems and leaves for data with various orders of magnitude and ranges. For example:

- A stem-and-leaf display of quiz scores (out of 100) is probably okay if done in the obvious way – tens digit for the stems and units digits as leaves.
- A stem-and-leaf display of states visited might benefit from splitting the stems: 0 – 4, 5-9, etc. You can compare the distribution for males and females with back-to-back stemplots.
- A stem-and-leaf display of students' weights (in pounds) should split 145 as 14 (stem) and 5 (leaf), but a stem and-leaf display of SAT scores should split 520 as 5 (stem) and 2 (leaf). We might want to split the stems as well (into lower 500s and upper 500s). A stem-and-leaf display of heights, may split the "4," "5," and "6" stems into two or even five lines each.

- A stem-and-leaf display of weights of cattle (1000–2000 pounds) should probably split 1368 pounds as 13 (stem) and 6 (leaf), ignoring the 8. It's quicker to truncate than to round off, and it makes little difference in the shape.

3. Compare two histograms or dotplots, asking which has a larger SD.

- Compare the distributions of height for right-handed and left-handed students. For which group do we have a better idea of the center? Why?

4. Collect some data and plot them.

- (Yet another way to use M&Ms!) Buy a bunch of the little bags and count the number of candies and their colors. (Save these data for analysis in later chapters.)
- Get a good scale from your Science faculty and weigh the bags. If you get a really sensitive scale, weigh individual candies.
- Weigh pennies. The mint changed from copper to zinc in the early 1980s, so the distribution of weights is bimodal – a surprise. You could use these weights (in grams)—but the lesson is much more compelling if your class does it themselves.

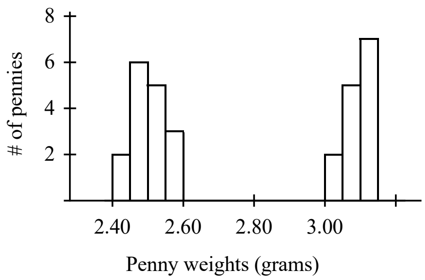
2.57	2.56	3.14	3.03	3.13	2.47	2.43	3.11	3.06	2.48
2.51	2.50	3.07	3.08	3.01	2.45	2.50	3.13	2.51	3.12
3.10	3.08	2.46	2.44	2.47	2.54	3.09	3.13	2.56	2.49

Solution:

Think

We are assuming that these pennies are a representative sample of all pennies minted.

Show



Group	Light	Heavy
Count	16	14
Min	2.43	3.01
Max	2.57	3.14
Mean	2.496	3.091
SD	0.044	0.039
Median	2.495	3.095
Q1	2.465	3.07
Q3	2.525	3.13
IQR	0.06	0.06

Tell

The distribution of weights of pennies is bimodal. About half the pennies are in the lighter cluster, which is roughly symmetric with a median weight of 2.495g. Half of the lighter pennies weigh between 2.465g and 2.525g. The rest of the pennies are in the heavier cluster, which is slightly skewed to the left, with a median weight of 3.095g. Half of the heavier pennies weigh between 3.07g and 3.13g. Each group has an interquartile range of 0.06g. A little research tells us that the mint stopped using copper and started using zinc in the early 1980s. What we thought was one population of pennies was actually two: those minted before the switch, and those minted after the switch. (Note that we used the median and IQR to describe the lighter cluster, even though it had a symmetric distribution. That's because we were comparing it to the heavier cluster, in which we had to use the median and IQR, since the distribution was skewed.)

- A look at home sales records at your city or county clerk's office, or a scan of the real estate ads can give you data about home values in your area. These data will almost certainly be skewed, with many low and moderately priced homes and fewer very expensive homes. Such data can often be made more symmetric with a logarithmic re-expression.

5. To help establish the concept of what standard deviation measures, have students calculate a few by hand. Consider this scenario: A class has been divided into groups of five students each. The groups have completed an independent study project, and at the end they take an individual 20-point quiz. Here are the scores, by group:

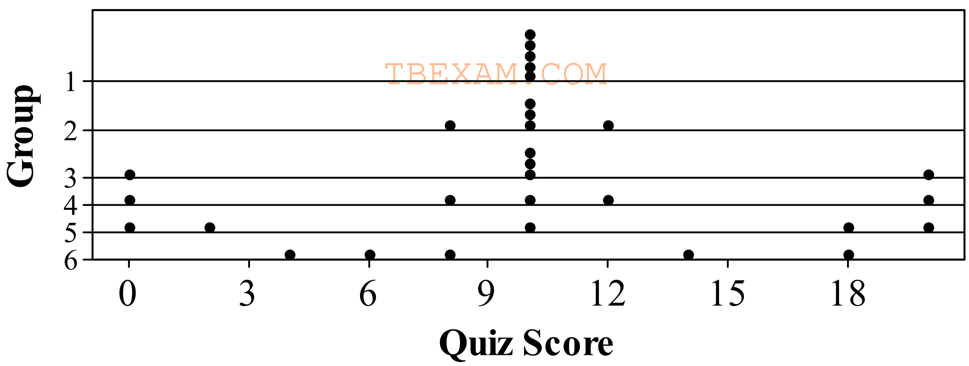
	Group					
	1	**2**	**3**	**4**	**5**	**6**
Quiz Scores (out of 20)	10	8	0	0	0	4
	10	10	10	8	2	6
	10	10	10	10	10	8
	10	10	10	12	18	14
	10	12	20	20	20	18

Note that all the means are 10. Center alone cannot describe the differences we see among these groups. Note also that three of the groups have ranges of 20, yet we see clear differences in how far the scores are spread from the mean. The SD for group 1 is certainly 0. Hand-calculate the SD for group 2. Note that it's only slightly larger, indicating a little more spread.

Have the class calculate the others and talk about how the SD reflects the spread in the scores.

Solution:

```
Variable     Group    Mean      StDev     Range
Quiz Score   1        10.000        0         0
             2        10.000    1.414     4.000
             3         10.00     7.07     20.00
             4         10.00     7.21     20.00
             5         10.00     9.06     20.00
             6         10.00     5.83     14.00
```



The standard deviation measures spread. Generally, distributions that are more spread out have higher standard deviations. For example, Group 4 is more spread out than Group 3, and therefore has a higher standard deviation. Standard deviation is a good measure of spread when distributions are roughly symmetric with no outliers. Standard deviation probably isn't a good summary of spread for Group 6.

Resources

Activity-Based Statistics

- "Matching Plots to Variables" defines a variable and asks students to guess what kind of shape the distribution might take.

Workshop Statistics

- "Features of Distributions" This In-Class Activity asks students to compare dotplots for several distributions of exam scores to spur discussion of shape, center, spread, and unusual features. Topic 3: Displaying and Describing Distributions offers many other useful activities.

- Topic 5: Measures of Spread

  "Guessing Standard Deviations" helps students get a feel for what the number means.

  "Hypothetical Manufacturing Processes" asks students to think about center and spread, and (later) to match standard deviations to dotplots.

Decisions Through Data

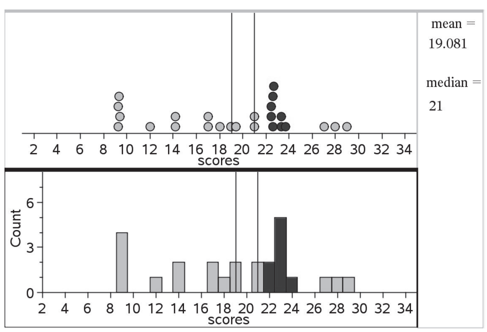
- Video Unit 2: Stemplots
- Video Unit 3: Histograms and Distributions
- Video Unit 4: Measures of Center
- Video Unit 5: Boxplots - looks at constructing boxplots and using them to compare several distributions.
- Video Unit 6: The Standard Deviation.

Meaning from Data: Statistics Made Clear

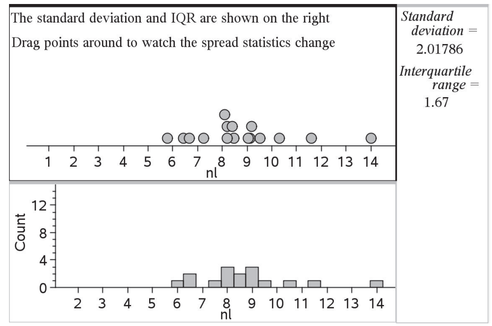
- Lecture 2: Data and Distributions – Getting the Picture
- Lecture 19: Misleading, Distorting and Lying

TI-Nspire™ Demonstrations

- Mean, median, and outliers.
  - Drag data points around to explore how outliers affect the mean and median.
  - Demonstrate that the mean is sensitive to extreme values while the median's resistance makes it a better summary statistic when outliers are present.



- Standard deviation, IQR and outliers.
  - Drag data points around to explore how outliers affect measures of spread
  - Demonstrate that the standard deviation is sensitive to extreme values while the IQR's resistance makes it a better summary statistics when outliers are present.
  - Compare the sizes of the IQR and the standard deviation.



For more information, see page 0-6 of this guide or the online instructor resources.

Web Links

- At the U.S. Census Bureau's International Data Base (CIDB), you can create population pyramids for any country and any year (even animate the changes) www.census.gov/ipc/www/idb/pyramids.html
- www.stat.tamu.edu/~west/ has an applet for playing with histogram bin widths.
- You can find an applet to explore the relationship between mean and median at www.stat.tamu.edu/~west/
- A nice collection of good and bad statistical graphs can be found at the Gallery of Data Visualization, www.math.yorku.ca/SCS/Gallery/

- For still more data sets, try the Data and Story Library, lib.stat.cmu.edu/DASL/.
- AP* Statistics teacher and AP* exam Reader Amy Hogan, from Brooklyn Technical High School, has created a wonderful site for her students which includes links to many free internet sources with access to real data and data sets which cover a wide range of interests including: Arts and Entertainment; Education; Health, Medicine and Biological Sciences; New York and Federal; Social Sciences; Sports; Weather and Physical Sciences; Data Libraries and Archives. http://alittlestats.blogspot.com/p/data-sources.html

Pearson Applet

- Explore the effect of outliers -

  http://www.pearsonhighered.com/mathstatsresources/
- Explore what standard deviation describes - http://www.pearsonhighered.com/mathstatsresources/

Assignments

Spend about 4 days on this chapter. A few pages of reading and 5-6 exercises a night is about right. As students read the chapter, they should do the TI Tips and pay attention to the WCGW section.

Four chapter quizzes are provided.

AP* Statistics Quiz A – Chapter 3 ***Name*** ________________________

1. A survey conducted in a college intro stats class asked students about the number of credit hours they were taking that quarter. The number of credit hours for a random sample of 16 students is given in the table.

10	10	12	14	15	15	15	15
17	17	19	20	20	20	20	22

a. Sketch a histogram of these data

b. Find the mean and standard deviation for the number of credit hours.

c. Find the median and IQR for the number of credit hours.

d. Is it more appropriate to use the mean and standard deviation or the median and IQR to summarize theses data? Explain.

2. Suppose that the student taking 22 credit hours in the data set in the previous question was actually taking 28 credit hours instead of 22 (so we would replace the 22 in the data set with 28). Indicate whether changing the number of credit hours for that student would make each of the following summary statistics increase, decrease, or stay about the same:

a. mean ____________________

b. median ____________________

c. range ____________________

d. IQR ____________________

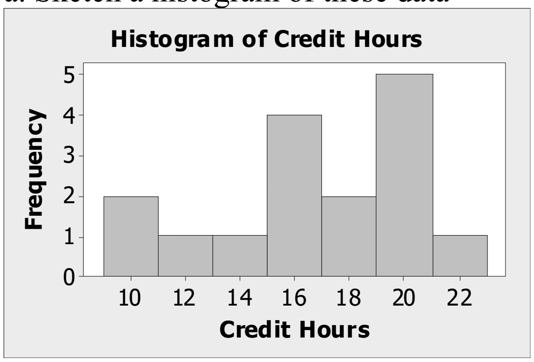
e. standard deviation ____________________

AP* Statistics Quiz A – Chapter 3 - KEY ***Name*** ____________________

1. A survey conducted in a college intro stats class asked students about the number of credit hours they were taking that quarter. The number of credit hours for a random sample of 16 students is given in the table.

10	10	12	14	15	15	15	15
17	17	19	20	20	20	20	22

a. Sketch a histogram of these data



b. Find the mean and standard deviation for the number of credit hours.

$\bar{x} = 16.3$ credit hours; $s = 3.7$ credit hours

c. Find the median and IQR for the number of credit hours.

The median is 16.0 credit hours.
IQR = Q3 – Q1 = 20 – 14.5 = 5.5 credit hours

d. Is it more appropriate to use the mean and standard deviation or the median and IQR to summarize theses data? Explain.

It is more appropriate to use the median and IQR to summarize these data, because these data are not unimodal and symmetric.

2. Suppose that the student taking 22 credit hours in the data set in the previous question was actually taking 28 credit hours instead of 22 (so we would replace the 22 in the data set with 28). Indicate whether changing the number of credit hours for that student would make each of the following summary statistics increase, decrease, or stay about the same:

a. mean — increase

b. median — stay about the same

c. range — increase

d. IQR — stay about the same

e. standard deviation — increase

AP* Statistics Quiz B – Chapter 3 ***Name*** ____________________________

1. The students in a biology class kept a record of the height (in centimeters) of plants for a class experiment.

49	67	38	55	62
54	36	41	56	43
48	75	44	60	48
52	48	53	59	32

a. Sketch a histogram for these data.

b. Find the mean and standard deviation of the plant heights.

c. Is it appropriate to use the mean and standard deviation to summarize these data? Explain.

d. Describe the distribution of plant heights.

2. All students in a physical education class completed a basketball free-throw shooting event and the highest number of shots made was 32. The next day a student who had just transferred into the school completed the event, making 35 shots. Indicate whether adding the new student's score to the rest of the data made each of these summary statistics increase, decrease, or stay about the same:

   a. mean ____________________

   b. median ____________________

   c. range ____________________

   d. IQR ____________________

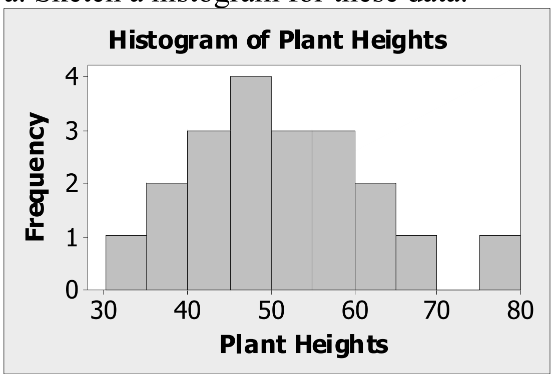
   e. standard deviation ____________________

AP Statistics Quiz B – Chapter 3 – KEY*

1. The students in a biology class kept a record of the height (in centimeters) of plants for a class experiment.

49	67	38	55	62
54	36	41	56	43
48	75	44	60	48
52	48	53	59	32

a. Sketch a histogram for these data.



b. Find the mean and standard deviation of the plant heights.

$\bar{x} = 51.0$ cm; $s = 10.6$ cm

c. Is it appropriate to use the mean and standard deviation to summarize these data? Explain.

Yes, the data are roughly unimodal and symmetric with no outliers.

d. Describe the distribution of plant heights.

