TT TEXTBOOK TESTBANKS

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INSTRUCTOR'S SOLUTIONS MANUAL

SAL SCIANDRA

Niagara County Community College

MATHEMATICS WITH APPLICATIONS
IN THE MANAGEMENT, NATURAL, AND SOCIAL SCIENCES
THIRTEENTH EDITION

FINITE MATHEMATICS WITH APPLICATIONS

IN THE MANAGEMENT, NATURAL, AND SOCIAL SCIENCES
THIRTEENTH EDITION

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Contents

Chapter 1 Algebra and Equations....................1

Chapter 2 Graphs, Lines, and Inequalities....................46

Chapter 3 Functions and Graphs....................92

Chapter 4 Exponential and Logarithmic Functions....................167

Chapter 5 The Mathematics of Personal Finance....................204

Chapter 6 Systems of Linear Equations and Matrices....................242

Chapter 7 Linear Programming....................323

Chapter 8 Sets and Probability....................428

Chapter 9 Counting, Probability Distributions, and Further Topics in Probability....................459

Chapter 10 Introduction to Statistics....................495

Chapter 11 Differential Calculus....................526

Chapter 12 Applications of the Derivative....................604

Chapter 13 Integral Calculus....................680

Chapter 14 Multivariate Calculus....................750

Chapter 1 Algebra and Equations

Section 1.1 The Real Numbers

1. True. This statement is true, since every integer can be written as the ratio of the integer and 1.
For example, $5 = \frac{5}{1}$.

2. False. For example, 5 is a real number, and $5 = \frac{10}{2}$ which is not an irrational number.

3. Answers vary with the calculator, but $\frac{2,508,429,787}{798,458,000}$ is the best.

4. $0 + (-7) = (-7) + 0$
This illustrates the commutative property of addition.

5. $6(t + 4) = 6t + 6 \cdot 4$
This illustrates the distributive property.

6. $3 + (-3) = (-3) + 3$
This illustrates the commutative property of addition.

7. $(-5) + 0 = -5$
This illustrates the identity property of addition.

8. $(-4)\bullet(\frac{-1}{4}) = 1$
This illustrates the multiplicative inverse property.

9. $8 + (12 + 6) = (8 + 12) + 6$
This illustrates the associative property of addition.

10. $1\bullet(-20) = -20$
This illustrates the identity property of multiplication.

11. Answers vary. One possible answer: The sum of a number and its additive inverse is the additive identity. The product of a number and its multiplicative inverse is the multiplicative identity.

12. Answers vary. One possible answer: When using the commutative property, the order of the addends or multipliers is changed, while the grouping of the addends or multipliers is changed when using the associative property.

For Exercises 13–16, let $p = -2$, $q = 3$ and $r = -5$.

13. $-3(p+5q) = -3[-2+5(3)] = -3[-2+15]$
$= -3(13) = -39$

14. $2(q-r) = 2(3+5) = 2(8) = 16$

15. $\frac{q+r}{q+p} = \frac{3+(-5)}{3+(-2)} = \frac{-2}{1} = -2$

16. $\frac{3q}{3p-2r} = \frac{3(3)}{3(-2)-2(-5)} = \frac{9}{-6+10} = \frac{9}{4}$

17. Let $r = 3.8$.
$\text{APR} = 12r = 12(3.8) = 45.6\%$

18. Let $r = 0.8$.
$\text{APR} = 12r = 12(0.8) = 9.6\%$

19. Let $APR = 11$.
$$\begin{aligned} APR &= 12r \\ 11 &= 12r \\ \frac{11}{12} &= r \\ r &\approx .9167\% \end{aligned}$$

20. Let $APR = 13.2$.
$$\begin{aligned} APR &= 12r \\ 13.2 &= 12r \\ \frac{13.2}{12} &= r \\ r &= 1.1\% \end{aligned}$$

21. $3 - 4 \cdot 5 + 5 = 3 - 20 + 5 = -17 + 5 = -12$

22. $8 - (-4)^2 - (-12)$
Take powers first.
$8 - 16 - (-12)$
Then add and subtract in order from left to right.
$8 - 16 + 12 = -8 + 12 = 4$

23. $(4-5)\cdot 6 + 6 = -1\cdot 6 + 6 = -6 + 6 = 0$

24. $\dfrac{2(3-7)+4(8)}{4(-3)+(-3)(-2)}$

Work above and below fraction bar. Do multiplications and work inside parentheses.

$= \dfrac{2(-4)+32}{-12+6} = \dfrac{-8+32}{-12+6} = \dfrac{24}{-6} = -4$

25. $8-4^2-(-12)$

Take powers first.

$8 - 16 - (-12)$

Then add and subtract in order from left to right.

$8 - 16 + 12 = -8 + 12 = 4$

26. $-(3-5)-\left[2-\left(3^2-13\right)\right]$

Take powers first.

$-(3 - 5) - [2 - (9 - 13)]$

Work inside brackets and parentheses.

$-(-2) - [2 - (-4)] = 2 - [2 + 4]$

$= 2 - 6 = -4$

27. $\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{\left(-\sqrt{16}\right)}}{\sqrt{64}-1}$

Work above and below fraction bar. Take roots.

$\dfrac{2(-3)+\frac{3}{(-2)}-\frac{2}{(-4)}}{8-1}$

Do multiplications and divisions.

$\dfrac{-6-\frac{3}{2}+\frac{1}{2}}{8-1}$

Add and subtract.

$\dfrac{-\frac{12}{2}-\frac{3}{2}+\frac{1}{2}}{7} = \dfrac{-\frac{14}{2}}{7} = \dfrac{-7}{7} = -1$

28. $\dfrac{6^2-3\sqrt{25}}{\sqrt{6^2+13}}$

Take powers and roots.

$\dfrac{36-3(5)}{\sqrt{36+13}} = \dfrac{36-15}{\sqrt{49}} = \dfrac{21}{7} = 3$

29. $\dfrac{2040}{523}, \dfrac{189}{37}, \sqrt{27}, \dfrac{4587}{691}, 6.735, \sqrt{47}$

30. $\dfrac{187}{63}, 2.9884, \sqrt{\sqrt{85}}, \pi, \sqrt{10}, \dfrac{385}{117}$

31. 12 is less than 18.5.

$12 < 18.5$

32. –2 is greater than –20.

$-2 > -20$

33. x is greater than or equal to 5.7.

$x \ge 5.7$

34. y is less than or equal to –5.

$y \le -5$

35. z is at most 7.5.

$z \le 7.5$

36. w is negative.

$w < 0$

37. $-6 < -2$

38. $3.14 < \pi$

39. $3/4 = .75$

40. $1/3 > .33$

41. a lies to the right of b or is equal to b.

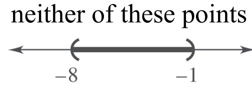
42. $b + c = a$

43. $c < a < b$

44. a lies to the right of 0

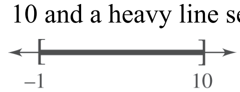
45. $(-8, -1)$

This represents all real numbers between –8 and –1, not including –8 and –1. Draw parentheses at –8 and –1 and a heavy line segment between them. The parentheses at –8 and –1 show that neither of these points belongs to the graph.



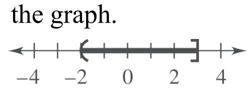
46. $[-1, 10]$

This represents all real numbers between –1 and 10, including –1 and 10. Draw brackets at –1 and 10 and a heavy line segment between them.



47. $(-2, 3]$

This represents all real numbers x such that $-2 < x \le 3$. Draw a heavy line segment from –2 to 3. Use a parenthesis at –2 since it is not part of the graph. Use a bracket at 3 since it is part of the graph.

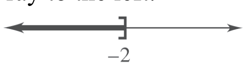


48. $[-2, 2)$
This represents all real numbers between –2 and 2, including –2, not including 2.
Draw a bracket at –2, a parenthesis at 2, and a heavy line segment between them.

49. $(-2, \infty)$

This represents all real numbers x such that $x > -2$. Start at –2 and draw a heavy line segment to the right. Use a parenthesis at –2 since it is not part of the graph.

50. $(-\infty, -2]$
This represents all real numbers less than or equal to –2. Draw a bracket at –2 and a heavy ray to the left.



51. $|-9| - |-12| = 9 - (12) = -3$

52. $|8| - |-4| = 8 - (4) = 4$

53. $-|-4| - |-1 - 14| = -(4) - |-15|$
$= -(4) - 15 = -19$

54. $-|6| - |-12 - 4| = -(6) - |-16| = -6 - (16) = -22$

55. $|5| \_\_\_ |-5|$
$5 \_\_ 5$
$5 = 5$

56. $-|-4| \_\_\_ |4|$
$-4 \_\_ 4$
$-4 < 4$

57. $|10 - 3| \_\_\_ |3 - 10|$
$|7| \_\_\_ |-7|$
$7 \_\_ 7$
$7 = 7$

58. $|6 - (-4)| \_\_\_ |-4 - 6|$
$|10| \_\_\_ |-10|$
$10 \_\_\_ 10$
$10 = 10$

59. $|-2 + 8| \_\_\_ |2 - 8|$
$|6| \_\_\_ |-6|$
$6 \_\_\_ 6$
$6 = 6$

60. $|3| \cdot |-5| \_\_\_ |3(-5)|$
$|3| \cdot |-5| \_\_\_ |-15|$
$3 \cdot 5 \_\_\_ 15$
$15 = 15$

61. $|3 - 5| \_\_\_ |3| - |5|$
$|-2| \_\_\_ 3 - 5$
$2 \_\_\_ -2$
$2 > -2$

62. $|-5 + 1| \_\_\_ |-5| + |1|$
$|-4| \_\_\_ 5 + 1$
$4 \_\_\_ 6$
$4 < 6$

63. When $a < 7$, $a - 7$ is negative.
So $|a - 7| = -(a - 7) = 7 - a$.

64. When $b \geq c$, $b - c$ is positive.
So $|b - c| = b - c$.

Answers will vary for exercises 65–67. Sample answers are given.

65. No, it is not always true that $|a + b| = |a| + |b|$. For example, let $a = 1$ and $b = -1$. Then,
$|a + b| = |1 + (-1)| = |0| = 0$, but
$|a| + |b| = |1| + |(-1)| = 1 + 1 = 2$.

66. Yes, if a and b are any two real numbers, it is always true that $|a - b| = |b - a|$. In general, $a - b = -(b - a)$. When we take the absolute value of each side, we get
$|a - b| = |-(b - a)| = |b - a|$.

67. For females, the height inequality will be: $|x-63.5|\le 4.4$.

68. For males, the height inequality will be: $|x-69.0|\le 4.8$

69. 0

70. 9; 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020

71. 10; 2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020

72. 4; 2013, 2015, 2016, 2020

73. 4; 2011, 2012, 2017, 2018

74. 8; 2011, 2012, 2013, 2014, 2016, 2017, 2018, 2019

75. $|73.7-(-2.7)|=|76.4|=76.4\%$

76. $|-31.1-12.5|=|-43.6|=43.6\%$

77. $|-41.9-73.7|=|-115.6|=115.6\%$

78. $|-31.1-0.1|=|-31.2|=31.2\%$

79. $|-41.9-(-31.1)|=|-10.8|=10.8\%$

80. $|12.5-73.7|=|-61.2|=61.2\%$

81. 5; 2013, 2014, 2017, 2018, 2019

82. 5; 2015, 2016, 2017, 2018, 2019

83. 4; 2016, 2017, 2018, 2019

84. 2; 2018, 2019

Section 1.2 Polynomials

1. $11.2^6 \approx 1{,}973{,}822.685$

2. $(-6.54)^{11} \approx -936{,}171{,}103.1$

3. $\left(-\frac{18}{7}\right)^6 \approx 289.0991339$

4. $\left(\frac{5}{9}\right)^7 \approx .0163339967$

5. -3^2 is negative, whereas $(-3)^2$ is positive. Both -3^3 and $(-3)^3$ are negative.

6. To multiply 4^3 and 4^5, add the exponents since the bases are the same. The product of 4^3 and 3^4 cannot be found in the same way since the bases are different. To evaluate the product, first do the powers, and then multiply the results.

7. $4^2\cdot 4^3 = 4^{2+3} = 4^5$

8. $(-4)^4\cdot(-4)^6 = (-4)^{4+6} = (-4)^{10}$

9. $(-6)^2\cdot(-6)^5 = (-6)^{2+5} = (-6)^7$

10. $(2z)^5\cdot(2z)^6 = (2z)^{5+6} = (2z)^{11}$

11. $\left[(5u)^4\right]^7 = (5u)^{4\cdot 7} = (5u)^{28}$

12. $(6y)^3\cdot\left[(6y)^5\right]^4 = (6y)^3\cdot(6y)^{20}$
$= (6y)^{23}$

13. degree 4; coefficients: 6.2, –5, 4, –3, 3.7; constant term 3.7.

14. degree 7; coefficients: 6, 4, 0, 0, –1, 0, 1, 0; constant term 0.

15. Since the highest power of x is 3, the degree is 3.

16. Since the highest power of x is 5, the degree is 5.

17. $(3x^3+2x^2-5x)+(-4x^3-x^2-8x)$
$=(3x^3-4x^3)+(2x^2-x^2)+(-5x-8x)$
$=-x^3+x^2-13x$

18. $(-2p^3-5p+7)+(-4p^2+8p+2)$
$=-2p^3-4p^2+(-5p+8p)+(7+2)$
$=-2p^3-4p^2+3p+9$

19. $\left(-4y^2-3y+8\right)-\left(2y^2-6y+2\right)$
$=\left(-4y^2-3y+8\right)+\left(-2y^2+6y-2\right)$
$=-4y^2-3y+8-2y^2+6y-2$
$=\left(-4y^2-2y^2\right)+(-3y+6y)+(8-2)$
$=-6y^2+3y+6$

20. $\left(7b^2+2b-5\right)-\left(3b^2+2b-6\right)$
$=\left(7b^2+2b-5\right)+\left(-3b^2-2b+6\right)$
$=\left(7b^2-3b^2\right)+(2b-2b)+(-5+6)$
$=4b^2+1$

21. $\left(2x^3+2x^2+4x-3\right)-\left(2x^3+8x^2+1\right)$
$=\left(2x^3+2x^2+4x-3\right)+\left(-2x^3-8x^2-1\right)$
$=2x^3+2x^2+4x-3-2x^3-8x^2-1$
$=\left(2x^3-2x^3\right)+\left(2x^2-8x^2\right)+(4x)+(-3-1)$
$=-6x^2+4x-4$

22. $\left(3y^3+9y^2-11y+8\right)-\left(-4y^2+10y-6\right)$
$=\left(3y^3+9y^2-11y+8\right)+\left(4y^2-10y+6\right)$
$=3y^3+\left(9y^2+4y^2\right)+(-11y-10y)+(8+6)$
$=3y^3+13y^2-21y+14$

23. $-9m\left(2m^2+6m-1\right)$
$=(-9m)\left(2m^2\right)+(-9m)(6m)+(-9m)(-1)$
$=-18m^3-54m^2+9m$

24. $2a\left(4a^2-6a+8\right)$
$=2a\left(4a^2\right)+2a(-6a)+2a(8)$
$=8a^3-12a^2+16a$

25. $(3z+5)\left(4z^2-2z+1\right)$
$=(3z)\left(4z^2-2z+1\right)+(5)\left(4z^2-2z+1\right)$
$=12z^3-6z^2+3z+20z^2-10z+5$
$=12z^3+14z^2-7z+5$

26. $(2k+3)\left(4k^3-3k^2+k\right)$
$=2k\left(4k^3-3k^2+k\right)+3\left(4k^3-3k^2+k\right)$
$=8k^4-6k^3+2k^2+12k^3-9k^2+3k$
$=8k^4+6k^3-7k^2+3k$

27. $(6k-1)(2k+3)$
$=(6k)(2k+3)+(-1)(2k+3)$
$=12k^2+18k-2k-3$
$=12k^2+16k-3$

28. $(8r+3)(r-1)$
Use FOIL.
$=8r^2-8r+3r-3$
$=8r^2-5r-3$

29. $(3y+5)(2y+1)$
Use FOIL.
$=6y^2+3y+10y+5$
$=6y^2+13y+5$

30. $(5r-3s)(5r-4s)$
$=25r^2-20rs-15rs+12s^2$

$=25r^2-35rs+12s^2$

31. $(9k+q)(2k-q)$
$=18k^2-9kq+2kq-q^2$
$=18k^2-7kq-q^2$

32. $(.012x-.17)(.3x+.54)$
$=(.012x)(.3x)+(.012x)(.54)$
$+(-.17)(.3x)+(-.17)(.54)$
$=.0036x^2+.00648x-.051x-.0918$
$=.0036x^2-.04452x-.0918$

33. $(6.2m-3.4)(.7m+1.3)$
$=4.34m^2+8.06m-2.38m-4.42$
$=4.34m^2+5.68m-4.42$

34. $2p-3[4p-(8p+1)]$
$=2p-3(4p-8p-1)$
$=2p-3(-4p-1)$
$=2p+12p+3$
$=14p+3$

35. $5k-[k+(-3+5k)]$
$=5k-[6k-3]$
$=5k-6k+3$
$=-k+3$

36. $(3x-1)(x+2)-(2x+5)^2$
$$=\left(3x^2+5x-2\right)-\left(4x^2+20x+25\right)$$
$$=3x^2+5x-2-4x^2-20x-25$$
$$=\left(3x^2-4x^2\right)+(5x-20x)+(-2-25)$$
$$=-x^2-15x-27$$

37. $R = 5\,(1000x) = 5000x$
$C = 200{,}000 + 1800x$
$P = (5000x) - (200{,}000 + 1800x)$
$= 3200x - 200{,}000$

38. $R = 8.50(1000x) = 8500x$
$C = 225{,}000 + 4200x$
$P = (8500x) - (225{,}000 + 4200x)$
$= 4300x - 225{,}000$

39. **a.** $R = 9.75(1000x) = 9750x$

b. $C = 260{,}000 + (-3x^2 + 3480x - 325)$
$= -3x^2 + 3480x + 259{,}675$

c. $P = (9750x) - (-3x^2 + 3480x + 259{,}675)$
$= 3x^2 + 6270x - 259{,}675$

40. **a.** $R = 23.50(1000x) = 23{,}500x$

b. $C = 145{,}000 + (-4.2x^2 + 3220x - 425)$
$= -4.2x^2 + 3220x + 144{,}575$

c. $P = (23{,}500x) - (-4.2x^2 + 3220x + 144{,}575)$
$= 4.2x^2 + 20{,}280x - 144{,}575.$

41. **a.** According to the bar graph, the net earnings in 2013 were \$147 billion.

b. Let $x = 13$.
$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(13)^3 + 3.77(13)^2 - 49.1(13) + 338$
$= 141.5167$
According to the polynomial, the net earnings in 2013 were approximately \$142 billion.

42. **a.** According to the bar graph, the net earnings in 2016 were \$152 billion.

b. Let $x = 16$.
$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(16)^3 + 3.77(16)^2 - 49.1(16) + 338$
$= 153.3856$

According to the polynomial, the net earnings in 2016 were approximately \$153 billion.

43. **a.** According to the bar graph, the net earnings in 2018 were \$160 billion.

b. Let $x = 18$.
$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(18)^3 + 3.77(18)^2 - 49.1(18) + 338$
$= 157.2152$

According to the polynomial, the net earnings in 2018 were approximately \$157 billion.

44. **a.** According to the bar graph, the net earnings in 2019 were \$156 billion.

b. Let $x = 19$.
$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(19)^3 + 3.77(19)^2 - 49.1(19) + 338$
$= 156.3049$

According to the polynomial, the net earnings in 2019 were approximately \$156 billion.

45. Let $x = 20$.
$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(20)^3 + 3.77(20)^2 - 49.1(20) + 338$
$= 152.8$
According to the polynomial, the net earnings in 2020 will be approximately \$153 billion.

46. Let $x = 21$.

$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(21)^3 + 3.77(21)^2 - 49.1(21) + 338$
$= 146.1671$

According to the polynomial, the net earnings in 2021 will be approximately \$146 billion.

47. Let $x = 22$.

$-.0889x^3 + 3.77x^2 - 49.1x + 338$
$= -.0889(22)^3 + 3.77(22)^2 - 49.1(22) + 338$
$= 135.8728$

According to the polynomial, the net earnings in 2022 will be approximately \$136 billion.

48. The figures for 2020 – 2022 seem to continuously decrease. It seems unlikely revenue would continue to keep dropping.

For exercises 49–52, we use the polynomial $.0013x^3 - .173x^2 + 6.0x - 42.3$.

49. Let $x = 15$.

$.0013(15)^3 - .173(15)^2 + 6.0(15) - 42.3$
$= 12.7125$

Thus, the costs were approximately \$12.7 billion in 2015. The statement is true.

50. Let $x = 18$.

$.0013(18)^3 - .173(18)^2 + 6.0(18) - 42.3$
$= 17.2296$

Thus, the costs were approximately \$17.2 billion in 2018. The statement is true.

51. Let $x = 20$.

$.0013(20)^3 - .173(20)^2 + 6.0(20) - 42.3$
$= 18.9$

Thus, the costs were approximately \$18.9 billion in 2020. The statement is false.

52. Let $x = 16$.

$.0013(16)^3 - .173(16)^2 + 6.0(16) - 42.3$
$= 14.7368$

Thus, the costs were approximately \$14.7 billion in 2016. The statement is true.

For exercises 53–58, we use the polynomial $-.0471x^3 + 2.089x^2 - 28.5x + 135.2$.

53. Let $x = 12$.

$-.0471x^3 + 2.089x^2 - 28.5x + 135.2$
$= -.0471(12)^3 + 2.089(12)^2 - 28.5(12) + 135.2$
$= 12.6272$

Thus, the revenue for the Starbuck's Corporation Inc in 2012 was approximately \$12.6 billion.

54. Let $x = 17$.

$-.0471x^3 + 2.089x^2 - 28.5x + 135.2$
$= -.0471(17)^3 + 2.089(17)^2 - 28.5(17) + 135.2$
$= 23.0187$

Thus, the revenue for the Starbuck's Corporation Inc in 2017 was approximately \$23.0 billion.

55. Let $x = 19$.

$-.0471x^3 + 2.089x^2 - 28.5x + 135.2$
$= -.0471(19)^3 + 2.089(19)^2 - 28.5(19) + 135.2$
$= 24.7701$

Thus, the revenue for the Starbuck's Corporation Inc in 2019 was approximately \$24.8 billion.

56. Let $x = 20$.

$-.0471x^3 + 2.089x^2 - 28.5x + 135.2$
$= -.0471(20)^3 + 2.089(20)^2 - 28.5(20) + 135.2$
$= 24$

Thus, the revenue for the Starbuck's Corporation Inc in 2020 was approximately \$24.0 billion.

57. By comparing the answers to problems 54 and 56, the profit was higher in 2020.

58. Let $x = 18$.

$-.0471x^3 + 2.089x^2 - 28.5x + 135.2$
$= -.0471(18)^3 + 2.089(18)^2 - 28.5(18) + 135.2$
$= 24.3488$

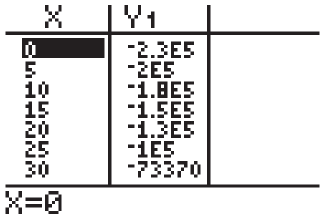
Thus, the revenue for the Starbuck's Corporation Inc in 2018 was approximately \$24.3 billion.

Let $x = 19$.

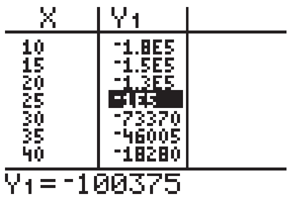
$-.0471x^3 + 2.089x^2 - 28.5x + 135.2$
$= -.0471(19)^3 + 2.089(19)^2 - 28.5(19) + 135.2$
$= 24.7701$

Thus, the revenue for the Starbuck's Corporation Inc in 2019 was approximately \$24.8 billion. Therefore, the profit was higher in 2019.

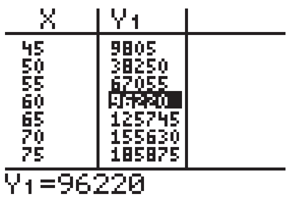
59. $P = 7.2x^2 + 5005x - 230,000$. Here is part of the screen capture.



For 25,000, the loss will be $100,375;



For 60,000, there profit will be $96,220.

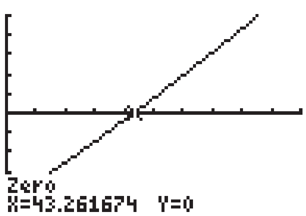


There is a loss at the beginning because of large fixed costs. When more items are made, these costs become a smaller portion of the total costs.

60. In order for the company to make a profit,

$P = 7.2x^2 + 5005x - 230,000 > 0$

Graph the function and locate a zero.



[0, 100] by [–250000, 250000]

The zero is at $x \approx 4$. Therefore, between 40,000 and 45,000 calculators must be sold for the company to make a profit.

61. Let x = 100 (in thousands)

$7.2(100)^2 + 5005(100) - 230,000 = 342,500$

The profit for selling 100,000 calculators is $342,500.

62. Let x = 150 (in thousands)

$7.2(150)^2 + 5005(150) - 230,000 = 682,750$

The profit for selling 150,000 calculators is $682,750.

Section 1.3 Factoring

1. $12x^2 - 24x = 12x \cdot x - 12x \cdot 2 = 12x(x-2)$

2. $5y - 65xy = 5y(1) - 5y(13x) = 5y(1-13x)$

3. $r^3 - 5r^2 + r = r\left(r^2\right) - r(5r) + r(1)$

$= r\left(r^2 - 5r + 1\right)$

4. $t^3 + 3t^2 + 8t = t\left(t^2\right) + t(3t) + t(8)$

$= t\left(t^2 + 3t + 8\right)$

5. $6z^3 - 12z^2 + 18z$

$= 6z\left(z^2\right) - 6z(2z) + 6z(3)$

$= 6z\left(z^2 - 2z + 3\right)$

6. $5x^3 + 55x^2 + 10x$

$= 5x\left(x^2\right) + 5x(11x) + 5x(2)$

$= 5x\left(x^2 + 11x + 2\right)$

7. $3(2y-1)^2 + 7(2y-1)^3$

$= (2y-1)^2(3) + (2y-1)^2 \cdot 7(2y-1)$

$= (2y-1)^2[3 + 7(2y-1)]$

$= (2y-1)^2(3 + 14y - 7)$

$= (2y-1)^2(14y - 4)$

$= 2(2y-1)^2(7y-2)$

8. $(3x+7)^5 - 4(3x+7)^3$
$= (3x+7)^3(3x+7)^2 - (3x+7)^3(4)$
$= (3x+7)^3\left[(3x+7)^2 - 4\right]$
$= (3x+7)^3\left(9x^2+42x+49-4\right)$
$= (3x+7)^3\left(9x^2+42x+45\right)$

9. $3(x+5)^4 + (x+5)^6$
$= (x+5)^4 \cdot 3 + (x+5)^4(x+5)^2$
$= (x+5)^4\left[3+(x+5)^2\right]$
$= (x+5)^4\left(3+x^2+10x+25\right)$
$= (x+5)^4\left(x^2+10x+28\right)$

10. $3(x+6)^2 + 6(x+6)^4$
$= 3(x+6)^2(1) + 3(x+6)^2\left[2(x+6)^2\right]$
$= 3(x+6)^2\left[1+2(x+6)^2\right]$
$= 3(x+6)^2\left[1+2\left(x^2+12x+36\right)\right]$
$= 3(x+6)^2\left(1+2x^2+24x+72\right)$
$= 3(x+6)^2\left(2x^2+24x+73\right)$

11. $x^2+5x+4 = (x+1)(x+4)$

12. $u^2+7u+6 = (u+1)(u+6)$

13. $x^2+7x+12 = (x+3)(x+4)$

14. $y^2+8y+12 = (y+2)(y+6)$

15. $x^2+x-6 = (x+3)(x-2)$

16. $x^2+4x-5 = (x+5)(x-1)$

17. $x^2+2x-3 = (x+3)(x-1)$

18. $y^2+y-12 = (y+4)(y-3)$

19. $x^2-3x-4 = (x+1)(x-4)$

20. $u^2-2u-8 = (u+2)(u-4)$

21. $z^2-9z+14 = (z-2)(z-7)$

22. $w^2-6w-16 = (w+2)(w-8)$

23. $z^2+10z+24 = (z+4)(z+6)$

24. $r^2+16r+60 = (r+6)(r+10)$

25. $2x^2-9x+4 = (2x-1)(x-4)$

26. $3w^2-8w+4 = (3w-2)(w-2)$

27. $15p^2-23p+4 = (3p-4)(5p-1)$

28. $8x^2-14x+3 = (4x-1)(2x-3)$

29. $4z^2-16z+15 = (2z-5)(2z-3)$

30. $12y^2-29y+15 = (3y-5)(4y-3)$

31. $6x^2-5x-4 = (2x+1)(3x-4)$

32. $12z^2+z-1 = (4z-1)(3z+1)$

33. $10y^2+21y-10 = (5y-2)(2y+5)$

34. $15u^2+4u-4 = (5u-2)(3u+2)$

35. $6x^2+5x-4 = (2x-1)(3x+4)$

36. $12y^2+7y-10 = (3y-2)(4y+5)$

37. $3a^2+2a-5 = (3a+5)(a-1)$

38. $6a^2-48a-120 = 6\left(a^2-8a-20\right)$
$= 6(a-10)(a+2)$

39. $x^2-81 = x^2-(9)^2 = (x+9)(x-9)$

40. $x^2+17xy+72y^2 = (x+8y)(x+9y)$

41. $9p^2-12p+4 = (3p)^2-2(3p)(2)+2^2$
$= (3p-2)^2$

42. $3r^2-r-2 = (3r+2)(r-1)$

43. $r^2 + 3rt - 10t^2 = (r - 2t)(r + 5t)$

44. $2a^2 + ab - 6b^2 = (2a - 3b)(a + 2b)$

45. $m^2 - 8mn + 16n^2 = (m)^2 - 2(m)(4n) + (4n)^2$
$= (m - 4n)^2$

46. $8k^2 - 16k - 10 = 2(4k^2 - 8k - 5)$
$= 2(2k + 1)(2k - 5)$

47. $4u^2 + 12u + 9 = (2u + 3)^2$

48. $9p^2 - 16 = (3p)^2 - 4^2 = (3p - 4)(3p + 4)$

49. $25p^2 - 10p + 4$
This polynomial cannot be factored

50. $10x^2 - 17x + 3 = (5x - 1)(2x - 3)$

51. $4r^2 - 9v^2 = (2r + 3v)(2r - 3v)$

52. $x^2 + 3xy - 28y^2 = (x + 7y)(x - 4y)$

53. $x^2 + 4xy + 4y^2 = (x + 2y)^2$

54. $16u^2 + 12u - 18 = 2(8u^2 + 6u - 9)$
$= 2(4u - 3)(2u + 3)$

55. $3a^2 - 13a - 30 = (3a + 5)(a - 6)$

56. $3k^2 + 2k - 8 = (3k - 4)(k + 2)$

57. $21m^2 + 13mn + 2n^2 = (7m + 2n)(3m + n)$

58. $81y^2 - 100 = (9y + 10)(9y - 10)$

59. $y^2 - 4yz - 21z^2 = (y - 7z)(y + 3z)$

60. $49a^2 + 9$
This polynomial cannot be factored.

61. $121x^2 - 64 = (11x + 8)(11x - 8)$

62. $4z^2 + 56zy + 196y^2$
$= 4(z^2 + 14zy + 49y^2)$
$= 4\left[z^2 + 2(z)(7y) + (7y)^2\right] = 4(z + 7y)^2$

63. $a^3 - 64 = a^3 - (4)^3 = (a - 4)(a^2 + 4a + 16)$

64. $b^3 + 216 = b^3 + 6^3 = (b + 6)(b^2 - 6b + 36)$

65. $8r^3 - 27s^3$
$= (2r)^3 - (3s)^3$
$= (2r - 3s)\left[(2r)^2 + (2r)(3s) + (3s)^2\right]$
$= (2r - 3s)(4r^2 + 6rs + 9s^2)$

66. $1000p^3 + 27q^3$
$= (10p)^3 + (3q)^3$
$= (10p + 3q)(100p^2 - 30pq + 9q^2)$

67. $64m^3 + 125$
$= (4m)^3 + (5)^3$
$= (4m + 5)\left[(4m)^2 - (4m)(5) + (5)^2\right]$
$= (4m + 5)(16m^2 - 20m + 25)$

68. $216y^3 - 343$
$= (6y)^3 - (7)^3$
$= (6y - 7)(36y^2 + 42y + 49)$

69. $1000y^3 - z^3$
$= (10y)^3 - (z)^3$
$= (10y - z)\left[(10y)^2 + (10y)(z) + (z)^2\right]$
$= (10y - z)(100y^2 + 10yz + z^2)$

70. $125p^3 + 8q^3 = (5p)^3 + (2q)^3$
$= (5p + 2q)(25p^2 - 10pq + 4q^2)$

71. $x^4 + 5x^2 + 6 = (x^2 + 2)(x^2 + 3)$

72. $y^4 + 7y^2 + 10 = (y^2 + 2)(y^2 + 5)$

73. $b^4 - b^2 = b^2\left(b^2 - 1\right) = b^2(b+1)(b-1)$

74. $$\begin{aligned} z^4 - 3z^2 - 4 &= \left(z^2 - 4\right)\left(z^2 + 1\right) \\ &= (z+2)(z-2)\left(z^2+1\right) \end{aligned}$$

75. $$\begin{aligned} x^4 - x^2 - 12 &= \left(x^2 - 4\right)\left(x^2 + 3\right) \\ &= (x+2)(x-2)\left(x^2+3\right) \end{aligned}$$

76. $$\begin{aligned} 4x^4 + 27x^2 - 81 &= \left(4x^2 - 9\right)\left(x^2 + 9\right) \\ &= (2x+3)(2x-3)\left(x^2+9\right) \end{aligned}$$

77. $$\begin{aligned} 16a^4 - 81b^4 &= \left(4a^2 - 9b^2\right)\left(4a^2 + 9b^2\right) \\ &= (2a+3b)(2a-3b)\left(4a^2+9b^2\right) \end{aligned}$$

78. $$\begin{aligned} x^6 - y^6 &= \left(x^2\right)^3 - \left(y^2\right)^3 \\ &= \left(x^2 - y^2\right)\left(x^4 + x^2y^2 + y^4\right) \\ &= (x+y)(x-y)\left(x^2+xy+y^2\right)\cdot \\ &\qquad \left(x^2 - xy + y^2\right) \end{aligned}$$

79. $$\begin{aligned} x^8 + 8x^2 &= x^2\left(x^6 + 8\right) = x^2\left(\left(x^2\right)^3 + 2^3\right) \\ &= x^2\left(x^2+2\right)\left(x^4 - 2x^2 + 4\right) \end{aligned}$$

80. $$\begin{aligned} x^9 - 64x^3 &= x^3\left(x^6 - 64\right) = x^3\left(x^6 - 2^6\right) \\ &= x^3\left[\left(x^3\right)^2 - \left(2^3\right)^2\right] \\ &= x^3\left(x^3 - 2^3\right)\left(x^3 + 2^3\right) \\ &= x^3(x-2)\left(x^2+2x+4\right)\cdot \\ &\qquad (x+2)\left(x^2 - 2x + 4\right) \end{aligned}$$

81. $6x^4 - 3x^2 - 3 = \left(2x^2+1\right)\left(3x^2-3\right)$ is not the correct complete factorization because $3x^2 - 3$ contains a common factor of 3. This common factor should be factored out as the first step. This will reveal a difference of two squares, which requires further factorization. The correct factorization is
$$\begin{aligned} 6x^4 - 3x^2 - 3 &= 3\left(2x^4 - x^2 - 1\right) \\ &= 3\left(2x^2+1\right)\left(x^2-1\right) \\ &= 3\left(2x^2+1\right)(x+1)(x-1) \end{aligned}$$

82. The sum of two squares can be factored when the terms have a common factor. An example is
$(3x)^2 + 3^2 = 9x^2 + 9 = 9(x^2+1)$

83. $$\begin{aligned} (x+2)^3 &= (x+2)(x+2)^2 \\ &= (x+2)(x^2+4x+4) \\ &= x^3 + 4x^2 + 2x^2 + 8x + 4x + 8 \\ &= x^3 + 6x^2 + 12x + 8, \end{aligned}$$
which is not equal to $x^3 + 8$. The correct factorization is $x^3 + 8 = (x+2)(x^2 - 2x + 4)$.