The data are roughly symmetric with no outliers; however there is a small gap from 70 to 75 cm. The average plant height is 51.0 centimeters, with a standard deviation of 10.6 centimeters. The range of plant heights is 43 centimeters. The distribution of plant heights has a mode between 45 and 49 centimeters.

2. All students in a physical education class completed a basketball free-throw shooting event and the highest number of shots made was 32. The next day a student who had just transferred into the school completed the event, making 35 shots. Indicate whether adding the new student's score to the rest of the data made each of these summary statistics increase, decrease, or stay about the same:

a. mean — increase

b. median — stay about the same

c. range — increase

d. IQR — stay about the same

e. standard deviation — increase

AP* Statistics Quiz C – Chapter 3 ***Name*** ____________________________

1. A brake and muffler shop reported the repair bills, in dollars, for their customers yesterday.

88	283	312	290	172
154	400	381	346	181
203	118	143	252	227
56	192	292	213	422

a. Sketch a histogram for these data.

b. Find the mean and standard deviation of the repair costs.

c. Is it appropriate to use the mean and standard deviation to summarize these data? Explain.

d. Describe the distribution of repair costs.

2. On Monday, a class of students took a big test, and the highest score was 92. The next day, a student who had been absent made up the test, scoring 100. Indicate whether adding that student's score to the rest of the data made each of these summary statistics increase, decrease, or stay about the same:

a. mean ____________________

b. median ____________________

c. range ____________________

d. IQR ____________________

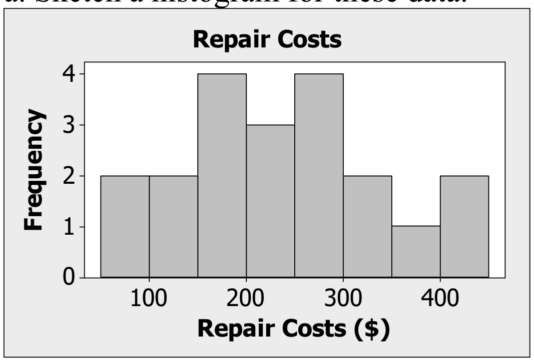
e. standard deviation ____________________

AP Statistics Quiz C – Chapter 3 – KEY*

1. A brake and muffler shop reported the repair bills, in dollars, for their customers yesterday.

88	283	312	290	172
154	400	381	346	181
203	118	143	252	227
56	192	292	213	422

a. Sketch a histogram for these data.



b. Find the mean and standard deviation of the repair costs.

$\bar{x} = \$236.25$; $s = \$103.43$

c. Is it appropriate to use the mean and standard deviation to summarize these data? Explain.

Yes, the data are roughly unimodal and symmetric with no outliers.

d. Describe the distribution of repair costs.

The repair costs averaged $236.25, ranging from $56 to $422 with a standard deviation of $103.43. The distribution was approximately symmetric, with typical repair costs clustered between $150 and $300.

2. On Monday, a class of students took a big test, and the highest score was 92. The next day, a student who had been absent made up the test, scoring 100. Indicate whether adding that student's score to the rest of the data made each of these summary statistics increase, decrease, or stay about the same:

a. mean ______increase______

b. median ______same______

c. range ______increase______

d. IQR ______same______

e. standard deviation ______increase______

AP* Statistics Quiz D – Chapter 3 **_Name _________________________**

1. There are 18 roller coasters in Virginia for which the lengths (in feet) were reported. Those lengths are listed in the table at the right.

1385	2757	2835	2231	3828	1312
3369	3240	5100	3157	2700	600
3150	3108	2705	4210	4882	1960

a. Sketch a histogram for these data.

b. Find the mean and standard deviation of the roller coaster lengths.

c. Is it appropriate to use the mean and standard deviation to summarize these data? Explain.

d. Describe the distribution of roller coaster lengths.

2. There is a proposal to replace the shortest roller coaster above with one that has a length of 1,200 ft. Indicate whether changing that roller coaster's length would make each of these summary statistics increase, decrease, or stay about the same

a. mean ____________________

b. median ____________________

c. range ____________________

d. IQR ____________________

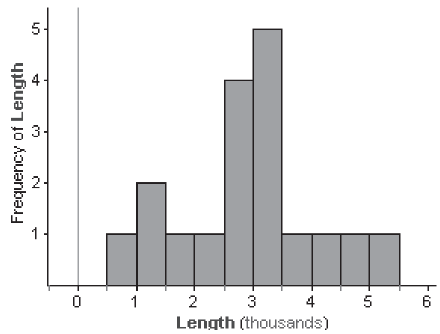
e. standard deviation ____________________

AP Statistics Quiz D – Chapter 3 – KEY*

1. There are 18 roller coasters in Virginia for which the lengths (in feet) were reported. Those lengths are listed in the table at the right.

1385	2757	2835	2231	3828	1312
3369	3240	5100	3157	2700	600
3150	3108	2705	4210	4882	1960

a. Sketch a histogram for these data.



b. Find the mean and standard deviation of the roller coaster lengths.

$\bar{x} = 2918.3\,\text{ft},\ s = 1172.92\,\text{ft}$

c. Is it appropriate to use the mean and standard deviation to summarize these data? Explain.

Yes, the data are roughly unimodal and symmetric with no outliers.

d. Describe the distribution of roller coaster lengths.

The mean roller coaster length is 2918 ft, ranging from 600 ft to 5100 ft. The distribution is roughly symmetric, with typical lengths clustered between 2000 ft and 4000 ft.

2. There is a proposal to replace the shortest roller coaster above with one that has a length of 1,200 ft. Indicate whether changing that roller coaster's length would make each of these summary statistics increase, decrease, or stay about the same

a. mean ____Increase____

b. median ____Same____

c. range ____Increase____

d. IQR ____Same____

e. standard deviation ____Decrease____

Dollars for Students

In 2008 the U.S. Census Bureau published *Public Education Finances*, reporting the average amount (dollars per student) spent by public schools in each state during the 2006 school year. (The table seen here divides states according to whether they lie east or west of the Mississippi River).

Write a report describing the amounts states spent to educate their children.

Eastern State	$ per Student	Western State	$ per Student
Alabama	7,646	Alaska	11,460
Connecticut	12,323	Arizona	6,472
Delaware	11,633	Arkansas	7,927
D.C.	13,446	California	8,486
Florida	7,759	Colorado	8,057
Georgia	8,565	Hawaii	9,876
Illinois	9,149	Idaho	6,440
Indiana	8,793	Iowa	8,360
Kentucky	7,662	Kansas	8,392
Maine	10,586	Louisiana	8,402
Maryland	10,670	Minnesota	9,138
Massachusetts	11,981	Missouri	8,107
Michigan	9,572	Montana	8,581
Mississippi	7,221	Nebraska	8,736
New Hampshire	10,079	Nevada	7,345
New Jersey	14,630	New Mexico	8,086
New York	14,884	North Dakota	8,603
North Carolina	7,388	Oklahoma	6,961
Ohio	9,598	Oregon	8,545
Pennsylvania	11,028	South Dakota	7,651
Rhode Island	11,769	Texas	7,561
South Carolina	8,091	Utah	5,437
Tennessee	6,883	Washington	7,830
Vermont	12,614	Wyoming	11,197
Virginia	9,447		
West Virginia	9,352		
Wisconsin	9,970		

A complete report will include a visual display (stem-and-leaf plot), appropriate statistics, and a well-written description of the expenditures (in context, of course).

AP* Statistics - Investigative Task A ***Chapter 3***

	Components	**Comments**
Think	Demonstrates clear understanding of statistical concepts, vocabulary, and procedures in analyzing and describing these data.	
Show	**Visual/Numerical**: o stem-and-leaf plot o plot well-labeled o plot correctly constructed o summary statistics correct	
Tell	**Verbal**: Describes the distribution of expenditures *in context*, including… o shape (skewed right by higher spending in the East) o center (median) o spread (IQR) The written analysis… o also interprets at least one quartile, or the max or min *in context* o identifies the W's o uses statistical vocabulary correctly o avoids speculation	

Components are scored as **E**ssentially correct, **P**artially correct, or **I**ncorrect

1: Visual/Numerical

E – Has all 4 features

P – Has only 3 of the 4 features, but attempts an appropriate graph (ex: histogram)

I – Graph is not appropriate (ex: bar chart), has many errors, or is missing

2: Shape

E – Identifies skewness to the right, attributed to generally higher spending in the East.

P – Mis-identifies skewness OR overlooks East/West spending difference

I – Description of shape is missing or incorrect

3: Center and Spread

E – Correctly interprets median and IQR in context

P – Correctly interprets only one of median/IQR OR lacks context OR uses mean and SD

I – Has more than one of the three shortcomings described in "P"

4: Written Analysis

E – Has all 4 listed properties.

P – Has only 2 or 3 of the listed properties.

I – Has fewer than of the properties.

Scoring

- E's count 1 point, P's are 1/2
- AP* Score = sum of 4 components; rounding based on overall communication quality
- Grade: A = 4, B = 3, etc., with +/- based on rounding (ex: 3.5 rounded to 3 is a B+)

Name ________________________________ ***AP* Score*** ____ ***Grade*** ____

NOTE: We present a model solution with some trepidation. This is not a scoring key, just an example. Many other approaches could fully satisfy the requirements outlined in the scoring rubric. That (not this) is the standard by which student responses should be evaluated.

Model Solution – Investigative Task – Dollars for Students

The U.S. Census published *Public Education Finances*, a collection of educational information in 2008. Among the data reported was information on average educational spending by state during the 2006 school year, in dollars per student.
Spending by many states was near the nationwide median of $8,581 per student. The middle 50% of states spent between $7,759 and $10,586 per student, an interquartile range of $2,827. The distribution of the average number of education dollars spent per student was skewed to the right. Many Eastern States spent in excess of $12,000 per student, including the maximum of $14,884 per student in New York. Utah spent the least, $5,437 per student.

Dollars Spent per Student

5	4
6	4489
7	2335666789
8	00013344555677
9	11345589
10	056
11	014679
12	36
13	4
14	68

Key : 6|7 = 6000 - 6999 dollars spent per student

Tongue-Rolling and Gender

A biology class in an urban high school was studying genetics. They conducted a survey of a random sample of students to determine whether there is a link between gender and the ability to roll one's tongue. They surveyed 75 students and the data are below.

Gender	Roll Tongue?	Gender	Roll Tongue?	Gender	Roll Tongue?
F	Yes	F	Yes	M	Yes
F	No	M	Yes	M	No
F	Yes	F	Yes	M	Yes
F	Yes	F	Yes	M	Yes
F	Yes	M	Yes	M	Yes
M	Yes	F	No	F	Yes
F	No	F	Yes	M	No
M	Yes	F	Yes	F	Yes
M	Yes	F	No	F	Yes
M	Yes	M	Yes	M	No
F	Yes	F	Yes	M	No
M	Yes	F	Yes	M	Yes
F	Yes	F	Yes	F	Yes
F	Yes	F	Yes	F	No
M	Yes	F	No	F	No
F	Yes	F	Yes	F	Yes
M	Yes	M	Yes	F	No
F	No	F	Yes	F	No
M	Yes	F	Yes	M	Yes
F	Yes	F	Yes	M	No
F	Yes	F	No	F	Yes
M	No	M	Yes	M	Yes
F	No	F	No	M	No
M	Yes	M	No	M	Yes
M	Yes	F	No	M	Yes

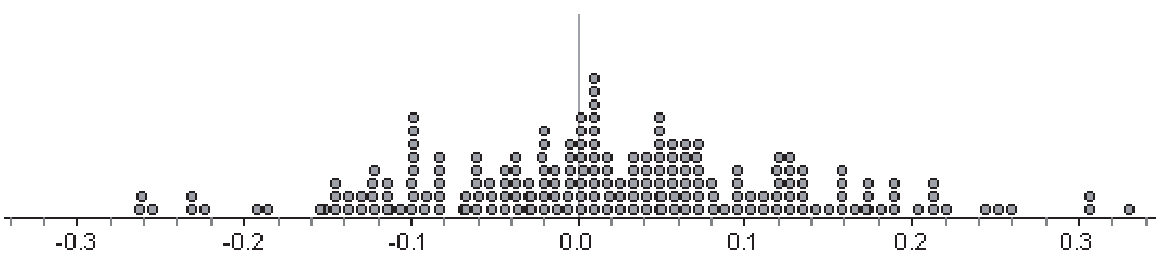
1. Organize the data into a two-way table.

2. Give the proportion of males in the sample that can roll their tongue, the proportion of females in the sample that can roll their tongue, and the difference in proportions (female – male). Based on these proportions, comment on whether there is an association between gender and the ability to roll one's tongue in this sample.

To see if this is evidence that gender and tongue-rolling are associated in the school population, students used a simulation. As a first step, they assumed that the proportion of males and females that can roll their tongue is the same.

3. Based on the sample, what is the best estimate of the proportion of all students that can roll their tongue?

Students simulated sampling 32 males and 43 females, using that best estimate as the proportion of people who can roll their tongue, and calculated the difference in the proportions. The results of 200 trials are shown in the plot below.



Difference in Proportion (F – M)

4. How many of these simulated differences were as far from zero as the difference in proportions you calculated from the sample data in question 2? Comment on whether this provides evidence that there is a difference in the proportion of females and the proportion of males at the school that can roll their tongue.

AP Statistics Task B* *Chapter 3*

Components are scored as **E**ssentially correct, **P**artially correct, or **I**ncorrect

1: Organizing raw categorical data into a two-way table.

E – The table is set up with one variable on each axis, and the frequencies are correct (except for possible counting errors). The grand total must be 75 students and the marginal frequencies must match the frequencies in the table. The table axes must be clearly labeled.

P – The table is set up with one variable on each axis, but the numbers in the table are incorrect (beyond counting errors, or the table is correct but axes are not labeled.

I – Table is set up incorrectly.

2: Calculating proportions

E – Gives correct proportions and difference (consistent with their table from part 1), and concludes that there is an association in this sample with explicit reference to the calculations.

P – Proportions are correct based on their table, and either they conclude that there is no association or they conclude that there is an association but makes no connection to the computations.

I – Proportions are not correct or conclusion is incorrect and makes no attempt to use the calculations to support the conclusion.

3: Computations: This component consists of Question 3 and the first part of Question 4

E – Correctly uses the marginal frequency of tongue-rollers and the grand total from their table to calculate the estimate with computation shown, AND correctly counts the number of dots based on their calculation in Question 2.

P – One of the two parts is correct.

I – Both parts are incorrect.

4: Conclusion

E – Calculates the relative frequency of trials with a difference at least as large as their difference from Question 2, and makes a decision based on this relative frequency. (At this point in the course, they are not expected to know about alpha levels, etc. so any conclusion would be acceptable if it is based on the relative frequency. The idea of a result occurring in less than 5% of trials will be developed later.)

P – Calculates the relative frequency but the decision is not linked to this or communication is weak.

I – Does not compute a relative frequency or computes the wrong one.

Scoring

- E's count 1 point, P's are 1/2
- AP* score = sum of 4 components; rounding based on overall communication quality
- Grade: A = 4, B = 3, etc., with +/- based on rounding (ex: 3.5 rounded to 3 is a B+)

***Name* ________________________________ *AP* Score* ____ *Grade* ____**

NOTE: We present a model solution with some trepidation. This is not a scoring key, just an example. Students are being asked to do things they haven't exactly been asked to do before. That is the spirit of investigative tasks: students must sometimes apply what they've learned in different situations, or make connections that might not have been explicitly described previously. The scoring guide (not this solution) is the standard by which student responses should be evaluated.

Model Solution – Investigative Task B – Tongue-Rolling and Gender

1.

		Roll_tongue		Row Summary
		No	Yes	
Gender	F	14	29	43
	M	8	24	32
Column Summary		22	53	75

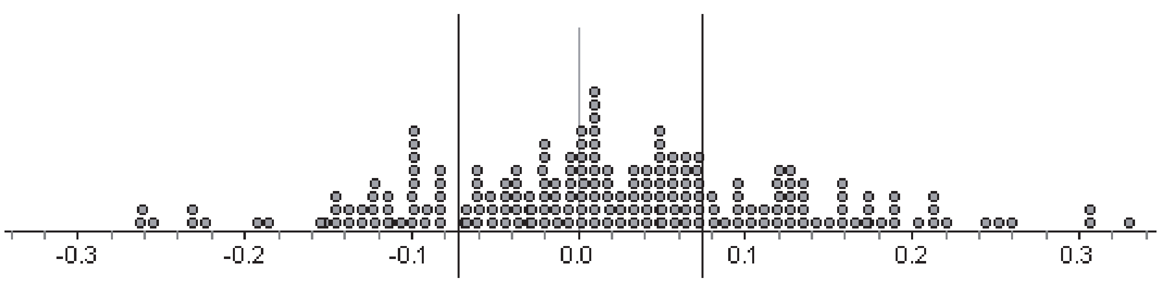
2. Proportion of males in the sample that can roll their tongue = $\dfrac{24}{32} = 0.75$

   Proportion of females in the sample that can roll their tongue = $\dfrac{29}{43} \approx 0.674$

   The difference in proportions (Female – Male) = 0.674 – 0.75 = –0.076
   There is an association between gender and tongue-rolling ability in this sample because the proportion of males who can roll their tongue and the proportion of females who can roll their tongue are not the same.

3. $\dfrac{53}{75} \approx 0.71$

4. Outside the vertical bars at ±0.076 are about 91 points. (Answers will surely vary on this, depending on where exactly the bars are placed. And miscounting is easy and forgivable.)



Difference in Proportion (F – M)

About $\dfrac{91}{200} \approx 0.455$ of the samples had a difference of 0.076 or more when there is no difference in proportions. Because this occurred quite frequently, random variation alone could account for the difference in proportions in our sample. Therefore we don't have evidence that there is an association between tongue-rolling and gender in the school population.

Chapter 4
Understanding and Comparing Distributions

What's It About?

In Chapter 3, students summarized the distribution of a single variable by displaying and describing its shape and by finding numerical summaries of center and spread. In Chapter 4, they get to compare distributions and look for trends and patterns in centers and spreads. Throughout the book, we'll follow this habit, presenting the fundamental idea for describing a single variable, and then generalizing them to talk about relationships, differences, and trends.

Here we want to whet students' appetites. There's a lot to be learned just from comparing groups. The story of this chapter is even more intriguing because we look at the same data at different levels of aggregation, seeing something new each time. So behind the new methods is the lesson that how you structure your data can matter a great deal.

We examine the average daily wind speed and other weather data recorded in a single year in the Hopkins Forest of western Massachusetts. We start with the overall distribution, noting its shape, center, and spread as we learned to do in Chapter 3.

We split the year into two halves and compare the distributions within the two groups. Then we split things still further into months to see annual patterns in both center and spread. This aggregation level reveals outliers, one of which we can identify as a major blizzard. Finally, we get down to the individual observations and consider patterns and trends over time.

The chapter ends with an introduction to data re-expression. At this point, we mostly want to show that it works, without discussing how or why.

TI Tips show students how to create boxplots.

Looking Ahead

Whenever we compare groups, it is natural to use the spread as a background measure of how big a difference is "real". Encourage that line of thinking; it will be fundamental throughout the course (and indeed, throughout all of Statistics).

When we compare boxplots, students should discuss several aspects. The first one they'll see is how the centers compare. Perhaps more important is how the spreads change. And, of course, they should note and discuss any outliers. Later in the course, we'll use boxplots when we test independent groups for differences between means, and these will be precisely the aspects we'll care about. Students who go on to learn about analysis of variance (for example, if you choose to teach a little more after the AP*, using the online chapter), will *always* make boxplots before computing the ANOVA.

Class Do's

Insist that when students compare groups with boxplots, they discuss the centers, the spreads, any outliers, and the overall shapes in their *Tell* step. It is important that they use words of comparison such as *greater than*, *about the same as*, and so forth. It is not enough to list characteristics of the two distributions separately. The comparison is more important than listing numerical values.

When we re-express a skewed variable, points that seemed at first to be outliers may, in the re-expressed form, be simply part of the tail of the distribution. Were they not "really" outliers?

Generally, an outlier is a point that genuinely doesn't belong with the rest of the data. Of course, that means we must define "rest of the data" appropriately. If the data stretch out in a consistent (but thinning) pattern from the third quartile to the maximum, and the maximum is just barely beyond the fence, but not otherwise unusual, then you'd probably elect not to call it an outlier. On the other hand, if the maximum is widely separated from the other data values, yet just inside the fence, by all means, it's an outlier. An outlier that is no longer extraordinary after the data are re-expressed to be more nearly symmetric is often not really different. When we can identify a point as truly different – as we do with the blizzard days in the wind speed data – then we can be reasonably sure it should not be analyzed along with the other data values.

The Importance of What You Don't Say

- *Don't use the word "trend" if time is not a variable.* It can help to keep in mind the difference between "trend" and "tend". There is a clear trend of increasing numbers of girls involved in sports since the passage of Title IX – it is changing over time. Nonetheless, in general men tend to be stronger physically than women. That's a tendency, not a trend, since there's no time element involved.

What If…?

This *What If* brings up the idea of statistical significance. Again, this is not a concept students need to master at this point, but laying the groundwork now will make things easier later. Similar to the *What If* in Chapter 2, we have set up a statistical model, and we compare our actual data with that model. This time, however, we see that the model says that what happened would be really rare from chance alone if the type of roller coaster didn't affect the speed. Here are a couple questions to bring up with your students.

- What does each case in the histogram represent? (Each case represents one possible random splitting of the speeds into two groups – wooden and steel coasters. Its value is the difference in median speeds for that particular split.)
- Based on the histogram, we can say that random splitting of the groups generally results in a difference in medians between ________ and ________.
- What does *statistically significant* mean? (It means that the result would be quite unusual from a random process. Often, if something occurs in less than 5% of random trials, we call it statistically significant.)

Class Examples

1. If you ever need an example of skewed data, just take a look at baseball salaries. The mean salary for players in 2008 was $3 million, but the median was $1 million. Also, Alex Rodriguez and Jason Giambi each made more money ($28 million and 23.4 million, respectively) than the entire Florida Marlins team ($21.8 million).
2. Use the data you collected at the start of the year—states visited or number of siblings. Find 5-number summaries, make boxplots, check for outliers, and compare the mean and median.
3. You can also use your data on the count and weight of M&Ms.
4. Try the Mooseburgers/McTofu class activity. The data provide an opportunity to find summary statistics, sketch parallel boxplots, and see the impact of outliers on summary statistics. Students practice writing sentences comparing the distributions, and must provide a statistical explanation for their decision about where they would prefer to work.
5. Imagine we were to examine data for the number of inches of snowfall in December in Bismarck, ND, for the past 100 years. What information could we learn from looking at a histogram that would be difficult to see in a timeplot? What information could we see in a timeplot that would not show up in a histogram?

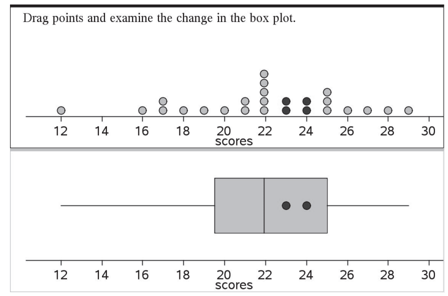
   Solution:

   Using a histogram, we could learn about the shape, center, spread, and unusual features of the distribution of snowfall in December in Bismarck, North Dakota. We would be able to see the average snowfall over the 100-year period, how spread out the distribution of snowfalls was, and whether the distribution was unimodal, bimodal, skewed, or uniform. Using a timeplot, we could learn whether or not December snowfall totals are increasing, decreasing, or staying the same year to year over the 100-year period.
6. Spend some time matching histograms or dotplots to box-plots; it's a good way to help students see what these displays say.
7. Similarly, compare two histograms or two dotplots, asking which has a larger SD.
8. If you haven't done so yet, it's time to get students working with statistical software. By now they can create pie charts, histograms, and boxplots, and they can calculate summary statistics. We offer a *Data Desk* lab activity that uses some of the class data. Some aspects of our instructions may be system specific and hence require modification for your computers – or for different software or a different dataset, of course. Our lab can at least provide you with some ideas, we hope. We also offer some *Data Desk* tips on the back of the lab, to help students use the software with their own data (the Task?). They won't need all of the tips now, but the rest will be useful later on.

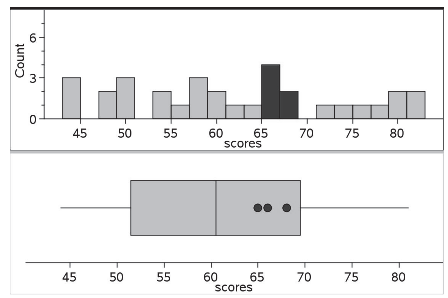
Resources

TI-Nspire™ Demonstrations

- Boxplots and Dotplots
  - Drag data points around to explore what a boxplot shows (and doesn't).
  - Help students understand boxplots by comparing a boxplot to the dotplot.
  - See how the boxplot displays center, spread, and outliers.
  - See how well (or poorly) the boxplot displays shape.



- Histograms and Boxplots
  - See that the shape of a distribution is not always evident in a boxplot.
  - Demonstrate the change (or lack of change) in a boxplot when the histogram becomes skewed, bimodal, or symmetric.
  - Try to match a boxplot with a histogram.



Activity-Based Statistics

- "Matching Statistics to Plots" asks students to match histograms to boxplots, and also asks them to match summary statistics to histograms. The latter is a challenge best used after students have had lots of experience with summary statistics, especially standard deviation.

Workshop Statistics

- Topic 6: Comparing Distributions

  "Lifetimes of Notables", "Governor's Salaries", "Cars' Fuel Efficiencies", and "*Star Trek* Episodes" all provide practice reading boxplots and comparing distributions.

Web Links

- You can get more weather data from the Williams College Center for Environmental Studies at http://web.williams.edu/weather/archives.php
- The *Chance* Data Sets provide data for the number of births on each day of the year. Plotted as a histogram, the distribution is bimodal. As a timeplot, another pattern emerges. (You'll find many other data sets there, too!) www.dartmouth.edu/~chance/teaching_aids/data.html
- The Data and Story Library (lib.stat.cmu.edu/DASL/) has a large number of complex data sets suitable for comparing several groups.

Assignments

Again, have the students read the chapter. It works well in about 3 sections: first comparing distributions with boxplots and histograms; then timeplots and re-expressing data, and finally the What Can Go Wrong and summary sections. Discuss outliers throughout the chapter. There are a wide variety of exercises, and it's reasonable to assign about 6 a night, cycling through the various types. Remember not to overdo the re-expression discussion. This is just an introduction. We'll spend much more time on it in Chapter 9.

Exercises 29 and 30 ask students to use a cumulative relative frequency polygon (aka "ogive") to find medians and quartiles; they have not seen these before. Continue to assign one or two exercises from Chapter 4 as you move on into Chapter 5.

The Ithaca Times graph in the What Can Go Wrong section is a beaut!

Encourage students to be on the lookout for bad graphs in print. During the year they'll turn up several. (If any are really fun, please send them to us!)

The Investigative Task dealing with Auto Safety gives students the chance to compare several distributions. An alternate version of the task deals with SUV insurance ratings, relating to overall insurance losses (meaning what the company has to pay!) Another task compares the ages of US Presidents to British Prime Ministers. This is a great time to give students some experience dealing with data on the computer, if your school has the resources. Work through the *Data Desk* Lab with them. With knowledge gained here (and with the help of the *Data Desk* tips on the back), students have all the skills they need to analyze the automobile safety ratings on their own. Don't assign all three, just pick your favorite! Grading rubrics and sample solutions are provided.

As always, four chapter quizzes are provided as well.

AP Statistics – Classwork* *Chapter 4*

Here are the weekly payrolls for two imaginary restaurants, Mooseburgers and McTofu.

Mooseburgers		McTofu	
Al	$123	Ken	$110
Boris	$136	Latisha	$115
Connie	$144	Maria	$130
Dwight	$150	Nate	$100
Ernie	$110	Otto	$120
Francois	$131	Pablo	$146
Gloria	$140	Quentin	$117
Horace	$160	Rosa	$129
Isaac	$120	Sally	$360
Juan	$130	Ted	$132
		Uta	$107

1. Find the 5-number summaries.

Statistic	M-burgers	McTofu
Min		
Q1		
Median		
Q3		
Max		

2. Create parallel boxplots. Label your graph clearly.

3. Write a few sentences comparing the distributions.

4. Which restaurant pays the higher average salary? ______________________

5. Why is the mean salary misleading?

6. At which restaurant would you rather work? Give a sound statistical justification for your decision.

AP Statistics – Classwork* **KEY** *Chapter 4*

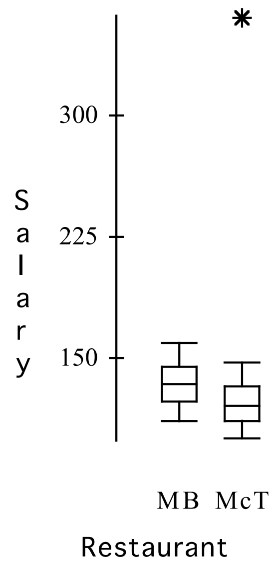
Here are the weekly payrolls for two imaginary restaurants, Mooseburgers and McTofu.

1. Find the 5-number summaries.

Statistic	M-burgers	McTofu
Min	110	100
Q1	123	110
Median	133.5	120
Q3	144	132
Max	160	360

2. Create parallel boxplots. Label your graph clearly.

Boxplots are at the right.



3. Write a few sentences comparing the distributions.

The distribution of salaries at Mooseburgers is symmetric, with a typical salary of about $134.50. The distribution of salaries at McTofu is also symmetric, with the exception of one very high salary, Sally's $360. A typical salary at McTofu is lower, around $120. The distributions of salaries are fairly compact at both restaurants, with interquartile ranges of $21 at Mooseburgers and $20.50 at McTofu. In fact, with the exception of the outlier, the distribution of salaries at McTofu is similar to the distribution of salaries at Mooseburgers, but generally about $15 lower.

4. Which restaurant pays the higher average salary? McTofu

5. Why is the mean salary misleading?

The mean salary at McTofu is pulled up by a salary that is an outlier. Sally's salary of $360 is much higher than the salary of a typical McTofu employee.

6. At which restaurant would you rather work? Give a sound statistical justification for your decision.

Even though McTofu had a higher mean salary, $142.36, compared to Mooseburgers' mean of $134.40, I would rather work at Mooseburgers. Salaries are typically higher there.