84. Factoring and multiplication are inverse operations. If we factor a polynomial and then multiply the factors, we get the original polynomial. For example, we can factor $x^2 - x - 6$ to get $(x-3)(x+2)$. Then if we multiply the factors, we get
$(x-3)(x+2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$

Section 1.4 Rational Expressions

1. $\dfrac{8x^2}{56x} = \dfrac{x\cdot 8x}{7\cdot 8x} = \dfrac{x}{7}$

2. $\dfrac{27m}{81m^3} = \dfrac{27m}{27m\cdot 3m^2} = \dfrac{1}{3m^2}$

3. $\dfrac{25p^2}{35p^3} = \dfrac{5\cdot 5p^2}{7p\cdot 5p^2} = \dfrac{5}{7p}$

4. $\dfrac{18y^4}{24y^2} = \dfrac{6y^2\cdot 3y^2}{6y^2\cdot 4} = \dfrac{3y^2}{4}$

5. $\frac{5m+15}{4m+12} = \frac{5(m+3)}{4(m+3)} = \frac{5}{4}$

6. $\frac{10z+5}{20z+10} = \frac{5(2x+1)}{5(2x+1)\cdot 2} = \frac{1}{2}$

7. $\frac{4(w-3)}{(w-3)(w+6)} = \frac{4}{w+6}$

8. $\frac{-6(x+2)}{(x+4)(x+2)} = \frac{-6}{x+4}$ or $-\frac{6}{x+4}$

9. $\frac{3y^2-12y}{9y^3} = \frac{3y(y-4)}{3y\left(3y^2\right)}$
$= \frac{y-4}{3y^2}$

10. $\frac{15k^2+45k}{9k^2} = \frac{15k(k+3)}{3k\cdot 3k}$
$= \frac{3k\cdot 5(k+3)}{3k\cdot 3k}$
$= \frac{5(k+3)}{3k}$

11. $\frac{m^2-4m+4}{m^2+m-6} = \frac{(m-2)(m-2)}{(m+3)(m-2)} = \frac{m-2}{m+3}$

12. $\frac{r^2-r-6}{r^2+r-12} = \frac{(r-3)(r+2)}{(r+4)(r-3)} = \frac{r+2}{r+4}$

13. $\frac{x^2+2x-3}{x^2-1} = \frac{(x+3)(x-1)}{(x+1)(x-1)} = \frac{x+3}{x+1}$

14. $\frac{z^2+4z+4}{z^2-4} = \frac{(z+2)^2}{(z+2)(z-2)} = \frac{z+2}{z-2}$

15. $\frac{3a^2}{64}\cdot\frac{8}{2a^3} = \frac{3a^2\cdot 8}{64\cdot 2a^3} = \frac{3}{16a}$

16. $\frac{2u^2}{8u^4}\cdot\frac{10u^3}{9u} = \frac{2u^2\cdot 10u^3}{8u^4\cdot 9u} = \frac{5}{18}$

17. $\frac{7x}{11}\div\frac{14x^3}{66y} = \frac{7x}{11}\cdot\frac{66y}{14x^3} = \frac{7x\cdot 66y}{11\cdot 14x^3} = \frac{3y}{x^2}$

18. $\frac{6x^2y}{2x}\div\frac{21xy}{y} = \frac{6x^2y}{2x}\cdot\frac{y}{21xy} = \frac{y}{7}$

19. $\frac{2a+b}{3c}\cdot\frac{15}{4(2a+b)} = \frac{(2a+b)\cdot 15}{(2a+b)\cdot 12c} = \frac{15}{12c} = \frac{5}{4c}$

20. $\frac{4(x+2)}{w}\cdot\frac{3w^2}{8(x+2)} = \frac{4w^2(x+2)\cdot 3}{4w(x+2)\cdot 2} = \frac{3w}{2}$

21. $\frac{15p-3}{6}\div\frac{10p-2}{3} = \frac{15p-3}{6}\cdot\frac{3}{10p-2}$
$= \frac{3(5p-1)\cdot 3}{3\cdot 2\cdot 2\cdot(5p-1)}$
$= \frac{3(5p-1)\cdot 3}{3(5p-1)\cdot 2\cdot 2} = \frac{3}{4}$

22. $\frac{2k+8}{6}\div\frac{3k+12}{3} = \frac{2k+8}{6}\cdot\frac{3}{3k+12}$
$= \frac{2(k+4)}{6}\cdot\frac{3}{3(k+4)}$
$= \frac{6(k+4)}{18(k+4)} = \frac{6}{18} = \frac{1}{3}$

23. $\frac{9y-18}{6y+12}\cdot\frac{3y+6}{15y-30} = \frac{9(y-2)}{6(y+2)}\cdot\frac{3(y+2)}{15(y-2)}$
$= \frac{27(y-2)(y+2)}{90(y+2)(y-2)} = \frac{27}{90} = \frac{3}{10}$

24. $\frac{12r+24}{36r-36}\div\frac{6r+12}{8r-8} = \frac{12(r+2)}{36(r-1)}\div\frac{6(r+2)}{8(r-1)}$
$= \frac{r+2}{3(r-1)}\div\frac{3(r+2)}{4(r-1)}$
$= \frac{r+2}{3(r-1)}\cdot\frac{4(r-1)}{3(r+2)} = \frac{4}{9}$

25. $\frac{4a+12}{2a-10}\div\frac{a^2-9}{a^2-a-20} = \frac{4a+12}{2a-10}\cdot\frac{a^2-a-20}{a^2-9}$
$= \frac{4(a+3)}{2(a-5)}\cdot\frac{(a-5)(a+4)}{(a+3)(a-3)}$
$= \frac{4(a+3)(a-5)(a+4)}{2(a-5)(a+3)(a-3)}$
$= \frac{2(a+4)}{a-3}$

26. $\dfrac{6r-18}{9r^2+6r-24}\cdot\dfrac{12r-16}{4r-12}$

$=\dfrac{6(r-3)}{3\left(3r^2+2r-8\right)}\cdot\dfrac{4(3r-4)}{4(r-3)}=\dfrac{2(3r-4)}{\left(3r^2+2r-8\right)}$

$=\dfrac{2(3r-4)}{(3r-4)(r+2)}=\dfrac{2}{r+2}$

27. $\dfrac{k^2-k-6}{k^2+k-12}\cdot\dfrac{k^2+3k-4}{k^2+2k-3}$

$=\dfrac{(k-3)(k+2)}{(k+4)(k-3)}\cdot\dfrac{(k+4)(k-1)}{(k+3)(k-1)}$

$=\dfrac{(k-3)(k+2)(k+4)(k-1)}{(k+4)(k-3)(k+3)(k-1)}=\dfrac{k+2}{k+3}$

28. $\dfrac{n^2-n-6}{n^2-2n-8}\div\dfrac{n^2-9}{n^2+7n+12}$

$=\dfrac{(n-3)(n+2)}{(n-4)(n+2)}\div\dfrac{(n-3)(n+3)}{(n+3)(n+4)}$

$=\dfrac{n-3}{n-4}\div\dfrac{n-3}{n+4}=\dfrac{n-3}{n-4}\cdot\dfrac{n+4}{n-3}=\dfrac{n+4}{n-4}$

Answers will vary for exercises 29 and 30. Sample answers are given.

29. To find the least common denominator for two fractions, factor each denominator into prime factors, multiply all unique prime factors raising each factor to the highest frequency it occurred.

30. To add three rational expressions, first factor each denominator completely. Then, find the least common denominator and rewrite each expression with that denominator. Next, add the numerators and place over the common denominator. Finally, simplify the resulting expression and write it in lowest terms.

31. The common denominator is $35z$.

$\dfrac{2}{7z}-\dfrac{1}{5z}=\dfrac{2\cdot5}{7z\cdot5}-\dfrac{1\cdot7}{5z\cdot7}=\dfrac{10}{35z}-\dfrac{7}{35z}=\dfrac{3}{35z}$

32. The common denominator is $12z$.

$\dfrac{4}{3z}-\dfrac{5}{4z}=\dfrac{4\cdot4}{3z\cdot4}-\dfrac{5\cdot3}{4z\cdot3}=\dfrac{16}{12z}-\dfrac{15}{12z}=\dfrac{1}{12z}$

33. $\dfrac{r+2}{3}-\dfrac{r-2}{3}=\dfrac{(r+2)-(r-2)}{3}$

$=\dfrac{r+2-r+2}{3}=\dfrac{4}{3}$

34. $\dfrac{3y-1}{8}-\dfrac{3y+1}{8}=\dfrac{(3y-1)-(3y+1)}{8}=\dfrac{-2}{8}=-\dfrac{1}{4}$

35. The common denominator is $5x$.

$\dfrac{4}{x}+\dfrac{1}{5}=\dfrac{4\cdot5}{x\cdot5}+\dfrac{1\cdot x}{5\cdot x}=\dfrac{20}{5x}+\dfrac{x}{5x}=\dfrac{20+x}{5x}$

36. The common denominator is $4r$.

$\dfrac{6}{r}-\dfrac{3}{4}=\dfrac{6\cdot4}{r\cdot4}-\dfrac{3\cdot r}{4\cdot r}=\dfrac{24}{4r}-\dfrac{3r}{4r}$

$=\dfrac{24-3r}{4r}=\dfrac{3(8-r)}{4r}$

37. The common denominator is $m(m-1)$.

$\dfrac{1}{m-1}+\dfrac{2}{m}=\dfrac{m\cdot1}{m\cdot(m-1)}+\dfrac{(m-1)\cdot2}{(m-1)\cdot m}$

$=\dfrac{m}{m(m-1)}+\dfrac{2(m-1)}{m(m-1)}$

$=\dfrac{m+2(m-1)}{m(m-1)}=\dfrac{m+2m-2}{m(m-1)}$

$=\dfrac{3m-2}{m(m-1)}$

38. The common denominator is $(y+2)y$.

$\dfrac{8}{y+2}-\dfrac{3}{y}=\dfrac{8y}{(y+2)y}-\dfrac{3(y+2)}{y(y+2)}=\dfrac{8y-3(y+2)}{(y+2)y}$

$=\dfrac{8y-3y-6}{(y+2)y}=\dfrac{5y-6}{(y+2)y}$ or $\dfrac{5y-6}{y(y+2)}$

39. The common denominator is $5(b+2)$.

$\dfrac{7}{b+2}+\dfrac{2}{5(b+2)}=\dfrac{7\cdot5}{(b+2)\cdot5}+\dfrac{2}{5(b+2)}$

$=\dfrac{35+2}{5(b+2)}=\dfrac{37}{5(b+2)}$

40. The common denominator is $3(k+1)$.

$\dfrac{4}{3(k+1)}+\dfrac{3}{k+1}=\dfrac{4}{3(k+1)}+\dfrac{3\cdot3}{3(k+1)}$

$=\dfrac{4+9}{3(k+1)}=\dfrac{13}{3(k+1)}$

41. The common denominator is $20(k-2)$.

$\dfrac{2}{5(k-2)}+\dfrac{5}{4(k-2)}=\dfrac{8}{20(k-2)}+\dfrac{25}{20(k-2)}$

$=\dfrac{8+25}{20(k-2)}=\dfrac{33}{20(k-2)}$

42. The common denominator is $6(p+4)$.

$$\frac{11}{3(p+4)}-\frac{5}{6(p+4)}=\frac{22}{6(p+4)}-\frac{5}{6(p+4)}$$
$$=\frac{22-5}{6(p+4)}=\frac{17}{6(p+4)}$$

43. First factor the denominators in order to find the common denominator.

$$x^2-4x+3=(x-3)(x-1)$$
$$x^2-x-6=(x-3)(x+2)$$

The common denominator is $(x-3)(x-1)(x+2)$.

$$\frac{2}{x^2-4x+3}+\frac{5}{x^2-x-6}$$
$$=\frac{2}{(x-3)(x-1)}+\frac{5}{(x-3)(x+2)}$$
$$=\frac{2(x+2)}{(x-3)(x-1)(x+2)}+\frac{5(x-1)}{(x-3)(x+2)(x-1)}$$
$$=\frac{2(x+2)+5(x-1)}{(x-3)(x+2)(x-1)}=\frac{2x+4+5x-5}{(x-3)(x-1)(x+2)}$$
$$=\frac{7x-1}{(x-3)(x-1)(x+2)}$$

44. First factor the denominators in order to find the common denominator.

$$m^2-3m-10=(m-5)(m+2)$$
$$m^2-m-20=(m-5)(m+4)$$

The common denominator is $(m-5)(m+2)(m+4)$.

$$\frac{3}{m^2-3m-10}+\frac{7}{m^2-m-20}$$
$$=\frac{3}{(m-5)(m+2)}+\frac{7}{(m-5)(m+4)}$$
$$=\frac{3(m+4)}{(m-5)(m+2)(m+4)}+\frac{7(m+2)}{(m-5)(m+4)(m+2)}$$
$$=\frac{3m+12+7m+14}{(m-5)(m+2)(m+4)}=\frac{10m+26}{(m-5)(m+2)(m+4)}$$

45. First factor the denominators in order to find the common denominator.

$$y^2+7y+12=(y+3)(y+4)$$
$$y^2+5y+6=(y+3)(y+2)$$

The common denominator is $(y+4)(y+3)(y+2)$.

$$\frac{2y}{y^2+7y+12}-\frac{y}{y^2+5y+6}$$
$$=\frac{2y}{(y+4)(y+3)}-\frac{y}{(y+3)(y+2)}$$
$$=\frac{2y(y+2)}{(y+4)(y+3)(y+2)}-\frac{y(y+4)}{(y+4)(y+3)(y+2)}$$
$$=\frac{2y(y+2)-y(y+4)}{(y+4)(y+3)(y+2)}=\frac{2y^2+4y-y^2-4y}{(y+4)(y+3)(y+2)}$$
$$=\frac{y^2}{(y+4)(y+3)(y+2)}$$

46. First factor the denominators in order to find the common denominator.

$$r^2-10r+16=(r-8)(r-2)$$
$$r^2+2r-8=(r+4)(r-2)$$

The common denominator is $(r-8)(r-2)(r+4)$.

$$\frac{-r}{r^2-10r+16}-\frac{3r}{r^2+2r-8}$$
$$=\frac{-r}{(r-8)(r-2)}-\frac{3r}{(r+4)(r-2)}$$
$$=\frac{-r(r+4)}{(r-8)(r-2)(r+4)}-\frac{3r(r-8)}{(r+4)(r-2)(r-8)}$$
$$=\frac{-r^2-4r-\left(3r^2-24r\right)}{(r-8)(r-2)(r+4)}=\frac{-r^2-4r-3r^2+24r}{(r-8)(r-2)(r+4)}$$
$$=\frac{-4r^2+20r}{(r-8)(r-2)(r+4)}$$

47. $\dfrac{1+\frac{1}{x}}{1-\frac{1}{x}}$

Multiply both numerator and denominator of this complex fraction by the common denominator, x.

$$\frac{1+\frac{1}{x}}{1-\frac{1}{x}}=\frac{x\left(1+\frac{1}{x}\right)}{x\left(1-\frac{1}{x}\right)}=\frac{x\cdot 1+x\left(\frac{1}{x}\right)}{x\cdot 1-x\left(\frac{1}{x}\right)}=\frac{x+1}{x-1}$$

48. $\dfrac{2-\frac{2}{y}}{2+\frac{2}{y}}$

Multiply both numerator and denominator by the common denominator, y.

$$\frac{2-\frac{2}{y}}{2+\frac{2}{y}}=\frac{y\left(2-\frac{2}{y}\right)}{y\left(2+\frac{2}{y}\right)}=\frac{2y-2}{2y+2}=\frac{2(y-1)}{2(y+1)}=\frac{y-1}{y+1}$$

49. $\dfrac{\frac{1}{x+h}-\frac{1}{x}}{h}$

The common denominator in the numerator is $x(x+h)$

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\frac{\frac{x-x-h}{x(x+h)}}{h}=\frac{\frac{-h}{x(x+h)}}{h}$$
$$=\frac{-h}{x(x+h)}\div h=\frac{-h}{x(x+h)}\cdot\frac{1}{h}$$
$$=\frac{-1}{x(x+h)}\text{ or }-\frac{1}{x(x+h)}$$

50. $\dfrac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$

The common denominator of the numerator is $(x+h)^2x^2$.

$$\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}=\frac{\frac{x^2}{(x+h)^2x^2}-\frac{(x+h)^2}{(x+h)^2x^2}}{\frac{h}{1}}$$
$$=\frac{x^2-(x+h)^2}{(x+h)^2x^2}\cdot\frac{1}{h}$$
$$=\frac{x^2-\left(x^2+2xh+h^2\right)}{(x+h)^2x^2}\cdot\frac{1}{h}$$
$$=\frac{-2xh-h^2}{(x+h)^2x^2h}=\frac{h(-2x-h)}{(x+h)^2x^2h}$$
$$=\frac{-2x-h}{(x+h)^2x^2}$$

51. The length of each side of the dartboard is $2x$, so the area of the dartboard is $4x^2$. The area of the shaded region is πx^2.

a. The probability that a dart will land in the shaded region is $\dfrac{\pi x^2}{4x^2}$.

b. $\dfrac{\pi x^2}{4x^2}=\dfrac{\pi}{4}$

52. The radius of the dartboard is $x + 2x + 3x = 6x$, so the area of the dartboard is $\pi(6x)^2=36\pi x^2$. The area of the shaded region is πx^2.

a. The probability that a dart will land in the shaded region is $\dfrac{\pi x^2}{36\pi x^2}$.

b. $\dfrac{\pi x^2}{36\pi x^2}=\dfrac{1}{36}$

53. The length of each side of the dartboard is $5x$, so the area of the dartboard is $25x^2$. The area of the shaded region is x^2.

a. The probability that a dart will land in the shaded region is $\dfrac{x^2}{25x^2}$.

b. $\dfrac{x^2}{25x^2}=\dfrac{1}{25}$

54. The length of each side of the dartboard is $3x$, so the area of the dartboard is $9x^2$. The area of the shaded region is $\frac{1}{2}x^2$.

a. The probability that a dart will land in the shaded region is $\dfrac{\frac{1}{2}x^2}{9x^2}=\dfrac{x^2}{18x^2}$.

b. $\dfrac{x^2}{18x^2}=\dfrac{1}{18}$

55. Average cost = total cost C divided by the number of calculators produced.

$$\frac{-7.2x^2+6995x+230{,}000}{1000x}$$

56. Let $x = 20$ (in thousands).

$$\frac{-7.2(20)^2+6995(20)+230{,}000}{1000(20)}=\$18.35$$

Let $x = 50$ (in thousands).

$$\frac{-7.2(50)^2+6995(50)+230{,}000}{1000(50)}=\$11.24$$

Let $x = 125$ (in thousands).

$$\frac{-7.2(125)^2+6995(125)+230{,}000}{1000(125)}=\$7.94$$

57. Let $x = 16$. Then

$$\frac{.013(16)^2+8.49(16)-58.7}{16+1}\approx 4.7$$

The ad cost approximately \$4.7 million in 2016.

58. Let $x = 20$. Then

$$\frac{.013(20)^2+8.49(20)-58.7}{20+1}\approx 5.5$$

The ad cost approximately \$5.5 million in 2020.

59. Let $x = 22$. Then

$$\frac{.013(22)^2+8.49(22)-58.7}{22+1}\approx 5.8$$

The cost of an ad will not reach \$7 million in 2022.

60. Let $x = 24$. Then

$$\frac{.013(24)^2+8.49(24)-58.7}{24+1}\approx 6.1$$

The cost of an ad will not reach \$8 million in 2024.

61. Let $x = 15$. Then

$$\frac{72.392(15)^2+1354.5(15)+2197.5}{15+2}\approx 2282.54$$

The average cost per patient per day in 2015 was approximately \$2,283.

62. Let $x = 19$. Then

$$\frac{72.392(19)^2+1354.5(19)+2197.5}{19+2}\approx 2574.60$$

The average cost per patient per day in 2019 was approximately \$2,575.

63. Let $x = 22$. Then

$$\frac{72.392(22)^2+1354.5(22)+2197.5}{22+2}\approx 2793.09$$

The average cost per patient per day in 2022 will be approximately \$2,793 if the trend continues.

64. Let $x = 23$. Then

$$\frac{72.392(23)^2+1354.5(23)+2197.5}{23+2}\approx 2865.85$$

The average cost per patient per day in 2023 will be approximately \$2,866 if the trend continues. Therefore, the average cost per patient per day will not reach \$3000.

Section 1.5 Exponents and Radicals

1. $\dfrac{7^5}{7^3}=7^{5-3}=7^2=49$

2. $\dfrac{(-6)^{14}}{(-6)^6}=(-6)^8=1,679,616$

3. $(4c)^2=4^2c^2=16c^2$

4. $(-2x)^4=(-2)^4x^4=16x^4$

5. $\left(\dfrac{2}{x}\right)^5=\dfrac{2^5}{x^5}=\dfrac{32}{x^5}$

6. $\left(\dfrac{5}{xy}\right)^3=\dfrac{5^3}{x^3y^3}=\dfrac{125}{x^3y^3}$

7. $\left(3u^2\right)^3\left(2u^3\right)^2=\left(27u^6\right)\left(4u^6\right)=108u^{12}$

8. $\dfrac{\left(5v^2\right)^3}{(2v)^4}=\dfrac{125v^6}{16v^4}=\dfrac{125v^2}{16}$

9. $7^{-1}=\dfrac{1}{7^1}=\dfrac{1}{7}$

10. $10^{-3}=\dfrac{1}{10^3}=\dfrac{1}{1000}$

11. $-6^{-5}=-\dfrac{1}{6^5}=-\dfrac{1}{7776}$

12. $(-x)^{-4}=\dfrac{1}{(-x)^4}=\dfrac{1}{x^4}$

13. $(-y)^{-3}=\dfrac{1}{(-y)^3}=-\dfrac{1}{y^3}$

14. $(-y)^{-3} = \dfrac{1}{(-y)^3} = -\dfrac{1}{y^3}$

$\left(\dfrac{1}{6}\right)^{-2} = \left(\dfrac{6}{1}\right)^2 = 6^2 = 36$

15. $\left(\dfrac{4}{3}\right)^{-2} = \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

16. $\left(\dfrac{x}{y^2}\right)^{-2} = \left(\dfrac{y^2}{x}\right)^2 = \dfrac{y^4}{x^2}$

17. $\left(\dfrac{a}{b^3}\right)^{-1} = \left(\dfrac{b^3}{a}\right)^1 = \dfrac{b^3}{a}$

18. $-2^{-4} = -\dfrac{1}{2^4} = -\dfrac{1}{16}$, but

$(-2)^{-4} = \dfrac{1}{(-2)^4} = \dfrac{1}{16}$

19. $49^{1/2} = 7$ because $7^2 = 49$.

20. $8^{1/3} = 2$ because $2^3 = 8$.

21. $(5.71)^{1/4} = (5.71)^{.25} \approx 1.55$ Use a calculator.

22. $12^{5/2} = \left(12^{1/2}\right)^5 \approx 498.83$

23. $-64^{2/3} = -\left(64^{1/3}\right)^2 = -(4)^2 = -16$

24. $-64^{3/2} = -\left[\left(64^{1/2}\right)^3\right] = -\left(8^3\right) = -512$

25. $\left(\dfrac{8}{27}\right)^{-4/3} = \left(\dfrac{27^{1/3}}{8^{1/3}}\right)^4 = \left(\dfrac{3}{2}\right)^4 = \dfrac{3^4}{2^4} = \dfrac{81}{16}$

26. $\left(\dfrac{27}{64}\right)^{-1/3} = \left(\dfrac{64}{27}\right)^{1/3} = \dfrac{4}{3}$

27. $\dfrac{5^{-3}}{4^{-2}} = \dfrac{4^2}{5^3}$

28. $\dfrac{7^{-4}}{7^{-3}} = 7^{-4} \cdot 7^3 = 7^{-1} = \dfrac{1}{7}$

29. $4^{-3} \cdot 4^6 = 4^3$

30. $9^{-9} \cdot 9^{10} = 9^1 = 9$

31. $\dfrac{4^{10} \cdot 4^{-6}}{4^{-4}} = 4^{10} \cdot 4^{-6} \cdot 4^4 = 4^8$

32. $\dfrac{5^{-4} \cdot 5^6}{5^{-1}} = 5^{-4} \cdot 5^6 \cdot 5^1 = 5^3$

33. $\dfrac{z^6 \cdot z^2}{z^5} = \dfrac{z^8}{z^5} = z^{8-5} = z^3$

34. $\dfrac{k^6 \cdot k^9}{k^{12}} = \dfrac{k^{15}}{k^{12}} = k^{15-12} = k^3$

35. $\dfrac{3^{-1}\left(p^{-2}\right)^3}{3p^{-7}} = \dfrac{3^{-1}p^{-6}}{3^1 p^{-7}} = 3^{-1-1}p^{-6-(-7)}$

$= 3^{-2}p^1 = \dfrac{1}{3^2} \cdot p = \dfrac{p}{9}$

36. $\dfrac{\left(5x^3\right)^{-2}}{x^4} = \dfrac{5^{-2}\left(x^3\right)^{-2}}{x^4} = \dfrac{5^{-2}x^{-6}}{x^4}$

$= 5^{-2}x^{-10} = \dfrac{1}{25x^{10}}$

37. $\left(q^{-5}r^3\right)^{-1} = q^5 r^{-3} = q^5 \cdot \dfrac{1}{r^3} = \dfrac{q^5}{r^3}$

38. $\left(2y^2z^{-2}\right)^{-3} = 2^{-3}\left(y^2\right)^{-3}\left(z^{-2}\right)^{-3}$

$= 2^{-3}y^{-6}z^6 = \dfrac{z^6}{2^3y^6} = \dfrac{z^6}{8y^6}$

39. $\left(2p^{-1}\right)^3 \cdot \left(5p^2\right)^{-2} = 2^3\left(p^{-1}\right)^3(5)^{-2}\left(p^2\right)^{-2}$

$= 2^3\left(p^{-3}\right)\left(\dfrac{1}{5^2}\right)\left(p^{-4}\right)$

$= 2^3\left(\dfrac{1}{p^3}\right)\left(\dfrac{1}{5^2}\right)\left(\dfrac{1}{p^4}\right)$

$= \dfrac{8}{25p^7}$

40. $\left(4^{-1}x^3\right)^{-2}\cdot\left(3x^{-3}\right)^4$
$= \left(4^{-1}\right)^{-2}\cdot\left(x^3\right)^{-2}\cdot(3)^4\cdot\left(x^{-3}\right)^4$
$= 4^2\cdot x^{-6}\cdot 3^4\cdot x^{-12} = 1296x^{-18} = \dfrac{1296}{x^{18}}$

41. $(2p)^{1/2}\cdot\left(2p^3\right)^{1/3} = 2^{1/2}p^{1/2}\cdot 2^{1/3}\cdot\left(p^3\right)^{1/3}$
$= 2^{1/2}p^{1/2}\cdot 2^{1/3}\cdot p^1$
$= 2^{5/6}p^{3/2}$

42. $\left(5k^2\right)^{3/2}\cdot\left(5k^{1/3}\right)^{3/4} = 5^{3/2}\left(k^2\right)^{3/2}\cdot 5^{3/4}\cdot\left(k^{1/3}\right)^{3/4}$
$= 5^{\frac{3}{2}+\frac{3}{4}}k^3k^{1/4} = 5^{9/4}k^{13/4}$

43. $p^{2/3}\left(2p^{1/3}+5p\right) = p^{2/3}\left(2p^{1/3}\right)+p^{2/3}(5p)$
$= 2p+5p^{5/3}$

44. $3x^{3/2}\left(2x^{-3/2}+x^{3/2}\right) = 3x^{3/2}\cdot 2x^{-3/2}+3x^{3/2}\cdot x^{3/2}$
$= 6x^0+3x^{6/2} = 6+3x^3$

45. $\dfrac{\left(x^2\right)^{1/3}\left(y^2\right)^{2/3}}{3x^{2/3}y^2} = \dfrac{(x)^{2/3}(y)^{4/3}}{3x^{2/3}y^2}$
$= \dfrac{1}{3y^{2-4/3}} = \dfrac{1}{3y^{2/3}}$

46. $\dfrac{\left(c^{1/2}\right)^3\left(d^3\right)^{1/2}}{\left(c^3\right)^{1/4}\left(d^{1/4}\right)^3} = \dfrac{\left(c^{3/2}\right)\left(d^{3/2}\right)}{\left(c^{3/4}\right)\left(d^{3/4}\right)}$
$= c^{(3/2)-(3/4)}d^{(3/2)-(3/4)}$
$= c^{3/4}d^{3/4}$

47. $\dfrac{(7a)^2(5b)^{3/2}}{(5a)^{3/2}(7b)^4} = \dfrac{7^2a^25^{3/2}b^{3/2}}{5^{3/2}a^{3/2}7^4b^4} = \dfrac{a^{2-\frac{3}{2}}}{7^2b^{4-\frac{3}{2}}}$
$= \dfrac{a^{1/2}}{49b^{5/2}}$

48. $\dfrac{(4x)^{1/2}\sqrt{xy}}{x^{3/2}y^2} = \dfrac{(4x)^{1/2}(xy)^{1/2}}{x^{3/2}y^2} = \dfrac{4^{1/2}x^{1/2}x^{1/2}y^{1/2}}{x^{3/2}y^2}$
$= 2xy^{1/2}x^{-3/2}y^{-2} = 2x^{-1/2}y^{-3/2}$
$= \dfrac{2}{x^{1/2}y^{3/2}}$

49. $x^{1/2}\left(x^{2/3}-x^{4/3}\right) = x^{1/2}x^{2/3}-x^{1/2}x^{4/3}$
$= x^{7/6}-x^{11/6}$

50. $x^{1/2}\left(3x^{3/2}+2x^{-1/2}\right) = 3x^{1/2}x^{3/2}+2x^{1/2}x^{-1/2}$
$= 3x^2+2$

51. $\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right) = \left(x^{1/2}\right)^2-\left(y^{1/2}\right)^2$
$= x-y$

52. $\left(x^{1/3}+y^{1/2}\right)\left(2x^{1/3}-y^{3/2}\right)$
$= x^{1/3}\cdot 2x^{1/3}-x^{1/3}y^{3/2}+2x^{1/3}y^{1/2}$
$-y^{1/2}y^{3/2}$
$= 2x^{2/3}+2x^{1/3}y^{1/2}-x^{1/3}y^{3/2}-y^2$

53. $(-3x)^{1/3} = \sqrt[3]{-3x}$, (f)

54. $-3x^{1/3} = -3\sqrt[3]{x}$, (b)

55. $(-3x)^{-1/3} = \dfrac{1}{(-3x)^{1/3}} = \dfrac{1}{\sqrt[3]{-3x}}$, (h)

56. $-3x^{-1/3} = \dfrac{-3}{x^{1/3}} = \dfrac{-3}{\sqrt[3]{x}}$, (d)

57. $(3x)^{1/3} = \sqrt[3]{3x}$, (g)

58. $3x^{-1/3} = \dfrac{3}{x^{1/3}} = \dfrac{3}{\sqrt[3]{x}}$, (a)

59. $(3x)^{-1/3} = \dfrac{1}{(3x)^{1/3}} = \dfrac{1}{\sqrt[3]{3x}}$, (c)

60. $3x^{1/3} = 3\sqrt[3]{x}$, (e)

61. $\sqrt[3]{125} = 125^{1/3} = 5$

62. $\sqrt[6]{64} = 64^{1/6} = 2$

63. $\sqrt[4]{625} = 625^{1/4} = 5$

64. $\sqrt[7]{-128} = (-128)^{1/7} = -2$

65. $\sqrt{63}\cdot\sqrt{7} = 3\sqrt{7}\cdot\sqrt{7} = 3\cdot 7 = 21$

66. $\sqrt[3]{81}\cdot\sqrt[3]{9} = \sqrt[3]{729} = 9$

67. $\sqrt{81-4}=\sqrt{77}$

68. $\sqrt{49-16}=\sqrt{33}$

69. $\sqrt{5}\sqrt{15}=\sqrt{75}=\sqrt{25\cdot 3}=\sqrt{25}\sqrt{3}=5\sqrt{3}$

70. $\sqrt{8}\sqrt{96}=\sqrt{8}\sqrt{8\cdot 12}=\sqrt{8}\sqrt{8}\sqrt{12}=8\sqrt{4\cdot 3}$
$=8\sqrt{4}\sqrt{3}=8\cdot 2\sqrt{3}=16\sqrt{3}$

71. $\sqrt{50}-\sqrt{72}=5\sqrt{2}-6\sqrt{2}=-\sqrt{2}$

72 $\sqrt{75}+\sqrt{192}=5\sqrt{3}+8\sqrt{3}=13\sqrt{3}$

73. $5\sqrt{20}-\sqrt{45}+2\sqrt{80}$
$=5\cdot 2\sqrt{5}-3\sqrt{5}+2\cdot 4\sqrt{5}$
$=10\sqrt{5}-3\sqrt{5}+8\sqrt{5}=15\sqrt{5}$

74. $\left(\sqrt{3}+2\right)\left(\sqrt{3}-2\right)=\left(\sqrt{3}\right)^2-2^2=3-4=-1$

75. $\left(\sqrt{5}+\sqrt{2}\right)\left(\sqrt{5}-\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$
$=5-2=3$

76. $\sqrt[3]{4}\cdot\sqrt[3]{4}=4$. A correct statement would be $\sqrt[3]{4}\cdot\sqrt[3]{4}=\sqrt[3]{16}$.