Student Body

Learn to use *Data Desk* by following these instructions. Check off each step as you go.

1. Do-it-yourself:

☐ Create a *New Datafile*. Let's examine the shoe sizes we collected the first day.

☐ To enter our class *Data*, create *New Blank Variables* named *Gender* and *ShoeSize*. Enter the data by tabbing <u>across</u> each row, <u>not</u> down the columns as on the TI-83.

☐ Create these displays:

- a pie chart of the genders;
- a histogram of the shoe sizes;
- parallel boxplots of shoe sizes by gender;
- summary statistics comparing shoe sizes of males and females.

☐ Create a word processing document with your name on it. Cut and paste the 4 displays onto one page, resizing them if necessary. Print your report.

Data Desk Tips

To get started: **File – New Datafile.**

To enter data: **Data – New Blank Variable.**

- You must create *all* your variables first.
- Remember that you are creating a single data table with variables as columns and individuals as rows.
- Different groups will be identified by categorical variables. (Ex: to enter heights and weights of males and females requires three variables: a variable named *gender* that would have an entry of M or F in each row, and variables named *height* and *weight* that would contain all the heights and weights for both males and females.)
- If you want to analyze proportions, enter success/failure as 1's and 0's.
- Once the table structure is set up, enter the data by going *across the rows* using the tab key after each value is entered. (You cannot go down the columns like you do on the TI-83. For the height/weight example, you would enter M (tab) 70 (tab) 155 (tab) F (tab) 66 (tab) 128 (tab) and so on.)

To access your data:

- double click on the File folder, then the Data table icon. You need to see the little rectangular icons for the variables themselves.
- you always must select the variables you want to work with: click for *Y*, shift-click for *X*.

To make graphs: **Plot** any graph you want.

- Note that selecting *height* as *Y* and *gender* as *X* allows you to create parallel boxplots with **Boxplot Y by X**.
- Remember the hypermenus (little triangles in the title bars) have lots of goodies. You can change the plot scales, get a frequency distribution for categories in a pie chart, add a regression line to a scatterplot, etc.

To find your statistics: **Calc**

- Set up what you want with **Calc Options – select summary statistics**.
- Create **Calc Summaries** of an entire variable or **Calc Summaries by Groups** for *Y* broken into categories of *X*.
- Summarize the association between categorical variables with **Calc Contingency table**
- Find the equation of the regression line in your scatterplot's hypermenu.

To change the data: **Manip**

- **Transform** offers lots of reexpressions (log y, x/y, etc) or you can create your own using **New derived variable**.
- **Split into variables by groups** allows you to separate, say, boys' heights from girl's heights so you can make comparative histograms or create a confidence interval for the difference.

To do inference:

- Select z-tests, t-tests, confidence intervals, etc. under **Calc - Test**
- Find t-test for linear regression in your scatterplot's hypermenu.
- Find the chi-square test in your contingency table's hypermenu.

AP* Statistics Quiz A – Chapter 4 ***Name*** ________________________

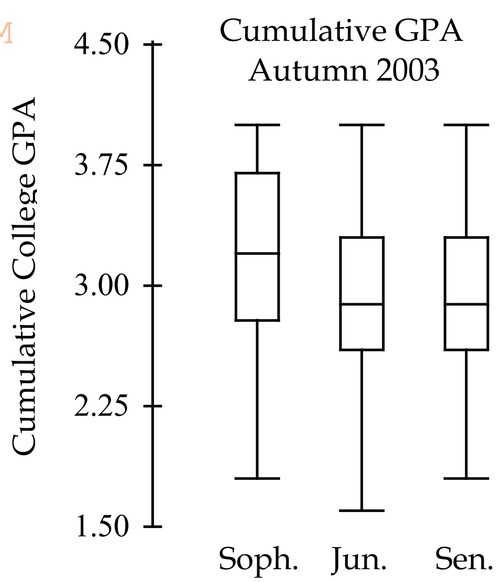
1. The five-number summary for midterm scores (number of points; the maximum possible score was 50 points) from an intro stats class is:

Min	Q1	Median	Q3	Max
16.5	32	39	43.5	48.5

   a. Would you expect the mean midterm score of all students who took the midterm to be higher or lower than the median? Explain.

   b. Based on the five-number summary, are any of the midterm scores outliers? Explain.

2. The side-by-side boxplots show the cumulative college GPAs for sophomores, juniors, and seniors taking an intro stats course in Autumn 2003.



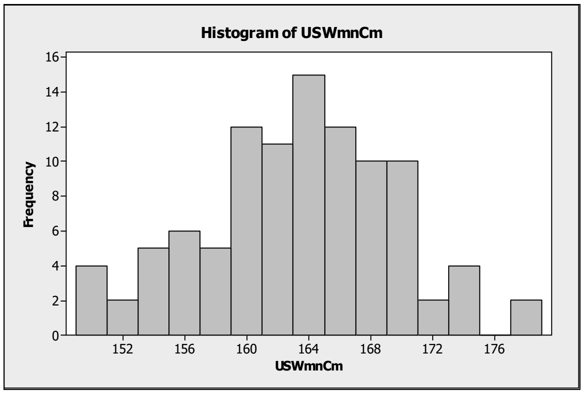
   a. Which class (sophomore, junior, or senior) had the lowest cumulative college GPA? What is the approximate value of that GPA?

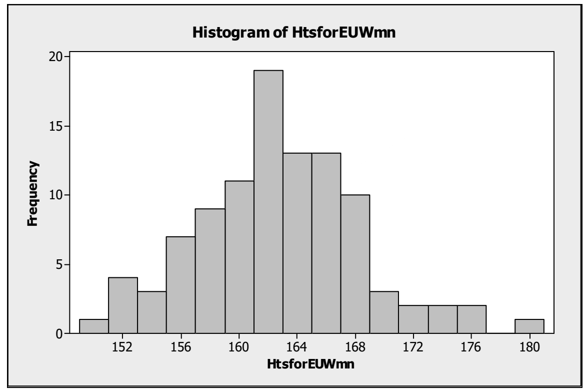
   b. Which class has the highest median GPA, and what is that GPA?

   c. Which class has the largest range for GPA, and what is it?

   d. Which class has the most symmetric set of GPAs? The most skewed set of GPAs?

3. The following are histograms for the heights of 100 US women and the heights of 100 European women:





Compare the two distributions of the women's heights. Be sure to talk about shape, center, and spread.

4. While the scales histograms are the same, there is something that could be improved so that we could compare these two distributions better. Identify this improvement and explain why it would be better.

AP* Statistics Quiz B – Chapter 4 ***Name*** ____________________

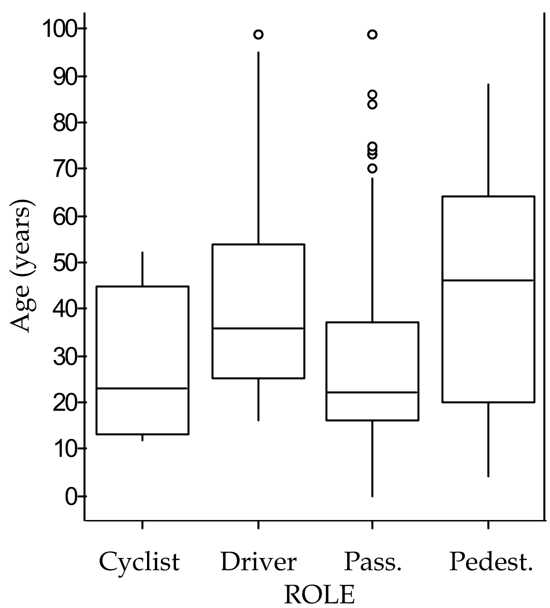
1. The body temperature of students is taken each time a student goes to the nurse's office. The five-number summary for the temperatures (in degrees Fahrenheit) of students on a particular day is:

Min	Q1	Median	Q3	Max
96.6º	97.85º	98.25º	98.6º	101.8º

   a. Would you expect the mean temperature of all students who visited the nurse's office to be higher or lower than the median? Explain.

   b. After the data were picked up in the afternoon, three more students visited the nurse's office with temperatures of 96.7º, 98.4º, and 99.2º. Were any of these students outliers? Explain.

2. The boxplots show the age of people involved in accidents according to their role in the accident.



   a. Which role involved the youngest person, and what is the age?

   ____________ ____________

   b. Which role had the lowest median age, and what is the age?

   ____________ ____________

   c. Which role had smallest range of ages, and what is it?

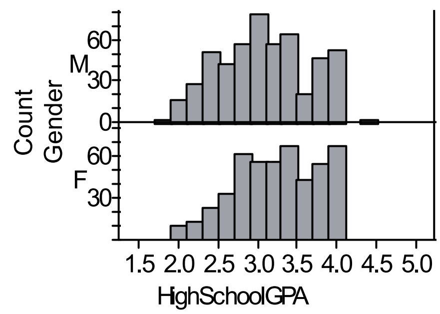
   ____________ ____________

   d. Which role had the largest IQR of ages, and what is it?

   ____________ ____________

   e. Which role generally involves the oldest people? Explain.

3. One thousand students from a local university were sampled to gather information such as gender, high school GPA, college GPA, and total SAT scores. The results were used to create histograms displaying high school grade point averages (GPA's) for both males and females. Compare the grade distribution of males and females.



AP Statistics Quiz C – Chapter 4* *Name* ______________________

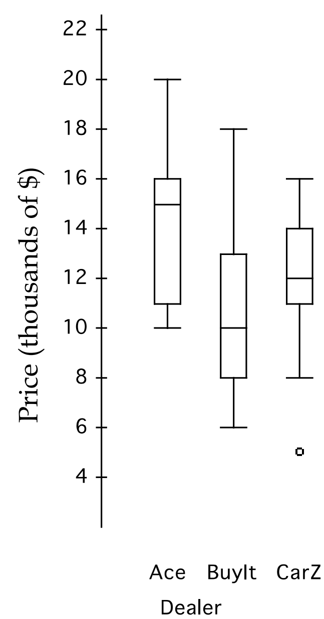
1. The five-number summary for the weights (in pounds) of fish caught in a bass tournament is:

Min	Q1	Median	Q3	Max
2.3	2.8	3.0	3.3	4.5

a. Would you expect the mean weight of all fish caught to be higher or lower than the median? Explain.

b. You caught 3 bass weighing 2.3 pounds, 3.9 pounds, and 4.2 pounds. Were any of your fish outliers? Explain.

2. The boxplots show prices of used cars (in thousands of dollars) advertised for sale at three different car dealers.



a. Which dealer offers the cheapest car offered, and at what price?

______________ ______________

b. Which dealer has the lowest median price, and how much is it?

______________ ______________

c. Which dealer has the smallest price range, and what is it?

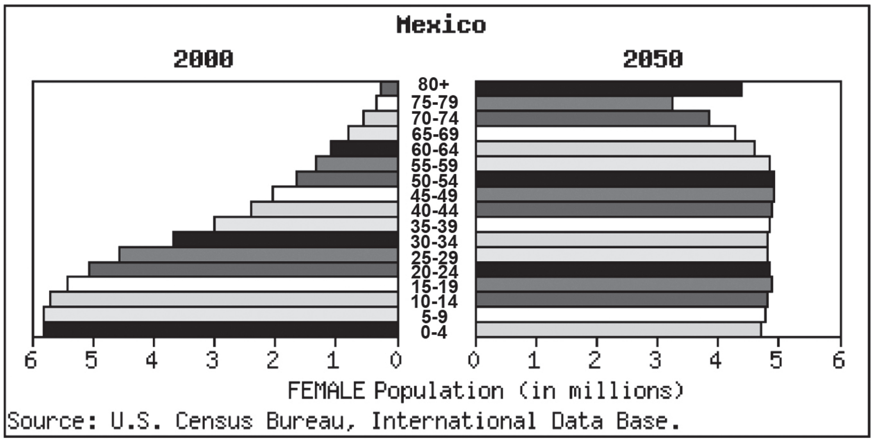
______________ ______________

d. Which dealer's prices have the smallest IQR, and what is it?

______________ ______________

e. Which dealer generally sells cars cheapest? Explain.

3. At www.census.gov you can create a "population pyramid" for any country. These pyramids are back-to-back histograms. This pyramid shows Mexico's 2000 female population and the census bureau's projection for 2050. Write a few sentences summarizing the changes that are forecast.



AP* Statistics Quiz D – Chapter 4 ***Name*** ________________________

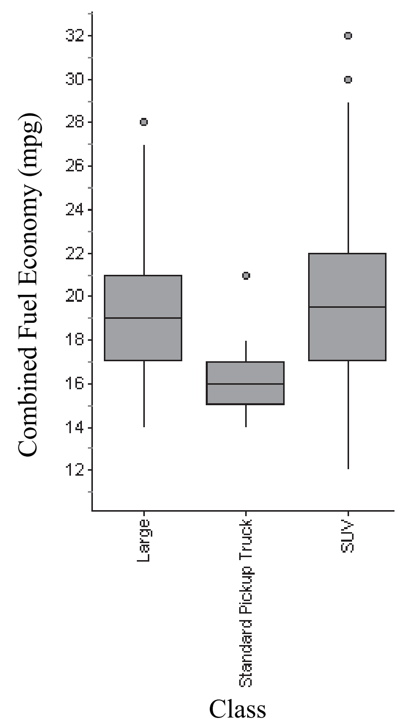
1. The five-number summary for the fuel economy (in miles per gallon) of year 2011 midsize cars is:

Min	Q1	Median	Q3	Max
13	21	23	26	50

   a. Would you expect the mean gas mileage of all midsize cars to be higher or lower than the median? Explain.

   b. One model of Volkswagen gets 34 mpg, one model of Toyota gets 28 mpg, and one model of Bentley gets 13 mpg. Are any of these cars outliers? Explain.

2. The boxplots show fuel economy of 2011 model cars for the classes shown.



   a. Which class offers the car with the best gas mileage, and what is the mileage of that car?

   _______________ _______________

   b. Which class has the highest median gas mileage, and how much is it?

   _______________ _______________

   c. Which class of car has the smallest range of fuel economy, and what is it?

   _______________ _______________

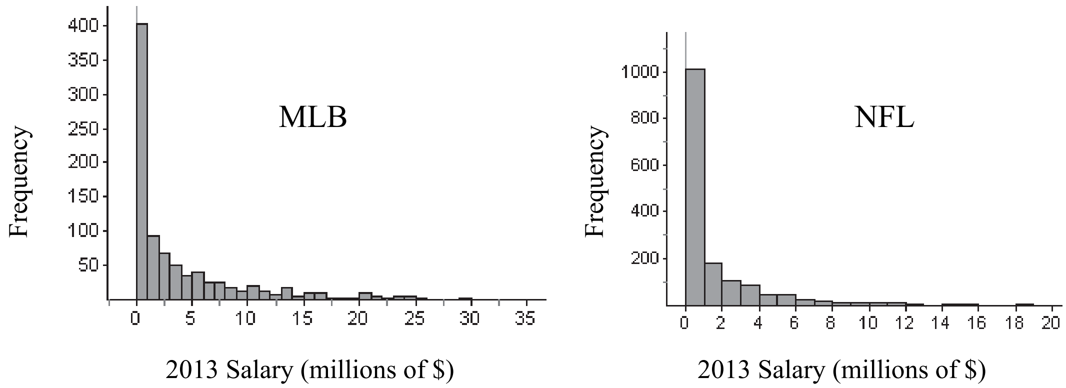
   d. Which class of car has the smallest IQR, and what is it?

   _______________ _______________

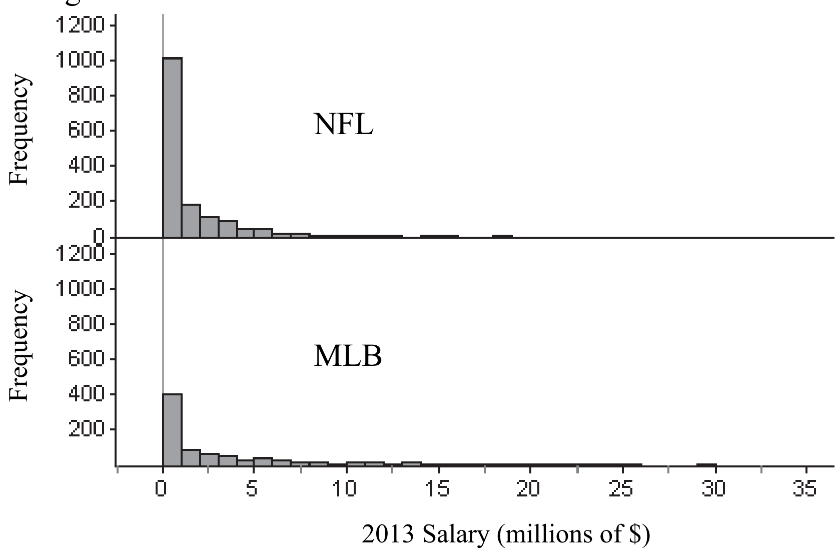
   e. Which class of car generally gets the best mileage? Explain.

3. How do sports salaries compare? Two sets of histograms below show the distributions of salaries for Major League Baseball and the National Football League. Give two reasons why the second set of histograms makes it easier to compare the distributions. Then write a few sentences comparing the salary distributions for the two sports.

Histogram set 1



Histogram set 2



Keys to Chapter 4 Quizzes A and B

Quiz A Key:

1. a. The mean midterm scores of all students would probably be lower than the median. Using the 5-number summary, it appears that the data are skewed to the left.
   b. IQR = 43.5 – 32 =11.5 Q1 – 1.5IQR = 32 – 1.5(11.5) = 14.75
   Q3 + 1.5IQR = 43.5 +1.5(11.5) = 60.75
   Since both the maximum and minimum scores fall between these "fences", there are not outliers in this data set.
2. a. The junior class had the lowest cumulative GPA, around 1.6.
   b. The sophomore class had the highest median cumulative GPA, around 3.2.
   c. The junior class had the largest range for GPA, about 2.4.
   d. The senior class had the most symmetric set of GPAs. The sophomore class had the most skewed set of GPAs, skewed to the left.
3. Both distributions are unimodal and roughly symmetric. Each distribution appears to be centered around 164 cm. The heights for the US women appear to be more spread out than those for the European women.
4. The y-axes should be made according to the same scale for easier comparison.

Quiz B Key:

1. a. The mean temperature of all students would probably be higher than the median. Using the five-number summary, it appears the data are skewed to the right.
   b. IQR = 98.6º – 97.85º = 0.75º. Since 1.5(IQR) = 1.125º, the fences are 97.85º – 1.125º = 96.725º and 98.6º + 1.125º = 99.725º. The lowest temperature (96.7º) being added to the data set is smaller than the lower fence (96.725º) so it is an outlier on the low end. The highest temperature (99.2º) being added to the data set is not above the upper fence (99.725º) so it is not an outlier on the high end.
2. a. Passenger, less than 1 year. b. Passenger, 21 yrs c. Cyclist, 40 yrs d. Pedestrian, 44 yrs
   e. Pedestrian. While the oldest person involved in an accident is not a pedestrian, the median age for pedestrians is almost 45 years, while the median age in the other groups are between 22 and 35 years old. The oldest 50% of the Pedestrian group, from 45 to 87 years, is generally older than the youngest 75% of two groups – Cyclist and Passenger, and only the Driver group has any of its middle 50% as old. The driver and passenger groups have a few people older than the pedestrian group.
3. The distributions of high school GPA for both males and females are skewed to the left, and both distributions appear to be centered at a GPA of about 3.0. The distribution of male GPA appears slightly more spread out than the distribution of female GPA.

Keys to Chapter 4 Quizzes C and D

Quiz C Key:

1. a. Probably higher. The data appear to be skewed to the right.
   b. IQR = 3.3 – 2.8 = 0.5. Since 1.5(IQR) = 0.75, the fences are 2.8 – 0.75 = 2.05 and 3.3 + 0.75 = 4.05. The fish weighing 4.2 pounds is more than 1.5 IQRs outside the quartiles, so it could be considered an outlier.
2. a. Car Z: $5000 b. BuyIt: $10,000 c. Ace: $10,000 d. CarZ: $3000
   e. BuyIt; half of their cars are cheaper than any of the cars at Ace, and 25% of their cars are cheaper than all but one car at CarZ. The third quartile of their prices is well below the third quartile at CarZ, and below even the median price at Ace.

3 The Census Bureau projects dramatic changes in the female population of Mexico over the next 50 years. The current distribution of ages is strongly skewed to the right with most of the women under 30 and far fewer 50 and above. By 2050 the population will become more uniform across age groups from 0 to 60, and we anticipate an unusually large number of women over 80.

Quiz D Key:

1. a. Probably higher. The data appear to be skewed to the right.
   b. IQR = 26 – 21 = 5. Since 1.5(IQR) = 7.5, the fences are 21 – 7.5 = 13.5 and 26 + 7.5 = 33.5. The Volkswagon with 34 mpg is more than 1.5 IQRs above Q3 and the Bentley with 13 mpg is more than 1.5 IQR below Q1, so they could both be considered outliers.
2. a. SUV: 32 mpg b. SUV: 19.5 mpg c. Pickup: 7 mpg d. Pickup: 2 mpg
   e. SUV; ¾ of SUVs get better mileage than ¾ of the Pickup Trucks. SUVs have the highest median, third quartile, and maximum. The first quartile is the same as that of Large cars, and the minimum is the lowest of all three, but overall SUVs appear to have the best mileage.

3 The second set of histograms is stacked vertically, and they use the same scales on the axes. Both distributions are strongly skewed to the right. MLB salaries are higher (the median is higher, and there are fewer low salaries and the highest salaries are much higher.) The MLB salaries also have much more variability (greater spread).

AP Statistics - Investigative Task A* *Chapter 4*

Auto Safety

You work for an automobile insurance company. Your boss has assigned you the task of reviewing recent auto safety records and thinking about how that information may be relevant to your company. You found these data on the web.

HIGHWAY LOSS DATA INSTITUTE (www.hwysafety.org)
AUTOMOBILE INSURANCE INJURY LOSSES (2009-11)

SMALL CARS		MID-SIZE CARS		LARGE CARS	
Model	**Rating**	**Model**	**Rating**	**Model**	**Rating**
Chevrolet Cruze	108	Acura TSX	93	Buick LaCrosse	85
Ford Focus	132	Chevrolet Malibu	115	Buick Lucerne	91
Honda Civic	130	Chrysler 200	115	Buick Regal	93
Honda Civic hybrid	110	Dodge Avenger	167	Chevrolet Impala	116
Honda Civic Si	135	Ford Fusion	107	Ford Taurus	90
Honda Insight hybrid	100	Ford Fusion 4WD	103	Hyundai Azera	67
Hyundai Elantra	129	Ford Fusion hybrid	75	Toyota Avalon	92
Kia Forte	151	Honda Accord	107		
Mazda 3	107	Hyundai Sonata	115		
Mitsubishi Lancer	154	Kia Optima	121		
Nissan Juke 4WD	109	Mazda 6	103		
Nissan Sentra	162	Mercury Milan	104		
Nissan Versa	173	Mitsubishi Galant	179		
Nissan Versa hatchback	143	Nissan Altima	140		
Subaru Impreza 4WD	103	Nissan Altima hybrid	120		
Subaru Impreza WRX 4WD	107	Nissan Maxima	126		
Suzuki SX4	187	Subaru Legacy 4WD	86		
Toyota Corolla	149	Toyota Camry	142		
Toyota Prius hybrid	82	Toyota Camry hybrid	99		
Volkswagen Golf	69	Volkswagen Jetta	112		
Volkswagen GTI	65	Volkswagen Passat CC	98		
		Volvo S40	84		

Insurance injury loss results are stated in relative terms. 100 represents average result for all cars.
Lower numbers are better. For example, a rating of 122 means 22% worse than average.

Write a report to your boss, including:

- appropriate comparative plots and summary statistics;
- descriptions of the injury ratings for each group of cars;
- a comparison of injury ratings for the three sizes of cars;
- your recommendation to your boss about your company's insurance policies.

AP Statistics - Investigative Task A* *Chapter 4*

	Components	Comments
Think	Demonstrates clear understanding of statistical concepts and techniques in comparing the three distributions	
Show	o uses parallel boxplots o has consistent scale (any kind of graphs) o graphs are correct and clearly labeled o 5 # summaries and IQR's are correct	
Tell	Compares centers: o numerically (probably medians) o compares groups to each other o compares each group to average (100) o discusses all three groups Compares variability: o notes differences in IQR's o correctly interprets those differences o notes outliers o notes that groups overlap (ex: ??% of small cars safer than median of midsize) States conclusion: o in context (W's, insurance co. memo) o interprets the ratings properly o makes a recommendation o avoids speculation (drivers, accidents, etc)	

4 Components are scored as **E**ssentially correct, **P**artially correct, or **I**ncorrect

1: Graph Boxplots, on the same scale, clearly labeled with correct numerical summaries.
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

2: Compare the centers Correctly compares all three groups to each other and to the overall average (100), with proper use of numerical summaries (medians).
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

3: Compare the spreads Compares variability within groups (IQRs), noting consistent safety in large cars and greater variability elsewhere. Notes outliers. Discusses overlap between groups, probably using medians and quartiles.
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

4: General conclusion Clearly written, in the proper context, the general conclusion correctly interprets the ratings and recommends some course of action (perhaps lower premiums for large cars).
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

Scoring:

- E's count 1 point, P's are 1/2
- AP* Score = sum of components; rounding based on quality of "Partial" responses.
- Grade: A = 4, B = 3, etc., with +/- based on rounding (ex: 3.5 rounded to 3 is a B+)

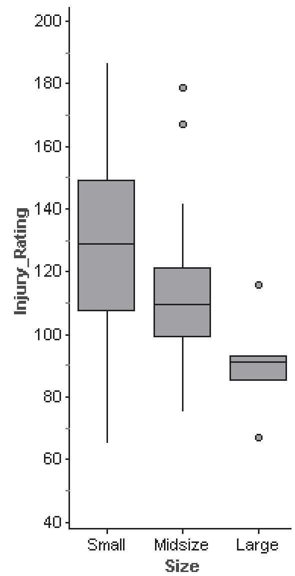
Name ________________________________ ***AP* Score*** ____ ***Grade*** ____

NOTE: We present a model solution with some trepidation. This is not a scoring key, just an example. Many other approaches could fully satisfy the requirements outlined in the scoring rubric. That (not this) is the standard by which student responses should be evaluated.

Chapter 4 – Investigative Task A – Model Solution – Auto Safety

To: Boss From: Employee Re: Automobile insurance injury losses and rates.

The Highway Data Institute has collected data on automobile insurance injury losses for 2009-2011. These losses are reported in relative terms, with a rating of 100 being average for all cars. For instance, the Hyundai Azera, with a rating of 67, was 100 – 67 = 33% better than average. The Toyota Corolla, with a rating of 149, was 149 – 100 = 49% worse than average. These ratings, when viewed with the size of the car in mind, will have a great impact on the rates we charge to insure cars.



The summary statistics for the insurance injury losses are organized in the table below. I have chosen to use 5-number summaries and interquartile range as a measure of variability, since two of the distributions of ratings for the small cars contained outliers. The presence of outliers makes means and standard deviations unreliable statistics. Since the ratings are measuring relative safety, parallel boxplots will be useful for comparing the ratings of the three different sizes of cars.

Group	Count	Min	Q1	Med	Q3	Max	IQR
small	21	65	107	129	149	187	42
mid-size	22	75	99	109.5	121	179	22
large	7	67	85	91	93	116	8

The median of the distribution of large car ratings is 91, meaning the typical larger car has a rating that is 9% better than average. The median of the distributions of mid-size car ratings is 109.5, which means that a typical mid-size cars has a rating that is about 9.5% worse than average for all cars. The median of the distribution of ratings of small cars is 129, which means that a typical small car is expected to be 29% worse than average with regards to injury losses. When comparing the cars by median rating, large cars are safest, followed by mid-size cars. Small cars are the least safe.

In addition to being safer in general than mid-size and small cars, large cars are also *consistently* safe. The interquartile range, which measures the spread of the middle 50% of ratings, is only 8. Small and mid-size cars have much more variability in their ratings, with IQRs of 42 and 22, respectively. It should also be noted that several small cars, the Suzuki SX4, the Nissan Versa, and the Nissan Sentra, had extremely high injury losses, the highest of which was 87% worse than average. Two Mid-size cars, the Dodge Avenger and Mitsubishi Galant, also had very high injury ratings. Using the parallel boxplots to compare the quartiles, we can see that all but one large car is safer than the first quartile mid-size car and small car. Also, the safest 50% of mid-size cars are rated safer than almost 75% of small cars.

Our company can expect to pay more in claims for cars with higher insurance injury losses. Owners of small cars should pay the highest insurance premiums, with premiums for mid-size car owners slightly lower. Large car owners should have the lowest premiums, since their cars generally have much lower insurance injury losses. Additionally some attention should be paid to individual models, since some models are very safe, while others are quite unsafe.

SUV Insurance

You work for an automobile insurance company. Your boss has assigned you the task of reviewing recent SUV insurance records and thinking about how that information may be relevant to your company. You found these data on the web.

HIGHWAY LOSS DATA INSTITUTE (www.hwysafety.org)
OVERALL INSURANCE LOSSES (2005-07)

SMALL SUVS		MID-SIZE SUVS				LARGE SUVS	
Model	**Rating**	**Model**	**Rating**	**Model**	**Rating**	**Model**	**Rating**
CR-V 4WD	71	Rendezvous	65	Liberty 4WD	87	Sequoia 4WD	79
Escape 4WD	72	Freestyle 4WD	69	Pacifica	89	Sequoia	87
Forester 4WD	72	Freestyle	70	Murano 4WD	89	Tahoe 4WD	90
CR-V	73	Pilot 4WD	70	Envoy	91	Yukon 4WD	94
Vue 4WD	73	Pilot	73	Grand Cherokee 4WD	91	Commander 4WD	94
Mariner 4WD	74	Highlander 4WD	73	Sorento 4WD	91	Pathfinder Armada 4WD	95
Tucson 4WD	81	Explorer 4WD	74	Pathfinder	91	Touareg 4WD	99
Vue	82	4Runner 4WD	75	Murano	92	Yukon	103
Equinox 4WD	83	Xterra 4WD	76	FJ Cruiser 4WD	95	Commander	103
Sportage 4WD	83	Wrangler 4WD	77	CX-7	96	Pathfinder Armada	106
Mariner	83	Highlander	78	Xterra	97	Tahoe	107
Escape	85	Envoy 4WD	79	Highlander Hybrid	97	Durango 4WD	107
Escape Hyb. 4WD	85	Santa Fe	82	TrailBlazer	98	Durango	122
Element 4WD	85	Tribeca 4WD	82	Sorento	98		
Torrent	85	TrailBlazer 4WD	85	Hummer H3 4WD	106		
RAV4 4WD	85	Explorer	85	Endeavor 4WD	108		
RAV4	86	4Runner	85	Grand Cherokee	110		
Escape Hybrid	88	Highlander Hyb. 4WD	85	Liberty	110		
Tucson	89	Pathfinder 4WD	86	Endeavor	114		
Sportage	91	Pacifica 4WD	87				
Equinox	97						
Element	98						
Grand Vitara	119						

Insurance loss results are stated in relative terms. 100 represents average result for all cars.
Lower numbers are better. For example, a rating of 122 means insurance losses 22% worse than average.