77. $\dfrac{3}{1-\sqrt{2}}=\dfrac{3}{1-\sqrt{2}}\cdot\dfrac{1+\sqrt{2}}{1+\sqrt{2}}=\dfrac{3\left(1+\sqrt{2}\right)}{(1)^2-\left(\sqrt{2}\right)^2}$
$=\dfrac{3\left(1+\sqrt{2}\right)}{1-2}=\dfrac{3\left(1+\sqrt{2}\right)}{-1}$
$=-3\left(1+\sqrt{2}\right)=-3-3\sqrt{2}$

78. $\dfrac{2}{1+\sqrt{5}}=\dfrac{2}{1+\sqrt{5}}\cdot\dfrac{1-\sqrt{5}}{1-\sqrt{5}}=\dfrac{2\left(1-\sqrt{5}\right)}{1-5}$
$=\dfrac{2\left(1-\sqrt{5}\right)}{-4}=\dfrac{1-\sqrt{5}}{-2}\cdot\dfrac{-1}{-1}=\dfrac{\sqrt{5}-1}{2}$

79. $\dfrac{9-\sqrt{3}}{3-\sqrt{3}}=\dfrac{9-\sqrt{3}}{3-\sqrt{3}}\cdot\dfrac{3+\sqrt{3}}{3+\sqrt{3}}=\dfrac{27+9\sqrt{3}-3\sqrt{3}-3}{3^2-\left(\sqrt{3}\right)^2}$
$=\dfrac{24+6\sqrt{3}}{9-3}=\dfrac{24+6\sqrt{3}}{6}=4+\sqrt{3}$

80. $\dfrac{\sqrt{3}-1}{\sqrt{3}-2}=\dfrac{\sqrt{3}-1}{\sqrt{3}-2}\cdot\dfrac{\sqrt{3}+2}{\sqrt{3}+2}=\dfrac{\left(\sqrt{3}-1\right)\left(\sqrt{3}+2\right)}{3-4}$
$=\dfrac{3+2\sqrt{3}-\sqrt{3}-2}{-1}=\dfrac{1+\sqrt{3}}{-1}=-1-\sqrt{3}$

81. $\dfrac{3-\sqrt{2}}{3+\sqrt{2}}=\dfrac{3-\sqrt{2}}{3+\sqrt{2}}\cdot\dfrac{3+\sqrt{2}}{3+\sqrt{2}}$
$=\dfrac{9-2}{9+6\sqrt{2}+2}=\dfrac{7}{11+6\sqrt{2}}$

82. $\dfrac{1+\sqrt{7}}{2-\sqrt{3}}=\dfrac{1+\sqrt{7}}{2-\sqrt{3}}\cdot\dfrac{1-\sqrt{7}}{1-\sqrt{7}}$
$=\dfrac{1-7}{2-2\sqrt{7}-\sqrt{3}+\sqrt{3}\sqrt{7}}$
$=\dfrac{-6}{2-2\sqrt{7}-\sqrt{3}+\sqrt{21}}$

83. $x=\sqrt{\dfrac{kM}{f}}$

Note that because x represents the number of units to order, the value of x should be rounded to the nearest integer.

a. $k=\$1, f=\$500, M=100{,}000$

$x=\sqrt{\dfrac{1\cdot 100{,}000}{500}}=\sqrt{200}\approx 14.1$

The number of units to order is 14.

b. $k=\$3, f=\$7, M=16{,}700$

$x=\sqrt{\dfrac{3\cdot 16{,}700}{7}}\approx 84.6$

The number of units to order is 85.

c. $k=\$1, f=\$5, M=16{,}800$

$x=\sqrt{\dfrac{1\cdot 16{,}800}{5}}=\sqrt{3360}\approx 58.0$

The number of units to order is 58.

84. $h=12.3T^{1/3}$

If $T=216$, find h.

$h=12.3(216)^{1/3}=73.8$

A height of 73.8 in. corresponds to a threshold weight of 216 lb.

For exercises 85–88, we use the model revenue $= 10.8x^{.467}$, $10 \le x \le 19$, $x = 10$ corresponds to 2010.

85. Let $x = 11$. Then $10.8(11)^{.467} \approx 33.09$
The movie revenue for 2011 was about \$33.1 billion.

86. Let $x = 15$. Then $10.8(15)^{.467} \approx 38.25$
The movie revenue for 2015 was about \$38.3 billion.

87. Let $x = 18$. Then $10.8(18)^{.467} \approx 41.65$
The movie revenue for 2018 was about \$41.7 billion.

88. Let $x = 19$. Then $10.8(19)^{.467} \approx 42.72$
The movie revenue for 2019 was about \$42.7 billion.

For exercises 89–92, we use the model revenue $= 3.6x^{.707}$, $x \ge 10$, $x = 10$ corresponds to the year 2010.

89. Let $x = 15$. Then $3.6(15)^{.707} \approx 24.42$
The alcohol spirits revenue in the year 2015 was approximately \$24.4 billion.

90. Let $x = 19$. Then $3.6(19)^{.707} \approx 28.87$
The alcohol spirits revenue in the year 2019 was approximately \$28.9 billion.

91. Let $x = 21$. Then $3.6(21)^{.707} \approx 30.98$
The alcohol spirits revenue in the year 2021 was approximately \$31.0 billion.

92. Let $x = 23$. Then $3.6(23)^{.707} \approx 33.04$
The alcohol spirits revenue in the year 2023 will be approximately \$33.04 billion.

For exercises 93–96, we use the model revenue $= 5.6x^{.861}; x \ge 10, x = 10$ corresponds to 2010.

93. Let $x = 13$. Then $5.6(13)^{.861} \approx 50.97$
According to the model, the annual revenue for Intel Corp in 2013 was approximately \$51.0 billion.

94. Let $x = 16$. Then $5.6(16)^{.861} \approx 60.94$
According to the model, the annual revenue for Intel Corp in 2016 was approximately \$60.9 billion.

95. Let $x = 20$. Then $5.6(20)^{.861} \approx 73.85$
According to the model, the annual revenue for Intel Corp in 2020 was approximately \$73.9 billion.

96. Let $x = 12$. Then $5.6(22)^{.861} \approx 80.17$
According to the model, the annual revenue for Intel Corp in 2022 was approximately \$80.2 billion.

Section 1.6 First-Degree Equations

1.
$$\begin{aligned} 3x+8 &= 20 \\ 3x+8-8 &= 20-8 \\ 3x &= 12 \\ \frac{1}{3}(3x) &= \frac{1}{3}(12) \\ x &= 4 \end{aligned}$$

2. $4 - 5y = 19$
$$\begin{aligned} 4-5y+(-4) &= 19+(-4) \\ -5y &= 15 \\ -\frac{1}{5}(-5y) &= -\frac{1}{5}(15) \\ y &= -3 \end{aligned}$$

3.
$$\begin{aligned} .6k-.3 &= .5k+.4 \\ .6k-.5k-.3 &= .5k-.5k+.4 \\ .1k-.3 &= .4 \\ .1k-.3+.3 &= .4+.3 \\ .1k &= .7 \\ \frac{.1k}{.1} &= \frac{.7}{.1} \Rightarrow k = 7 \end{aligned}$$

4.
$$\begin{aligned} 2.5+5.04m &= 8.5-.06m \\ 2.5+5.04m+.06m &= 8.5-.06m+.06m \\ 2.5+5.1m &= 8.5 \\ 2.5+5.1m+(-2.5) &= 8.5+(-2.5) \\ 5.1m &= 6.0 \\ \frac{5.1m}{5.1} &= \frac{6.0}{5.1} \\ m &= \frac{6.0}{5.1} \Rightarrow m \approx 1.18 \end{aligned}$$

5.
$$\begin{aligned}
2a-1&=4(a+1)+7a+5\\
2a-1&=4a+4+7a+5\\
2a-1&=11a+9\\
2a-2a-1&=11a-2a+9\\
-1&=9a+9\\
-1-9&=9a+9-9\\
-10&=9a\\
\frac{-10}{9}&=\frac{9a}{9}\Rightarrow -\frac{10}{9}=a
\end{aligned}$$

6.
$$\begin{aligned}
3(k-2)-6&=4k-(3k-1)\\
3k-6-6&=4k-3k+1\\
3k-12&=k+1\\
3k-12+(-k)&=k+1+(-k)\\
2k-12&=1\\
2k-12+12&=1+12\\
2k&=13\\
\frac{2k}{2}&=\frac{13}{2}\Rightarrow k=\frac{13}{2}
\end{aligned}$$

7.
$$\begin{aligned}
2[x-(3+2x)+9]&=3x-8\\
2(x-3-2x+9)&=3x-8\\
2(-x+6)&=3x-8\\
-2x+12&=3x-8\\
12&=5x-8\\
20&=5x\Rightarrow 4=x
\end{aligned}$$

8.
$$\begin{aligned}
-2\left[4(k+2)-3(k+1)\right]&=14+2k\\
-2(4k+8-3k-3)&=14+2k\\
-2(k+5)&=14+2k\\
-2k-10&=14+2k\\
-2k-10+2k&=14+2k+2k\\
-10&=14+4k\\
-10-14&=14+4k-14\\
-24&=4k\\
\frac{-24}{4}&=\frac{4k}{4}\Rightarrow -6=k
\end{aligned}$$

9. $\frac{3x}{5}-\frac{4}{5}(x+1)=2-\frac{3}{10}(3x-4)$

Multiply both sides by the common denominator, 10.

$$\begin{aligned}
&10\left(\frac{3x}{5}\right)-10\left(\frac{4}{5}\right)(x+1)\\
&=(10)(2)-(10)\left(\frac{3}{10}\right)(3x-4)\\
2(3x)-8(x+1)&=20-3(3x-4)\\
6x-8x-8&=20-9x+12\\
-2x-8&=32-9x\\
-2x+9x&=32+8\\
7x&=40\\
\frac{1}{7}(7x)&=\frac{1}{7}(40)\Rightarrow x=\frac{40}{7}
\end{aligned}$$

10.
$$\begin{aligned}
\frac{4}{3}(x-2)-\frac{1}{2}&=2\left(\frac{3}{4}x-1\right)\\
\frac{4}{3}x-\frac{8}{3}-\frac{1}{2}&=\frac{3}{2}x-2\\
\frac{4}{3}x-\frac{19}{6}&=\frac{3}{2}x-2\\
\frac{4}{3}x-\frac{19}{6}-\frac{4}{3}x&=\frac{3}{2}x-2-\frac{4}{3}x\\
-\frac{19}{6}&=\frac{1}{6}x-2\\
-\frac{19}{6}+2&=\frac{1}{6}x-2+2\\
-\frac{7}{6}&=\frac{1}{6}x\\
6\left(-\frac{7}{6}\right)&=6\left(\frac{1}{6}\right)x\Rightarrow -7=x
\end{aligned}$$

11.
$$\begin{aligned}
\frac{5y}{6}-8&=5-\frac{2y}{3}\\
6\left(\frac{5y}{6}-8\right)&=6\left(5-\frac{2y}{3}\right)\\
6\left(\frac{5y}{6}\right)-6(8)&=6(5)-6\left(\frac{2y}{3}\right)\\
5y-48&=30-4y\\
9y-48&=30\\
9y&=78\\
y&=\frac{78}{9}=\frac{26}{3}
\end{aligned}$$

12. $\frac{x}{2}-3=\frac{3x}{5}+1$

Multiply both sides by the common denominator, 10, to eliminate the fractions.

$$10\left(\frac{x}{2}-3\right)=10\left(\frac{3x}{5}+1\right)$$
$$5x-30=6x+10$$
$$5x-30-5x=6x+10-5x$$
$$-30=x+10$$
$$-30-10=x+10-10$$
$$-40=x$$

13. $$\frac{m}{2}-\frac{1}{m}=\frac{6m+5}{12}$$
$$12m\left(\frac{m}{2}-\frac{1}{m}\right)=12m\left(\frac{6m+5}{12}\right)$$
$$(12m)\left(\frac{m}{2}\right)-(12m)\left(\frac{1}{m}\right)=m(6m)+m(5)$$
$$6m^2-12=6m^2+5m$$
$$-12=5m$$
$$\frac{1}{5}(-12)=\frac{1}{5}(5m)\Rightarrow-\frac{12}{5}=m$$

14. $-\frac{3k}{2}+\frac{9k-5}{6}=\frac{11k+8}{k}$

Multiply both sides by the common denominator, $6k$ to eliminate the fractions.

$$6k\left(-\frac{3k}{2}+\frac{9k-5}{6}\right)=6k\left(\frac{11k+8}{k}\right)$$
$$6k\left(-\frac{3k}{2}\right)+6k\left(\frac{9k-5}{6}\right)=6k\left(\frac{11k}{k}\right)+6k\left(\frac{8}{k}\right)$$
$$-9k^2+k(9k-5)=6(11k)+6(8)$$
$$-9k^2+9k^2-5k=66k+48$$
$$-5k=66k+48$$
$$-5k-66k=66k+48-66k$$
$$-71k=48$$
$$\frac{-71k}{-71}=\frac{48}{-71}\Rightarrow k=-\frac{48}{71}$$

15. $\frac{4}{x-3}-\frac{8}{2x+5}+\frac{3}{x-3}=0$
$$\frac{4}{x-3}+\frac{3}{x-3}-\frac{8}{2x+5}=0$$
$$\frac{7}{x-3}-\frac{8}{2x+5}=0$$

Multiply each side by the common denominator, $(x-3)(2x+5)$.

$$(x-3)(2x+5)\left(\frac{7}{x-3}\right)-(x-3)(2x+5)\left(\frac{8}{2x+5}\right)=(x-3)(2x+5)(0)$$
$$7(2x+5)-8(x-3)=0$$
$$14x+35-8x+24=0$$
$$6x+59=0$$
$$6x=-59\Rightarrow x=-\frac{59}{6}$$

16. $$\frac{5}{2p+3}-\frac{3}{p-2}=\frac{4}{2p+3}$$
$$\frac{5}{2p+3}-\frac{3}{p-2}-\frac{5}{2p+3}=\frac{4}{2p+3}-\frac{5}{2p+3}$$
$$-\frac{3}{p-2}=-\frac{1}{2p+3}$$

Multiply both sides by the common denominator, $(2p+3)(p-2)$.

$$\left(-\frac{3}{p-2}\right)(p-2)(2p+3)=\left(-\frac{1}{2p+3}\right)(p-2)(2p+3)$$
$$-3(2p+3)=-1(p-2)$$
$$-6p-9=-p+2$$
$$-5p=11\Rightarrow p=-\frac{11}{5}$$

17. $\frac{3}{2m+4}=\frac{1}{m+2}-2$
$$\frac{3}{2(m+2)}=\frac{1}{m+2}-2$$
$$2(m+2)\left(\frac{3}{2(m+2)}\right)=2(m+2)\left(\frac{1}{m+2}\right)-2(m+2)(2)$$
$$3=2-4(m+2)$$
$$3=2-4m-8$$
$$3=-6-4m\Rightarrow 9=-4m\Rightarrow m=-\frac{9}{4}$$

18. $\frac{8}{3k-9}-\frac{5}{k-3}=4$

Multiply both sides by the common denominator, $3k-9$.

$$(3k-9)\left[\frac{8}{3k-9}-\frac{5}{k-3}\right]=(3k-9)4$$
$$(3k-9)\left(\frac{8}{3k-9}\right)+3(k-3)\left(-\frac{5}{k-3}\right)=12k-36$$
$$8-15=12k-36$$
$$-7=12k-36$$
$$29=12k\Rightarrow\frac{29}{12}=k$$

19.
$$9.06x+3.59(8x-5)=12.07x+.5612$$
$$9.06x+28.72x-17.95=12.07x+.5612$$
$$9.06x+28.72x-12.07x=17.95+.5612$$
$$25.71x=18.5112$$
$$x=\frac{18.5112}{25.71}=.72$$

20.
$$-5.74(3.1-2.7p)=1.09p+5.2588$$
$$-17.794+15.498p=1.09p+5.2588$$
$$15.498p-1.09p=5.2588+17.794$$
$$14.408p=23.0528$$
$$p=\frac{23.0528}{14.408}=1.6$$

21. $\frac{2.63r-8.99}{1.25}-\frac{3.90r-1.77}{2.45}=r$

Multiply by the common denominator (1.25)(2.45) to eliminate the fractions.

$$(2.45)(2.63r-8.99)-(1.25)(3.90r-1.77)=(2.45)(1.25)r$$
$$6.4435r-22.0255-4.875r+2.2125=3.0625r$$
$$1.5685r-19.813=3.0625r$$
$$-19.813=1.494r$$
$$-\frac{19.813}{1.494}=\frac{1.494r}{1.494}$$
$$r\approx-13.26$$

22. $\frac{8.19m+2.55}{4.34}-\frac{8.17m-9.94}{1.04}=4m$

$$(1.04)(8.19m+2.55)-(4.34)(8.17m-9.94)=4m(1.04)(4.34)$$
$$8.5176m+2.652-35.4578m+43.1396=18.0544m$$
$$-26.9402m+45.7916=18.0544m$$
$$45.7916=44.9946m$$
$$m=\frac{45.7916}{44.9946}\Rightarrow m\approx1.02$$

23.
$$4(a+x)=b-a+2x$$
$$4a+4x=b-a+2x$$
$$4a=b-a-2x$$
$$5a-b=-2x$$
$$\frac{5a-b}{-2}=\frac{-2x}{-2}$$
$$-\frac{5a-b}{2}=x \text{ or } x=\frac{b-5a}{2}$$

24.
$$(3a-b)-bx=a(x-2)$$
$$3a-b-bx=ax-2a$$
$$3a-b=ax-2a+bx$$
$$5a-b=ax+bx$$
$$5a-b=(a+b)x\Rightarrow\frac{5a-b}{a+b}=x$$

25.
$$5(b-x)=2b+ax$$
$$5b-5x=2b+ax$$
$$5b=2b+ax+5x$$
$$3b=ax+5x$$
$$3b=(a+5)x$$
$$\frac{3b}{a+5}=\frac{(a+5)x}{a+5}\Rightarrow\frac{3b}{a+5}=x$$

26. $bx-2b=2a-ax$

Isolate terms with x on the left.

$$bx+ax=2a+2b$$
$$ax+bx=2a+2b$$
$$(a+b)x=2(a+b)$$
$$x=\frac{2(a+b)}{a+b}\Rightarrow x=2$$

27. $PV=k$ for V

$$\frac{1}{P}(PV)=\frac{1}{P}(k)\Rightarrow V=\frac{k}{P}$$

28. $i = prt$ for p

$$\frac{i}{rt} = p$$

29. $V = V_0 + gt$ for g

$$V - V_0 = gt$$
$$\frac{V - V_0}{t} = \frac{gt}{t} \Rightarrow \frac{V - V_0}{t} = g$$

30. $S = S_0 + gt^2 + k$

$$S - S_0 - k = gt^2$$
$$\frac{S - S_0 - k}{t^2} = \frac{gt^2}{t^2} \Rightarrow \frac{S - S_0 - k}{t^2} = g$$

31. $A = \frac{1}{2}(B + b)h$ for B

$$A = \frac{1}{2}Bh + \frac{1}{2}bh$$
$$2A = Bh + bh \quad \text{Multiply by } 2.$$
$$2A - bh = Bh$$
$$\frac{2A - bh}{h} = \frac{Bh}{h} \quad \text{Multiply by } \frac{1}{h}.$$
$$\frac{2A - bh}{h} = \frac{2A}{h} - b = B$$

32. $C = \frac{5}{9}(F - 32)$ for F

$$\frac{9}{5}C = F - 32 \Rightarrow \frac{9}{5}C + 32 = F$$

33. $|2h - 1| = 5$

$2h - 1 = 5$ or $2h - 1 = -5$
$2h = 6$ or $2h = -4$
$h = 3$ or $h = -2$

34. $|4m - 3| = 12$

$4m - 3 = 12$ or $4m - 3 = -12$
$4m = 15$ or $4m = -9$
$m = \frac{15}{4}$ or $m = -\frac{9}{4}$

35. $|6 + 2p| = 10$

$6 + 2p = 10$ or $6 + 2p = -10$
$2p = 4$ or $2p = -16$
$p = 2$ or $p = -8$

36. $|-5x + 7| = 15$

$-5x + 7 = 15$ or $-5x + 7 = -15$
$-5x = 8$ or $-5x = -22$
$x = -\frac{8}{5}$ or $x = \frac{22}{5}$

37. $\left|\frac{5}{r - 3}\right| = 10$

$\frac{5}{r - 3} = 10$ or $\frac{5}{r - 3} = -10$
$5 = 10(r - 3)$ or $5 = -10(r - 3)$
$5 = 10r - 30$ or $5 = -10r + 30$
$35 = 10r$ or $-25 = -10r$
$\frac{35}{10} = \frac{7}{2} = r$ or $\frac{-25}{-10} = \frac{5}{2} = r$

38. $\left|\frac{3}{2h - 1}\right| = 4$

$\frac{3}{2h - 1} = 4$ or $\frac{3}{2h - 1} = -4$
$3 = 4(2h - 1)$ or $3 = -4(2h - 1)$
$3 = 8h - 4$ or $3 = -8h + 4$
$7 = 8h$ or $-1 = -8h$
$\frac{7}{8} = h$ or $\frac{1}{8} = h$

39. $-5 = \frac{5}{9}(F - 32)$

$$-5\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$$
$$-9 = F - 32 \Rightarrow 23 = F$$

The temperature –5°C = 23°F.

40. $-15 = \frac{5}{9}(F - 32)$

$$-15\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$$
$$-27 = F - 32 \Rightarrow 5 = F$$

The temperature –15°C = 5°F.

41. $22 = \frac{5}{9}(F - 32)$

$$22\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F - 32)$$
$$39.6 = F - 32 \Rightarrow 71.6 = F$$

The temperature 22°C = 71.6°F.

42. $$36 = \frac{5}{9}(F-32)$$
$$36\left(\frac{9}{5}\right) = \left(\frac{9}{5}\right)\left(\frac{5}{9}\right)(F-32)$$
$$64.8 = F - 32 \Rightarrow 96.8 = F$$
The temperature 36°C = 96.8°F.

43. Let x =20.

$$y = .98x + 3.85$$
$$y = .98(20) + 3.85$$
$$y = 23.45$$

Therefore, the gross federal debt in 2020 was \$23.45 trillion.

44. Let x = 23.

$$y = .98x + 3.85$$
$$y = .98(23) + 3.85$$
$$y = 26.39$$

Therefore, the gross federal debt in 2023 will be \$26.39 trillion.

45. $y = .98x + 3.85$
Substitute 24.43 for y.
$$24.43 = .98x + 3.85$$
$$20.58 = .98x \Rightarrow 21 = x$$
Therefore, the federal deficit was \$24.43 trillion in 2021.

46. $y = .98x + 3.85$
Substitute 27.37 for y.
$$27.37 = .98x + 3.85$$
$$23.52 = .98x \Rightarrow 24 = x$$
Therefore, the federal deficit will be \$27.37 trillion in 2024.

47. $y = .98x + 3.85$
Substitute 28.35 for y.
$$28.35 = .98x + 3.85$$
$$24.5 = .98x \Rightarrow 25 = x$$
Therefore, the federal deficit will be \$28.35 trillion in 2025.

48. $y = .98x + 3.85$
Substitute 31.29 for y.
$$31.29 = .98x + 3.85$$
$$27.44 = .98x \Rightarrow 28 = x$$
Therefore, the federal deficit will be \$31.29 trillion in 2028.

49. $E = .35x + 4.23$
Substitute \$11.58 in for E.
$$11.58 = .35x + 4.23$$
$$7.35 = .35x \Rightarrow 21 = x$$
The health care expenditures were \$11.58 thousand per capita in 2021.

50. $E = .35x + 4.23$
Substitute \$12.63 in for E.
$$12.63 = .35x + 4.23$$
$$8.4 = .35x \Rightarrow 24 = x$$
The health care expenditures will be \$12.63 thousand per capita in 2024.

51. $E = .35x + 4.23$
Substitute \$12.98 in for E.
$$12.98 = .35x + 4.23$$
$$8.75 = .35x \Rightarrow 25 = x$$
The health care expenditures will be \$12.98 thousand per capita in 2025.

52. $E = .35x + 4.23$
Substitute \$14.03 in for E.
$$14.03 = .35x + 4.23$$
$$9.8 = .35x \Rightarrow 28 = x$$
The health care expenditures will be \$14.03 thousand per capita in 2028.

53. $51.65(x-2010) = 5y - 545$
Substitute 2017 for x and solve for y.
$$51.65(2017-2010) = 5y - 545$$
$$361.55 = 5y - 545$$
$$906.55 = 5$$
$$181.31 = y$$
The amount of expenditures was \$181.31 billion in 2017.

54. $51.65(x-2010) = 5y - 545$
Substitute 2019 for x and solve for y.
$$51.65(2019-2010) = 5y - 545$$
$$464.85 = 5y - 545$$
$$1009.85 = 5$$
$$201.97 = y$$
The amount of expenditures was \$201.97 billion in 2019.

55. $51.65(x-2010)=5y-545$

Substitute 222.63 for y and solve for x.

$$\begin{aligned} 51.65(x-2010) &= 5(222.63)-545 \\ 51.65x-103{,}816.5 &= 568.15 \\ 51.65x &= 104{,}384.65 \\ x &= 2021 \end{aligned}$$

The amount of expenditures was $222.63 billion in 2021.

56. $51.65(x-2010)=5y-545$

Substitute 253.62 for y and solve for x.

$$\begin{aligned} 51.65(x-2010) &= 5(253.62)-545 \\ 51.65x-103{,}816.5 &= 723.1 \\ 51.65x &= 104{,}539.6 \\ x &= 2024 \end{aligned}$$

The amount of expenditures will be $253.62 billion in 2024.

57. $51.65(x-2010)=5y-545$

Substitute 263.95.63 for y and solve for x.

$$\begin{aligned} 51.65(x-2010) &= 5(263.95)-545 \\ 51.65x-103{,}816.5 &= 774.75 \\ 51.65x &= 104{,}591.25 \\ x &= 2025 \end{aligned}$$

The amount of expenditures will be $263.95 billion in 2025.

58. $51.65(x-2010)=5y-545$

Substitute 305.27 for y and solve for x.

$$\begin{aligned} 51.65(x-2010) &= 5(305.27)-545 \\ 51.65x-103{,}816.5 &= 981.35 \\ 51.65x &= 104{,}797.85 \\ x &= 2029 \end{aligned}$$

The amount of expenditures will be $305.27 billion in 2029.

59. Let x = the number invested at 5%.
Then 52,000 – x = the amount invested at 4%.
Since the total interest is $2290, we have

$$\begin{aligned} .05x+.04(52{,}000-x) &= 2290 \\ .05x+2080-.04x &= 2290 \\ .01x+2080 &= 2290 \\ .01x+2080-2080 &= 2290-2080 \\ .01x &= 210 \\ \frac{.01x}{.01x} &= \frac{210}{.01} \\ x &= 21{,}000 \end{aligned}$$

Pablo invested $21,000 at 5%.

60. Let x represent the amount invested at 4%. Then 20,000 – x is the amount invested at 6%. Since the total interest is $1040, we have

$$\begin{aligned} .04x+.06(20{,}000-x) &= 1040 \\ .04x+1200-.06x &= 1040 \\ -.02x+1200 &= 1040 \\ -.02x &= -160 \\ x &= 8000 \end{aligned}$$

She invested $8000 at 4%.

61. Let x = price of first plot.
Then 120,000 – x = price of second plot.
$.15x$ = profit from first plot
–.10(120,000 – x) = loss from second plot.

$$\begin{aligned} .15x-.10(120{,}000-x) &= 5500 \\ .15x-12{,}000+.10x &= 5500 \\ .25x &= 17{,}500 \\ x &= 70{,}000 \end{aligned}$$

Da' Rae paid $70,000 for the first plot and 120,000 – 70,000, or $50,000 for the second plot.

62. Let x represent the amount invested at 4%. $20,000 invested at 5% (or .05) plus x dollars invested at 4% (or .04) must equal 4.8% (or .048) of the total investment (or 20,000 + x). Solve this equation.

$$\begin{aligned} .05(20{,}000)+.04x &= .048(20{,}000+x) \\ 1000+.04x &= 960+.048x \\ 1000 &= 960+.008x \\ 40 &= .008x \Rightarrow x = 5000 \end{aligned}$$

$5000 should be invested at 4%.

63. Let x = the number of liters of 94 octane gas;
200 = the number of liters of 99 octane gas;
200 + x = the number of liters of 97 octane gas.

$$\begin{aligned} 94x+99(200) &= 97(200+x) \\ 94x+19{,}800 &= 19{,}400+97x \\ 400 &= 3x \\ \frac{400}{3} &= x \end{aligned}$$

Thus, $\frac{400}{3}$ liters of 94 octane gas are needed.

64. Let x be the amount of 92 octane gasoline. Then $12 - x$ is the amount of 98 octane gasoline. A mixture of the two must yield 12 L of 96 octane gasoline, so

$$92x + 98(12 - x) = 96(12)$$
$$92x + 1176 - 98x = 1152$$
$$-6x = -24$$
$$x = 4$$

$12 - x = 12 - 4 = 8.$

Mix 4 L of 92 octane gasoline with 8 L of 98 octane gasoline.

Section 1.7 Quadratic Equations

1. $(x + 4)(x - 14) = 0$

$x + 4 = 0$ or $x - 14 = 0$

$x = -4$ or $x = 14$

The solutions are –4 and 14.

2. $(p - 16)(p - 5) = 0$

$p - 16 = 0$ or $p - 5 = 0$

$p = 16$ or $p = 5$

The solutions are 16 and 5.

3. $x(x + 6) = 0$

$x = 0$ or $x + 6 = 0$

$x = -6$

The solutions are 0 and –6.

4. $x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ or $x - 2 = 0$

$x = 2$

The solutions are 0 and 2.

5. $2z^2 = 4z$

$2z^2 - 4z = 0$

$2z(z - 2) = 0$

$2z = 0$ or $z - 2 = 0$

$z = 0$ or $z = 2$

The solutions are 0 and 2.

6. $x^2 - 64 = 0$

$(x - 8)(x + 8) = 0$

$x - 8 = 0$ or $x + 8 = 0$

$x = 8$ or $x = -8$

The solutions are 8 and –8.

7. $y^2 + 15y + 56 = 0$

$(y + 7)(y + 8) = 0$

$y + 7 = 0$ or $y + 8 = 0$

$y = -7$ or $y = -8$

The solutions are –7 and –8.

8. $k^2 - 4k - 5 = 0$

$(k + 1)(k - 5) = 0$

$k + 1 = 0$ or $k - 5 = 0$

$k = -1$ or $k = 5$

The solutions are –1 and 5.

9. $2x^2 = 7x - 3$

$2x^2 - 7x + 3 = 0$

$(2x - 1)(x - 3) = 0$

$2x - 1 = 0$ or $x - 3 = 0$

$x = \frac{1}{2}$ or $x = 3$

The solutions are $\frac{1}{2}$ and 3.

10. $2 = 15z^2 + z$

$0 = 15z^2 + z - 2$

$0 = (3z - 1)(5z + 2)$

$3z - 1 = 0$ or $5z + 2 = 0$

$z = \frac{1}{3}$ or $z = -\frac{2}{5}$

The solutions are $\frac{1}{3}$ and $-\frac{2}{5}$.

11.
$$6r^2 + r = 1$$
$$6r^2 + r - 1 = 0$$
$$(3r-1)(2r+1) = 0$$
$$3r - 1 = 0 \quad \text{or} \quad 2r + 1 = 0$$
$$r = \frac{1}{3} \quad \text{or} \quad r = -\frac{1}{2}$$
The solutions are $\frac{1}{3}$ and $-\frac{1}{2}$.

12.
$$3y^2 = 16y - 5$$
$$3y^2 - 16y + 5 = 0$$
$$(3y-1)(y-5) = 0$$
$$3y - 1 = 0 \quad \text{or} \quad y - 5 = 0$$
$$y = \frac{1}{3} \quad \text{or} \quad y = 5$$
The solutions are $\frac{1}{3}$ and 5.

13.
$$2m^2 + 20 = 13m$$
$$2m^2 - 13m + 20 = 0$$
$$(2m-5)(m-4) = 0$$
$$2m - 5 = 0 \quad \text{or} \quad m - 4 = 0$$
$$m = \frac{5}{2} \quad \text{or} \quad m = 4$$
The solutions are $\frac{5}{2}$ and 4.

14.
$$6a^2 + 17a + 12 = 0$$
$$(2a+3)(3a+4) = 0$$
$$2a + 3 = 0 \quad \text{or} \quad 3a + 4 = 0$$
$$a = -\frac{3}{2} \quad \text{or} \quad a = -\frac{4}{3}$$
The solutions are $-\frac{3}{2}$ and $-\frac{4}{3}$.

15.
$$m(m+7) = -10$$
$$m^2 + 7m + 10 = 0$$
$$(m+5)(m+2) = 0$$
$$m + 5 = 0 \quad \text{or} \quad m + 2 = 0$$
$$m = -5 \quad \text{or} \quad m = -2$$
The solutions are –5 and –2.