Write a report to your boss, including:

- appropriate comparative plots and summary statistics;
- descriptions of the loss ratings for each group of SUVs;
- a comparison of loss ratings for the three sizes of SUVs;
- your recommendation to your boss about your company's insurance policies.

AP Statistics - Investigative Task B* *Chapter 4*

	Components	Comments
Think	Demonstrates clear understanding of statistical concepts and techniques in comparing the three distributions	
Show	o uses parallel boxplots o has consistent scale (any kind of graphs) o graphs are correct and clearly labeled o 5 # summaries and IQR's are correct	
Tell	Compares centers: o numerically (probably medians) o compares groups to each other o compares each group to average (100) o discusses all three groups Compares variability: o notes differences in IQR's o correctly interprets those differences o notes outlier o notes that groups overlap (ex: ??% of small SUVs have losses lower than median of large) States conclusion: o in context (W's, insurance co. memo) o interprets the ratings properly o makes a recommendation o avoids speculation (drivers, accidents, etc)	

4 Components are scored as **E**ssentially correct, **P**artially correct, or **I**ncorrect

1: **Graph** Boxplots, on the same scale, clearly labeled with correct numerical summaries.
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

2: **Compare the centers** Correctly compares all three groups to each other and to the overall average (100), with proper use of numerical summaries (medians).
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

3: Compare the spreads Compares variability within groups (IQRs), noting consistently lower losses in small SUVs and consistently higher losses in large SUVs. Notes outlier. Discusses overlap between groups, probably using medians and quartiles.
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

4: General conclusion Clearly written, in the proper context, the general conclusion correctly interprets the ratings and recommends some course of action (perhaps lower premiums for small SUVs).
E – All four requirements. P – Only 2 or 3. I – Fewer than 2

Scoring:

- E's count 1 point, P's are 1/2
- AP* Score = sum of components; rounding based on quality of "Partial" responses.
- Grade: A = 4, B = 3, etc., with +/- based on rounding (ex: 3.5 rounded to 3 is a B+)

Name ________________________________ ***AP* Score*** ____ ***Grade*** ____

NOTE: We present a model solution with some trepidation. This is not a scoring key, just an example. Many other approaches could fully satisfy the requirements outlined in the scoring rubric. That (not this) is the standard by which student responses should be evaluated.

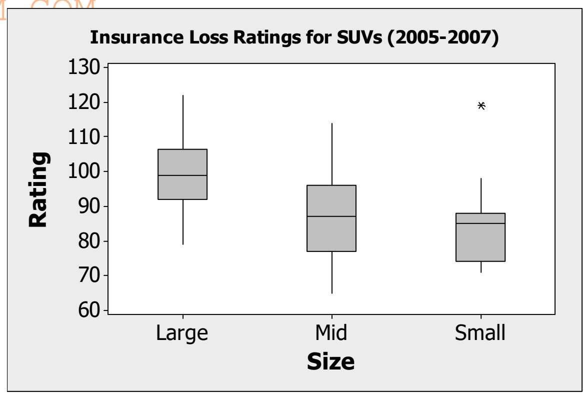
Chapter 4 – Investigative Task B – Model Solution – SUV Insurance

To: Boss From: Employee Re: Overall SUV insurance losses and rates.

The Highway Data Institute has collected data on overall insurance losses for 2005-2007. These losses are reported in relative terms, with a rating of 100 being average for all cars. For instance, the Ford Escape, with a rating of 85, was 100 – 85 = 15% better than average. The Suzuki Grand Vitara, with a rating of 119, was 119 – 100 = 19% worse than average. These ratings, when viewed with the size of the SUV in mind, will have a great impact on the rates we charge to insure SUVs.

The summary statistics for the overall insurance losses are organized in the table below. I have chosen to use 5-number summaries and interquartile range as a measure of variability, since one of the distributions of ratings for the small SUVs contained an outlier. The presence of outliers makes means and standard deviations unreliable statistics. Since the ratings are measuring relative losses, parallel boxplots will be useful for comparing the ratings of the three different sizes of SUVs.

Size	Count	Min	Q1	Median	Q3	Max	IQR
Large	13	79.00	92.00	99.00	106.50	122.00	14.50
Mid	39	65.00	77.00	87.00	96.00	114.00	19.00
Small	23	71.00	74.00	85.00	88.00	119.00	14.00



The median of the distribution of large SUV ratings is 99, meaning the typical large SUV has a rating that is about average. The median of the distribution of mid-size SUV ratings is 87, which means that a typical mid-size SUV has a rating that is 13% better than average for all cars. The median of the distribution of ratings of small SUVs is 85, which means that a typical small SUV is expected to be 15% better than average with regards to overall losses. When comparing the SUVs by median rating, small and mid-size SUVs have the relatively low losses. Large SUVs have the largest losses, making them riskier for us to insure.

It should be noted that small SUVs have *consistently* better ratings. The interquartile range, which measures the spread of the middle 50% of ratings, is only 14. Mid-size SUVs have slightly more variability in rating, with an IQR of 19. Large SUVs, with an IQR of 14.5, are consistent also, but consistently worse in general. It should also be noted one SUV, the Suzuki Grand Vitara had an extremely high insurance losses, for a small SUV. Using the parallel boxplots to compare the quartiles, the most expensive 25% of small SUVs are rated less expensive than about 75% of large SUVs.

Our company can expect to pay more in claims for cars with higher insurance losses. Owners of large SUVs should pay the highest insurance premiums, with premiums for mid-size SUV owners slightly lower. Small SUV owners should have the lowest premiums, since their cars generally have lower insurance losses. Additionally some attention should be paid to individual models, since some models are expensive, while others are inexpensive, regardless of size.

Brits Vs Yanks

In 1776, the United States won independence from the British. How do our leaders compare? The British were the first to have a female leader – Margaret Thatcher became prime minister in 1979, while the United States was the first to have an African American leader – Barack Obama became president in 2009.

Write a report comparing the ages of all the country leaders in the two countries.

United Kingdom – the Brits			
Prime Minister	**Age at start of first term**	**Prime Minister**	**Age at start of first term**
Robert Walpole	44 years	Henry Temple	70 years
Spencer Compton	68 years	Benjamin Disraeli	63 years
Henry Pelham	48 years	William Ewart Gladstone	58 years
Thomas Pelham-Holles	60 years	Robert Cecil	55 years
William Cavendish	36 years	Archibald Primrose	46 years
John Stuart	49 years	Arthur Balfour	53 years
George Grenville	50 years	Henry Campbell-Bannerman	69 years
Charles Watson-Wentworth	35 years	Herbert Asquith	55 years
William Pitt the Elder	57 years	David Lloyd George	53 years
Augustus FitzRoy	33 years	Bonar Law	64 years
Frederick North	37 years	Stanley Baldwin	55 years
William Petty	45 years	Ramsay MacDonald	57 years
William Cavendish-Bentinck	44 years	Neville Chamberlain	68 years
William Pitt the Younger	24 years	Winston Churchill	65 years
Henry Addington	43 years	Clement Attlee	62 years
William Grenville	46 years	Anthony Eden	57 years
Spencer Perceval	46 years	Harold Macmillan	62 years
Robert Jenkinson	42 years	Alec Douglas-Home	60 years
George Canning	56 years	Harold Wilson	48 years
Frederick Robinson	44 years	Edward Heath	53 years
Arthur Wellesley	58 years	James Callaghan	64 years
Charles Grey	66 years	Margaret Thatcher	53 years
William Lamb	55 years	John Major	47 years
Robert Peel	46 years	Tony Blair	43 years
John Russell	53 years	Gordon Brown	56 years
Edward Smith-Stanley	52 years	David Cameron	43 years
George Hamilton-Gordon	68 years		

United States – the Yanks			
President	**Age at start of first term**	**President**	**Age at start of first term**
George Washington	57 years	Benjamin Harrison	55 years
John Adams	61 years	Grover Cleveland	55 years
Thomas Jefferson	57 years	William McKinley	54 years
James Madison	57 years	Theodore Roosevelt	42 years
James Monroe	58 years	William Howard Taft	51 years
John Quincy Adams	57 years	Woodrow Wilson	56 years
Andrew Jackson	61 years	Warren G. Harding	55 years
Martin Van Buren	54 years	Calvin Coolidge	51 years
William Henry Harrison	68 years	Herbert Hoover	54 years
John Tyler	51 years	Franklin D. Roosevelt	51 years
James K. Polk	49 years	Harry S. Truman	60 years
Zachary Taylor	64 years	Dwight D. Eisenhower	62 years
Millard Fillmore	50 years,	John F. Kennedy	43 years
Franklin Pierce	48 years	Lyndon B. Johnson	55 years
James Buchanan	65 years	Richard Nixon	56 years
Abraham Lincoln	52 years	Gerald Ford	61 years
Andrew Johnson	56 years	Jimmy Carter	52 years
Ulysses S. Grant	46 years	Ronald Reagan	69 years
Rutherford B. Hayes	54 years	George H. W. Bush	64 years
James A. Garfield	49 years	Bill Clinton	46 years
Chester A. Arthur	51 years	George W. Bush	54 years
Grover Cleveland	47 years	Barack Obama	47 years

A complete report will include a visual display (back to back stem plot), appropriate summary calculations, and a well-written comparison of the ages (in context, of course).

AP* Statistics - Investigative Task C ***Chapter 4***

	Components	***Comments***
Think	Demonstrates clear understanding of statistical concepts, vocabulary, and procedures in analyzing and describing these data.	
Show	**Visual/Numerical**: o Back to back stem plot with split stems o plot well-labeled including key o plot correctly constructed o summary parameters correct	
Tell	**Verbal**: Compares the distributions of ages *in context*, including… o shape (both symmetrical) o center (very close – US only slightly higher based on median and mean) o spread (range and IQR much higher for Britain) The written analysis… o also interprets at least one quartile, or the max or min *in context* o identifies the W's o uses statistical vocabulary correctly o avoids speculation	

<u>Components are scored as **E**ssentially correct, **P**artially correct, or **I**ncorrect</u>

1: Visual/Numerical

E – Has all 4 features
P – Has only 3 of the 4 features, but attempts an appropriate graphical comparison (ex: 2 histograms)
I – Graph is not appropriate (ex: bar chart), has many errors, or is missing

2: Shape and Center

E – Identifies both shapes as symmetrical and correctly compares either medians or means
P – Compares only 1 (shape OR center) correctly or lacks context
I – Has both of the shortcomings described in "P"

3: Spread

E – Correctly compares range and IQR in context
P – Correctly compares only 1 (range or IQR) correctly or lacks context
I – Has both of the shortcomings described in "P"

4: Written Analysis

E – Has all 4 listed properties.
P – Has only 2 or 3 of the listed properties.
I – Has fewer than of the properties.

Scoring

- E's count 1 point, P's are 1/2
- AP* Score = sum of 4 components; rounding based on overall communication quality
- Grade: A = 4, B = 3, etc., with +/- based on rounding (ex: 3.5 rounded to 3 is a B+)

Name ________________________________ ***AP* Score*** ____ ***Grade*** ____

4-32

NOTE: We present a model solution with some trepidation. This is not a scoring key, just an example. Many other approaches could fully satisfy the requirements outlined in the scoring rubric. That (not this) is the standard by which student responses should be evaluated.

Model Solution – Investigative Task B – Brits Vs Yanks

	2	4
	2	
	3	3
	3	5 6 7
3 2	4	2 3 3 3 4 4 4
9 9 8 7 7 6 6	4	5 6 6 6 6 7 8 8 9
4 4 4 4 4 2 2 1 1 1 1 1 0	5	0 2 3 3 3 3 3
8 7 7 7 7 6 6 6 5 5 5 5	5	5 5 5 5 6 6 7 7 7 8 8
4 4 2 1 1 1 0	6	0 0 2 2 3 4 4
9 8 5	6	5 6 8 8 8 9
	7	0
	7	

KEY: 6|4|5 46 years old British Prime Minister
45 years old US President

The United States was originally governed by the Brits but declared independence in 1776. How different are the leaders we select? Britain was the first of the two countries to elect a woman prime minister, and the United States was the first to select an African American president. A back to back stemplot can be created to display the ages of all the country leaders to date in each country.

The summary calculations are parameters since we are working with population data, not summary data. Britain has had 53 prime ministers and the United States has had 44 presidents. The shapes of the two distributions are similar since both distributions are fairly symmetrical. The ages of Britain's leaders have much more variability than those in the US as measured by range [Britain 70 – 24 = 46; US 69 – 42 = 27] as well as IQR [Britain 60-45.5 = 14.5; US 57.5-51 = 6.5]. Britain has both the youngest leader, William Pitt the Younger, [Min: Britain 24; US 42] as well as the oldest leader, Henry Temple [Max: Britain 70; US 69]. The centers of the distributions are very close as measured by median [Britain 53; US 54.50] and mean [Britain 52.53; US 54.66] with United States presidents only slightly older than British prime ministers on average.

Chapter 5
The Standard Deviation as Ruler and the Normal Model

What's It About?

Chapter 5 looks at the effects of shifting and scaling on summary statistics. We explore further the importance of standard deviation as a measure of spread, showing that it serves as a universal yardstick for measuring variables. And we introduce the Normal curve as a useful model for some variables whose distribution is unimodal and symmetric. TI Tips help students work with Normal curve probabilities and create Normal probability plots.

This chapter concludes Part I of the text. We provide a Quick Review of the concepts encountered so far, and a large collection of review exercises in preparation for the first big test. The first comprehensive Practice Exam follows the review exercises. This one is short, since we haven't covered too many topics yet, but the Practice Exams will get longer as we progress through the course.

Comments

Although students need to develop some mechanical skills, the underlying concepts in this chapter should be your main focus. They'll show up repeatedly throughout the rest of the course.

The first concept involves changing numerical data by adding (or subtracting) a constant and/or by multiplying (or dividing) by one. We do this often, changing units or re-expressing our data. We need to know how these changes will affect the summary statistics, and recognize that location and spread are not affected the same way.

The second concept, standardizing, allows us—literally—to compare apples to oranges. In Chapter 4, students learned how to calculate a standard deviation and to recognize that it describes the spread of the data. We now show how using the standard deviation as a unit of measurement provides us with a universal yardstick. This yardstick enables comparisons of widely different measurements. More important later on, it allows us to decide if the results of a study are unusually different from what we might have expected—the very essence of statistical inference.