16.
$$z(2z+7) = 4$$
$$2z^2 + 7z - 4 = 0$$
$$(2z-1)(z+4) = 0$$
$$2z - 1 = 0 \quad \text{or} \quad z + 4 = 0$$
$$z = \frac{1}{2} \quad \text{or} \quad z = -4$$
The solutions are $\frac{1}{2}$ and –4.

17.
$$9x^2 - 16 = 0$$
$$(3x+4)(3x-4) = 0$$
$$3x + 4 = 0 \quad \text{or} \quad 3x - 4 = 0$$
$$3x = -4 \qquad 3x = 4$$
$$x - \frac{4}{3} \quad \text{or} \quad x = \frac{4}{3}$$
The solutions are $-\frac{4}{3}$ and $\frac{4}{3}$.

18.
$$36y^2 - 49 = 0$$
$$(6y-7)(6y+7) = 0$$
$$6y - 7 = 0 \quad \text{or} \quad 6y + 7 = 0$$
$$y = \frac{7}{6} \quad \text{or} \quad y = -\frac{7}{6}$$
The solutions are $\frac{7}{6}$ and $-\frac{7}{6}$.

19.
$$16x^2 - 16x = 0$$
$$16x(x-1) = 0$$
$$16x = 0 \quad \text{or} \quad x - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = 1$$
The solutions are 0 and 1.

20.
$$12y^2 - 48y = 0$$
$$12y(y-4) = 0$$
$$y = 0 \quad \text{or} \quad y - 4 = 0$$
$$y = 0 \quad \text{or} \quad y = 4$$
The solutions are 0 and 4.

21.
$$(r-2)^2 = 7$$
$$r - 2 = \sqrt{7} \quad \text{or} \quad r - 2 = -\sqrt{7}$$
$$r = 2 + \sqrt{7} \quad \text{or} \quad r = 2 - \sqrt{7}$$
We abbreviate the solutions as $2 \pm \sqrt{7}$.

22. $(b+4)^2 = 27$

$b+4=\sqrt{27}$ or $b+4=-\sqrt{27}$

$b+4=3\sqrt{3}$ or $b+4=-3\sqrt{3}$

$b=-4+3\sqrt{3}$ or $b=-4-3\sqrt{3}$

We abbreviate the solutions as $-4\pm 3\sqrt{3}$.

23. $(4x-1)^2 = 20$

Use the square root property.

$4x-1=\sqrt{20}$ or $4x-1=-\sqrt{20}$

$4x-1=2\sqrt{5}$ or $4x-1=-2\sqrt{5}$

$4x=1+2\sqrt{5}$ or $4x=1-2\sqrt{5}$

The solutions are $\dfrac{1\pm 2\sqrt{5}}{4}$.

24. $(3t+5)^2 = 11$

$3t+5=\sqrt{11}$ or $3t+5=-\sqrt{11}$

$3t=-5+\sqrt{11}$ or $3t=-5-\sqrt{11}$

$t=\dfrac{-5+\sqrt{11}}{3}$ or $t=\dfrac{-5\pm\sqrt{11}}{3}$

The solutions are $\dfrac{-5\pm\sqrt{11}}{3}$.

25. $2x^2+7x+1=0$

Use the quadratic formula with $a = 2$, $b = 7$, and $c = 1$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-7\pm\sqrt{7^2-4(2)(1)}}{2(2)}$$

$$=\frac{-7\pm\sqrt{49-8}}{4}=\frac{-7\pm\sqrt{41}}{4}$$

The solutions are $\dfrac{-7+\sqrt{41}}{4}$ and $\dfrac{-7-\sqrt{41}}{4}$, which are approximately –.1492 and –3.3508.

26. $3x^2-x-7=0$

Use the quadratic formula with $a = 3$, $b = -1$, and $c = -7$.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(3)(-7)}}{2(3)}$$

$$=\frac{1\pm\sqrt{1+84}}{6}=\frac{1\pm\sqrt{85}}{6}$$

The solutions $\dfrac{1\pm\sqrt{85}}{6}$ are approximately 1.7033 and –1.3699.

27. $4k^2+2k=1$

Rewrite the equation in standard form.

$4k^2+2k-1=0$

Use the quadratic formula with $a = 4$, $b = 2$, and $c = -1$.

$$k=\frac{-2\pm\sqrt{2^2-4(4)(-1)}}{2(4)}=\frac{-2\pm\sqrt{4+16}}{8}$$

$$=\frac{-2\pm\sqrt{20}}{8}=\frac{-2\pm 2\sqrt{5}}{8}=\frac{2\left(-1\pm\sqrt{5}\right)}{2\cdot 4}$$

$$k=\frac{-1\pm\sqrt{5}}{4}$$

The solutions are $\dfrac{-1+\sqrt{5}}{4}$ and $\dfrac{-1-\sqrt{5}}{4}$, which are approximately .3090 and –.8090.

28. $r^2=3r+5$

$r^2-3r-5=0$

Use the quadratic formula with $a = 1$, $b = -3$, and $c = -5$.

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$r=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(-5)}}{2(1)}$$

$$=\frac{3\pm\sqrt{9+20}}{2}=\frac{3\pm\sqrt{29}}{2}$$

The solutions $\dfrac{3\pm\sqrt{29}}{2}$ are approximately 4.1926 and –1.1926.

29. $5y^2 + 5y = 2$

$5y^2 + 5y - 2 = 0$

$a = 5, b = 5, c = -2$

$$y = \frac{-5 \pm \sqrt{5^2 - 4(5)(-2)}}{2(5)} = \frac{-5 \pm \sqrt{25+40}}{10}$$

$$= \frac{-5 \pm \sqrt{65}}{10} = \frac{-5 \pm \sqrt{65}}{10}$$

The solutions are $\frac{-5+\sqrt{65}}{10}$ and $\frac{-5-\sqrt{65}}{10}$, which are approximately .3062 and –1.3062.

30. $2z^2 + 3 = 8z$

$2z^2 - 8z + 3 = 0$

$a = 2$, $b = -8$, and $c = 3$.

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$

$$= \frac{8 \pm \sqrt{64-24}}{4} = \frac{8 \pm \sqrt{40}}{4} = \frac{8 \pm 2\sqrt{10}}{4}$$

$$= \frac{2\left(4 \pm \sqrt{10}\right)}{4} = \frac{4 \pm \sqrt{10}}{2}$$

The solutions $\frac{4 \pm \sqrt{10}}{2}$, are approximately 3.5811 and .4189.

31. $6x^2 + 6x + 4 = 0$

$a = 6, b = 6, c = 4$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(6)(4)}}{2(6)} = \frac{-6 \pm \sqrt{36-96}}{12}$$

$$= \frac{-6 \pm \sqrt{-60}}{12}$$

Because $\sqrt{-60}$ is not a real number, the given equation has no real number solutions.

32. $3a^2 - 2a + 2 = 0$

$a = 3$, $b = -2$, and $c = 2$.

$$a = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(2)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{4-24}}{6} = \frac{2 \pm \sqrt{-20}}{6}$$

Since $\sqrt{-20}$ is not a real number, the given equation has no real number solutions.

33. $2r^2 + 3r - 5 = 0$

$a = 2, b = 3, c = -5$

$$r = \frac{-(3) \pm \sqrt{9 - 4(2)(-5)}}{2(2)} = \frac{-3 \pm \sqrt{9+40}}{4}$$

$$= \frac{-3 \pm \sqrt{49}}{4} = \frac{-3 \pm 7}{4}$$

$$r = \frac{-3+7}{4} = \frac{4}{4} = 1 \text{ or } r = \frac{-3-7}{4} = \frac{-10}{4} = \frac{-5}{2}$$

The solutions are $-\frac{5}{2}$ and 1.

34. $8x^2 = 8x - 3$

$8x^2 - 8x + 3 = 0$

$a = 8$, $b = -8$, and $c = 3$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(8)(3)}}{2(8)}$$

$$= \frac{8 \pm \sqrt{64-96}}{16} = \frac{8 \pm \sqrt{-32}}{16}$$

Because $\sqrt{-32}$ is not a real number, there are no real number solutions.

35. $2x^2 - 7x + 30 = 0$

$a = 2, b = -7, c = 30$

$$x = \frac{-(-7) \pm \sqrt{49 - 4(2)(30)}}{2(2)} = \frac{7 \pm \sqrt{-191}}{4}$$

Since $\sqrt{-191}$ is not a real number, there are no real solutions.

36. $3k^2 + k = 6$

$3k^2 + k - 6 = 0$

$a = 3$, $b = 1$, and $c = -6$.

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k = \frac{-1 \pm \sqrt{1^2 - 4(3)(-6)}}{2(3)} = \frac{-1 \pm \sqrt{1+72}}{6}$$

$$k = \frac{-1 \pm \sqrt{73}}{6}$$

The solutions $\frac{-1 \pm \sqrt{73}}{6}$ are approximately 1.2573 and –1.5907.

37. $1+\dfrac{7}{2a}=\dfrac{15}{2a^2}$

To eliminate fractions, multiply both sides by the common denominator, $2a^2$.

$2a^2+7a=15 \Rightarrow 2a^2+7a-15=0$

$a=2, b=7, c=-15$

$$a=\frac{-7\pm\sqrt{7^2-4(2)(-15)}}{2(2)}=\frac{-7\pm\sqrt{49+120}}{4}$$

$$=\frac{-7\pm\sqrt{169}}{4}\Rightarrow a=\frac{-7+13}{4}=\frac{6}{4}=\frac{3}{2} \text{ or}$$

$$a=\frac{-7-13}{4}=\frac{-20}{4}=-5$$

The solutions are $\frac{3}{2}$ and -5.

38. $5-\dfrac{4}{k}-\dfrac{1}{k^2}=0$

Multiply both sides by k^2.

$5k^2-4k-1=0$

$a=5, b=-4, c=-1$

$$k=\frac{-(-4)\pm\sqrt{(-4)^2-4(5)(-1)}}{2(5)}$$

$$=\frac{4\pm\sqrt{16+20}}{10}=\frac{4\pm\sqrt{36}}{10}=\frac{4\pm 6}{10}$$

$$k=\frac{4+6}{10}=\frac{10}{10}=1 \text{ or } k=\frac{4-6}{10}=\frac{-2}{10}=-\frac{1}{5}$$

The solutions are $-\frac{1}{5}$ and 1.

39. $25t^2+49=70t$

$25t^2-70t+49=0$

$$b^2-4ac=(-70)^2-4(25)(49)$$
$$=4900-4900$$
$$=0$$

The discriminant is 0.
There is one real solution to the equation.

40. $9z^2-12z=1$

$9z^2-12z-1=0$

$$b^2-4ac=(-12)^2-4(9)(-1)$$
$$=144+36$$
$$=180$$

The discriminant is positive.
There are two real solutions to the equation.

41. $13x^2+24x-5=0$

$$b^2-4ac=(24)^2-4(13)(-5)$$
$$=576+260=836$$

The discriminant is positive.
There are two real solutions to the equation.

42. $20x^2+19x+5=0$

$$b^2-4ac=(19)^2-4(20)(5)$$
$$=361-400=-39$$

The discriminant is negative.
There are no real solutions to the equation.

For Exercises 43–46 use the quadratic formula:
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

43. $4.42x^2-10.14x+3.79=0$

$$x=\frac{-(-10.14)\pm\sqrt{(-10.14)^2-4(4.42)(3.79)}}{2(4.42)}$$
$$\approx\frac{10.14\pm 5.9843}{8.84}\approx .4701 \text{ or } 1.8240$$

44. $3x^2-82.74x+570.4923=0$

$$x=\frac{-(-82.74)\pm\sqrt{(-82.74)^2-4(3)(570.4923)}}{2(3)}$$
$$=\frac{82.74}{6}=13.79$$

45. $7.63x^2+2.79x=5.32$

$7.63x^2+2.79x-5.32=0$

$$x=\frac{-2.79\pm\sqrt{(2.79)^2-4(7.63)(-5.32)}}{2(7.63)}$$
$$\approx\frac{-2.79\pm 13.0442}{15.26}\approx -1.0376 \text{ or } .6720$$

46. $8.06x^2+25.8726x=25.047256$

$8.06x^2+25.8726x-25.047256=0$

$$x=\frac{-25.8726\pm\sqrt{(25.8726)^2-4(8.06)(-25.047256)}}{2(8.06)}$$
$$\approx\frac{-25.8726\pm 38.4307}{16.12}\approx -3.9890 \text{ or } .7790$$

47. Let $x = 15$.

$$F = 4.402x^2 - 146.2x + 1278$$
$$F = 4.402(15)^2 - 146.2(15) + 1278$$
$$F = 75.45$$

The net U.S. farm income in 2015 was $75.45 billion.

48. Let $x = 19$.

$$F = 4.402x^2 - 146.2x + 1278$$
$$F = 4.402(19)^2 - 146.2(19) + 1278$$
$$F = 89.322$$

The net U.S. farm income in 2019 was $89.322 billion.

49. Let $F = 72$.

$$F = 4.402x^2 - 146.2x + 1278$$
$$72 = 4.402x^2 - 146.2x + 1278$$
$$0 = 4.402x^2 - 146.2x + 1206$$
$$x = \frac{146.2 \pm \sqrt{(-146.2)^2 - 4(4.402)(1206)}}{2(4.402)}$$
$$\approx 15.3 \text{ or } 17.9$$

The first full year the net U.S. farm income was below $72 billion was 2016.

50. Let $F = 69$.

$$F = 4.402x^2 - 146.2x + 1278$$
$$69 = 4.402x^2 - 146.2x + 1278$$
$$0 = 4.402x^2 - 146.2x + 1209$$
$$x = \frac{146.2 \pm \sqrt{(-146.2)^2 - 4(4.402)(1209)}}{2(4.402)}$$
$$\approx 15.6 \text{ or } 17.7$$

The first full year the net U.S. farm income recovered to be above $69 billion was 2018.

51. Let $F = 84$.

$$F = 4.402x^2 - 146.2x + 1278$$
$$84 = 4.402x^2 - 146.2x + 1278$$
$$0 = 4.402x^2 - 146.2x + 1194$$
$$x = \frac{146.2 \pm \sqrt{(-146.2)^2 - 4(4.402)(1194)}}{2(4.402)}$$
$$\approx 14.5 \text{ or } 18.7$$

The first full year the net U.S. farm income recovered to be above $84 billion was 2019.

52. Let $F = 90$.

$$F = 4.402x^2 - 146.2x + 1278$$
$$90 = 4.402x^2 - 146.2x + 1278$$
$$0 = 4.402x^2 - 146.2x + 1188$$
$$x = \frac{146.2 \pm \sqrt{(-146.2)^2 - 4(4.402)(1188)}}{2(4.402)}$$
$$\approx 14.2 \text{ or } 19.0$$

The first full year the net U.S. farm income recovered to be above $90 billion was 2020.

53. Let $x = 11$.

$$R = -.081x^2 + 2.176x - 5.325$$
$$R = -.081(11)^2 + 2.176(11) - 5.325$$
$$R = 8.81$$

The annual revenue for Gamestop in 2011 was $8.81 billion.

54. Let $x = 19$.

$$R = -.081x^2 + 2.176x - 5.325$$
$$R = -.081(19)^2 + 2.176(19) - 5.325$$
$$R = 6.778$$

The annual revenue for Gamestop in 2019 was $6.778 billion.

55. Let $R = 8.2$.

$$R = -.081x^2 + 2.176x - 5.325$$
$$8.2 = -.081x^2 + 2.176x - 5.325$$
$$0 = .081x^2 - 2.176x + 13.525$$
$$x = \frac{2.176 \pm \sqrt{(-2.176)^2 - 4(.081)(13.525)}}{2(.081)}$$
$$\approx 9.8 \text{ or } 17.1$$

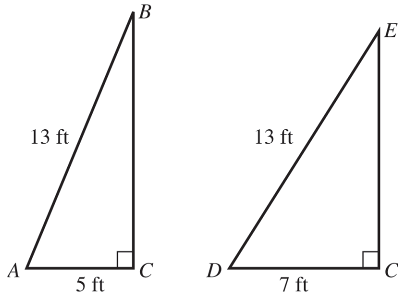
The first full year that revenue for Gamestop hit $8.2 billion was 2010.

56. Let $R = 7.8$.

$$R = -.081x^2 + 2.176x - 5.325$$
$$7.8 = -.081x^2 + 2.176x - 5.325$$
$$0 = .081x^2 - 2.176x + 13.125$$
$$x = \frac{2.176 \pm \sqrt{(-2.176)^2 - 4(.081)(13.125)}}{2(.081)}$$
$$\approx 9.1 \text{ or } 17.7$$

The first full year that revenue for Gamestop fell below $7.8 billion was 2018.

57. Triangle ABC represents the original position of the ladder, while triangle DEC represents the position of the ladder after it was moved.



Use the Pythagorean theorem to find the distance from the top of the ladder to the ground.
In triangle ABC,

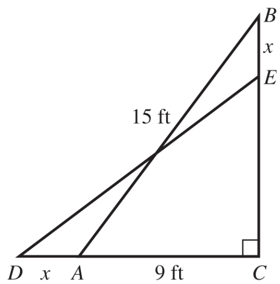
$13^2 = 5^2 + BC^2 \Rightarrow 169 - 25 = BC^2 \Rightarrow$
$144 = BC^2 \Rightarrow 12 = BC$

Thus, the top of the ladder was originally 12 feet from the ground.
In triangle DEC,

$13^2 = 7^2 + EC^2 \Rightarrow 169 - 49 = EC^2 \Rightarrow$
$120 = EC^2 \Rightarrow EC = \sqrt{120}$

The top of the ladder was $\sqrt{120}$ feet from the ground after the ladder was moved. Therefore, the ladder moved down $12 - \sqrt{120} \approx 1.046$ feet.

58. Triangle ABC shows the original position of the ladder against the wall, and triangle DEC is the position of the ladder after it was moved.



In triangle ABC,

$15^2 = BC^2 + 9^2 \Rightarrow BC^2 = 15^2 - 9^2 = 144 \Rightarrow$
$BC = 12$

In triangle DEC

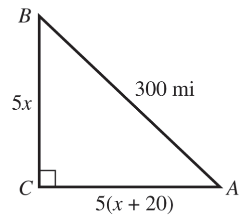
$15^2 = (12 - x)^2 + (9 + x)^2 \Rightarrow$
$225 = 144 - 24x + x^2 + 81 + 18x + x^2 \Rightarrow$
$0 = -6x + 2x^2 \Rightarrow 2x(x - 3) = 0 \Rightarrow$
$2x = 0 \Rightarrow x = 0$ or $x - 3 = 0 \Rightarrow x = 3$

The bottom of the ladder should be pulled 3 feet away from the wall.

59. a. The eastbound train travels at a speed of $x + 20$.

b. The northbound train travels a distance of $5x$ in 5 hours.
The eastbound train travels a distance of $5(x + 20) = 5x + 100$ in 5 hours.

c.



By the Pythagorean theorem,
$(5x)^2 + (5x + 100)^2 = 300^2$

d. Expand and combine like terms.

$25x^2 + 25x^2 + 1000x + 10,000 = 90,000$
$50x^2 + 1000x - 80,000 = 0$

Factor out the common factor, 50, and divide both sides by 50.

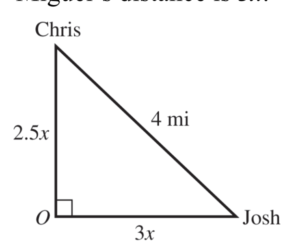
$50(x^2 + 20x - 1600) = 0$
$x^2 + 20x - 1600 = 0$

Now use the quadratic formula to solve for x.

$$x = \frac{-20 \pm \sqrt{20^2 - 4(1)(-1600)}}{2(1)}$$
≈ -51.23 or 31.23

Since x cannot be negative, the speed of the northbound train is $x \approx 31.23$ mph, and the speed of the eastbound train is $x + 20 \approx 51.23$ mph.

60. Let x represent the length of time they will be able to talk to each other.
Since $d = rt$, Tyrone' distance is $2.5x$ and Miguel's distance is $3x$.



This is a right triangle, so
$(2.5x)^2 + (3x)^2 = 4^2 \Rightarrow 6.25x^2 + 9x^2 = 16 \Rightarrow$
$15.25x^2 = 16 \Rightarrow x^2 = \frac{16}{15.25} \Rightarrow$
$x = \pm\sqrt{\frac{16}{15.25}} \approx \pm 1.024$
Since time must be nonnegative, they will be able to talk to each other for about 1.024 hr or approximately 61 min.

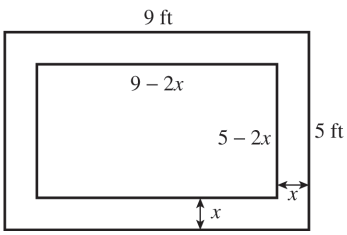
61. **a.** Let x represent the length. Then, $\frac{300-2x}{2}$ or $150 - x$ represents the width.

b. Use the formula for the area of a rectangle.
$LW = A \Rightarrow x(150 - x) = 5000$

c. $150x - x^2 = 5000$
Write this quadratic equation in standard form and solve by factoring.
$0 = x^2 - 150x + 5000$

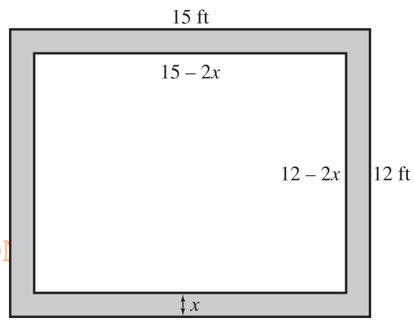
$x^2 - 150x + 5000 = 0$
$(x - 50)(x - 100) = 0$
$x - 50 = 0$ or $x - 100 = 0$
$x = 50$ or $x = 100$
Choose $x = 100$ because the length is the larger dimension. The length is 100 m and the width is $150 - 100 = 50$ m.

62. Let x represent the width of the border.



The area of the flower bed is $9 \cdot 5 = 45$. The area of the center is $(9 - 2x)(5 - 2x)$. Therefore,
$45 - (9 - 2x)(5 - 2x) = 24$
$45 - \left(45 - 28x + 4x^2\right) = 24$
$45 - 45 + 28x - 4x^2 = 24$
$0 = 4x^2 - 28x + 24$
$0 = 4\left(x^2 - 7x + 6\right)$
$0 = 4(x - 1)(x - 6)$
$x - 1 = 0$ or $x - 6 = 0$
$x = 1$ or $x = 6$
The solution $x = 6$ is impossible since both $9 - 2x$ and $5 - 2x$ would be negative. Therefore, the width of the border is 1 ft.

63. Let x = the width of the uniform strip around the rug.



The dimensions of the rug are $15 - 2x$ and $12 - 2x$. The area, 108, is the length times the width.

Solve the equation.

$$(15-2x)(12-2x)=108$$
$$180-54x+4x^2=108$$
$$4x^2-54x+72=0$$
$$2x^2-27x+36=0$$
$$(x-12)(2x-3)=0$$
$$x-12=0 \quad \text{or} \quad 2x-3=0$$
$$x=12 \quad \text{or} \quad x=\frac{3}{2}$$

Discard $x = 12$ since both $12 - 2x$ and $15 - 2x$ would be negative.

If $x=\frac{3}{2}$, then

$$15-2x=15-2\left(\frac{3}{2}\right)=12$$

and $12-2x=12-2\left(\frac{3}{2}\right)=9$.

The dimensions of the rug should be 9 ft by 12 ft.

64. Let x = Haroud's speed. Then $x + 92$ = Franchitti's speed. From the formula $d = rt$, we have Haroud's time $=\frac{500}{x}$ and Franchitti's time $=\frac{500}{x+92}$. Then, we have

$$\frac{500}{x+92}=\frac{500}{x}-3.72 \Rightarrow$$
$$500x=500(x+92)-3.72x(x+92) \Rightarrow$$
$$500x=500x+46,000-3.72x^2-342.24x \Rightarrow$$
$$0=-3.72x^2-342.24x+46,000$$

Use the quadratic formula to solve for x.

$$x=\frac{-(-342.24)\pm\sqrt{(-342.24)^2-4(-3.72)(46,000)}}{2(-3.72)}$$
$$\approx -166.3 \text{ or } 74.3$$

The negative value is not applicable. Haroud's speed was 74.3 mph and Franchitti's speed was $74.3 + 92 = 166.3$ mph.

For exercises 65–70, use the formula $h=-16t^2+v_0t+h_0$, where h_0 is the height of the object when $t = 0$, and v_0 is the initial velocity at time $t = 0$.

65. $v_0=0,\ h_0=625,\ h=0$

$$0=-16t^2+(0)t+625 \Rightarrow 16t^2-625=0 \Rightarrow$$
$$(4t-25)(4t+25)=0 \Rightarrow t=\frac{25}{4}=6.25 \text{ or}$$
$$t=-\frac{25}{4}=-6.25$$

The negative solution is not applicable. It takes 6.25 seconds for the baseball to reach the ground.

66. When the ball has fallen 196 feet, it is $625 - 196 = 429$ feet above the ground.

$v_0=0,\ h_0=625,\ h=429$

$$429=-16t^2+(0)t+625 \Rightarrow 16t^2-196=0 \Rightarrow$$
$$(4t-14)(4t+14)=0 \Rightarrow t=\frac{14}{4}=3.5 \text{ or}$$
$$t=-\frac{14}{4}=-3.5$$

The negative solution is not applicable. It takes 3.5 seconds for the ball to fall 196 feet.

67. a. $v_0=0,\ h_0=200,\ h=0$

$$0=-16t^2-(0)t+200 \Rightarrow 16t^2=200 \Rightarrow$$
$$t^2=\frac{200}{16} \Rightarrow t=\pm\frac{\sqrt{200}}{4}\approx \pm 3.54$$

The negative solution is not applicable. It will take about 3.54 seconds for the rock to reach the ground if it is dropped.

b. $v_0=-40,\ h_0=200,\ h=0$

$$0=-16t^2-40t+200$$

Using the quadratic formula, we have

$$t=\frac{-(-40)\pm\sqrt{(-40)^2-4(-16)(200)}}{2(-16)}$$
$$=-5 \text{ or } 2.5$$

The negative solution is not applicable. It will take about 2.5 seconds for the rock to reach the ground if it is thrown with an initial velocity of 40 ft/sec.

c. $v_0=-40,\ h_0=200,\ t=2$

$$h=-16(2)^2-40(2)+200=56$$

After 2 seconds, the rock is 56 feet above the ground. This means it has fallen $200 - 56 = 144$ feet.

68. **a.** $v_0 = 800,\ h_0 = 0,\ h = 3200$

$3200 = -16t^2 + 800t + 0 \Rightarrow$
$16t^2 - 800t + 3200 = 0 \Rightarrow$
$t^2 - 50t + 200 = 0$

Using the quadratic formula, we have

$$t = \frac{-(-50) \pm \sqrt{(-50)^2 - 4(1)(200)}}{2(1)} \Rightarrow$$

$t \approx 4.38$ or $t \approx 45.62$

The rocket rises to 3200 feet after about 4.38 seconds. It also reaches 3200 feet as it falls back to the ground after about 45.62 seconds. Because we are asked how long to rise to 3200 feet, the answer will be about 4.38 sec.

b. $v_0 = 800,\ h_0 = 0,\ h = 0$

$0 = -16t^2 + 800t + 0 \Rightarrow$
$16t^2 - 800t = 0 \Rightarrow 16t(t-50) = 0 \Rightarrow$
$t = 0$ or $t = 50$

The rocket hits the ground after 50 seconds.

69. **a.** $v_0 = 64,\ h_0 = 0,\ h = 64$

$64 = -16t^2 + 64t + 0 \Rightarrow$
$16t^2 - 64t + 64 = 0 \Rightarrow$
$t^2 - 4t + 4 = 0 \Rightarrow (t-2)^2 = 0 \Rightarrow t = 2$

The ball will reach 64 feet after 2 seconds.

b. $v_0 = 64,\ h_0 = 0,\ h = 39$

$39 = -16t^2 + 64t + 0 \Rightarrow$
$16t^2 - 64t + 39 = 0 \Rightarrow (4t-13)(4t-3) \Rightarrow$
$t = \frac{13}{4} = 3.25$ or $t = \frac{3}{4} = .75$

The ball will reach 39 feet after .75 seconds and after 3.25 seconds.

c. Two answers are possible because the ball reaches the given height twice, once on the way up and once on the way down.

70. **a.** $v_0 = 100,\ h_0 = 0,\ h = 50$

$50 = -16t^2 + 100t + 0 \Rightarrow$
$16t^2 - 100t + 50 = 0 \Rightarrow$
$8t^2 - 50t + 25 = 0 \Rightarrow t \approx .55$ or 5.7

The ball will reach 50 feet on the way up after approximately 0.55 seconds.

b. $v_0 = 100,\ h_0 = 0,\ h = 35$

$35 = -16t^2 + 100t + 0 \Rightarrow$
$16t^2 - 100t + 35 = 0 \Rightarrow$
$t \approx .37$ or 5.88

The ball will reach 35 feet on the way up after approximately 0.37 seconds.

In exercises 71–76, we discard negative roots since all variables represent positive real numbers.

71. $S = \frac{1}{2}gt^2$ for t

$$2S = gt^2$$
$$\frac{2S}{g} = t^2$$
$$\sqrt{\frac{2S}{g}} \cdot \frac{\sqrt{g}}{\sqrt{g}} = t$$
$$\frac{\sqrt{2Sg}}{g} = t$$

72. Solve for r.

$$a = \pi r^2$$
$$\frac{a}{\pi} = r^2$$
$$\sqrt{\frac{a}{\pi}} \cdot \frac{\sqrt{\pi}}{\sqrt{\pi}} = r \quad (r > 0)$$
$$r = \sqrt{\frac{a}{\pi}} = \frac{\sqrt{\pi a}}{\pi}$$

73. $L = \frac{d^4k}{h^2}$ for h

$$Lh^2 = d^4k$$
$$h^2 = \frac{d^4k}{L}$$
$$h = \sqrt{\frac{d^4k}{L}} \cdot \frac{\sqrt{L}}{\sqrt{L}} = \frac{\sqrt{d^4kL}}{L}$$
$$h = \frac{d^2\sqrt{kL}}{L}$$

74. Solve for v.

$$F = \frac{kMv^2}{r}$$

$$\frac{Fr}{kM} = v^2$$

$$\sqrt{\frac{Fr}{kM}} \cdot \frac{\sqrt{kM}}{\sqrt{kM}} = v \; (v > 0)$$

$$\sqrt{\frac{Fr}{kM}} = \frac{\sqrt{FrkM}}{kM} = v$$

75. $$P = \frac{E^2R}{(r+R)^2} \text{ for } R$$

$$P(r+R)^2 = E^2R$$

$$P\left(r^2 + 2rR + R^2\right) = E^2R$$

$$Pr^2 + 2PrR + PR^2 = E^2R$$

$$PR^2 + \left(2Pr - E^2\right)R + Pr^2 = 0$$

Solve for R by using the quadratic formula with $a = P,\ b = 2\text{Pr} - E^2$, and $c = \text{Pr}^2$.

$$R = \frac{-\left(2\text{Pr} - E^2\right) \pm \sqrt{\left(2\text{Pr} - E^2\right)^2 - 4P \cdot \text{Pr}^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{4P^2r^2 - 4\,\text{Pr}\,E^2 + E^4 - 4P^2r^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{E^4 - 4\,\text{Pr}\,E^2}}{2P}$$

$$= \frac{-2\text{Pr} + E^2 \pm \sqrt{E^2\left(E^2 - 4\text{Pr}\right)}}{2P}$$

$$R = \frac{-2\text{Pr} + E^2 \pm E\sqrt{E^2 - 4\text{Pr}}}{2P}$$

76. Solve for r.

$S = 2\pi rh + 2\pi r^2$

Write as a quadratic equation in r.

$(2\pi)r^2 + (2\pi h)r - S = 0$

Solve for r using the quadratic formula with $a = 2\pi$, $b = 2\pi h$, and $c = -S$.

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-S)}}{2(2\pi)}$$

$$= \frac{-2\pi h \pm \sqrt{4\pi^2h^2 + 8\pi S}}{4\pi}$$

$$= \frac{-2\pi h \pm 2\sqrt{\pi^2h^2 + 2\pi S}}{4\pi}$$

$$= \frac{2\left(-\pi h \pm \sqrt{\pi^2h^2 + 2\pi S}\right)}{4\pi}$$

$$r = \frac{-\pi h \pm \sqrt{\pi^2h^2 + 2\pi S}}{2\pi}$$

Chapter 1 Review Exercises

1. 0 and 6 are whole numbers.

2. $-12, -6, -\sqrt{4}$, 0, and 6 are integers.

3. $-12, -6, -\frac{9}{10}, -\sqrt{4}, 0, \frac{1}{8}$, and 6 are rational numbers.

4. $-\sqrt{7}, \frac{\pi}{4}, \sqrt{11}$ are irrational numbers.

5. $9[(-3)4] = 9[4(-3)]$
Commutative property of multiplication

6. $7(4 + 5) = (4 + 5)7$
Commutative property of multiplication

7. $6(x + y - 3) = 6x + 6y + 6(-3)$
Distributive property

8. $11 + (5 + 3) = (11 + 5) + 3$
Associative property of addition

9. x is at least 9.
$x \geq 9$

10. x is negative.
$x < 0$

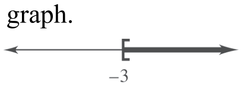
11. $|6-4| = 2,\ -|-2| = -2,\ |8+1| = |9| = 9,$
$-|3-(-2)| = -|3+2| = -|5| = -5$
Since $-5, -2, 2, 9$ are in order, then
$-|3-(-2)|,\ -|-2|,\ |6-4|,\ |8+1|$ are in order.

12. $-\left|\sqrt{16}\right| = -4,\ -\sqrt{8},\ \sqrt{7},\ \left|-\sqrt{12}\right| = \sqrt{12}$

13. $7 - |-8| = 7 - 8 = -1$

14. $|-3| - |-9+6| = 3 - |-3| = 3 - 3 = 0$

15. $x \geq -3$
Start at -3 and draw a ray to the right. Use a bracket at -3 to show that -3 is a part of the graph.



16. $-4 < x \leq 6$
Put a parenthesis at -4 and a bracket at 6. Draw a line segment between these two endpoints.

17. $\dfrac{-9+(-6)(-3)\div 9}{6-(-3)} = \dfrac{-9+18\div 9}{6+3} = \dfrac{-9+2}{9} = -\dfrac{7}{9}$

18. $\dfrac{20\div 4\cdot 2\div 5-1}{-9-(-3)-12\div 3} = \dfrac{5\cdot 2\div 5-1}{-9-(-3)-4}$
$= \dfrac{10\div 5-1}{-9+3-4} = \dfrac{2-1}{-6-4} = -\dfrac{1}{10}$

19. $\left(3x^4 - x^2 + 5x\right) - \left(-x^4 + 3x^2 - 6x\right)$
$= 3x^4 - x^2 + 5x + x^4 - 3x^2 + 6x$
$= \left(3x^4 + x^4\right) + \left(-x^2 - 3x^2\right) + (5x + 6x)$
$= 4x^4 - 4x^2 + 11x$

20. $\left(-8y^3 + 8y^2 - 5y\right) - \left(2y^3 + 4y^2 - 10\right)$
$= -8y^3 + 8y^2 - 5y - 2y^3 - 4y^2 + 10$
$= -8y^3 - 2y^3 + 8y^2 - 4y^2 - 5y + 10$
$= -10y^3 + 4y^2 - 5y + 10$

21. $(5k-2h)(5k+2h) = (5k)^2 - (2h)^2 = 25k^2 - 4h^2$

22. $(2r-5y)(2r+5y) = (2r)^2 - (5y)^2$
$= 4r^2 - 25y^2$

23. $(3x+4y)^2 = (3x)^2 + 2(3x)(4y) + (4y)^2$
$= 9x^2 + 24xy + 16y^2$

24. $(2a-5b)^2 = (2a)^2 - 2(2a)(5b) + (5b)^2$
$= 4a^2 - 20ab + 25b^2$

25. $2kh^2 - 4kh + 5k = k\left(2h^2 - 4h + 5\right)$

26. $2m^2n^2 + 6mn^2 + 16n^2 = 2n^2\left(m^2 + 3m + 8\right)$

27. $5a^4 + 12a^3 + 4a^2 = a^2\left(5a^2 + 12a + 4\right)$
$= a^2(5a+2)(a+2)$

28. $24x^3+4x^2-4x=4x\left(6x^2+x-1\right)$
$=4x(3x-1)(2x+1)$

29. $144p^2-169q^2=(12p)^2-(13q)^2$
$=(12p-13q)(12p+13q)$

30. $81z^2-25x^2=(9z)^2-(5x)^2$
$=(9z+5x)(9z-5x)$

31. $27y^3-1=(3y)^3-1^3$
$=(3y-1)\left[(3y)^2+(3y)(1)+1^2\right]$
$=(3y-1)\left(9y^2+3y+1\right)$

32. $125a^3+216=(5a)^3+(6)^3$
$=(5a+6)\left[(5a)^2-5a(6)+6^2\right]$
$=(5a+6)\left(25a^2-30a+36\right)$

33. $\frac{3x}{5}\cdot\frac{45x}{12}=\frac{3x\cdot 45x}{5\cdot 12}=\frac{3\cdot 5\cdot 3\cdot 3x^2}{4\cdot 5\cdot 3}=\frac{9x^2}{4}$

34. $\frac{5k^2}{24}-\frac{70k}{36}=\frac{5k^2\cdot 3}{24\cdot 3}-\frac{70k\cdot 2}{36\cdot 2}=\frac{15k^2}{72}-\frac{140k}{72}$
$=\frac{15k^2-140k}{72}=\frac{5k(3k-28)}{72}$

35. $\frac{c^2-3c+2}{2c(c-1)}\div\frac{c-2}{8c}=\frac{(c-1)(c-2)}{2c(c-1)}\cdot\frac{8c}{(c-2)}$
$=\frac{8c(c-1)(c-2)}{2c(c-1)(c-2)}=\frac{8}{2}=4$

36. $\frac{p^3-2p^2-8p}{3p\left(p^2-16\right)}\div\frac{p^2+4p+4}{9p^2}$
$=\frac{p\left(p^2-2p-8\right)}{3p(p+4)(p-4)}\cdot\frac{9p^2}{(p+2)(p+2)}$
$=\frac{p(p-4)(p+2)\cdot 9p^2}{3p(p+4)(p-4)(p+2)(p+2)}$
$=\frac{3p(p-4)(p+2)\cdot 3p^2}{3p(p-4)(p+2)\cdot(p+4)(p+2)}$
$=\frac{3p^2}{(p+4)(p+2)}$

37. $\frac{2m^2-4m+2}{m^2-1}\div\frac{6m+18}{m^2+2m-3}$
$=\frac{2\left(m^2-2m+1\right)}{(m+1)(m-1)}\cdot\frac{m^2+2m-3}{6m+18}$
$=\frac{2(m-1)^2}{(m+1)(m-1)}\cdot\frac{(m+3)(m-1)}{6(m+3)}$
$=\frac{2(m-1)(m+3)\cdot(m-1)^2}{2(m-1)(m+3)\cdot 3(m+1)}=\frac{(m-1)^2}{3(m+1)}$

38. $\frac{x^2+6x+5}{4\left(x^2+1\right)}\cdot\frac{2x(x+1)}{x^2-25}$
$=\frac{(x+5)(x+1)\cdot 2x(x+1)}{4\left(x^2+1\right)\cdot(x+5)(x-5)}$
$=\frac{2(x+5)\cdot x(x+1)^2}{2(x+5)\cdot 2\left(x^2+1\right)(x-5)}$
$=\frac{x(x+1)^2}{2\left(x^2+1\right)(x-5)}$

39. $5^{-3}=\frac{1}{5^3}$ or $\frac{1}{125}$

40. $10^{-2}=\frac{1}{10^2}$ or $\frac{1}{100}$

41. $-8^0=-\left(8^0\right)=-1$

42. $\left(-\frac{5}{6}\right)^{-2}=\left(-\frac{6}{5}\right)^2=\frac{36}{25}$

43. $4^6\cdot 4^{-3}=4^{6+(-3)}=4^3$

44. $7^{-5}\cdot 7^{-2}=7^{-5+(-2)}=7^{-7}=\frac{1}{7^7}$

45. $\frac{8^{-5}}{8^{-4}}=8^{-5-(-4)}=8^{-5+4}=8^{-1}=\frac{1}{8}$

46. $\frac{6^{-3}}{6^4}=6^{-3-4}=6^{-7}=\frac{1}{6^7}$

47. $5^{-1}+2^{-1}=\frac{1}{5}+\frac{1}{2}=\frac{7}{10}$

48. $5^{-2}+5^{-1}=\frac{1}{5^2}+\frac{1}{5}=\frac{1}{25}+\frac{1}{5}=\frac{6}{25}$

49. $\frac{5^{1/3}5^{1/2}}{5^{3/2}}=5^{1/3+1/2-3/2}=5^{-2/3}=\frac{1}{5^{2/3}}$

50. $\frac{2^{3/4}\cdot 2^{-1/2}}{2^{1/4}}=\frac{2^{1/4}}{2^{1/4}}=1$

51. $\left(3a^2\right)^{1/2}\cdot\left(3^2a\right)^{3/2}=3^{1/2}a\cdot 3^3a^{3/2}=3^{7/2}a^{5/2}$

52.
$$\begin{aligned}(4p)^{2/3}\cdot\left(2p^3\right)^{3/2}&=4^{2/3}p^{2/3}\cdot 2^{3/2}\cdot p^{9/2}\\&=\left(2^2\right)^{2/3}p^{2/3}\cdot 2^{3/2}p^{9/2}\\&=2^{4/3}\cdot 2^{3/2}p^{2/3}p^{9/2}\\&=2^{17/6}p^{31/6}\end{aligned}$$

53. $\sqrt[3]{27}=3$

54. $\sqrt[6]{-64}$ is not a real number.

55.
$$\begin{aligned}\sqrt[3]{54p^3q^5}&=\sqrt[3]{27\cdot 2p^3q^3q^2}\\&=\sqrt[3]{27p^3q^3}\cdot\sqrt[3]{2q^2}=3pq\sqrt[3]{2q^2}\end{aligned}$$

56. $\sqrt[4]{64a^5b^3}=\sqrt[4]{16a^4}\cdot\sqrt[4]{4ab^3}=2a\sqrt[4]{4ab^3}$

57.
$$\begin{aligned}3\sqrt{3}-12\sqrt{12}&=3\sqrt{3}-12\sqrt{4\cdot 3}=3\sqrt{3}-12\cdot 2\sqrt{3}\\&=3\sqrt{3}-24\sqrt{3}=-21\sqrt{3}\end{aligned}$$

58.
$$\begin{aligned}8\sqrt{7}+2\sqrt{63}&=8\sqrt{7}+2\sqrt{9\cdot 7}\\&=8\sqrt{7}+6\sqrt{7}=14\sqrt{7}\end{aligned}$$

59.
$$\begin{aligned}\frac{\sqrt{3}}{1+\sqrt{2}}&=\frac{\sqrt{3}\left(1-\sqrt{2}\right)}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)}=\frac{\sqrt{3}-\sqrt{6}}{1-2}\\&=\frac{\sqrt{3}-\sqrt{6}}{-1}=\sqrt{6}-\sqrt{3}\end{aligned}$$

60.
$$\begin{aligned}\frac{4+\sqrt{2}}{4-\sqrt{5}}&=\frac{\left(4+\sqrt{2}\right)}{\left(4-\sqrt{5}\right)}\cdot\frac{\left(4+\sqrt{5}\right)}{\left(4+\sqrt{5}\right)}\\&=\frac{16+4\sqrt{2}+4\sqrt{5}+\sqrt{10}}{16-\left(\sqrt{5}\right)^2}\\&=\frac{16+4\sqrt{2}+4\sqrt{5}+\sqrt{10}}{11}\end{aligned}$$

61.
$$\begin{aligned}3x-4\left(x-2\right)&=2x+9\\3x-4x+8&=2x+9\\-x+8&=2x+9\\-1&=3x\Rightarrow x=-\frac{1}{3}\end{aligned}$$

62.
$$\begin{aligned}4y+9&=-3(1-2y)+5\\4y+9&=-3+6y+5\\4y+9&=2+6y\\-2y&=-7\Rightarrow y=\frac{7}{2}\end{aligned}$$

63.
$$\begin{aligned}\frac{2m}{m-3}&=\frac{6}{m-3}+4\\2m&=6+4(m-3)\\2m&=6+4m-12\\2m&=4m-6\\6&=2m\Rightarrow 3=m\end{aligned}$$

Because $m = 3$ would make the denominators of the fractions equal to 0, making the fractions undefined, the given equation has no solution.

64. $\frac{15}{k+5}=4-\frac{3k}{k+5}$

Multiply both sides of the equation by the common denominator $k + 5$.

$15=4\left(k+5\right)-3k$

$15=4k+20-3k\Rightarrow k=-5$

If $k = -5$, the fractions would be undefined, so the given equation has no solution.

65.
$$\begin{aligned}8ax-3&=2x\\8ax-2x&=3\\x\left(8a-2\right)&=3\\x&=\frac{3}{8a-2}\end{aligned}$$

66.
$$\begin{aligned}b^2x-2x&=4b^2\\\left(b^2-2\right)x&=4b^2\\x&=\frac{4b^2}{b^2-2}\end{aligned}$$

67. $\left|\dfrac{2-y}{5}\right| = 8$

$\dfrac{2-y}{5} = 8$ or $\dfrac{2-y}{5} = -8$

$5\left(\dfrac{2-y}{5}\right) = 5(8)$ or $5\left(\dfrac{2-y}{5}\right) = 5(-8)$

$2-y = 40$ or $2-y = -40$

$-y = 38$ or $-y = -42$

$y = -38$ or $y = 42$

The solutions are –38 and 42.

68. $|4k+1| = |6k-3|$

$4k+1 = 6k-3$ or $4k+1 = -(6k-3)$

$4k+1 = 6k-3$ or $4k+1 = -6k+3$

$-2k = -4$ or $10k = 2$

$k = 2$ or $k = \dfrac{1}{5}$

The solutions are 2 and $\dfrac{1}{5}$.

69. $(b+7)^2 = 5$

Use the square root property to solve this quadratic equation.

$b+7 = \sqrt{5}$ or $b+7 = -\sqrt{5}$

$b = -7+\sqrt{5}$ or $b = -7-\sqrt{5}$

The solutions are $-7+\sqrt{5}$ and $-7-\sqrt{5}$, which we abbreviate as $-7 \pm \sqrt{5}$.

70. $(2p+1)^2 = 7$

Solve by the square root property.

$2p+1 = \sqrt{7}$ or $2p+1 = -\sqrt{7}$

$2p = -1+\sqrt{7}$ or $2p = -1-\sqrt{7}$

$p = \dfrac{-1+\sqrt{7}}{2}$ or $p = \dfrac{-1-\sqrt{7}}{2}$

The solutions are $\dfrac{-1 \pm \sqrt{7}}{2}$.

71. $2p^2 + 3p = 2$

Write the equation in standard form and solve by factoring.

$2p^2 + 3p - 2 = 0$

$(2p-1)(p+2) = 0$

$2p-1 = 0$ or $p+2 = 0$

$p = \dfrac{1}{2}$ or $p = -2$

The solutions are $\dfrac{1}{2}$ and –2.

72. $2y^2 = 15 + y$

Write the equation in standard form and solve by factoring.

$2y^2 - y - 15 = 0$

$(y-3)(2y+5) = 0$

$y = 3$ or $y = -\dfrac{5}{2}$

The solutions are 3 and $-\dfrac{5}{2}$.

73. $2q^2 - 11q = 21 \Rightarrow 2q^2 - 11q - 21 = 0 \Rightarrow$

$(2q+3)(q-7) = 0$

$2q+3 = 0$ or $q-7 = 0$

$q = -\dfrac{3}{2}$ or $q = 7$

The solutions are $-\dfrac{3}{2}$ and 7.

74. $3x^2 + 2x = 16 \Rightarrow 3x^2 + 2x - 16 = 0 \Rightarrow$

$(3x+8)(x-2) = 0$

$3x+8 = 0$ or $x-2 = 0$

$x = -\dfrac{8}{3}$ or $x = 2$

The solutions are $-\dfrac{8}{3}$ and 2.

75. $6k^4 + k^2 = 1 \Rightarrow 6k^4 + k^2 - 1 = 0$

Let $p = k^2$, so $p^2 = k^4$.

$6p^2 + p - 1 = 0$

$(3p-1)(2p+1) = 0$

$3p-1 = 0$ or $2p+1 = 0$

$p = \dfrac{1}{3}$ or $p = -\dfrac{1}{2}$

If $p = \frac{1}{3}$, $k^2 = \frac{1}{3} \Rightarrow k = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

If $p = -\frac{1}{2}$, $k^2 = -\frac{1}{2}$ has no real number solution.

The solutions are $\pm\frac{\sqrt{3}}{3}$.

76. $21p^4 = 2 + p^2 \Rightarrow 21p^4 - p^2 - 2 = 0$

Let $u = p^2$; then $u^2 = p^4$.

$$21u^2 - u - 2 = 0$$
$$(3u-1)(7u+2) = 0$$
$$3u - 1 = 0 \quad \text{or} \quad 7u + 2 = 0$$
$$x = \frac{1}{3} \quad \text{or} \quad x = -\frac{2}{7}$$
$$p^2 = \frac{1}{3} \quad \text{or} \quad p^2 = -\frac{2}{7}$$

If $x = -\frac{2}{7}$, $p^2 = -\frac{2}{7}$ has no real number solution.

$$p = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$

The solutions are $\pm\frac{\sqrt{3}}{3}$.

77. $p = \frac{E^2R}{(r+R)^2}$ for r.

$$p(r+R)^2 = E^2R$$
$$p\left(r^2 + 2rR + R^2\right) = E^2R$$
$$pr^2 + 2rpR + R^2p = E^2R$$
$$pr^2 + 2rpR + R^2p - E^2R = 0$$

Use the quadratic formula to solve for r.

$$r = \frac{-2pR \pm \sqrt{4p^2R^2 - 4p\left(R^2p - E^2R\right)}}{2p}$$
$$r = \frac{-2pR \pm \sqrt{4pE^2R}}{2p} = \frac{-pR \pm E\sqrt{pR}}{p}$$

78. $p = \frac{E^2R}{(r+R)^2}$ for E.

$$p(r+R)^2 = E^2R \Rightarrow E^2 = \frac{p(r+R)^2}{R} \Rightarrow$$
$$E = \pm\sqrt{\frac{p(r+R)^2}{R}} = \frac{\pm(r+R)\sqrt{pR}}{R}$$

79. $K = s(s-a)$ for s.

$$K = s^2 - as \Rightarrow s^2 - as - K = 0$$

Use the quadratic formula.

$$s = \frac{a \pm \sqrt{a^2 - 4(-K)}}{2} = \frac{a \pm \sqrt{a^2 + 4K}}{2}$$

80. $kz^2 - hz - t = 0$ for z.

Use the quadratic formula with $a = k$, $b = -h$, and $c = -t$.

$$z = \frac{-(-h) \pm \sqrt{(-h)^2 - 4(k)(-t)}}{2k} = \frac{h \pm \sqrt{h^2 + 4kt}}{2k}$$

81. $|82.3 - (-33.9)| = |116.2| = 116.2\%$

82. $|-3.5 - (-14.7)| = |-11.2| = 11.2\%$

83. Let x = the original price

$$x - .2x = 1229$$
$$.8x = 1229$$
$$x = 1536.25$$

The original price of the laptop was $1536.25.

84. Let x = the original price

$$x - .25x = 740$$
$$0.75x = 740$$
$$x = 986.67$$

The original price of the phone was $986.67.

85. Let $x = 7.2$

$$A = .24(7.2) + .85$$
$$A = 2.578$$

The percent of consumer expenditures for alcohol when the country spent 7.2% on food was 2.578%.

86. Let $x = 11.5$

$$A = .24(11.5) + .85$$
$$A = 3.61$$

The percent of consumer expenditures for alcohol when the country spent 11.5% on food was 3.61%.

87. Let $x = 9.5$

$A = .24(9.5) + .85$

$A = 3.13$

The percent of consumer expenditures for alcohol when the country spent 9.5% on food was 3.13%.

88. Let $x = 12.1$

$A = .24(12.1) + .85$

$A = 3.754$

The percent of consumer expenditures for alcohol when the country spent 12.1% on food was 3.754%.

89. Let $A = 4.09$

$4.09 = .24x + .85$

$3.24 = .24x$

$13.5 = x$

When the percent of consumer expenditures for alcohol was 4.09%, the country spent 13.5% on food.

90. Let $A = 2.77$

$2.77 = .24x + .85$

$1.92 = .24x$

$8 = x$

When the percent of consumer expenditures for alcohol was 2.77%, the country spent 8.0% on food.

91. Let $A = 1.96$

$1.96 = .24x + .85$

$1.11 = .24x$

$4.625 = x$

When the percent of consumer expenditures for alcohol was 1.96%, the country spent 4.625% on food.

92. Let $A = 3.85$

$3.85 = .24x + .85$

$3 = .24x$

$12.5 = x$

When the percent of consumer expenditures for alcohol was 3.85%, the country spent 12.5% on food.

93. Let $x = 3$

$$V = \frac{83.1995(3)^2 + 490(3) + 1917.3}{3+1} \approx 1034.02$$

The total number of new motor vehicles sold in the U.S. in March 2020 was approximately 1034 thousand.

94. Let $x = 10$

$$V = \frac{83.1995(10)^2 + 490(10) + 1917.3}{10+1} \approx 1376.11$$

The total number of new motor vehicles sold in the U.S. in October 2020 was approximately 1376 thousand.

95. Let $V = 1200$.

$$V = \frac{83.1995x^2 + 490x + 1917.3}{x+1}$$

$$1200 = \frac{83.1995x^2 + 490x + 1917.3}{x+1}$$

$$1200(x+1) = 83.1995x^2 + 490x + 1917.3$$

$$0 = 83.1995x^2 - 710x + 717.3$$

$$x = \frac{710 \pm \sqrt{(-710)^2 - 4(83.1995)(717.3)}}{2(83.1995)}$$

$$\approx 1.2 \text{ or } 7.3$$

The number of new vehicles sold in the second half of 2020 surpassed 1200 thousand in August.

96. Let $V = 1400$.

$$V = \frac{83.1995x^2 + 490x + 1917.3}{x+1}$$

$$1400 = \frac{83.1995x^2 + 490x + 1917.3}{x+1}$$

$$1400(x+1) = 83.1995x^2 + 490x + 1917.3$$

$$0 = 83.1995x^2 - 910x + 517.3$$

$$x = \frac{910 \pm \sqrt{(-910)^2 - 4(83.1995)(517.3)}}{2(83.1995)}$$

$$\approx 0.6 \text{ or } 10.3$$

The number of new vehicles sold in the second half of 2020 surpassed 1400 thousand in November.

97. Let $x = 17$

$R = 24.7(17)^{.4} \approx 76.71$

The total amount spent on non-defense research and development in the U.S. in 2017 was approximately \$76.7 billion.

98. Let $x = 19$

$R = 24.7(19)^{.4} \approx 80.20$

The total amount spent on non-defense research and development in the U.S. in 2019 was approximately \$80.2 billion.

99. Let x = the single interest rate

$2000(.12) + 500(.07) = 2500x$

$275 = 2500x$

$.11 = x$

Therefore, a single interest rate of 11% will yield the same results.

100. Let x = the amount of beef. Then $30 - x$ = the amount of pork.

$$2.8x + 3.25(30 - x) = 3.10(30)$$
$$2.8x + 97.5 - 3.25x = 93$$
$$-.45x = -4.5$$
$$x = 10$$

Therefore the butcher should use 10 pounds of beef and 30 – 10 = 20 pounds of pork.

101. Let $x = 15$

$$V = .104x^2 - 3.22x + 32.8$$
$$V = .104(15)^2 - 3.22(15) + 32.8$$
$$V = 7.9$$

The vacancy rate at U.S. malls in 2015 was approximately 7.9%.

102. Let $x = 20$

$$V = .104x^2 - 3.22x + 32.8$$
$$V = .104(20)^2 - 3.22(20) + 32.8$$
$$V = 10$$

The vacancy rate at U.S. malls in 2020 was approximately 10%.

103. Let $V = 8.3\%$.

$$V = .104x^2 - 3.22x + 32.8$$
$$8.3 = .104x^2 - 3.22x + 32.8$$
$$0 = .104x^2 - 3.22x + 24.5$$
$$x = \frac{3.22 \pm \sqrt{(-3.22)^2 - 4(.104)(24.5)}}{2(.104)}$$
$$\approx 13.46 \text{ or } 17.5$$

The most recent time the vacancy rate at U.S. malls exceeded 8.3% was in 2018.

104. Let $V = 9.2\%$.

$$V = .104x^2 - 3.22x + 32.8$$
$$9.2 = .104x^2 - 3.22x + 32.8$$
$$0 = .104x^2 - 3.22x + 23.6$$
$$x = \frac{3.22 \pm \sqrt{(-3.22)^2 - 4(.104)(23.6)}}{2(.104)}$$
$$\approx 11.91 \text{ or } 19.05$$

The most recent time the vacancy rate at U.S. malls exceeded 9.2% was in 2020.

105. Let x = the width of the walk.

$$\text{The area } = (10 + 2x)(15 + 2x) - 10(15)$$
$$= 150 + 20x + 30x + 4x^2 - 150$$
$$= 4x^2 + 50x$$

To use all of the cement, solve

$$200 = 4x^2 + 50x$$
$$0 = 4x^2 + 50x - 200$$

Using the quadratic formula, we have

$$x = \frac{-50 \pm \sqrt{(50)^2 - 4(4)(-200)}}{2(4)}$$
$$= \frac{-50 \pm \sqrt{5700}}{2(4)} \approx -15.6875 \text{ or } 3.1875$$

The width cannot be negative, so the solution is approximately 3.2 feet.

106. Let x = the length of the yard. Then $160 - x$ = the width of the yard. Area is equal to length times width.

$$\text{The area } = lw$$
$$4000 = (x)(160 - x)$$
$$0 = x^2 - 160x + 4000$$

Using the quadratic formula, we have

$$x = \frac{160 \pm \sqrt{(-160)^2 - 4(1)(4000)}}{2(1)}$$
$$= \frac{160 \pm \sqrt{9600}}{2(1)} \approx 31.01 \text{ or } 128.99$$

The length is longer than the width, so the length is approximately 129 feet and the width is approximately 160 – 129 = 31 feet.

107. $v_0 = 150,\ h_0 = 0,\ h = 200$

$$200 = -16t^2 + 150t + 0 \Rightarrow$$
$$16t^2 - 150t + 200 = 0 \Rightarrow$$
$$8t^2 - 75t + 100 = 0 \Rightarrow t \approx 1.61 \text{ or } 7.77$$

The ball will reach 200 feet on its downward trip after approximately 7.77 seconds.

108. $v_0 = 55,\ h_0 = 700,\ h = 0$

$$0 = -16t^2 - 55t + 700 \Rightarrow$$
$$16t^2 + 55t - 700 = 0 \Rightarrow t \approx -8.55 \text{ or } 5.12$$

The ball will reach the ground after approximately 5.12 seconds

Case 1 Consumers Often Defy Common Sense

1. The total cost to buy and run this electric hot water tank for x years is $E = 218 + 508x$.

2. The total cost to buy and run this gas hot water tank for x years is $G = 328 + 309x$.

3. Over 10 years, the electric hot water tank costs $218 + 508(10) = 5298$ or \$5298, and the gas hot water tank costs $328 + 309(10) = 3418$ or \$3418. The electric hot water tank costs \$1880 more over 10 years.

4. The total costs for the two hot water tanks will be equal when $218 + 508x = 328 + 309x \Rightarrow$ $199x = 110 \Rightarrow x \approx 0.55.$
   The costs will be equal within the first year.

5. The total cost to buy and run this Maytag refrigerator for x years is $M = 1529.10 + 50x$.

6. The total cost to buy and run this LG refrigerator for x years is $L = 1618.20 + 44x$.

7. Over 10 years, the Maytag refrigerator costs $1529.10 + 50(10) = 2029.10$ or \$2029.10, and the LG refrigerator costs $1618.20 + 44(10) = 2058.20$ or \$2058.20. The LG refrigerator costs \$29.10 more over 10 years.

8. The total costs for the two refrigerators will be equal when $1529.10 + 50x = 1618.20 + 44x \Rightarrow$ $6x = 89.10 \Rightarrow x \approx 14.85.$
   The costs will be equal in the 14^{th} year.

Chapter 2 Graphs, Lines, and Inequalities

Section 2.1 Graphs, Lines, and Inequalities

1. (1, –2) lies in quadrant IV
(–2, 1)lies in quadrant II
(3, 4) lies in quadrant I
(–5, –6) lies in quadrant III

2. $(\pi, 2)$ lies in quadrant I
$\left(3, -\sqrt{2}\right)$ lies in quadrant IV
$(4, 0)$ lies in no quadrant
$(-\sqrt{3}, \sqrt{3})$ lies in quadrant II

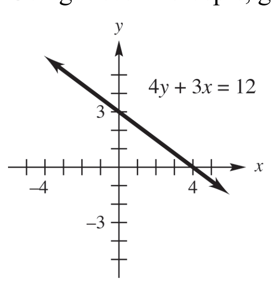
3. (1, –3) is a solution to $3x - y - 6 = 0$ because $3(1) - (-3) - 6 = 0$ is a true statement.

4. (2, –1) is a solution to $x^2 + y^2 - 6x + 8y = -15$ because $(2)^2 + (-1)^2 - 6(2) + 8(-1) = -15$ is a true statement.

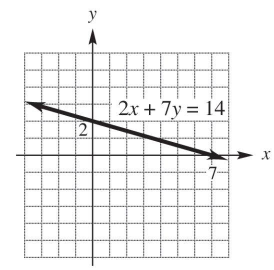
5. (3, 4) is not a solution to $(x-2)^2 + (y+2)^2 = 6$ because $(3-2)^2 + (4+2)^2 = 37$, not 6.

6. (1, –1) is not a solution to $\frac{x^2}{2} + \frac{y^2}{3} = -4$ because $\frac{1^2}{2} + \frac{(-1)^2}{3} = \frac{5}{6}$, not -4.

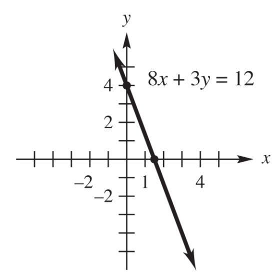
7. $4y + 3x = 12$
Find the y-intercept. If $x = 0$,
$4y = -3(0) + 12 \Rightarrow 4y = 12 \Rightarrow y = 3$
The y-intercept is 3.
Next find the x-intercept. If $y = 0$,
$4(0) + 3x = 12 \Rightarrow 3x = 12 \Rightarrow x = 4$
The x-intercept is 4.
Using these intercepts, graph the line.



8. $2x + 7y = 14$
Find the y-intercept. If $x = 0$,
$2(0) + 7y = 14 \Rightarrow 7y = 14 \Rightarrow y = 2$
The y-intercept is 2.
Next find the x-intercept. If $y = 0$,
$2x + 7(0) = 14 \Rightarrow 2x = 14 \Rightarrow x = 7$
The x-intercept is 7.
Using these intercepts, graph the line.

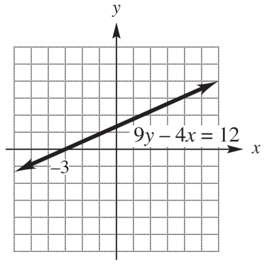


9. $8x + 3y = 12$
Find the y-intercept. If $x = 0$,
$3y = 12 \Rightarrow y = 4$
The y-intercept is 4.
Next, find the x-intercept. If $y = 0$,
$8x = 12 \Rightarrow x = \frac{12}{8} = \frac{3}{2}$
The x-intercept is $\frac{3}{2}$.
Using these intercepts, graph the line.



10. $9y - 4x = 12$
Find the y-intercept. If $x = 0$,
$9y - 4(0) = 12 \Rightarrow 9y = 12 \Rightarrow y = \frac{12}{9} = \frac{4}{3}$
The y-intercept is $\frac{4}{3}$.
Next find the x-intercept. If $y = 0$,
$9(0) - 4x = 12 \Rightarrow -4x = 12 \Rightarrow x = -3$
The x-intercept is –3.

Using these intercepts, graph the line.



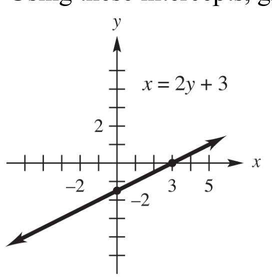
11. $x = 2y + 3$
Find the y-intercept. If $x = 0$,

$$0 = 2y + 3 \Rightarrow 2y = -3 \Rightarrow y = -\frac{3}{2}$$

The y-intercept is $-\frac{3}{2}$.

Next, find the x-intercept. If $y = 0$,
$x = 2(0) + 3 \Rightarrow x = 3$

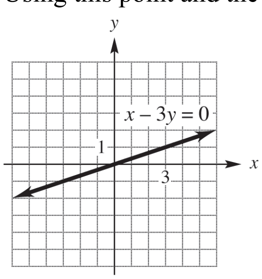
The x-intercept is 3.
Using these intercepts, graph the line.



12. $x - 3y = 0$
Find the y-intercept. If $x = 0$,
$-3y = 0 \Rightarrow 0$

The y-intercept is 0. Since the line passes through the origin, the x-intercept is also 0. Find another point on the line by arbitrarily choosing a value for x. Let $x = 3$. Then,
$-3y = -3 \Rightarrow y = 1$

The point with coordinates (3, 1) is on the line. Using this point and the origin, graph the line.



13. The x-intercepts are where the rays cross the x-axis, –2.5 and 3. The y-intercept is where the ray crosses the y-axis, 3.

14. The x-intercept is 3; the y-intercept is 1.

15. The x-intercepts are –1 and 2. The y-intercept is –2.

16. The x-intercept is 1. There is no y-intercept.

17. $3x + 4y = 12$
To find the x-intercept, let $y = 0$:
$3x + 4(0) = 12 \Rightarrow 3x = 12 \Rightarrow x = 4$

The x-intercept is 4.
To find the y-intercept, let $x = 0$:
$3(0) + 4y = 12 \Rightarrow 4y = 12 \Rightarrow y = 3$

The y-intercept is 3.

18. $x - 2y = 5$
To find the x-intercept, let $y = 0$:
$x - 2(0) = 5 \Rightarrow x = 5$

The x-intercept is 5.
To find the y-intercept, let $x = 0$.

$$0 - 2y = 5 \Rightarrow -2y = 5 \Rightarrow y = -\frac{5}{2}$$

The y-intercept is $-\frac{5}{2}$.

19. $2x - 3y = 24$
To find the x-intercept, let $y = 0$:
$2x - 3(0) = 24 \Rightarrow 2x = 24 \Rightarrow x = 12$

The x-intercept is 12.
To find the y-intercept, let $x = 0$:
$2(0) - 3y = 24 \Rightarrow -3y = 24 \Rightarrow y = -8$

The y-intercept is –8.

20. $3x + y = 4$
To find the x-intercept, let $y = 0$:

$$3x + 0 = 4 \Rightarrow 3x = 4 \Rightarrow x = \frac{4}{3}$$

The x-intercept is $\frac{4}{3}$.

To find the y-intercept, let $x = 0$:
$3(0) + y = 4 \Rightarrow y = 4$

The y-intercept is 4.

21. $y = x^2 - 9$
To find the x-intercepts, let $y = 0$:
$0 = x^2 - 9 \Rightarrow x^2 = 9 \Rightarrow x = \pm\sqrt{9} = \pm 3$

The x-intercepts are 3 and –3.
To find the y-intercept, let $x = 0$:
$y = 0 - 9 = -9$

The y-intercept is –9.

22. $y = x^2 + 4$

To find the x-intercepts, let $y = 0$:

$0 = x^2 + 4 \Rightarrow x^2 = -4 \Rightarrow$

$x = \pm\sqrt{-4}$ not a real number

There is no x-intercept.

To find the y-intercept, let $x = 0$:

$y = 0^2 + 4 = 4$

The y-intercept is 4.

23. $y = x^2 + x - 20$

To find the x-intercepts, let $y = 0$:

$0 = x^2 + x - 20 \Rightarrow 0 = (x+5)(x-4) \Rightarrow$

$x + 5 = 0 \Rightarrow x = -5$ or $x - 4 = 0 \Rightarrow x = 4$

The x-intercepts are –5 and 4.