At its heart, inference involves looking at sample data and trying to guess what's actually going on in a population. It is vitally important that students be able to see that such comparisons involve known, observed data on the one hand and unknown—indeed, unknowable—facts about the world on the other. To help make that distinction we have adopted the term "model" to represent these images of theoretical perfection, as distinguished from what we observe. In class, begin now to draw the lines of distinction between these two worlds. The difference between what we see in actual data and conjectures we make about the world is a central issue throughout this course, and something students often confuse. It will appear again when we build linear models to fit scatterplots, when we do simulations, when we formally discuss probability, and throughout inference.

Start now to establish the idea that statisticians talk about models, and that these models differ from real data. We will build on this understanding over and over. It is one of the central unifying themes of the book. AP* Exam readers see that student failure to make this distinction leads to all kinds of misinterpretations.

We don't spend much time on the Normal tables. There is a table (Table Z) at the back of the book, but there is little need to drill students extensively in its use. Calculators serve the purpose more conveniently, as does any major statistics package. The book calls the Normal table a "desert island" method—something we might do if we were stranded far from technology and desperately needed a Normal percentile. In fact, the people who still use Normal tables fall into two categories:

1. Students in introductory Statistics classes, and
2. Teachers writing homework exercises for (1)

Emphasize instead the 68-95-99.7 Rule and encourage your students to use technology otherwise.

Looking Ahead

We hardly have to promise to get back to *z*-scores and the Normal model. But the really important concept here is the standard deviation as a ruler. That idea is central to all of Statistics, whether in this course, or in a course in multivariate statistical methods. Be sure to let students know that they'll see it again and again. And, if they understand it now, they'll find many things that come up later look just like something they already have under their belts.

It's important to require students to justify the use of a Normal model. The assumption that a Normal model is appropriate should be specified in the exercise or verified by checking the corresponding Nearly Normal Condition that the distribution of the data is roughly unimodal and symmetric. We'll encounter many situations where we must check some conditions to verify underlying assumptions before going forward with some statistical procedure. Failure to do so is one of the common mistakes students make when writing solutions to inference questions on the AP* Exam. Reinforce the importance of checking assumptions and conditions now.

Class Do's

With a little thought, students will see how changes in scale affect summary statistics. It boils down to whether we are merely shifting the data (a translation) or rescaling it (a dilation). Don't be afraid to use geometric or algebraic intuition. Translating any geometric shape or a function to the left or right leaves an object the same shape and size but just positioned elsewhere. The same is true of a distribution of data (think histogram). Sliding it a few units left or right relocates the center, max, min, and quartiles but leaves its shape and size (spread) the same. A dilation of a triangle or a function from the origin preserves its shape, but both relocates it and changes its size. A rescaling of a histogram therefore resizes everything about it, now including its spread.

Emphasize the usefulness of standard deviations as units of measurement. It's commonly said that one can't compare apples to oranges. But actually we can, and all it takes is a standard deviation. An apple and an orange might each weigh 12 ounces, but if that makes the apple 2 standard deviations above the mean weight for apples while the orange is only 1 standard deviation above the mean weight for oranges, then the apple is comparatively bigger. Most students know this intuitively. The idea that we can standardize deviations from mean values is very powerful, and essential to the rest of Statistics. You'll want to spend enough time working on this idea to be sure students grasp the concept—not just the mechanics! Just finding a *z*-score of –1.5 is not enough; have students *Tell* you that such-and-such is $1\frac{1}{2}$ standard deviations below the mean.

Do use the "model" terminology. We carry the models theme throughout the text. Emphasize the difference between a distribution and a model.

Distribution	Model
real data	theoretical values
observed	imagined
histogram	mathematical curve
statistics	parameters
center $\bar{y}$	center μ
spread s	spread σ

To quote George Box, "All models are wrong, but some are useful." One very useful model is the Normal model. No data are ever Normally distributed, but when data are roughly unimodal and symmetric, a Normal model may provide some useful insights. Make students justify their use of the Normal model (with a histogram that is unimodal and symmetric, or a reasonably straight Normal probability plot) before they apply it.

Spend time with the 68-95-99.7 Rule. Having students draw and label models can help them develop a feel for what standard deviations tell them about distributions for which Normal models are appropriate. Don't be in a great rush to get to tables, statistical software, or the calculator.

Have students work with *z*-scores. Always. Understanding what a *z*-score is and developing a feel for how *z*-scores relate to probabilities are skills that provide valuable insights into much of what we do later in the course (including correlation and regression, confidence intervals and hypothesis testing). Although the calculator's `normalcdf` function allows students to bypass *z*-scores, don't let your students use the shortcut of entering μ and σ in the `normalcdf` function. Make them work with the *z*-score as we demonstrate in the TI Tips. This pays big dividends later on.

Remember our "make a picture" rule. Every problem students do that involves a Normal model—including all the exercises that want them to use *z*-scores to determine probabilities, means, standard deviations, or cutoffs—needs an accompanying picture. They will understand what they are doing sooner and better and spot errors more easily if you insist on this. And we will insist on it later in the book when we work with models that require Normally distributed data or errors. (Glance ahead at the Step-by-Steps in Chapter 22, for example.)

Again, expect students to read the chapter. We hope by now that they are engaged in reading the book and have seen how helpful it can be. This chapter has many new ideas and important skills. The narrative, the TI Tips, and the Step-By-Step worked examples are all intended to be part of the process of building understanding.

The Importance of What You Don't Say

This chapter touches on a number of topics that you will probably recognize as central to Statistics. But it isn't time to discuss them yet. Please don't distract students with advanced concepts that they are not yet ready for. We have important principles to establish now. We'll get to the more advanced application of these principles in due time.

Don't talk about density. We have in fact introduced a Normal density function, but that formal concept isn't actually needed in this course and is not part of the AP* syllabus. By contrast, the concept of a *model* is fundamental. Push *that* term instead. Discussing the Normal model reminds students of its role as an idealized model.

Don't say "Empirical Rule". The name "68-95-99.7 Rule" is more useful because it carries the important numbers. The rule is "empirical" only for historical reasons; that name carries no useful information.

Don't mention "probability". We are still talking about relative frequencies or areas under a curve. We'll get to probability, but not for a number of chapters yet. We don't yet have a good place for students to "hang" the concepts they'll need.

Don't say "critical value". That's an idea that shows up in inference, but it doesn't belong here.

Don't say "The data are Normal." No data are ever or can ever be Normal. The Normal model can serve as a useful way of thinking about data, but the model is not the data. Always make this distinction clear.

Finally, as we said earlier, try not to get hung up making students learn to use the Normal table. It's a nice backup when the calculator fails, but it's unlikely they'll ever use one otherwise. (They don't use log or trig tables either! You needn't mention Table Z at all!) That said, don't use all the calculator shortcuts. We want students to develop an understanding of *z*-scores.

What If…?

The *What If* in Chapter 5 gives some insight into one place we often use a Normal model: the sampling distribution of the sample mean. We don't call it that. Not yet. It also illustrates the importance of making sure a Normal model is appropriate before using it. Still, it is important that students understand what they are looking at in these plots. Here are some questions you might want to raise with your students.

- What is a single case in each histogram? *A sample of 25 numbers from the population.* (Indicate the far right bar in one of the histograms.) What does it mean if a case is over here? *That set of 25 values had a particularly high mean (or max).*
- The mean of the plot of sample means is 550, and the standard deviation of the sample means is 50. Use the histogram to estimate the proportion of cases in which the sample mean is within one standard deviation of the mean sample mean. Does that match with what we'd expect from a Normal model?
- The mean of the plot of sample maxima is 962.8 and the standard deviation of the sample maxima is 30.74. Use the histogram to estimate the proportion of cases in which the sample maximum is within one standard deviation of the mean sample maximum. Does that match with what we'd expect from a Normal model? *No! And we should not expect it to because a Normal model is not appropriate.*

Class Examples

1. Suppose the class took a 40-point quiz. Results show a mean score of 30, median 32, IQR 8, SD 6, min 12, and Q1 27. (Suppose YOU got a 35.) What happens to each of the statistics if…

   - I decide to weight the quiz as 50 points, and will add 10 points to every score. Your score is now 45.
   - I decide to weight the quiz as 80 points, and double each score. Your score is now 70.
   - I decide to count the quiz as 100 points; I'll double each score and add 20 points. Your score is now 90.

   Solution:

   Measures of center and position are affected by addition and multiplication. Measures of spread are only affected by multiplication.

Statistic	Original (y)	$y+10$	$2y$	$2y+20$
mean	30	40	60	80
median	32	42	64	84
IQR	8	8	16	16
SD	6	6	12	12
minimum	12	22	24	44
Q1	27	37	54	74
your score	35	45	70	90

2. Let's talk about scoring the decathlon. Silly example, but suppose two competitors tie in each of the first eight events. In the ninth event, the high jump, one clears the bar 1 in. higher. Then in the 1500-meter run the other one runs 5 seconds faster. Who wins? It boils down to knowing whether it is harder to jump an inch higher or run 5 seconds faster. We have to be able to compare two fundamentally different activities involving different units. Standard deviations to the rescue! If we knew the mean performance (by world-class athletes) in each event, and the standard deviation, we could compute how far each performance was from the mean in SD units (called z-scores). So consider the three athletes' performances shown below in a three-event competition. Note that each placed first, second, and third in an event. Who gets the gold medal? Who turned in the most remarkable performance of the competition?

	Event		
Competitor	**100 m Dash**	**Shot Put**	**Long Jump**
A	10.1 sec	66'	26'
B	9.9 sec	60'	27'
C	10.3 sec	63'	27'3"
Mean	10 sec	60'	26'
St Dev	0.2 sec	3'	6"

Solution:

Think

We want to compare athletic performance across several different events, so we need some way to compare without units. We will compare *z*-scores from each event. Also, don't forget that lower times are *better* in the 100-m dash!

Show

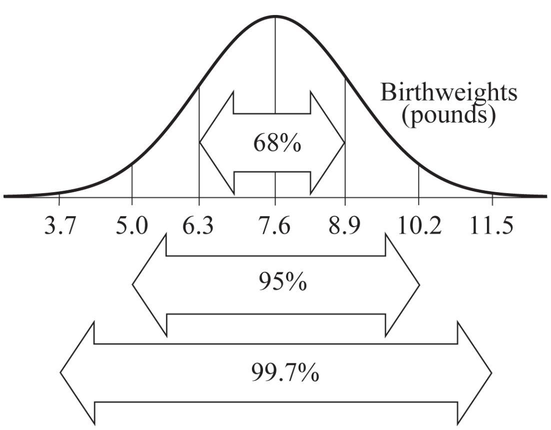
Competitor	100-m dash	Shot put	Long jump
A	10.1 sec	66'	26'
z-scores (Total = 1.5)	$z = -\left(\frac{10.1 - 10.0}{0.2}\right) = -0.5$	$z = \frac{66 - 60}{3} = 2$	$z = \frac{26 - 26}{0.5} = 0$
B	9.9 sec	60'	27'
z-scores (Total = 2.5)	$z = -\left(\frac{9.9 - 10.0}{0.2}\right) = 0.5$	$z = \frac{60 - 60}{3} = 0$	$z = \frac{27 - 26}{0.5} = 2$
C	10.3 sec	63'	27'3"
z-scores (Total = 2.0)	$z = -\left(\frac{10.3 - 10.0}{0.2}\right) = -1.5$	$z = \frac{63 - 60}{3} = 1$	$z = \frac{27.25 - 26}{0.5} = 2.5$

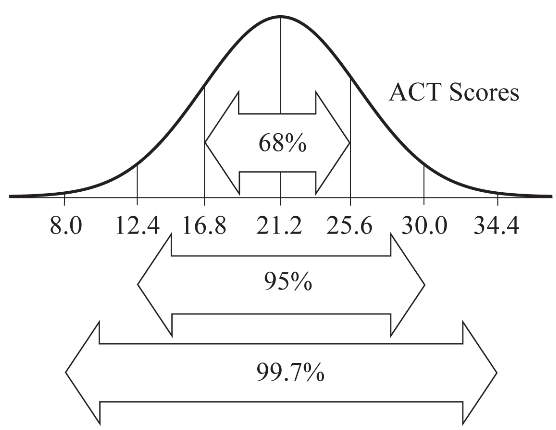
Tell

B was the winner, with a total of 2.5 standard deviations above the mean performance. C came in second and A third, with 2.0 and 1.5 standard deviations above the mean, respectively. And C' long jump was the most startling feat: 2.5 standard deviations longer than average.

3. Sketch Normal models using the 68-95-99.7 Rule:
   - Birth weights of babies, N(7.6 lb, 1.3 lb)
   - ACT scores at a certain college, N(21.2, 4.4)

Solution:





4. Use our knowledge of Normal models to guess plausible standard deviations for common variables – height of 16-yr old girls, weight of high school boys, current price of gasoline. If appropriate, you can base such estimates on the assumption that the distributions are likely to be unimodal and symmetric, then estimate that the range (excepting very extreme cases) is probably about ±3 standard deviations wide. Why can't we do this for the data we collected about the number of states students visited?

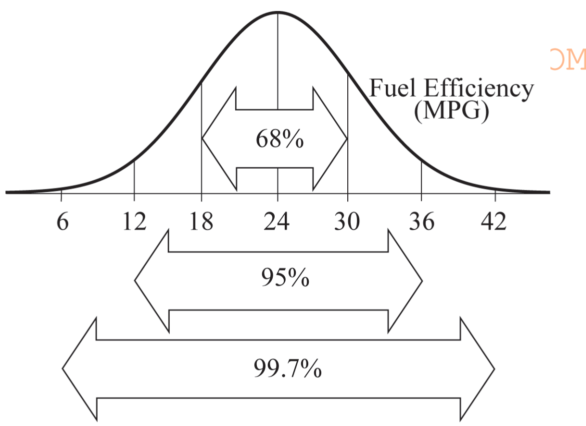
   Solution:

   If virtually all 16-year-old girls have heights between 58" and 74", a reasonable guess for the standard deviation might be (76 – 58)/6 = 3". Most high school boys would weigh between 90 and 250 pounds, so the standard deviation in weight might be about 25 pounds. If gasoline prices in a city varied from $2.00 to $2.25 per gallon, the standard deviation in price might be around $0.04 per gallon. It is reasonable to use the ±3 standard deviation estimate in these examples, since the distributions are plausibly Normal. One could not use the ±3 standard deviation estimate for the number of states students have visited. Since this distribution is likely to be skewed to the right, a Normal model is not appropriate.

5. Examine your M&Ms data. Are the number of candies or the weights plausibly Normal?

6. Suppose a Normal model describes the fuel efficiency of cars currently registered in your state. The mean is 24 mpg, with a standard deviation of 6 mpg.

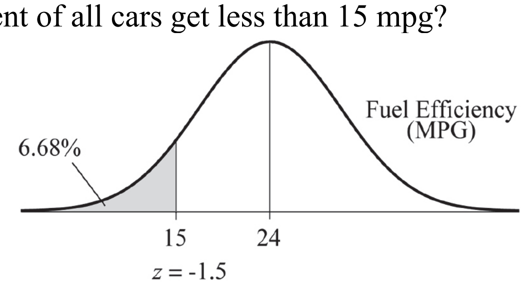
   - Sketch the Normal model.