To find the y-intercept, let $x = 0$:

$y = 0^2 + 0 - 20 = -20$

The y-intercept is –20.

24. $y = 5x^2 + 6x + 1$

To find the x-intercepts, let $y = 0$:

$0 = 5x^2 + 6x + 1 \Rightarrow 0 = (5x+1)(x+1) \Rightarrow$

$5x + 1 = 0 \Rightarrow x = -\frac{1}{5}$ or $x + 1 = 0 \Rightarrow x = -1$

The x-intercepts are $-\frac{1}{5}$ and –1.

To find the y-intercept, let $x = 0$:

$y = 5(0)^2 + 6(0) + 1 = 1$

The y-intercept is 1.

25. $y = 2x^2 - 5x + 7$

To find the x-intercepts, let $y = 0$:

$0 = 2x^2 - 5x + 7$

This equation does not have real solutions, so there are no x-intercepts.

To find the y-intercept, let $x = 0$:

$y = 2(0)^2 - 5(0) + 7 = 7$

The y-intercept is 7.

26. $y = 3x^2 + 4x - 4$

To find the x-intercepts, let $y = 0$:

$0 = 3x^2 + 4x - 4 \Rightarrow 0 = (3x-2)(x+2) \Rightarrow$

$3x - 2 = 0 \Rightarrow x = \frac{2}{3}$ or $x + 2 = 0 \Rightarrow x = -2$

The x-intercepts are -2 and $\frac{2}{3}$.

To find the y-intercept, let $x = 0$:

$y = 3(0)^2 + 4(0) - 4 = -4$

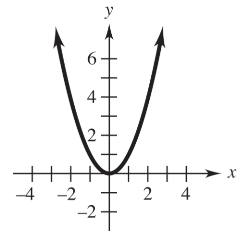
The y-intercept is –4.

27. $y = x^2$

x-intercept: $0 = x^2 \Rightarrow x = 0$

y-intercept: $y = 0$

x	y
–2	4
–1	1
0	0
1	1
2	4



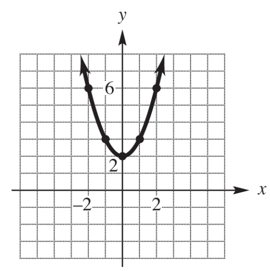
28. $y = x^2 + 2$

x-intercept: $0 = x^2 + 2 \Rightarrow x^2 = -2 \Rightarrow$

$x = \pm\sqrt{-2}$ not a real number

y-intercept: $y = 0^2 + 2 \Rightarrow y = 2$

x	y
–2	6
–1	3
0	2
1	3
2	6

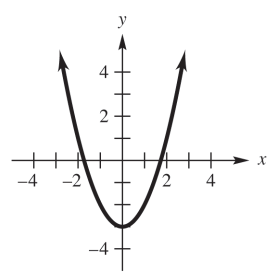


29. $y = x^2 - 3$

x-intercepts: $0 = x^2 - 3 \Rightarrow x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

y-intercepts: $y = 0^2 - 3 = -3$

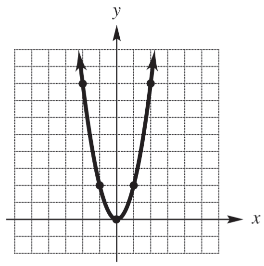
x	y
–3	6
–1	–2
0	–3
1	–2
3	6



30. $y = 2x^2$

x-intercept: $0 = 2x^2 \Rightarrow x = 0$
y-intercept: $y = 2(0) = 0$

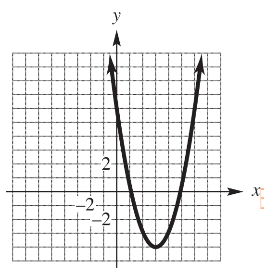
x	y
–2	8
–1	2
0	0
1	2
–2	8



31. $y = x^2 - 6x + 5$

x-intercept:
$0 = x^2 - 6x + 5 \Rightarrow 0 = (x-1)(x-5) \Rightarrow$
$x - 1 = 0 \Rightarrow x = 1$ or $x - 5 = 0 \Rightarrow x = 5$
y-intercept: $y = (0)^2 - 6(0) + 5 = 5$

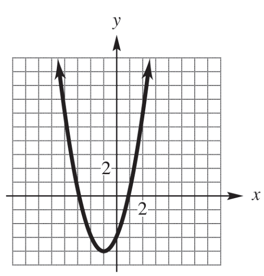
x	y
–2	21
–1	12
0	5
1	0
2	–3



32. $y = x^2 + 2x - 3$

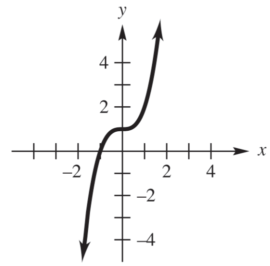
x-intercept:
$0 = x^2 + 2x - 3 \Rightarrow 0 = (x+3)(x-1) \Rightarrow$
$x + 3 = 0 \Rightarrow x = -3$ or $x - 1 = 0 \Rightarrow x = 1$
y-intercept: $y = (0)^2 + 2(0) - 3 = -3$

x	y
–3	0
–2	–3
–1	–4
0	–3
1	0



33. $y = x^3 + 1$
x-intercept:
$0 = x^3 + 1 \Rightarrow x^3 = -1 \Rightarrow x = \sqrt[3]{-1} = -1$
y-intercept: $y = 0^3 + 1 = 1$

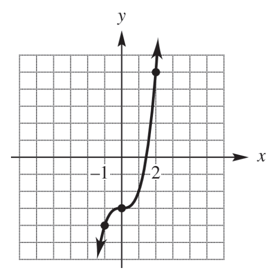
x	y
–2	–7
–1	0
0	1
1	2
2	9



34. $y = x^3 - 3$

x-intercept: $0 = x^3 - 3 \Rightarrow x^3 = 3 \Rightarrow x = \sqrt[3]{3}$
y-intercept: $y = 0^3 - 3 \Rightarrow y = -3$

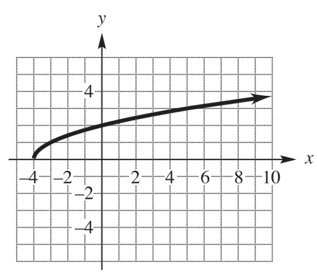
x	y
–2	–11
–1	–4
0	–3
1	–2
2	5



35. $y = \sqrt{x+4}$

x-intercept: $0 = \sqrt{x+4} \Rightarrow 0 = x + 4 \Rightarrow x = -4$
y-intercept: $y = \sqrt{0+4} = \sqrt{4} = 2$

x	y
–2	$\sqrt{2} \approx 1.4$
–1	$\sqrt{3} \approx 1.7$
0	2
2	$\sqrt{6} \approx 2.4$
5	3

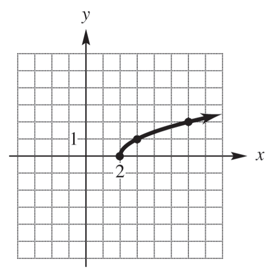


36. $y = \sqrt{x-2}$

x-intercept: $0 = \sqrt{x-2} \Rightarrow 0 = x-2 \Rightarrow x = 2$

y-intercept: $y = \sqrt{0-2} = \sqrt{-2}$, not a real number

x	y
2	0
3	1
6	2
11	3



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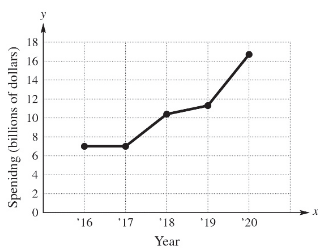
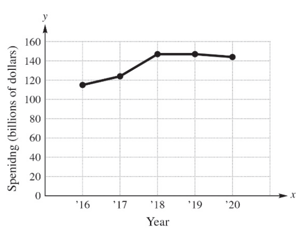


38.



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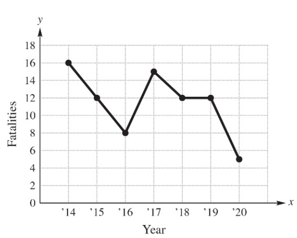
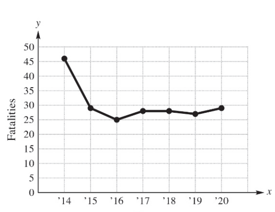


40.



41. 2015

42. 2018

43. 2017, 2018, 2019

44. 2017, 2018, 2019

45. **(a)** about $1,250,000

(b) about $1,750,000

(c) about $4,250,000

46. **(a)** about $1,000,000

(b) about $2,250,000.

(c) about $3,250,000.

47. **(a)** about $500,000

(b) about $1,000,000.

(c) about $1,500,000.

48. **(a)** about $250,000

(b) about $1,250,000

(c) about $1,500,000.

49. beef, about 56 pounds; chicken, about 90 pounds; pork, about 50 pounds

50. beef, about 57 pounds; chicken, about 93 pounds; pork, about 51 pounds

51. 2019

52. 2015

53. about $45

54. about $40

55. days 9 and 12

56. days 1, 6, and 13 – 19

57. Day 9; about $46.40

58. Day 12; about $41.80

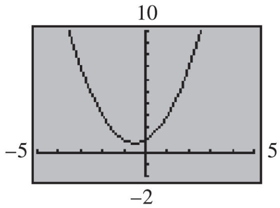
59. Lower

60. Lower

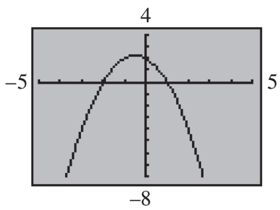
61. Day 9; about $4.90

62. Day 15; about \$1.30

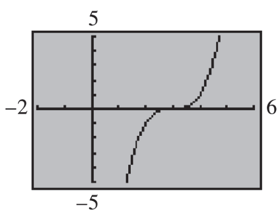
63. $y = x^2 + x + 1$



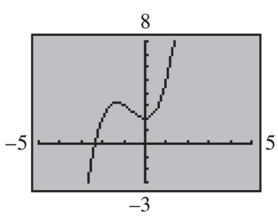
64. $y = 2 - x - x^2$



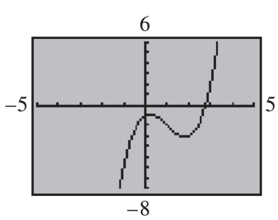
65. $y = (x-3)^3$



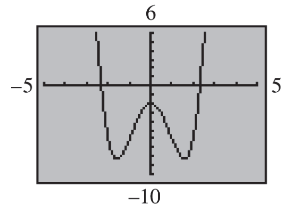
66. $y = x^3 + 2x^2 + 2$



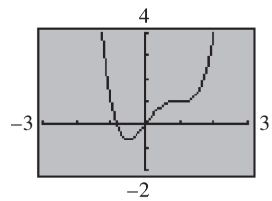
67. $y = x^3 - 3x^2 + x - 1$



68. $y = x^4 - 5x^2 - 2$

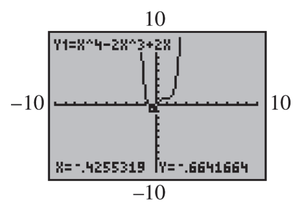


69. $y = x^4 - 2x^3 + 2x$



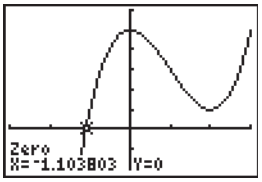
The "flat" part of the graph near $x = 1$ looks like a horizontal line segment, but it is not. The y values increase slightly as you trace along the segment from left to right.

70. **(a)** $y = x^4 - 2x^3 + 2x$



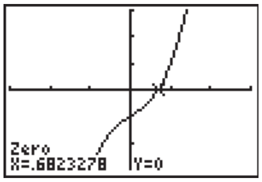
(b) The lowest point on the graph is approximately at (–.5, –.6875). Answers vary.

71. $x \approx -1.1038$



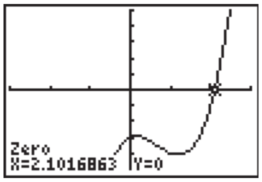
[–3, 3] by [–2, 6]

72. $x \approx .6823$



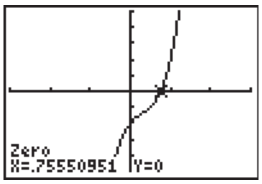
[–3, 3] by [–3, 3]

73. $x \approx 2.1017$



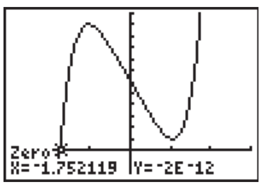
[–3, 3] by [–5, 5]

74. $x \approx .7555$



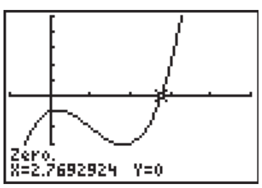
[–3, 3] by [–5, 5]

75. $x \approx -1.7521$



[–3, 3] by [–2, 12]

76. $x \approx 2.7693$



[–1, 53] by [–5, 5]

Section 2.2 Equations of Lines

1. Through (2, 5) and (0, 8)

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{8-5}{0-2} = \frac{3}{-2} = -\frac{3}{2}$$

2. Through (9, 0) and (12, 12)

$$\text{slope} = \frac{12-0}{12-9} = \frac{12}{3} = 4$$

3. Through (–4, 14) and (3, 0)

$$\text{slope} = \frac{14-0}{-4-3} = \frac{14}{-7} = -2$$

4. Through (–5, –2) and (–4, 11)

$$\text{slope} = \frac{-2-11}{-5-(-4)} = \frac{-13}{-1} = 13$$

5. Through the origin and (–4, 10); the origin has coordinate (0, 0).

$$\text{slope} = \frac{10-0}{-4-0} = \frac{10}{-4} = -\frac{5}{2}$$

6. Through the origin, (0, 0), and (8, –2)

$$\text{slope} = \frac{-2-0}{8-0} = \frac{-2}{8} = -\frac{1}{4}$$

7. Through (–1, 4) and (–1, 6)

$$\text{slope} = \frac{6-4}{-1-(-1)} = \frac{2}{0}, \text{ not defined}$$

The slope is undefined.

8. Through (–3, 5) and (2, 5)

$$\text{slope} = \frac{5-5}{2-(-3)} = \frac{0}{5} = 0$$

9. $b = 5, m = 4$
$y = mx + b$
$y = 4x + 5$

10. $b = -3, m = -7$
$y = mx + b$
$y = -7x - 3$

11. $b = 1.5, m = -2.3$
$y = mx + b$
$y = -2.3x + 1.5$

12. $b = -4.5, m = 2.5$
$y = mx + b$
$y = 2.5x - 4.5$

13. $b = 4,\ m = -\frac{3}{4}$
$y = mx + b$
$y = -\frac{3}{4}x + 4$

14. $b = -3,\ m = \frac{4}{3}$
$y = mx + b$
$y = \frac{4}{3}x - 3$

15. $2x - y = 9$
Rewrite in slope-intercept form.
$-y = -2x + 9$
$y = 2x - 9$
$m = 2, b = -9.$

16. $x + 2y = 7$
Rewrite in slope-intercept form.
$$2y = -x + 7$$
$$y = -\frac{1}{2}x + \frac{7}{2}$$
$$m = -\frac{1}{2},\ b = \frac{7}{2}.$$

17. $6x = 2y + 4$
Rewrite in slope-intercept form.
$$2y = 6x - 4 \Rightarrow y = 3x - 2$$
$m = 3$, $b = -2$.

18. $4x + 3y = 24$
Rewrite in slope-intercept form.
$$3y = -4x + 24$$
$$y = -\frac{4}{3}x + 8$$
$$m = -\frac{4}{3},\ b = 8.$$

19. $6x - 9y = 16$
Write in slope-intercept form.
$$-9y = -6x + 16$$
$$9y = 6x - 16$$
$$y = \frac{2}{3}x - \frac{16}{9}$$
$$m = \frac{2}{3},\ b = -\frac{16}{9}.$$

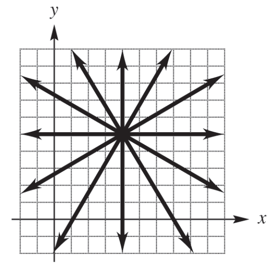
20. $4x + 2y = 0$
Rewrite in slope-intercept form.
$$2y = -4x \Rightarrow y = -2x$$
$m = -2$, $b = 0$.

21. $2x - 3y = 0$
Rewrite in slope-intercept form.
$$3y = 2x \Rightarrow y = \frac{2}{3}x$$
$$m = \frac{2}{3},\ b = 0.$$

22. $y = 7$ can be written as
$y = 0x + 7$
$m = 0$, $b = 7$.

23. $x = y - 5$
Rewrite in slope-intercept form.
$y = x + 5$
$m = 1$, $b = 5$

24. There are many correct answers, including:



25. **(a)** Largest value of slope is at C.

(b) Smallest value of slope is at B.

(c) Largest absolute value is at B

(d) Closest to 0 is at D

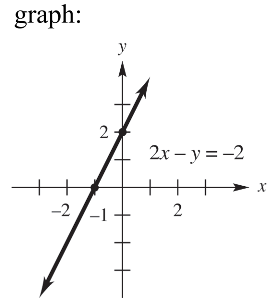
26. **(a)** $y = 3x + 2$
The slope is positive, and the y-intercept is positive. This is line D.

(b) $y = -3x + 2$
The slope is negative, and the y-intercept is positive. This is line B.

(c) $y = 3x - 2$
The slope is positive, and the y-intercept is negative. This is line A.

(d) $y = -3x - 2$
The slope is negative, and the y-intercept is negative. This is line C.

27. $2x - y = -2$
Find the x-intercept by setting $y = 0$ and solving for x: $2x - 0 = -2 \Rightarrow 2x = -2 \Rightarrow x = -1$
Find the y-intercept by setting $x = 0$ and solving for y: $2(0) - y = -2 \Rightarrow -y = -2 \Rightarrow y = 2$
Use the points (–1, 0) and (0, 2) to sketch the graph:

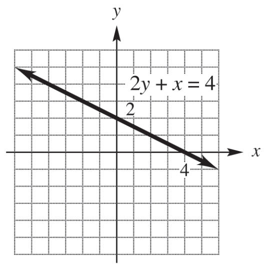


28. $2y + x = 4$

Find the x-intercept by setting $y = 0$ and solving for x: $2(0) + x = 4 \Rightarrow x = 4$

Find the y-intercept by setting $x = 0$ and solving for y: $2y + 0 = 4 \Rightarrow 2y = 4 \Rightarrow y = 2$

Use the points (4, 0) and (0, 2) to sketch the graph:

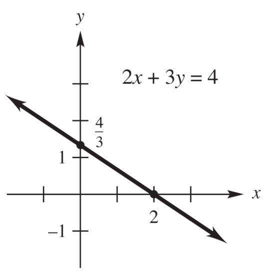


29. $2x + 3y = 4$

Find the x-intercept by setting $y = 0$ and solving for x: $2x + 3(0) = 4 \Rightarrow 2x = 4 \Rightarrow x = 2$

Find the y-intercept by setting $x = 0$ and solving for y: $2(0) + 3y = 4 \Rightarrow 3y = 4 \Rightarrow y = \frac{4}{3}$

Use the points (2, 0) and $\left(0, \frac{4}{3}\right)$ to sketch the graph:



30. $-5x + 4y = 3$

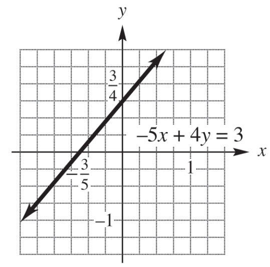
Find the x-intercept by setting $y = 0$ and solving for x:

$$-5x + 4(0) = 3 \Rightarrow -5x = 3 \Rightarrow x = -\frac{3}{5}$$

Find the y-intercept by setting $x = 0$ and solving for y:

$$-5(0) + 4y = 3 \Rightarrow 4y = 3 \Rightarrow y = \frac{3}{4}$$

Use the points $\left(-\frac{3}{5}, 0\right)$ and $\left(0, \frac{3}{4}\right)$ to sketch the graph:



31. $4x - 5y = 2$

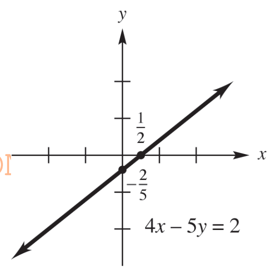
Find the x-intercept, by setting $y = 0$ and solving for x:

$$4x - 5(0) = 2 \Rightarrow 4x = 2 \Rightarrow x = \frac{1}{2}$$

Find the y-intercept by setting $x = 0$ and solving for y:

$$4(0) - 5y = 2 \Rightarrow -5y = 2 \Rightarrow y = -\frac{2}{5}$$

Use the points $\left(\frac{1}{2}, 0\right)$ and $\left(0, -\frac{2}{5}\right)$ to sketch the graph:



32. $3x + 2y = 8$

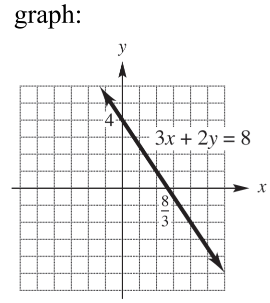
Find the x-intercept by setting $y = 0$ and solving for x:

$$3x + 2(0) = 8 \Rightarrow 3x = 8 \Rightarrow x = \frac{8}{3}$$

Find the y-intercept by setting $x = 0$ and solving for y:

$3(0) + 2y = 8 \Rightarrow 2y = 8 \Rightarrow y = 4$

Use the points $\left(\frac{8}{3}, 0\right)$ and (0, 4) to sketch the graph:



33. For $4x - 3y = 6$, solve for y.

$y = \frac{4}{3}x - 2$

For $3x + 4y = 8$, solve for y.

$y = -\frac{3}{4}x + 2$

The two slopes are $\frac{4}{3}$ and $-\frac{3}{4}$. Since

$\left(\frac{4}{3}\right)\left(-\frac{3}{4}\right) = -1$,

the lines are perpendicular.

34. $2x - 5y = 7$ and $15y - 5 = 6x$

Solve each equation for y to find the slope.

$2x - 5y = 7 \Rightarrow -5y = -2x + 7 \Rightarrow y = \frac{2}{5}x - \frac{7}{5}$

$15y - 5 = 6x \Rightarrow 15y = 6x + 5 \Rightarrow$

$y = \frac{6}{15}x + \frac{5}{15} = \frac{2}{5}x + \frac{1}{3}$

The slope of each line is $\frac{2}{5}$, so the lines are parallel.

35. For $3x + 2y = 8$, solve for y.

$y = -\frac{3}{2}x + 4$

For $6y = 5 - 9x$, solve for y.

$y = -\frac{3}{2}x + \frac{5}{6}$

Since the slopes are both $-\frac{3}{2}$, the lines are parallel.

36. $x - 3y = 4$ and $y = 1 - 3x$

Solve each equation for y to find the slope.

$x - 3y = 4 \Rightarrow -3y = -x + 4 \Rightarrow y = \frac{1}{3}x - \frac{4}{3}$

$y = 1 - 3x = -3x + 1$

The product of the slopes is $\left(\frac{1}{3}\right)(-3) = -1$, so the lines are perpendicular.

37. For $4x = 2y + 3$, solve for y.

$y = 2x - \frac{3}{2}$

For $2y = 2x + 3$, solve for y.

$y = x + \frac{3}{2}$

Since the two slopes are 2 and 1, the lines are neither parallel nor perpendicular.

38. $2x - y = 6$ and $x - 2y = 4$

Solve each equation for y to find the slope.

$2x - y = 6 \Rightarrow -y = -2x + 6 \Rightarrow y = 2x - 6$

$x - 2y = 4 \Rightarrow -2y = -x + 4 \Rightarrow y = \frac{1}{2}x - 2$

The slopes are not equal, and their product is $2\left(\frac{1}{2}\right) = 1$, not –1, so the lines are neither parallel nor perpendicular.

39. Triangle with vertices (9, 6), (–1, 2) and (1, –3).

a. Slope of side between vertices (9, 6) and (–1, 2):

$m = \frac{6-2}{9-(-1)} = \frac{4}{10} = \frac{2}{5}$

Slope of side between vertices (–1, 2) and (1, –3):

$m = \frac{2-(-3)}{-1-1} = \frac{5}{-2} = -\frac{5}{2}$

Slope of side between vertices (1, –3) and (9, 6):

$m = \frac{-3-6}{1-9} = \frac{-9}{-8} = \frac{9}{8}$

b. The sides with slopes $\frac{2}{5}$ and $-\frac{5}{2}$ are perpendicular, because $\frac{2}{5}\left(-\frac{5}{2}\right) = -1$. Thus, the triangle is a right triangle.

40. Quadrilateral with vertices (–5, –2), (–3, 1), (3, 0), and (1, –3):

a. Slope of side between vertices (–5, –2) and (–3, 1):

$m = \frac{-2-1}{-5-(-3)} = \frac{-3}{-2} = \frac{3}{2}$

Slope of side between vertices (–3, 1) and (3, 0):

$m = \frac{1-0}{-3-3} = \frac{1}{-6} = -\frac{1}{6}$

Slope of side between vertices (3, 0) and (1, –3):

$m = \frac{0-(-3)}{3-1} = \frac{3}{2}$

Slope of side between vertices (1, –3) and (–5, –2):

$m = \frac{-3-(-2)}{1-(-5)} = \frac{-3+2}{1+5} = -\frac{1}{6}$

b. Yes, the quadrilateral is a parallelogram because opposite sides have the same slope and are therefore parallel.

41. Use point-slope form with

$(x_1, y_1) = (-3, 2),\ m = -\frac{2}{3}$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= -\frac{2}{3}(x - (-3)) \\ y - 2 &= -\frac{2}{3}(x + 3) \\ y - 2 &= -\frac{2}{3}x - 2 \\ y &= -\frac{2}{3}x \end{aligned}$$

42. $(x_1, y_1) = (-5, -2),\ m = \frac{4}{5}$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-2) &= \frac{4}{5}(x - (-5)) \\ y + 2 &= \frac{4}{5}(x + 5) \\ y + 2 &= \frac{4}{5}x + 4 \\ y &= \frac{4}{5}x + 2 \end{aligned}$$

43. $(x_1, y_1) = (2, 3),\ m = 3$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= 3(x - 2) \\ y - 3 &= 3x - 6 \\ y &= 3x - 3 \end{aligned}$$

44. $(x_1, y_1) = (3, -4),\ m = -\frac{1}{4}$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-4) &= -\frac{1}{4}(x - 3) \\ y + 4 &= -\frac{1}{4}(x - 3) \\ 4y + 16 &= -x + 3 \\ 4y &= -x - 13 \\ y &= -\frac{1}{4}x - \frac{13}{4} \end{aligned}$$

45. $(x_1, y_1) = (10, 1),\ m = 0$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= 0(x - 10) \\ y - 1 &= 0 \Rightarrow y = 1 \end{aligned}$$

46. $(x_1, y_1) = (-3, -9),\ m = 0$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-9) &= 0(x - (-3)) \\ y + 9 &= 0 \Rightarrow y = -9 \end{aligned}$$

47. Since the slope is undefined, the equation is that of a vertical line through (–2, 12).
$x = -2$

48. Since the slope is undefined, the equation is that of a vertical line through (1, 1).
$x = 1$

49. Through (–1, 1) and (2, 7)
Find the slope.

$$m = \frac{7 - 1}{2 - (-1)} = \frac{6}{3} = 2$$

Use the point-slope form with $(2, 7) = (x_1, y_1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= 2(x - 2) \\ y - 7 &= 2x - 4 \\ y &= 2x + 3 \end{aligned}$$

50. Through (2, 5) and (0, 6)
Find the slope.

$$m = \frac{5 - 6}{2 - 0} = \frac{-1}{2}$$

Use the point-slope form with $(0, 6) = (x_1, y_1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 6 &= -\frac{1}{2}(x - 0) \\ y - 6 &= -\frac{1}{2}x \\ y &= -\frac{1}{2}x + 6 \end{aligned}$$

51. Through (1, 2) and (3, 9)
Find the slope.

$$m = \frac{9-2}{3-1} = \frac{7}{2}$$

Use the point-slope form with $(1, 2) = (x_1, y_1)$.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{7}{2}(x-1)$$
$$y - 2 = \frac{7}{2}x - \frac{7}{2}$$
$$y = \frac{7}{2}x - \frac{3}{2}$$
$$2y = 7x - 3$$

52. Through (–1, –2) and (2, –1)
Find the slope.

$$m = \frac{-2-(-1)}{-1-2} = \frac{-1}{-3} = \frac{1}{3}$$

Use the point-slope form with
$(-1, -2) = (x_1, y_1)$

$$y - y_1 = m(x - x_1)$$
$$y - (-2) = \frac{1}{3}(x - (-1))$$
$$y + 2 = \frac{1}{3}(x+1)$$
$$3y + 6 = x + 1$$
$$3y = x - 5$$

53. Through the origin with slope 5.
$(x_1, y_1) = (0, 0)$; $m = 5$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 5(x - 0) \Rightarrow y = 5x$$

54. Through the origin and horizontal.
A horizontal line has slope 0.
$(x_1, y_1) = (0, 0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 0(m - 0)$$
$$y = 0$$

55. Through (6, 8) and vertical.
A vertical line has undefined slope.
$(x_1, y_1) = (6, 8)$
$x = 6$

56. Through (7, 9) and parallel to $y = 6$.
The line $y = 6$ has slope 0.
$(x_1, y_1) = (7, 9)$

$$y - y_1 = m(x - x_1)$$
$$y - 9 = 0(x - 7)$$
$$y - 9 = 0 \Rightarrow y = 9$$

57. Through (3, 4) and parallel to $4x - 2y = 5$.
Find the slope of the given line because a line parallel to the line has the same slope.
$(x_1, y_1) = (3, 4)$

$$4x = 2y + 5$$
$$2y = 4x - 5$$
$$y = 2x - \frac{5}{2} \quad m = 2$$
$$y - y_1 = m(x - x_1)$$
$$y - 4 = 2(x - 3)$$
$$y - 4 = 2x - 6$$
$$y = 2x - 2$$

58. Through (6, 8) and perpendicular to $y = 2x - 3$.
The slope of the given line is 2, so the slope of a line perpendicular to the given line has the slope $m = -\frac{1}{2}$.

$(x_1, y_1) = (6, 8)$

$$y - y_1 = m(x - x_1)$$
$$y - 8 = -\frac{1}{2}(x - 6)$$
$$2y - 16 = -x + 6$$
$$2y = -x + 22$$

59. x-intercept 6; y-intercept –6
Through the points (6, 0) and (0, –6).

$$m = \frac{0-(-6)}{6-0} = \frac{6}{6} = 1$$

$(x_1, y_1) = (6, 0)$

$$y - y_1 = m(x - x_1)$$
$$y - 0 = 1(x - 6) \Rightarrow y = x - 6$$

60. Through (–5, 2) and parallel to the line through (1, 2) and (4, 3).
The slope of the given line is
$m = \frac{2-3}{1-4} = \frac{-1}{-3} = \frac{1}{3}$, so the slope of a line parallel to the line is also $\frac{1}{3}$.

$$(x_1, y_1) = (-5, 2)$$
$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{1}{3}(x - (-5))$$
$$3y - 6 = x + 5$$
$$3y = x + 11$$

61. Through (–1, 3) and perpendicular to the line through (0, 1) and (2, 3).
The slope of the given line is
$m_1 = \frac{1-3}{0-2} = \frac{-2}{-2} = 1$, so the slope of a line perpendicular to the line is $m_2 = \frac{-1}{1} = -1$.

$$(x_1, y_1) = (-1, 3)$$
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -1(x - (-1))$$
$$y - 3 = -x - 1$$
$$y = -x + 2$$

62. y-intercept 3 and perpendicular to $2x - y + 6 = 0$.
$2x - y + 6 = 0 \Rightarrow y = 2x + 6$
The slope of this line is 2, so the slope of a line perpendicular to the line is $-\frac{1}{2}$.

$$(x_1, y_1) = (0, 3)$$
$$y - y_1 = m(x - x_1)$$
$$y - 3 = -\frac{1}{2}(x - 0)$$
$$2y - 6 = -x$$
$$2y = -x + 6$$

63. Let cost x = 15,965 and life: 12 years. Find D.
$$D = \left(\frac{1}{n}\right)x = \frac{1}{12}(15{,}965) \approx 1330.42$$
The depreciation is $1330.42 per year.

64. Cost: $41,762; life: 15 years
$$D = \left(\frac{1}{n}\right)x = \frac{1}{15}(41{,}762) \approx 2784.13$$
The depreciation is $2784.13 per year.

65. Let cost x = $201,457; life: 30 years
$$D = \left(\frac{1}{n}\right)x = \frac{1}{30}(201{,}457) \approx 6715.23$$
The depreciation is $6715.23 per year.

66. Let x = the amount of the bonus. The manager received as a bonus .10(165,000 – x),so x = .10(165,000 – x).
Solve this equation.
$$x = 16{,}500 - .10x$$
$$1.10x = 16{,}500$$
$$x = \frac{16{,}500}{1.10} = 15{,}000$$
The bonus amounted to $15,000.

67. $x = 15$
$$y = 567(15) - 2459 = 6046$$
There were 6046 breweries in the United States in the year 2015.

68. $x = 20$

$$y = 567(20) - 2459 = 8881$$
There were 8881 breweries in the United States in the year 2020.

69. $y = 3000$
$$3000 = 567x - 2459$$
$$5459 = 567x$$
$$9.63 \approx x$$
The first full year that there were 3000 breweries in the United States was 2010.

70. $y = 6500$
$$6500 = 567x - 2459$$
$$8959 = 567x$$
$$15.80 \approx x$$
The first full year that there were 6500 breweries in the United States was 2016.

71. $x = 241$
$$y = .522(241) + .7 = 126.502$$
The size of Texas' wholesale trade economy was $126.502 billion when it's manufacturing economy was $241 billion.

72. $x = 71$
$y = .522(71) + .7 = 37.762$
The size of New York's wholesale trade economy was \$37.762 billion when it's manufacturing economy was \$71 billion.

73. $y = 50$
$50 = .522x + .7$
$49.3 = .522x$
$94.44 = x$
The size of a state's manufacturing economy needs to be approximately \$94.4 billion to achieve a wholesale trade economy of \$50 billion.

74. $y = 30$
$30 = .522x + .7$
$29.3 = .522x$
$56.13 = x$
The size of a state's manufacturing economy needs to be approximately \$56.1 billion to achieve a wholesale trade economy of \$30 billion.

75. $y = -.116(23) + 3.8 \approx 1.132$
The model predicts that the quarterly sales for the 23rd quarter will be about 1.132 trillion won. This actual sales were .468 trillion won higher than the model predicts.

76. The slope of –.166 represents the rate of declining sales every quarter. It is negative because the sales are generally decreasing as time progresses.

77. **a.** $(x_1, y_1) = (5, 8.6)$ and
$(x_2, y_2) = (20, 25.1)$
Find the slope.
$$m = \frac{25.1 - 8.6}{20 - 5} = \frac{16.5}{15} = 1.1$$
$$y - 8.6 = 1.1(x - 5)$$
$$y - 8.6 = 1.1x - 5.5$$
$$y = 1.1x + 3.1$$

b. The year 2015 corresponds to $x = 2015 - 2000 = 15$.
$y = 1.1(15) + 3.1 = 19.6$
In 2015, there were 19.6% of U.S. homeowners who lived in their homes for more than 20 years.

c. $y = 15$
$15 = 1.1x + 3.1$
$11.9 = 1.1x$
$10.82 \approx x$
The first full year that 15% of homeowners lived in their homes for more than 20 years was 2011.

78. **a.** $(x_1, y_1) = (1, 3.75)$ and
$(x_2, y_2) = (13, 2.65)$
Find the slope.
$$m = \frac{2.65 - 3.75}{13 - 1} = \frac{-.1.1}{12} = -.09$$
$$y - 3.75 = -.09(x - 1)$$
$$y - 3.75 = -.09x + .09$$
$$y = -.09x + 3.84$$

b. The month of July 2020 corresponds to $x = 7$.
$y = -.09(7) + 3.84 = 3.21$
In July 2020, the mortgage rate was 3.21%.

c. $y = 2.8$
$2.8 = -.09x + 3.84$
$-1.04 = -.09x$
$11.56 = x$
The first month that the mortgage rate fell below 2.8% was December 2020.

79. **a.** $(x_1, y_1) = (14, 4800)$ and
$(x_2, y_2) = (19, 6000)$
Find the slope.
$$m = \frac{6000 - 4800}{19 - 14} = \frac{1200}{5} = 240$$
$$y - 4800 = 240(x - 14)$$
$$y - 4800 = 240x - 3360$$
$$y = 240x + 1440$$

b. The year 2017 corresponds to $x = 2017 - 2000 = 17$.
$y = 240(17) + 1440 = 5520$
In 2017, the average annual cost to employees for employer family health coverage was \$5520.

c. $y = 6750$

$$6750 = 240x + 1440$$
$$5310 = 240x$$
$$22.125 \approx x$$

The first full year that the average annual cost to employees for employer family health coverage will exceed $6750 will be 2023.

80.a. $(x_1, y_1) = (9,9800)$ and $(x_2, y_2) = (19,14500)$

Find the slope.

$$m = \frac{14500 - 9800}{19 - 9} = \frac{4700}{10} = 470$$

$$y - 9800 = 470(x - 9)$$
$$y - 9800 = 470x - 4230$$
$$y = 470x + 5570$$

b. The year 2015 corresponds to $x = 2015 - 2000 = 15$.

$$y = 470(15) + 5570 = 12,620$$

In 2015, the average annual cost to employers for employer family health coverage was $12,620.

c. $y = 17,000$

$$17,000 = 470x + 5570$$
$$11,430 = 470x$$
$$24.32 \approx x$$

The first full year that the average annual cost to employers for employer family health coverage will exceed $17,000 will be 2025.

Section 2.3 Linear Models

1. a. Let (x_1, y_1) be (32, 0) and (x_2, y_2) be (68, 20).

Find the slope.

$$m = \frac{20 - 0}{68 - 32} = \frac{20}{36} = \frac{5}{9}$$

Use the point-slope form with (32, 0).

$$y - 0 = \frac{5}{9}(x - 32) \Rightarrow y = \frac{5}{9}(x - 32)$$

b. Let $x = 50$.

$$y = \frac{5}{9}(50 - 32) = \frac{5}{9}(18) = 10°\text{C}$$

Let $x = 75$.

$$y = \frac{5}{9}(75 - 32) = \frac{5}{9}(43) \approx 23.89°\text{C}$$

2. $y = \frac{5}{9}(x - 32) \Rightarrow C = \frac{5}{9}(F - 32)$

a. $F = 58°\text{F}$

$$C = \frac{5}{9}(58 - 32) = \frac{5}{9}(26) \approx 14.4°C$$

b. $C = 50°\text{C}$

$$50 = \frac{5}{9}(F - 32)$$
$$450 = 5F - 160$$
$$610 = 5F \Rightarrow F = 122°$$

c. $C = -10°\text{C}$

$$-10 = \frac{5}{9}(F - 32)$$
$$-90 = 5F - 160$$
$$70 = 5F \Rightarrow F = 14°$$

d. $F = -20°\text{F}$

$$C = \frac{5}{9}(-20 - 32) = \frac{5}{9}(-52) \approx -28.9°C$$

3. F = 867°

$$C = \frac{5}{9}(867 - 32) = \frac{5}{9}(835) \approx 463.89°C$$

4. When are Celsius and Fahrenheit temperatures numerically equal? Set $F = C$:

$$C = \frac{5}{9}(F - 32) = F \Rightarrow 9F = 5F - 160 \Rightarrow$$
$$4F = -160 \Rightarrow F = -40°$$

The temperatures are numerically equal at –40°.

5. Let $(x_1, y_1) = (16, 240.0)$ and $(x_2, y_2) = (20, 258.8)$. Find the slope.

$$m = \frac{258.8 - 240.0}{20 - 16} = 4.7$$

$$y - 240.0 = 4.7(x - 16)$$
$$y - 240 = 4.7x - 75.2$$
$$y = 4.7x + 164.8$$

To estimate the CPI in 2017, let $x = 17$.

$y = 4.7(17) + 164.8 \approx 244.7$

To estimate the CPI in 2022, let $x = 22$.

$y = 4.7(22) + 164.8 \approx 268.2$

6. Let $(x_1, y_1) = (9, 95.0)$ and $(x_2, y_2) = (19, 151.8)$. Find the slope.

$$m = \frac{151.8 - 95.0}{19 - 9} = \frac{56.8}{10} \approx 5.68$$

$$y - 95.0 = 5.68(x - 9)$$
$$y - 95 = 5.68x - 51.12$$
$$y = 5.68x + 43.88$$

To estimate the number of electronically filed returns in 2010, let $x = 2010 - 2000 = 10$.

$y = 5.68(10) + 43.88 = 100.68$

There were about 100.68 million electronically filed tax returns in 2010.

To estimate the number of electronically filed returns in 2018, let $x = 2018 - 2000 = 18$

$y = 5.68(18) + 43.88 \approx 146.12$

There were about 146.12 million electronically filed tax returns in 2018.

7. Let $(x_1, y_1) = (5, 12.3)$ and $(x_2, y_2) = (19, 14.2)$. Find the slope.

$$m = \frac{14.2 - 12.3}{19 - 5} = \frac{1.9}{14} \approx .14$$

$$y - 12.3 = .14(x - 5)$$
$$y - 12.3 = .14x - .7$$
$$y = .14x + 11.6$$

To estimate the number of employees working in educational services in 2017, let $x = 2017 - 2000 = 17$.

$y = .14(17) + 11.6 = 13.98$

The number of employees was estimated to be 13.98 million in 2017.

8. Let $(x_1, y_1) = (5, 12.1)$ and $(x_2, y_2) = (19, 14.6)$. Find the slope.

$$m = \frac{14.6 - 12.1}{19 - 5} = .18$$

$$y - 12.1 = .18(x - 5)$$
$$y - 12.1 = .18x - .9$$
$$y = .18x + 11.2$$

To estimate the number of employees working in the leisure and hospitality industries in 2016, let $x = 2016 - 2000 = 16$.

$y = .18(16) + 11.2 = 14.08$

The number of employees was about 14.08 million in 2016.

9. Find the slope of the line.

$(x_1, y_1) = (50, 320)$

$(x_2, y_2) = (80, 440)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{440 - 320}{80 - 50} = \frac{120}{30} = 4$$

Each mile per hour increase in the speed of the bat will make the ball travel 4 more feet.

10. $y = \frac{1120 \text{ gal}}{\text{min}} \cdot \frac{60 \text{ min}}{\text{hr}} \cdot \frac{12 \text{ hr}}{\text{day}} \cdot x = 806,400x$

After 30 days,

$y = 806,400(30) = 24,192,000$ gallons

11. **a.** $y = 1.5x + 26.7$

Data Point (x, y)	Model Point $(x, \hat{y})$	Residual $y - \hat{y}$	Squared Residual $(y - \hat{y})^2$
(11, 43.2)	(11, 43.2)	0	0
(13, 48.6)	(13, 46.2)	2.4	5.76
(15, 49.2)	(15, 49.2)	0	0
(17, 48.0)	(17, 52.2)	-4.2	17.64
(19, 51.9)	(19, 55.2)	-3.3	10.89

$y = 1.9x + 15.7$

Data Point (x, y)	Model Point $(x, \hat{y})$	Residual $y - \hat{y}$	Squared Residual $(y - \hat{y})^2$
(11, 43.2)	(11, 36.6)	6.6	43.56
(13, 48.6)	(13, 40.4)	8.2	67.24
(15, 49.2)	(15, 44.2)	5	25
(17, 48.0)	(17, 48)	0	0
(19, 51.9)	(19, 51.8)	.1	.01

Sum of the residuals for model 1 = –5.1
Sum of the residuals for model 2 = 19.9

b. Sum of the squares of the residuals for model 1 =34.29
Sum of the squares of the residuals for model 2 = 135.81

c. Model 1 is the better fit.

12. a. $y = 3.9x - 36.8$

Data Point (x, y)	Model Point $(x, \hat{y})$	Residual $y - \hat{y}$	Squared Residual $(y - \hat{y})^2$
(16, 28.1)	(16, 25.6)	2.5	6.25
(17, 29.5)	(17, 29.5)	0	0
(18, 33.4)	(18, 33.4)	0	0
(19, 33.3)	(19, 37.3)	–4	16
(20, 44.6)	(20, 41.2)	3.4	11.56

$y = 1.9x - 2.8$

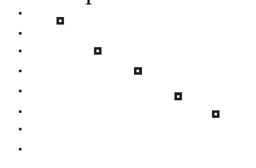
Data Point (x, y)	Model Point $(x, \hat{y})$	Residual $y - \hat{y}$	Squared Residual $(y - \hat{y})^2$
(16, 28.1)	(16, 27.6)	.5	.25
(17, 29.5)	(17, 29.5)	0	0
(18, 33.4)	(18, 31.4)	2	4
(19, 33.3)	(19, 33.3)	0	0
(20, 44.6)	(20, 35.2)	9.4	88.36

Sum of the residuals for equation 1 = 1.9
Sum of the residuals for equation 2 = 11.9

b. Sum of the squares of the residuals for equation 1= 33.81.
Sum of the squares of the residuals for equation 2= 92.61.

c. Equation 2 is the better fit.

13. Plot the points.



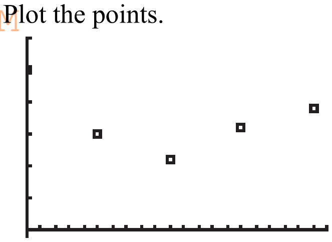
[14, 20] by [3000, 3800]

Visually, a straight line looks to be a good model for the data.

```
LinReg
 y=a+bx
 a=5535.8
 b=-119.4
 r²=.9924648093
 r=-.9962252804
```

The coefficient of correlation is $r \approx -.9962$, which indicates that the regression line is a good fit for the data.

14. Plot the points.



[0, 21] by [–1, 30]

Visually, a straight line looks to be a poor model for the data because it shows a great deal of curvature.

```
LinReg
 y=a+bx
 a=19.4
 b=-.22
 r²=.1112132353
 r=-.3334864844
```

The coefficient of correlation is $r \approx -.3335$, which indicates that the regression line is not a good fit for the data.

15. a. Using a graphing calculator, the regression-line model is $y = -119.4x + 5535.8$.

b. The year 2023 corresponds to $x = 23$. Using the regression-line model generated by a graphing calculator, we have $y = -119.4(23) + 5535.8 \approx 2789.6$, or about 2790 credit unions.

16. a. Using a graphing calculator, we find that the linear model is $y = .08x + .05$

b. Let $x = 23$. Using the regression-line model generated by a graphing calculator, we have $y = .08(23) + .05 \approx 1.89$, or about \$1.89 trillion.

17. a. Using a graphing calculator, the regression-line model is
$y = .11x - .35$.

b. The month of June 2021 corresponds to $x =$ 16. Using the regression-line model generated by a graphing calculator, we have $y = .11(18) - .35 \approx 1.63$, or about 1.63% yield.

18. a. Using a graphing calculator, the regression-line model is
$y = 85.4 + .842x$.

b. Using the regression-line model:
Let $x = 42$ years-old.
$y = 85.4 + .842(42) = 120.764$
Let $x = 53$ years-old.
$y = 85.4 + .842(53) = 130.026$
Let $x = 69$ years-old.
$y = 85.4 + .842(69) = 143.498$
The predicted values are very close to the actual data values.

c. Using the regression-line model:
Let $x = 45$ years-old.
$y = 85.4 + .842(45) = 123.29$
Let $x = 70$ years-old.
$y = 85.4 + .842(70) = 144.34$
The systolic blood pressure for a 45 year-old and a 70 year-old are predicted to be 123.29 and 144.34 respectively.

19. a. Using a graphing calculator, the regression line model for the value of the DJIA is given by $y = 600.46x + 22058.66$.

b. Let $x = 14$ (February 2021).
$y = 600.46(14) + 22058.66 \approx 30465.1$
The value of the DJIA was about 30465 in February 2021.

20. a. Using a graphing calculator, the regression-line model is $y = 3.86x - 28.2$.

b. $y = 3.86(23) - 28.2 \approx 60.587$. The revenue will be approximately \$60.587 billion in 2023.

c. Let $y = 65$.
$$65 = 3.86x - 28.2$$
$$93.2 = 3.86x$$
$$x \approx 24.14$$
There will be more than \$65 billion in revenue in the year 2025.

d. Using a graphing calculator, the coefficient of correlation is about .92.

21. a. Using a graphing calculator, the regression-line model is
$y = 2.07x + 91.96$.

b. $y = 2.07(11) + 91.96 \approx 114.73$. There was about \$114.73 billion in revenue for the third quarter of 2020.

c. Let $y = 120$.
$$120 = 2.07x + 91.96$$
$$28.04 = 2.07x$$
$$x \approx 13.55$$
There will be \$120 billion in revenue during the second quarter of 2021.

d. Using a graphing calculator, the coefficient of correlation is about .84.

22. a. Using a graphing calculator, the regression-line model is $y = .43x + 19.01$.

b. $y = .43(12) + 19.01 \approx 24.17$. There was about \$24.17 billion in revenue for the fourth quarter of 2020.

c. Let $y = 26$.
$$26 = .43x + 19.01$$
$$6.99 = .43x$$
$$x \approx 16.26$$
There will be \$26 billion in revenue during the first quarter of 2022.

d. Using a graphing calculator, the coefficient of correlation is about .54.

Section 2.4 Linear Inequalities

1. Use brackets if you want to include the endpoint, and parentheses if you want to exclude it.

2. (c). $-7 > -10$

3. $-8k \le 32$

Multiply both sides of the inequality by $-\frac{1}{8}$.

Since this is a negative number, change the direction of the inequality symbol.

$$-\frac{1}{8}(-8k) \ge -\frac{1}{8}(32) \Rightarrow k \ge -4$$

The solution is $[-4, \infty)$.

4. $-4a \le 36$

Multiply both sides by $-\frac{1}{4}$. Change the direction of the inequality symbol.

$$-\frac{1}{4}(-4a) \ge -\frac{1}{4}(36) \Rightarrow a \ge -9$$

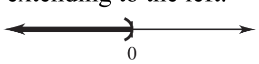
The solution is $[-9, \infty)$.

5. $-2b > 0$

Multiply both sides by $-\frac{1}{2}$.

$$-2b > 0 \Rightarrow -\frac{1}{2}(-2b) < -\frac{1}{2}(0) \Rightarrow b < 0$$

The solution is $(-\infty, 0)$. To graph this solution, put a parenthesis at 0 and draw an arrow extending to the left.



6. $6 - 6z < 0$

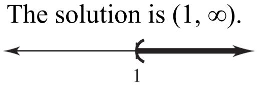
Add $6z$ to both sides.

$6 - 6z + 6z < 0 + 6z \Rightarrow 6 < 6z$

Multiply both sides by $\frac{1}{6}$.

$$\frac{1}{6}(6) < \frac{1}{6}(6z) \Rightarrow 1 < z \text{ or } z > 1$$

The solution is $(1, \infty)$.



7. $3x + 4 \le 14$

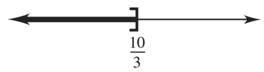
Subtract 4 from both sides.

$3x + 4 - 4 \le 14 - 4 \Rightarrow 3x \le 10$

Multiply each side by $\frac{1}{3}$.

$$\frac{1}{3}(3x) \le \frac{1}{3}(10) \Rightarrow x \le \frac{10}{3}$$

The solution is $\left(-\infty, \frac{10}{3}\right]$.



8. $2y - 7 < 9$

Add 7 to both sides.

$2y - 7 + 7 < 9 + 7 \Rightarrow 2y < 16$

Multiply both sides by $\frac{1}{2}$.

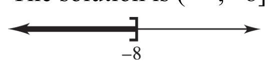
$$\frac{1}{2}(2y) < \frac{1}{2}(16) \Rightarrow y < 8$$

Solution is $(-\infty, 8)$.

For exercises 9–26, we give the solutions without additional explanation.

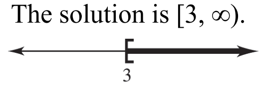
9.
$$\begin{aligned} -5 - p &\ge 3 \\ -5 + 5 - p &\ge 3 + 5 \\ -p &\ge 8 \\ (-1)(-p) &\le (-1)(8) \\ p &\le -8 \end{aligned}$$

The solution is $(-\infty, -8]$.



10.
$$\begin{aligned} 5 - 3r + (-5) &\le -4 + (-5) \\ -3r &\le -9 \\ -\frac{1}{3}(-3r) &\ge -9\left(-\frac{1}{3}\right) \\ r &\ge 3 \end{aligned}$$

The solution is $[3, \infty)$.

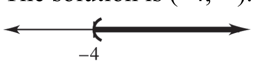


11.
$$\begin{aligned} 7m-5 &< 2m+10 \\ 5m-5 &< 10 \\ 5m &< 15 \\ \frac{1}{5}(5m) &< \frac{1}{5}(15) \\ m &< 3 \end{aligned}$$

The solution is $(-\infty, 3)$.

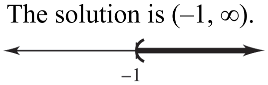
12.
$$\begin{aligned} 6x-2 &> 4x-10 \\ 6x-2-4x &> 4x-10-4x \\ 2x-2 &> -10 \\ 2x-2+2 &> -10+2 \\ 2x &> -8 \\ \frac{1}{2}(2x) &> \frac{1}{2}(-8) \\ x &> -4 \end{aligned}$$

The solution is $(-4, \infty)$.



13.
$$\begin{aligned} m-(4+2m)+3 &< 2m+2 \\ m-4-2m+3 &< 2m+2 \\ -1-m &< 2m+2 \\ -m-2m &< 2+1 \\ -3m &< 3 \\ -\frac{1}{3}(-3m) &> -\frac{1}{3}(3) \\ m &> -1 \end{aligned}$$

The solution is $(-1, \infty)$.

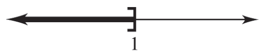


14.
$$\begin{aligned} 2p-(3-p) &\le -7p-2 \\ 2p-3+p &\le -7p-2 \\ 3p-3 &\le -7p-2 \\ 10p-3 &\le -2 \\ 10p &\le 1 \\ p &\le \frac{1}{10} \end{aligned}$$

The solution is $\left(-\infty, \frac{1}{10}\right]$.

15.
$$\begin{aligned} -2(3y-8) &\ge 5(4y-2) \\ -6y+16 &\ge 20y-10 \\ 16+10 &\ge 20y+6y \\ 26 &\ge 26y \\ 1 &\ge y \quad \text{or} \quad y \le 1 \end{aligned}$$

The solution is $(-\infty, 1]$.

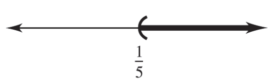


16.
$$\begin{aligned} 5r-(r+2) &\ge 3(r-1)+6 \\ 5r-r-2 &\ge 3r-3+6 \\ 4r-2 &\ge 3r+3 \\ r-2 &\ge 3 \\ r &\ge 5 \end{aligned}$$

The solution is $[5, \infty)$.

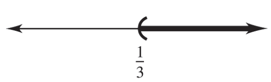
17.
$$\begin{aligned} 3p-1 &< 6p+2(p-1) \\ 3p-1 &< 6p+2p-2 \\ -1+2 &< 6p+2p-3p \\ 1 &< 5p \\ \frac{1}{5} &< p \quad \text{or} \quad p > \frac{1}{5} \end{aligned}$$

The solution is $\left(\frac{1}{5}, \infty\right)$.



18.
$$\begin{aligned} x+5(x+1) &> 4(2-x)+x \\ x+5x+5 &> 8-4x+x \\ 6x+5 &> 8-3x \\ 9x+5 &> 8 \\ 9x &> 3 \\ x &> \frac{1}{3} \end{aligned}$$

The solution is $\left(\frac{1}{3}, \infty\right)$.



19.
$$\begin{aligned} -7 &< y-2 < 5 \\ -7+2 &< y-2+2 < 5+2 \\ -5 &< y < 7 \end{aligned}$$

The solution is (–5, 7).

20.
$$\begin{aligned} -3 &< m+6 < 2 \\ -3+(-6) &< m+6+(-6) < 2+(-6) \\ -9 &< m < -4 \end{aligned}$$

The solution is (–9, –4).

21.
$$\begin{aligned} 8 &\le 3r+1 \le 16 \\ 8-1 &\le 3r \le 16-1 \\ 7 &\le 3r \le 15 \\ \frac{7}{3} &\le r \le 5 \end{aligned}$$

The solution is $\left[\frac{7}{3}, 5\right]$.

22.
$$\begin{aligned} -6 &< 2p-3 \le 5 \\ -6+3 &< 2p-3+3 \le 5+3 \\ -3 &< 2p \le 8 \\ \frac{1}{2}(-3) &< \frac{1}{2}(2p) \le \frac{1}{2}(8) \\ -\frac{3}{2} &< p \le 4 \end{aligned}$$

The solution is $\left(-\frac{3}{2}, 4\right]$.

23.
$$\begin{aligned} -4 &\le \frac{2k-1}{3} \le 2 \\ -4(3) &\le 3\left(\frac{2k-1}{3}\right) \le 2(3) \\ -12 &\le 2k-1 \le 6 \\ -12+1 &\le 2k \le 6+1 \\ -11 &\le 2k \le 7 \\ -\frac{11}{2} &\le k \le \frac{7}{2} \end{aligned}$$

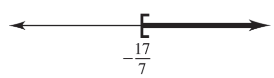
The solution is $\left[-\frac{11}{2}, \frac{7}{2}\right]$.

24.
$$\begin{aligned} -1 &\le \frac{5y+2}{3} \le 4 \\ 3(-1) &\le 3\left(\frac{5y+2}{3}\right) \le 3(4) \\ -3 &\le 5y+2 \le 12 \\ -5 &\le 5y \le 10 \\ -1 &\le y \le 2 \end{aligned}$$

The solution is [–1, 2].

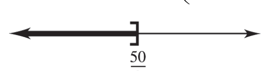
25.
$$\begin{aligned} \frac{3}{5}(2p+3) &\ge \frac{1}{10}(5p+1) \\ 10\cdot\frac{3}{5}(2p+3) &\ge 10\cdot\frac{1}{10}(5p+1) \\ 6(2p+3) &\ge 5p+1 \\ 12p+18 &\ge 5p+1 \\ 7p &\ge -17 \\ p &\ge -\frac{17}{7} \end{aligned}$$

The solution is $\left[-\frac{17}{7}, \infty\right)$.



26.
$$\begin{aligned} \frac{8}{3}(z-4) &\le \frac{2}{9}(3z+2) \\ \frac{8}{3}z - \frac{32}{3} &\le \frac{2}{3}z + \frac{4}{9} \\ \frac{6}{3}z - \frac{32}{3} &\le \frac{4}{9} \\ 2z &\le \frac{4}{9} + \frac{32}{3} \\ 2z &\le \frac{100}{9} \Rightarrow z \le \frac{50}{9} \end{aligned}$$

The solution is $\left(-\infty, \frac{50}{9}\right]$.



27. $x \ge 2$

28. $x < -2$

29. $-3 < x \le 5$

30. $-4 \le x \le 4$

31. $C = 50x + 6000$; $R = 65x$
To at least break even, $R \ge C$.
$65x \ge 50x + 6000$
$15x \ge 6000 \Rightarrow x \ge 400$
The number of units of wire must be in the interval $[400, \infty)$.

32. Given $C = 100x + 6000$; $R = 500x$.
Since $R \ge C$,
$500x \ge 100x + 6000 \Rightarrow 400x \ge 6000 \Rightarrow x \ge 15$
The number of units of squash must be in the interval $[15, \infty)$.

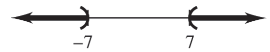
33. $C = 85x + 1000$; $R = 105x$
$R \ge C$
$105x \ge 85x + 1000$
$20x \ge 1000$
$x \ge \frac{1000}{20} \Rightarrow x \ge 50$
x must be in the interval $[50, \infty)$.

34. $C = 70x + 500$; $R = 60x$
$R \ge C$
$60x \ge 70x + 500$
$-10x \ge 500$
$10x \le -500$
$x \le -\frac{500}{10} \Rightarrow x \le -50$
It is impossible to break even.

35. $C = 1000x + 5000$; $R = 900x$
$R \ge C$
$900x \ge 1000x + 5000$
$-100x \ge 5000$
$x \le \frac{5000}{-100} \Rightarrow x \le -50$
It is impossible to break even.

36. $C = 25{,}000x + 21{,}700{,}000$; $R = 102{,}500x$
$R \ge C$
$102{,}500x \ge 25{,}000x + 21{,}700{,}000$
$77{,}500x \ge 21{,}700{,}000$
$x \ge \frac{21{,}700{,}000}{77{,}500} \Rightarrow x \ge 280$
x must be in the interval $[280, \infty)$.

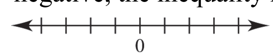
37. $|p| > 7$
$p < -7$ or $p > 7$
The solution is $(-\infty, -7)$ or $(7, \infty)$.



38. $|m| < 2 \Rightarrow -2 < m < 2$
The solution is $(-2, 2)$.

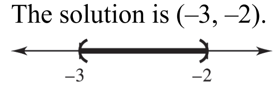
39. $|r| \le 5 \Rightarrow -5 \le r \le 5$
The solution is $[-5, 5]$.

40. $|a| < -2$
Since the absolute value of a number is never negative, the inequality has no solution.



41. $|b| > -5$
The absolute value of a number is always nonnegative. Therefore, $|b| > -5$ is always true, so the solution is the set of all real numbers.

42. $|2x + 5| < 1$
$-1 < 2x + 5 < 1$
$-1 - 5 < 2x < 1 - 5$
$-6 < 2x < -4$
$-3 < x < -2$
The solution is $(-3, -2)$.



43. $\left|x - \frac{1}{2}\right| < 2$
$-2 < x - \frac{1}{2} < 2$
$-\frac{3}{2} < x < \frac{5}{2}$
The solution is $\left(-\frac{3}{2}, \frac{5}{2}\right)$.

44. $|3z+1| \ge 4$

$3z+1 \ge 4$ or $3z+1 \le -4$

$3z \ge 4-1$ $\quad 3z \le -4-1$

$3z \ge 3$ $\quad 3z \le -5$

$z \ge 1$ $\quad z \le -\frac{5}{3}$

The solution is $\left(-\infty, -\frac{5}{3}\right]$ or $[1, \infty)$.

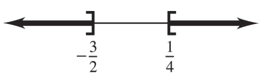
45. $|8b+5| \ge 7$

$8b+5 \le -7$ or $8b+5 \ge 7$

$8b \le -12$ or $8b \ge 2$

$b \le -\frac{3}{2}$ or $b \ge \frac{1}{4}$

The solution is $\left(-\infty, -\frac{3}{2}\right]$ or $\left[\frac{1}{4}, \infty\right)$.



46. $\left|5x+\frac{1}{2}\right| - 2 < 5$

$\left|5x+\frac{1}{2}\right| < 7$

$-7 < 5x + \frac{1}{2} < 7$

$-7 - \frac{1}{2} < 5x < 7 - \frac{1}{2}$

$-\frac{15}{2} < 5x < \frac{13}{2}$

$-\frac{15}{2} \cdot \frac{1}{5} < x < \frac{13}{2} \cdot \frac{1}{5}$

$-\frac{3}{2} < x < \frac{13}{10}$

The solution is $\left(-\frac{3}{2}, \frac{13}{10}\right)$.

47. $|T-83| \le 7$

$-7 \le T-83 \le 7$

$76 \le T \le 90$

48. $|T-63| \le 27$

$-27 \le T-63 \le 27$

$36 \le T \le 90$

49. $|T-61| \le 21$

$-21 \le T-61 \le 21$

$40 \le T \le 82$

50. $|T-43| \le 22$

$-22 \le T-43 \le 22$

$21 \le T \le 65$

51. $|P-36| \le 3$

$|P-36| \le \pm 3 \Rightarrow$

$-3 \le P-36 \le 3 \Rightarrow$

$33 \le P \le 39$

52. $|x-100| > 12$

53. $38 \le B \le 44$

54. $43 \le U \le 49$

55. The seven income ranges are:

$0 < x \le 9875$

$9875 < x \le 40,125$

$40,125 < x \le 85,525$

$85,525 < x \le 163,300$

$163,300 < x \le 207,350$

$207,350 < x \le 518,400$

$x > 518,400$

56. The eight income ranges are:

$0 < x \le 8500$

$8500 < x \le 11,700$

$11,700 < x \le 13,900$

$13,900 < x \le 21,400$

$21,400 < x \le 80,650$

$80,650 < x \le 215,400$

$215,400 < x \le 1,077,550$

$x > 1,077,550$

Section 2.5 Polynomial and Rational Inequalities

1. $(x+4)(2x-3) \le 0$

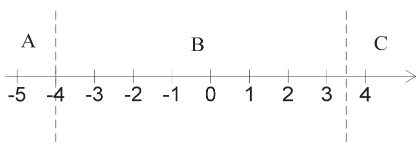
Solve the corresponding equation.

$(x+4)(2x-3) = 0$

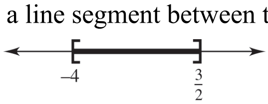
$x+4=0$ or $2x-3=0$

$x=-4$ $\quad x=\frac{3}{2}$

Note that because the inequality symbol is "≤," –4 and $\frac{3}{2}$ are solutions of the original inequality. These numbers separate the number line into three regions.



In region A, let $x = -6$:
$(-6+4)[2(-6)-3] = 30 > 0$.
In region B, let $x = 0$:
$(0+4)[2(0)-3] = -12 < 0$.
In region C, let $x = 2$:
$(2+4)[2(2)-3] = 6 > 0$.
The only region where $(x+4)(2x-3)$ is negative is region B, so the solution is $\left[-4, \frac{3}{2}\right]$. To graph this solution, put brackets at –4 and $\frac{3}{2}$ and draw a line segment between these two endpoints.



2. $(5y-1)(y+3) > 0$

Solve the corresponding equation.

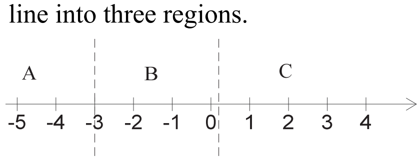
$(5y-1)(y+3) = 0$

$5y-1=0$ or $y+3=0$

$5y = 1$ $\quad y=-3$

$y = \frac{1}{5}$

Note that because the inequality symbol is "<", $\frac{1}{5}$ and -3 are not solutions of the original inequality. These numbers separate the number line into three regions.



In region A, let $x = -6$:
$[5(-6)-1][-6+3] = 93 > 0$.
In region B, let $x = 0$:
$[5(0)-1][0+3] = -3 < 0$.
In region C, let $x = 2$:
$[5(2)-1][2+3] = 45 > 0$
The regions where $(5y-1)(y+3)$ is positive are regions A and C, so the solutions are $(-\infty,-3)$ and $\left(\frac{1}{5},\infty\right)$. To graph this solution, put parentheses at –3 and $\frac{1}{5}$ and draw rays as shown below.

3. $r^2+4r > -3$

Solve the corresponding equation.

$r^2+4r=-3$

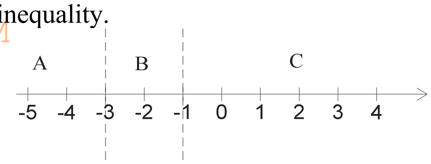
$r^2+4r+3=0$

$(r+1)(r+3)=0$

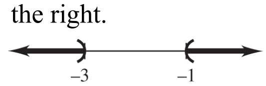
$r+1=0$ or $r+3=0$

$r=-1$ or $r=-3$

Note that because the inequality symbol is ">," –1 and –3 are not solutions of the original inequality.



In region A, let $r = -4$:
$(-4)^2+4(-4)=0>-3$.
In region B, let $r = -2$:
$(-2)^2+4(-2)=-4<-3$.
In region C, let $r = 0$:
$0^2+4(0)=0>-3$.
The solution is $(-\infty, -3)$ or $(-1, \infty)$.
To graph the solution, put a parenthesis at –3 and draw a ray extending to the left, and put a parenthesis at -1 and draw a ray extending to the right.



4. $z^2 + 6z > -8$

Solve the corresponding equation.

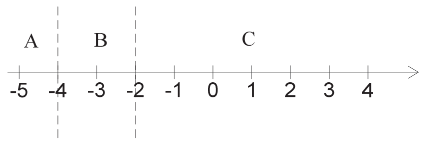
$z^2 + 6z = -8 \Rightarrow z^2 + 6z + 8 = 0$

$(z+2)(z+4) = 0$

$z + 2 = 0 \quad \text{or} \quad z + 4 = 0$

$z = -2 \quad \text{or} \quad z = -4$

Because the inequality symbol is ">", -2 and -4 are not solutions of the original inequality.