According to the Normal model, we expect 68% of cars to get between 18 and 30 mpg, 95% of cars to get between 12 and 36 mpg, and 99.7% of cars to get between 6 and 42 mpg.

   - What percent of all cars get less than 15 mpg?

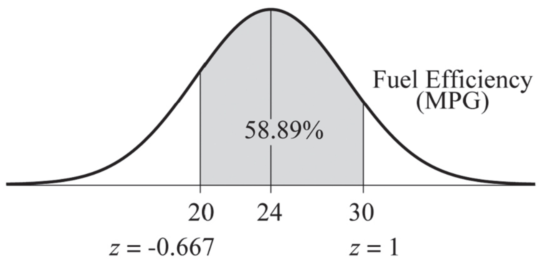
$$z = \frac{y-\mu}{\sigma}$$
$$z = \frac{15-24}{6}$$
$$z = -1.5$$



According to the Normal model, about 6.68% of cars are expected to get less than 15 mpg.

- What percent of all cars get between 20 and 30 mpg?

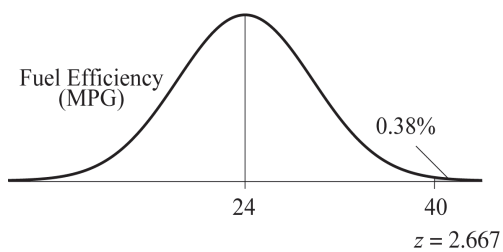
$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{20 - 24}{6}$$
$$z \approx -0.667$$



$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{30 - 24}{6}$$
$$z = 1$$

According to the Normal model, about 58.89% of cars are expected to get between 20 and 30 mpg.

- What percent of cars get more than 40 mpg?

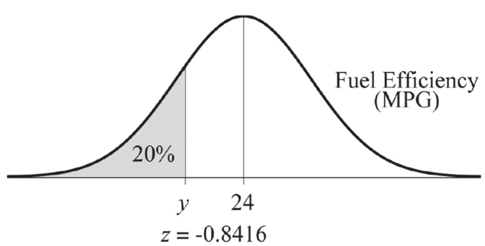


$$z = \frac{y - \mu}{\sigma}$$
$$z = \frac{40 - 24}{6}$$
$$z \approx 2.667$$

According to the Normal model, only about 0.38% of cars are expected to get more than 40 mpg.

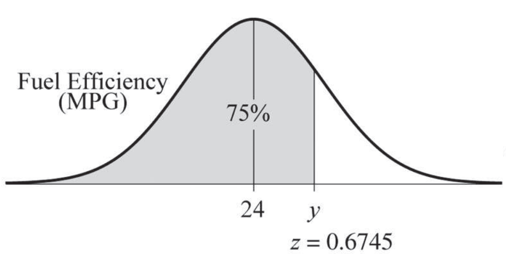
- Describe the fuel efficiency of the worst 20% of all cars.

$$z = \frac{y - \mu}{\sigma}$$
$$-0.8416 = \frac{y - 24}{6}$$
$$y \approx 18.95$$



According to the Normal model, the worst 20% of cars have fuel efficiency less than about 18.95 mpg.

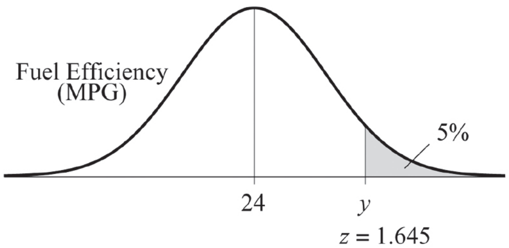
- What gas mileage represents the third quartile?



$$z = \frac{y - \mu}{\sigma}$$
$$0.6745 = \frac{y - 24}{6}$$
$$y \approx 28.05$$

According to the Normal model, the third quartile of the distribution of fuel efficiency is about 28.05 mpg.

- Describe the gas mileage of the most efficient 5% of all cars.



$$z = \frac{y-\mu}{\sigma}$$
$$1.645 = \frac{y-24}{6}$$
$$y \approx 33.87$$

According to the Normal model, we can expect the most efficient 5% of cars to have fuel efficiency in excess of 33.87 mpg.

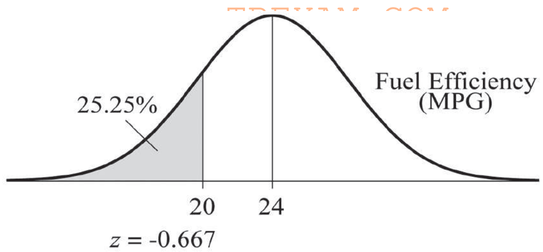
- What gas mileage would you consider unusual? Why?

Answer: It depends on your definition of unusual. If ±2 standard deviations is considered unusual, then, according to the Normal model, any gas mileage below 12 mpg or above 36 mpg would be considered unusual. If ±3 standard deviations is considered unusual, then, according to the Normal model, any gas mileage below 6 mpg or above 42 mpg would be considered unusual.

7. Consider the fuel economy model again: N(24,6). Sketch pictures as you answer these questions.

- What percent of cars get under 20 mpg?

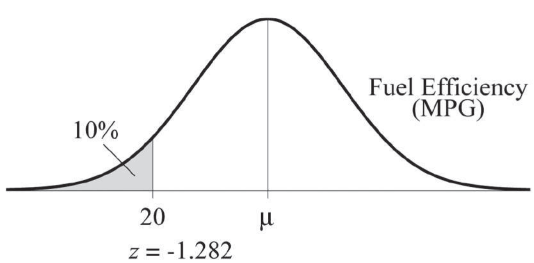
$$z = \frac{y-\mu}{\sigma}$$
$$z = \frac{20-24}{6}$$
$$z \approx -0.667$$



According to the Normal model, about 25.25% of cars have gas mileage less than 20 miles per gallon.

- An ecology group is lobbying for a national goal calling for no more than 10% of all cars to be under 20 mpg. If the standard deviation does not change what average fuel efficiency must be attained?

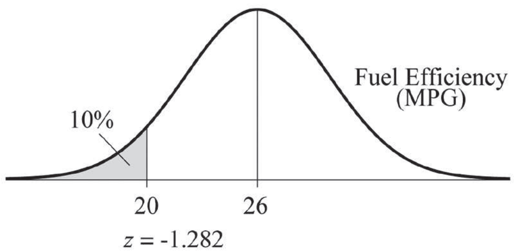
$$z = \frac{y-\mu}{\sigma}$$
$$-1.282 = \frac{20-\mu}{6}$$
$$\mu \approx 27.69$$



According to the Normal model, in order to meet the goal, the new mean mileage must be at least 27.69 mpg.

- Car manufacturers argue that they cannot raise the average that much – they believe they can only get to 26 mpg. What standard deviation would allow them to meet the "only 10% under 20 mpg" goal?

$$z = \frac{y - \mu}{\sigma}$$
$$-1.282 = \frac{20 - 26}{\sigma}$$
$$\sigma \approx 4.68$$



According to the Normal model, in order to meet the goal, the new standard deviation must be about 4.68 mpg.

- What change in the fuel economy of cars would achieving that standard deviation bring about? What are the advantages and disadvantages?
  Answer: A lower standard deviation would mean more consistent gas mileage. This would be advantageous, since the new goal of "only 10% below 20 mpg" would be met. However, it also means that only 10% of cars are expected to have gas mileage *higher than* 32 mpg.

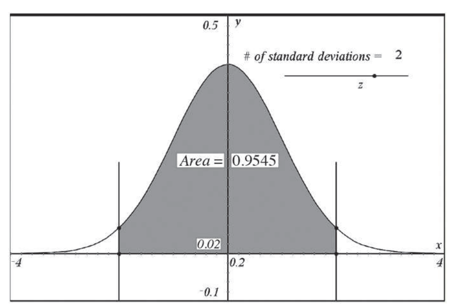
Resources

Web Links

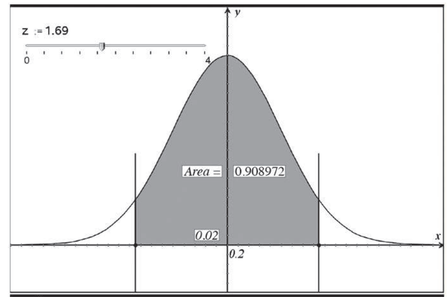
- If you want some math background on the derivation of the Normal model, look at the math talks on the NCSSM courses Web site. (courses.ncssm.edu/math/talks/index.htm)
- https://www.desmos.com/calculator/75plvl3xmp demonstrates how changes in mean and standard deviation impact Normal curve

TI-Nspire™ Demonstrations

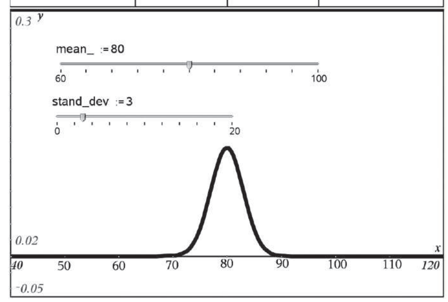
- The 68-95-99.7 Rule.
  - See it work for yourself.



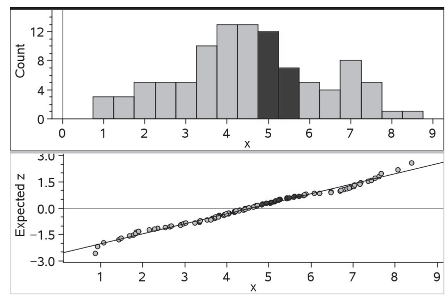
- Normal percentiles
  - Explore the relationship between *z*-scores and areas in a Normal model.
  - Demonstrate the change in the area between –*z* and +*z* as *z* changes.
  - Demonstrate the change in the area between –∞ and *z* as *z* changes.



- Normal Models
  - Watch the Normal model react as you change the mean and standard deviation.
  - Demonstrate the effect of increasing or decreasing the mean.
  - Demonstrate the effect of increasing or decreasing the standard deviation.



- Normal probability plots and histograms
  - See how the Normal probability plot responds as you change the shape of a distribution.
  - Determine the relationship between the skewness of a distribution and the shape of the Normal probability plot.
  - Demonstrate when is it reasonable to conclude that the distribution is approximately Normal, given a Normal probability plot.



Decisions Through Data

- Video Unit 7: Normal Distributions - provides a good introduction to Normal models and develops the 68-95-99.7 Rule. It includes an interview with the late Steven Jay Gould on using standardizing to explain the disappearance of .400 hitters in baseball.
- Video Unit 8: Normal Calculations - discusses standardizing, but looks at percentiles via the table. You might want to show only part of this.

Meaning from Data

- Lecture 4: Describing Dispersion or Measuring Spread
- Lecture 6: The Bell Curve
- Lecture 16: Sports – Who's Best of All Time?

Workshop Statistics

- Topic 15: Normal Distributions
  "Normal Curves" is an Activity that asks students to estimate means and standard deviations from the graphs of Normal models.
  "Random Normal Data" presents dotplots and asks students to decide whether it is reasonable to assume that a Normal model is appropriate.
  Several other activities deal with Normal models, the 68-95-99.7 Rule, and Normal percentiles.

Fifty Fathoms Demonstrations

- Demo 3: What Do Normal Data Look Like? – Normally-distributed data; The effect of changing mean and standard deviation

- Demo 4: Transforming the Mean and Standard Deviation – What happens to mean and standard deviation when you add a constant or every data value or multiply every data value by a constant

- Demo 5: The Mean is Least Squares, Too – Defining the mean as the place where the sum of squares of deviations is a minimum (just like the lease-squares line); the median and what it minimizes

Pearson Applet

- Play with Normal Model areas and cutpoints - http://www.pearsonhighered.com/mathstatsresources/

Assignments

Spread this chapter over about 5 days. First assign reading on shifting and scaling, and a few of the early exercises. Then move on to standardizing, having students read those sections and do about 4 or 5 exercises. Next use the 68-95-99.7 Rule to draw Normal models – again some reading and 4-5 exercises (cycling in the previous topics, too, as always). Finally, work on developing skills and understanding in working with Normal percentiles, which will take a couple of days. Start with some easy problems, then up the ante the second day. By now they will be doing 6-8 problems a night. Continue to assign a few of these exercises as you work on the Part I review in preparation for the Unit Test, coming soon. Four chapter quizzes are provided, but you may choose to use these as extra practice instead of quizzes.

Part I Review and Test

- Start with the Quick Review of Part I. Assign that as reading. Students can use it as a mental checklist.
- If they feel they need more clarification of issues in the Quick Review they can also read the Key Concepts and Skills sections in each chapter.
- And of course, reread parts of chapters about which they do not feel confident.
- Students should also look over old quizzes and tasks.
- You can start assigning a few problems from the review exercises as you are working through Chapter 5, and also get started on Chapter 7 as you continue to assign a few more.

About the Test

Students may not have time to finish the whole test, and may encounter questions they can't do. Warn them about this, and assure them that it's okay. They may not finish the AP* exam, and there may be questions there they cannot do. This is not an ordinary math course where all the questions look just like homework exercises and there's enough time to finish and check. If they can get 75% or better on the AP* exam they'll get a great score. The same is true of these tests.

Remember as you grade the test that numerical answers are just the beginning of a solution. Most questions will also ask for some kind of explanation or interpretation. As math teachers, we're used to grading calculations and equations, but less sure of ourselves when it comes to judging written work. One helpful suggestion appeared on the AP* Stat Teacher Community Discussion Board, courtesy of Dan Teague and Floyd Bullard (North Carolina School of Science and Mathematics):

> "I have found that one of the things that helps me most in making tests easier to grade is to decide very thoughtfully in advance what one or two things I want to focus on with a question…. And then I write a very simple rubric that usually assigns points to each idea on a scale of 0 to 3, where 0 = missed the idea entirely, 1 = mostly wrong, but there's at least some understanding there, 2 = mostly right, but there's still something important missing, and 3 = pretty much right on…. When I return the papers, I make the rubric known to the students."

AP Statistics – Classwork* *Chapter 5*

Life expectancy in the United States varies a bit from state to state. The lowest life expectancy at birth in 2012, 73.09 years, was in the District of Columbia. The highest, 80.23 years, was in Hawaii. The mean of all the state life expectancies is 77.19 years, with a standard deviation of 1.57 years.

1. a. Calculate the z-score for Washington, D.C.

   b. Interpret this z-score in context.

2. a. Connecticut has a z-score of 1.09. Interpret this z-score in context.

   b. Calculate the life expectancy at birth for Connecticut.

Life expectancy varies by race as well. Blacks in the United States tend to have shorter lives than whites. The mean of the state average life expectancies at birth for whites is 77.96 years, and the standard deviation is 1.29 years. For blacks, the mean is 72.33 years with a standard deviation of 1.46 years.

The state with the longest life expectancy for blacks is Massachusetts at 76.36 years. For whites, the longest life expectancy is in the District of Columbia at 81.54 years.

3. a. Which of these life expectancies is more extreme for the population?

   b. Use z-scores to explain your answer to part a.

4. Connecticut has a z-score of 1.70 for life expectancies for blacks and 1.09 for whites.

   a. What is the life expectancy at birth for blacks in Connecticut? For whites?

   b. You are asked what percentage of states have lower life expectancy for blacks than Connecticut. Explain why you don't have enough information to answer that question.