In region A, let $z = -5$:

$(-5)^2 + 6(-5) + 8 = 3 > 0$

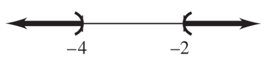
In region B, let $z = -3$:

$(-3)^2 + 6(-3) + 8 = -1 < 0$

In region C, let $z = 0$:

$(0)^2 + 6(0) + 8 = 8 > 0$

The solution is $(-\infty, -4)$ or $(-2, \infty)$.



5. $4m^2 + 7m - 2 \le 0$

Solve the corresponding equation.

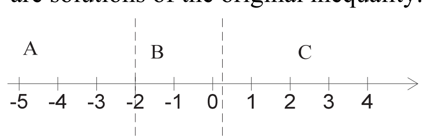
$4m^2 + 7m - 2 = 0$

$(4m-1)(m+2) = 0$

$4m - 1 = 0 \quad \text{or} \quad m + 2 = 0$

$m = \frac{1}{4} \quad \text{or} \quad m = -2$

Because the inequality symbol is "≤" $\frac{1}{4}$ and –2 are solutions of the original inequality.



In region A, let $m = -3$:

$4(-3)^2 + 7(-3) - 2 = 13 > 0$.

In region B, let $m = 0$:

$4(0)^2 + 7(0) - 2 = -2 < 0$.

In region C, let $m = 1$:

$4(1)^2 + 7(1) - 2 = 9 > 0$.

The solution is $\left[-2, \frac{1}{4}\right]$.

6. $6p^2 - 11p + 3 \le 0$

Solve the corresponding equation.

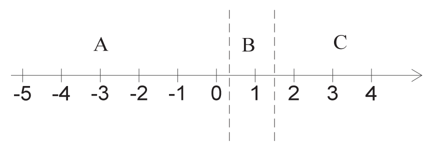
$6p^2 - 11p + 3 = 0$

$(3p-1)(2p-3) = 0$

$3p - 1 = 0 \quad \text{or} \quad 2p - 3 = 0$

$p = \frac{1}{3} \quad \text{or} \quad p = \frac{3}{2}$

Because the inequality symbol is "≤ ," $\frac{1}{3}$ and $\frac{3}{2}$ are solutions of the original inequality. These points divide a number line into three regions.



In region A, let $p = 0$.

$6(0)^2 - 11(0) + 3 = 3 > 0$

In region B, let $p = 1$.

$6(1)^2 - 11(1) + 3 = -2 < 0$

In region C, let $p = 10$.

$6(10)^2 - 11(10) + 3 = 493 > 0$

The numbers in regions A and C do not satisfy the inequality, so the solution is $\left[\frac{1}{3}, \frac{3}{2}\right]$.

7. $4x^2 + 3x - 1 > 0$

Solve the corresponding equation.

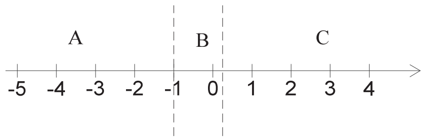
$4x^2 + 3x - 1 = 0$

$(4x-1)(x+1) = 0$

$4x - 1 = 0 \quad \text{or} \quad x + 1 = 0$

$x = \frac{1}{4} \quad \text{or} \quad x = -1$

Note that $\frac{1}{4}$ and –1 are not solutions of the original inequality.



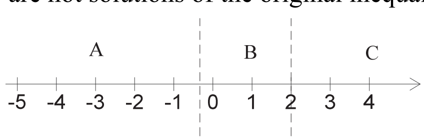
In region A, let $x = -2$:
$4(-2)^2 + 3(-2) - 1 = 9 > 0$.
In region B, let $x = 0$:
$4(0)^2 + 3(0) - 1 = -1 < 0$.
In region C, let $x = 1$:
$4(1)^2 + 3(1) - 1 = 6 > 0$.

The solution is $(-\infty, -1)$ or $\left(\frac{1}{4}, \infty\right)$.

8. $3x^2 - 5x > 2 \Rightarrow 3x^2 - 5x - 2 > 0$
Solve the corresponding equation.

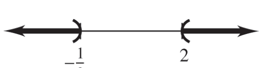
$$3x^2 - 5x - 2 = 0$$
$$(3x+1)(x-2) = 0$$
$$3x + 1 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -\frac{1}{3} \quad \text{or} \quad x = 2$$

Because the inequality symbol is ">," $-\frac{1}{3}$ and 2 are not solutions of the original inequality.

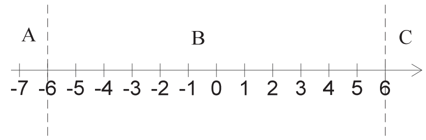


In region A, let $x = -1$.
$3(-1)^2 - 5(-1) = 8 > 2$
In region B, let $x = 0$.
$3(0)^2 - 5(0) = 0 < 2$
In region C, let $x = 10$.
$3(10)^2 - 5(10) = 250 > 2$

The numbers in regions A and C satisfy the inequality, so the solution is $\left(-\infty, -\frac{1}{3}\right)$ or $(2, \infty)$.

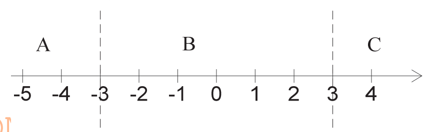


9. $x^2 \le 36$
Solve the corresponding equation.
$x^2 = 36 \Rightarrow x = \pm 6$

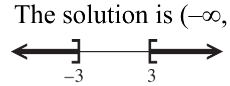


For region A, let $x = -7$: $(-7)^2 = 49 > 36$.
For region B, let $x = 0$: $0^2 = 0 < 36$.
For region C, let $x = 7$: $7^2 = 49 > 36$.
Both endpoints are included. The solution is $[-6, 6]$.

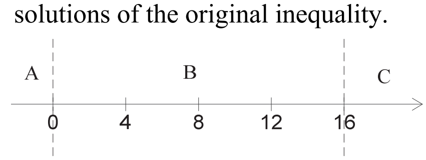
10. $y^2 \ge 9$
Solve the corresponding equation.
$y^2 = 9 \Rightarrow y = \pm 3$
Note that –3 and 3 are solutions to the original inequality.



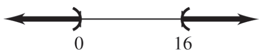
In region A, let $y = -4$:
$(-4)^2 = 16 > 9$
In region B, let $y = 0$:
$0^2 = 0 < 9$
In region C, let $y = 4$:
$(4)^2 = 16 > 9$
The solution is $(-\infty, -3]$ or $[3, \infty)$.



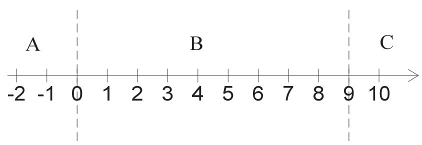
11. $p^2 - 16p > 0$
Solve the corresponding equation.
$p^2 - 16p = 0 \Rightarrow p(p - 16) = 0 \Rightarrow$
$p = 0$ or $p = 16$
Since the inequality is ">", 0 and 16 are not solutions of the original inequality.



For region A, let $p = -1$:
$(-1)^2 - 16(-1) = 17 > 0$.
For region B, let $p = 1$:
$1^2 - 16(1) = -15 < 0$.
For region C, let $p = 17$:
$17^2 - 16(17) = 17 > 0$.
The solution is $(-\infty, 0)$ or $(16, \infty)$.

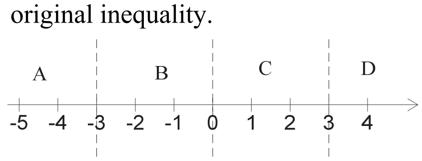


12. $r^2 - 9r < 0$
Solve the corresponding equation.
$r^2 - 9r = 0$
$r(r-9) = 0$
$r = 0$ or $r - 9 = 0$
$r = 0$ or $r = 9$
Note that 0 and 9 are not solutions to the original inequality.



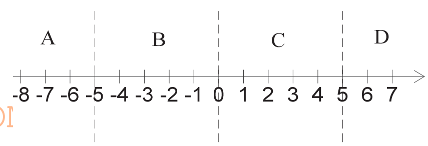
In region A, let $r = -1$:
$(-1)^2 - 9(-1) = 1 + 9 = 10 > 0$
In region B, let $r = 1$:
$(1)^2 - 9(1) = 1 - 9 = -8 < 0$
In region C, let $r = 10$:
$(10)^2 - 9(10) = 100 - 90 = 10 > 0$
The solution is $(0, 9)$.

13. $x^3 - 9x \ge 0$
Solve the corresponding equation.
$x^3 - 9x = 0$
$x\left(x^2 - 9\right) = 0$
$x(x+3)(x-3) = 0$
$x = 0$ or $x = -3$ or $x = 3$
Note that 0, –3, and 3 are all solutions of the original inequality.



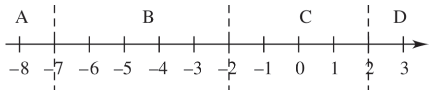
In region A, let $x = -4$:
$(-4)^3 - 9(-4) = -28 < 0$.
In region B, let $x = -1$:
$(-1)^3 - 9(-1) = 8 > 0$.
In region C, let $x = 1$:
$(1)^3 - 9(1) = -8 < 0$
In region D, let $x = 4$:
$4^3 - 9(4) = 28 > 0$.
The solution is $[-3, 0]$ or $[3, \infty)$.

14. $p^3 - 25p \le 0$
Solve the corresponding equation.
$p^3 - 25p = 0$
$p\left(p^2 - 25\right) = 0$
$p(p+5)(p-5) = 0$
$p = 0$ or $p = -5$ or $p = 5$
Because the inequality is "≤," 0, –5, and 5 are solutions of the original inequality. Locate these points and regions A, B, C, and D on a number line.



Test a number from each region in $p^3 - 25p \le 0$.
In region A, let $p = -10$.
$(-10)^3 - 25(-10) = -750 \le 0$
In region B, let $p = -1$.
$(-1)^3 - 25(-1) = 24 \ge 0$
In region C, let $p = 1$.
$1^3 - 25(1) = -24 \le 0$
In region D, let $p = 10$.
$10^3 - 25(10) = 750 \ge 0$
The numbers in regions A and C satisfy the inequality, so the solution is $(-\infty, -5]$ or $[0, 5]$.

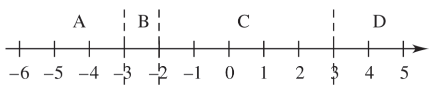
15. $(x+7)(x+2)(x-2) \ge 0$
Solve the corresponding equation.
$(x+7)(x+2)(x-2) = 0$
$x + 7 = 0$ or $x + 2 = 0$ or $x - 2 = 0$
$x = -7$ or $x = -2$ or $x = 2$
Note that –7, –2 and 2 are all solutions of the original inequality.



In region A, let $x = -8$:
$(-8 + 7)(-8 + 2)(-8 - 2) = -60 < 0$
In region B, let $x = -4$:
$(-4 + 7)(-4 + 2)(-4 - 2) = 36 > 0$
In region C, let $x = 0$:
$(0 + 7)(0 + 2)(0 - 2) = -28 < 0$
In region D, let $x = 3$:
$(3 + 7)(3 + 2)(3 - 2) = 50 > 0$
The solution is $[-7, -2]$ or $[2, \infty)$.

16. $(2x+4)\left(x^2-9\right) \le 0$

Solve the corresponding equation.
$(2x+4)(x^2-9) = 0$
$(2x+4)(x+3)(x-3) = 0$
$2x+4=0$ or $x+3=0$ or $x-3=0$
$x=-2$ or $x=-3$ or $x=3$
Note that –2, –3 and 3 are all solutions of the original inequality.

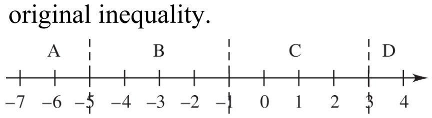


In region A, let $x = -5$:
$\left[2(-5)+4\right]\left[(-5)^2-9\right] = (-6)(16) = -96 < 0$

In region B, let $x = -2.5$:
$\left[2(-2.5)+4\right]\left[(-2.5)^2-9\right] = -1(-2.75)$
$= 2.75 > 0$
In region C, let $x = 0$:
$\left[2(0)+4\right]\left[(0)^2-9\right] = (4)(-9) = -36 < 0$
In region D, let $x = 4$:
$\left[2(4)+4\right]\left[(4)^2-9\right] = (12)(7) = 84 > 0$
The solution is $(-\infty, -3]$ or $[-2, 3]$.

17. $(x+5)\left(x^2-2x-3\right) < 0$

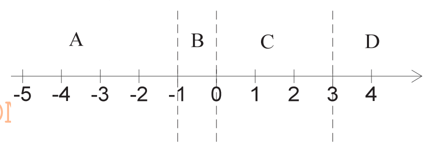
Solve the corresponding equation.
$(x+5)\left(x^2-2x-3\right) = 0$
$(x+5)(x+1)(x-3) = 0$
$x+5=0$ or $x+1=0$ or $x-3=0$
$x=-5$ or $x=-1$ or $x=3$
Note that –5, –1 and 3 are not solutions of the original inequality.



In region A, let $x = -6$:
$(-6+5)\left[(-6)^2-2(-6)-3\right] = (-1)(45)$
$= -45 < 0$
In region B, let $x = -2$:
$(-2+5)\left[(-2)^2-2(-2)-3\right] = 3(5) = 15 > 0$
In region C, let $x = 0$:
$(0+5)\left[(0)^2-2(0)-3\right] = 5(-3) = -15 < 0$
In region D, let $x = 4$:
$(4+5)\left[(4)^2-2(4)-3\right] = 9(5) = 45 > 0$
The solution is $(-\infty, -5)$ or $(-1, 3)$.

18. $x^3-2x^2-3x \le 0$

Solve the corresponding equation.
$x^3-2x^2-3x = 0$
$x\left(x^2-2x-3\right) = 0$
$x(x+1)(x-3) = 0$
$x = -1$ or $x = 0$ or $x = 3$
Note that –1, 0 and 3 are solutions of the original inequality.



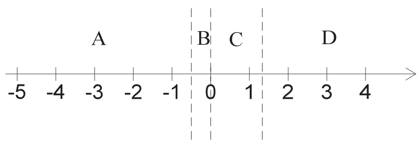
In region A, let $x = -2$:
$(-2)^3 - 2(-2)^2 - 3(-2) = -8-8+6 = -10 < 0$
In region B, let $x = -\frac{1}{2}$:
$\left(-\frac{1}{2}\right)^3 - 2\left(\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right)$
$= -\frac{1}{8} - \frac{1}{2} + \frac{3}{2} = \frac{7}{8} > 0$
In region C, let $x = 1$:
$1^3 - 2(1)^2 - 3(1) = 1-2-3 = -4 < 0$
In region D, let $x = 4$:
$4^3 - 2(4)^2 - 3(4) = 64-32-12 = 20 > 0$
The solution is $[-\infty, -1]$ or $[0, 3]$

19. $6k^3 - 5k^2 < 4k \Rightarrow 6k^3 - 5k^2 - 4k < 0$
Solve the corresponding equation.

$$6k^3 - 5k^2 - 4k = 0$$
$$k\left(6k^2 - 5k - 4\right) = 0$$
$$k(3k-4)(2k+1) = 0$$

k = 0 or $k = \frac{4}{3}$ or $k = -\frac{1}{2}$

Note that 0, $\frac{4}{3}$, and $-\frac{1}{2}$ are not solutions of the original inequality.



In region A, let $k = -1$:
$6(-1)^3 - 5(-1)^2 - 4(-1) = -7 < 0$

In region B, let $k = -\frac{1}{4}$:

$$6\left(-\frac{1}{4}\right)^3 - 5\left(-\frac{1}{4}\right)^2 - 4\left(-\frac{1}{4}\right) = \frac{19}{32} > 0;$$

In region C, let $k = 1$:
$6(1)^3 - 5(1)^2 - 4(1) = -3 < 0$

In region D, let $k = 10$:
$6(10)^3 - 5(10)^2 - 4(10) = 5460$

The given inequality is true in regions A and C.

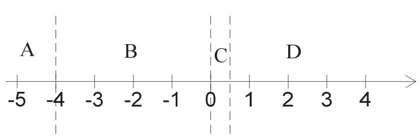
The solution is $\left(-\infty, -\frac{1}{2}\right)$ or $\left(0, \frac{4}{3}\right)$.

20. $2m^3 + 7m^2 > 4m \Rightarrow 2m^3 + 7m^2 - 4m > 0$
Solve the corresponding equation.

$$2m^3 + 7m^2 - 4m = 0$$
$$m\left(2m^2 + 7m - 4\right) = 0$$
$$m(2m-1)(m+4) = 0$$

$m = 0$ or $m = \frac{1}{2}$ or $m = -4$

Since the inequality is“>,” 0, $\frac{1}{2}$, and –4 are not solutions of the original inequality. Locate these points and regions, A, B, C, and D on a number line.



In region A, let $m = -10$.
$2(-10)^3 + 7(-10)^2 - 4(-10) = -1260 < 0$

In region B, let $m = -1$.
$2(-1)^3 + 7(-1)^2 - 4(-1) = 9 > 0$

In region C, let $m = \frac{1}{4}$.

$$2\left(\frac{1}{4}\right)^3 + 7\left(\frac{1}{4}\right)^2 - 4\left(\frac{1}{4}\right) = -\frac{17}{32} < 0$$

In region D, let $m = 1$.
$2(1)^3 + 7(1)^2 - 4(1) = 5 > 0$

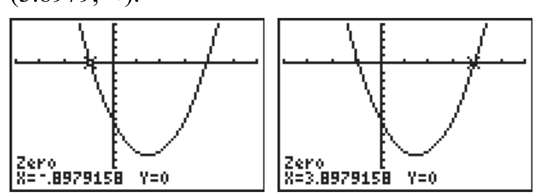
The numbers in regions B and D satisfy the inequality, so the solution is (–4, 0) or $\left(\frac{1}{2}, \infty\right)$.

21. The inequality $p^2 < 16$ should be rewritten as $p^2 - 16 < 0$ and solved by the method shown in this section for solving quadratic inequalities. This method will lead to the correct solution (–4, 4). The student’s method and solution are incorrect.

22. To solve $6x + 7 < 2x^2$, write the inequality as $2x^2 - 6x - 7 > 0$.

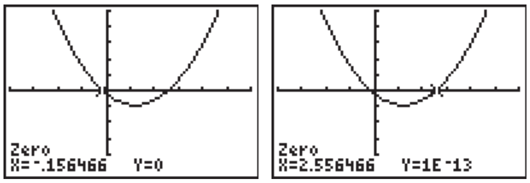
Graph the equation $y = 2x^2 - 6x - 7$.

Enter this equation as y_1 and use $-4 < x < 4$ and $-15 < y < 5$. On the CALC menu, use “zero” to find the x-values where the graph crosses the x-axis. These values are $x = -0.8979$ and $x = 3.8979$. The graph is above the x-axis to the left of – 0.8979 and to the right of 3.8979. The solution of the inequality is (–∞, –0.8979) or (3.8979, ∞).



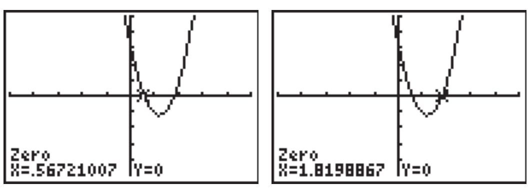
[–4, 6] by [–15,5]

23. To solve $.5x^2 - 1.2x < .2$,write the inequality as $.5x^2 - 1.2x - .2 < 0$. Graph the equation $y = .5x^2 - 1.2x - .2$. Enter this equation as y_1 and use $-4 \le x \le 6$ and $-5 \le y \le 5$. On the CALC menu, use "zero" to find the x-values where the graph crosses the x-axis. These values are $x = -.1565$ and $x = 2.5565$. The graph is below the x-axis between these two values. The solution of the inequality is $(-.1565, 2.5565)$.



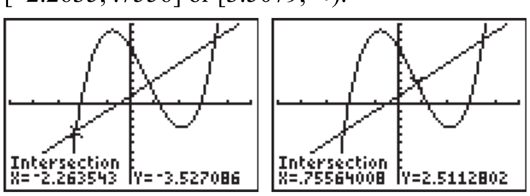
[–4, 6] by [–5,5]

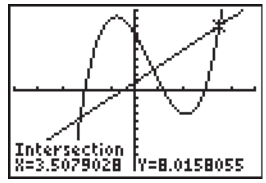
24. To solve $3.1x^2 - 7.4x + 3.2 > 0$, graph the equation $y = 3.1x^2 - 7.4x + 3.2$. Enter this equation as y_1 and use $-5 \le x \le 5$ and $-5 \le y \le 5$. On the CALC menu, use "zero" to find the x-values where the graph crosses the x-axis. These values are $x = .5672$ and $x = 1.8199$. The graph is above the x-axis to the left of .5672 and to the right of 1.8199. The solution of the inequality is $(-\infty, .5672)$ or $(1.8199, \infty)$.



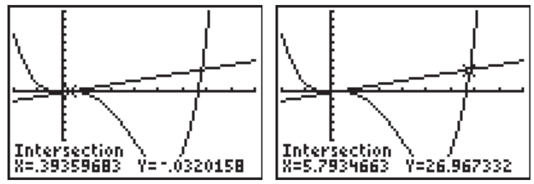
[–5, 5] by [–5, 5]

25. To solve $x^3 - 2x^2 - 5x + 7 \ge 2x + 1$, graph $y_1 = x^3 - 2x^2 - 5x + 7$ and $y_2 = 2x + 1$ in the window [–5, 5] by[–10, 10]. On the CALC menu, use "intersect" to find the x-values where the graphs intersect. These values are $x = -2.2635$, $x = .7556$ and $x = 3.5079$. The graph of y_1 is above the graph of y_2 for $[-2.2635, .7556]$ or $[3.5079, \infty)$.

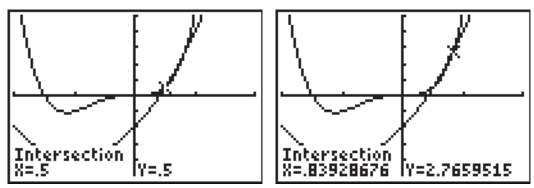




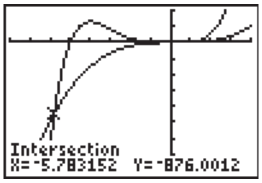
26. To solve $x^4 - 6x^3 + 2x^2 < 5x - 2$, graph $y_1 = x^4 - 6x^3 + 2x^2$ and $y_2 = 5x - 2$ in the window [–2, 8] by[–100, 100]. On the CALC menu, use "intersect" to find the x-values where the graphs intersect. These values are $x = .3936$ and $x = 5.7935$. The graph of y_1 is below the graph of y_2 for (.3936, 5.7935), so the solution is (.3936, 5.7935).

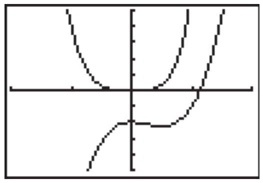


27. To solve $2x^4 + 3x^3 < 2x^2 + 4x - 2$, graph $y_1 = 2x^4 + 3x^3$ and $y_2 = 2x^2 + 4x - 2$ in the window [–2, 2] by[–5, 5]. On the CALC menu, use "intersect" to find the x-values where the graphs intersect. These values are $x = .5$ and $x = .8393$. The graph of y_1 is below the graph of y_2 to the right of .5 and to the left of .8393. The solution of the inequality is (.5, .8393).



28. To solve $x^5 + 5x^4 > 4x^3 - 3x^2 - 2$, graph $y_1 = x^5 + 5x^4$ and $y_2 = 4x^3 - 3x^2 - 2$ in the window [–8, 4] by[–1600, 400]. There is clearly one intersection near $x = -6$. On the CALC menu, use "intersect" to find this value, $x = -5.78323$. Next, change the window to [–2, 2] by [–5, 5] to examine the behavior of the graphs near the origin. From this view, it is clear that the graphs do not intersect, and y_1 is below the graph of y_2 The graph of y_1 is above the graph of y_2 for $(-5.78323, \infty)$.





Test a number from each region in the original inequality.

In region A, let $z = -6$.

$$\frac{-6+6}{-6+4} = 0 < 1$$

In region B, let $z = 0$.

$$\frac{0+6}{0+4} = \frac{3}{2} > 1$$

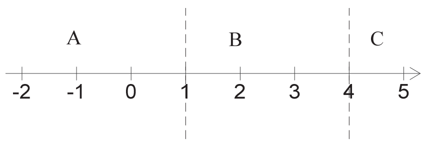
The numbers in region B satisfy the inequality, so the solution is $(-4, \infty)$.

29. $\frac{r-4}{r-1} \geq 0$

Solve the corresponding equation.

$$\frac{r-4}{r-1} = 0$$

The quotient can change sign only when the numerator is 0 or the denominator is 0. The numerator is 0 when $r = 4$. The denominator is 0 when $r = 1$. Note that 4 is a solution of the original inequality, but 1 is not.



In region A, let $r = 0$:

$$\frac{0-4}{0-1} = 4 > 0.$$

In region B, let $r = 2$:

$$\frac{2-4}{2-1} = -2 < 0.$$

In region C, let $r = 5$:

$$\frac{5-3}{5-1} = \frac{1}{4} > 0.$$

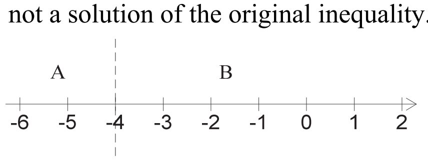
The given inequality is true in regions A and C, so the solution is $(-\infty, 1)$ or $[4, \infty)$.

30. $\frac{z+6}{z+4} > 1$

Solve the corresponding equation is $\frac{z+6}{z+4} = 1$.

$$\frac{z+6}{z+4} = 1 \Rightarrow \frac{z+6}{z+4} - 1 = 0 \Rightarrow$$
$$\frac{z+6}{z+4} - \frac{z+4}{z+4} = 0 \Rightarrow \frac{2}{z+4} = 0$$

Therefore, the function has no solutions. The denominator is zero when $z = -4$. Note that -4 is not a solution of the original inequality.

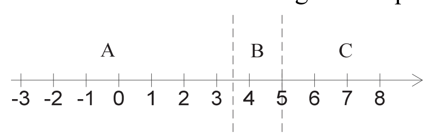


31. $\frac{a-2}{a-5} < -1$

Solve the corresponding equation.

$$\frac{a-2}{a-5} = -1$$
$$\frac{a-2}{a-5} + 1 = 0$$
$$\frac{a-2}{a-5} + \frac{a-5}{a-5} = 0$$
$$\frac{2a-7}{a-5} = 0$$

The numerator is 0 when $a = \frac{7}{2}$. The denominator is 0 when $a = 5$. Note that $\frac{7}{2}$ and 5 are not solutions of the original inequality.



In region A, let $a = 0$:

$$\frac{0-2}{0-5} = \frac{2}{5} > -1.$$

In region B, let $a = 4$:

$$\frac{4-2}{4-5} = \frac{2}{-1} = -2 < -1.$$

In region C, let $a = 10$:

$$\frac{10-2}{10-5} = \frac{8}{5} > -1.$$

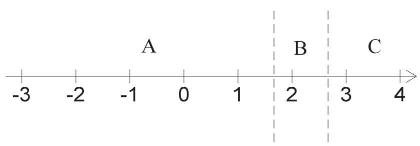
The solution is $\left(\frac{7}{2}, 5\right)$.

32. $\frac{1}{3k-5} < \frac{1}{3}$

Solve the corresponding equation

$$\frac{1}{3k-5} = \frac{1}{3}$$
$$\frac{1}{3k-5} - \frac{1}{3} = 0$$
$$\frac{3 \cdot 1}{3(3k-5)} - \frac{1(3k-5)}{3(3k-5)} = 0$$
$$\frac{3-(3k-5)}{3(3k-5)} = 0$$
$$\frac{3-3k+5}{3(3k-5)} = 0$$
$$\frac{8-3k}{3(3k-5)} = 0$$

The numerator is zero when $k = \frac{8}{3}$. The denominator is zero when $k = \frac{5}{3}$. Note that $\frac{8}{3}$ and $\frac{5}{3}$ are not solutions of the original inequality.



Test a number from each region in the original inequality.

In region A, let $k = 0$.

$$\frac{1}{3(0)-5} = -\frac{1}{5} < \frac{1}{3}$$

In region B, let $k = 2$.

$$\frac{1}{3(2)-5} = 1 > \frac{1}{3}$$

In region C, let $k = 3$.

$$\frac{1}{3(3)-5} = \frac{1}{4} < \frac{1}{3}$$

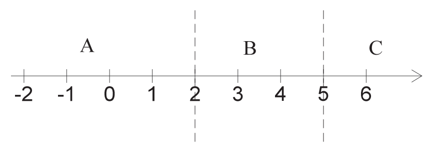
The numbers in regions A and C satisfy the inequality, so the solution is $\left(-\infty, \frac{5}{3}\right)$ or $\left(\frac{8}{3}, \infty\right)$.

33. $\frac{1}{p-2} < \frac{1}{3}$

Solve the corresponding equation.

$$\frac{1}{p-2} = \frac{1}{3}$$
$$\frac{1}{p-2} - \frac{1}{3} = 0$$
$$\frac{3-(p-2)}{3(p-2)} = 0$$
$$\frac{3-p+2}{3(p-2)} = 0$$
$$\frac{5-p}{3(p-2)} = 0$$

The numerator is 0 when $p = 5$. The denominator is 0 when $p = 2$. Note that 2 and 5 are not solutions of the original inequality.



In region A, let $p = 0$: $\frac{1}{0-2} = -\frac{1}{2} < \frac{1}{3}$.

In region B, let $p = 3$: $\frac{1}{3-2} = 1 > \frac{1}{3}$.

In region C, let $p = 6$: $\frac{1}{6-2} = \frac{1}{4} < \frac{1}{3}$.

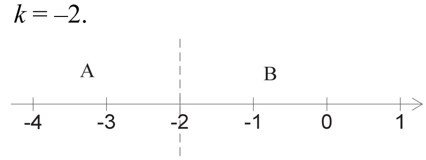
The solution is $(-\infty, 2)$ or $(5, \infty)$.

34. $\frac{7}{k+2} \ge \frac{1}{k+2}$

Solve the corresponding equation.

$$\frac{7}{k+2} = \frac{1}{k+2}$$
$$\frac{7}{k+2} - \frac{1}{k+2} = 0$$
$$\frac{6}{k+2} = 0$$

Therefore, the numerator is never zero, but the denominator is zero when $k + 2 = 0$ or $k = -2$, but the inequality is undefined when $k = -2$.

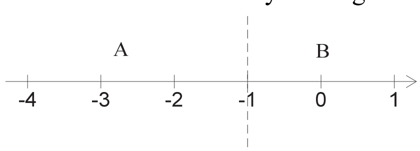


Test a number from each region in the original inequality.
For region A, let $k = -3$.
$\frac{7}{-3+2} = -7$ and $\frac{1}{-3+2} = -1$
Since $-7 \le -1$, -3 is not a solution of the inequality.
For region B, let $k = 0$.
$\frac{7}{0+2} = \frac{7}{2}$ and $\frac{1}{0+2} = \frac{1}{2}$
Since $\frac{7}{2} \ge \frac{1}{2}$, 0 is a solution of the inequality.
The numbers from region B satisfy the inequality, so the solution is $(-2, \infty)$.

35. $\frac{5}{p+1} > \frac{12}{p+1}$
Solve the corresponding equation.

$$\frac{5}{p+1} = \frac{12}{p+1}$$
$$\frac{5}{p+1} - \frac{12}{p+1} = 0$$
$$\frac{-7}{p+1} = 0$$

The numerator is never 0. The denominator is 0 when $p = -1$. Therefore, in this case, we separate the number line into only two regions.



In region A, let $p = -2$:

$$\frac{5}{-2+1} = -5$$
$$\frac{12}{-2+1} = -12$$
$$-5 > -12$$

In region B, let $p = 0$:

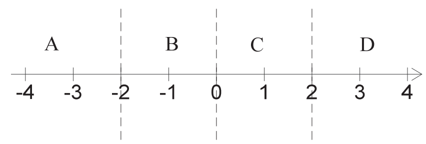
$$\frac{5}{0+1} = 5$$
$$\frac{12}{0+1} = 12$$
$$12 > 5$$

Therefore, the given inequality is true in region A. The only endpoint, -1, is not included because the symbol is ">." Therefore, the solution is $(-\infty, -1)$.

36. $\frac{x^2 - 4}{x} > 0$
Solve the corresponding equation.

$$\frac{x^2 - 4}{x} = 0$$
$$x^2 - 4 = 0 \quad \text{or} \quad x = 0$$
$$(x-2)(x+2) = 0 \quad \text{or} \quad x = 0$$
$$x - 2 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad x = 0$$
$$x = 2 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = 0$$

Note that -2, 0 and 2 are not solutions of the original inequality.

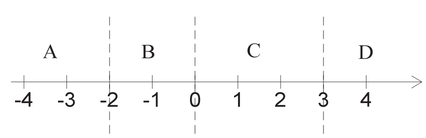


In region A, let $x = -3$,
$\frac{(-3)^2 - 4}{-3} = \frac{9-4}{-3} = \frac{-5}{3} < 0$.
In region B, let $x = -1$:
$\frac{(-1)^2 - 4}{-1} = \frac{1-4}{-1} = 3 > 0$.
In region C, let $x = 1$:
$\frac{1^2 - 4}{1} = \frac{-3}{1} = -3 < 0$.
In region D, let $x = 3$:
$\frac{3^2 - 4}{3} = \frac{9-4}{3} = \frac{5}{3} > 0$.
The solution is $(-2, 0)$ or $(2, \infty)$.

37. $\frac{x^2 - x - 6}{x} < 0$
Solve the corresponding equation.

$$\frac{x^2 - x - 6}{x} = 0$$
$$x^2 - x - 6 = 0 \quad \text{or} \quad x = 0$$
$$(x-3)(x+2) = 0 \quad \text{or} \quad x = 0$$
$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad x = 0$$
$$x = 3 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = 0$$

Note that -2, 0 and 3 are not solutions of the original inequality.



In region A, let $x = -3$:

$$\frac{(-3)^2-(-3)-6}{-3} = \frac{9+3-6}{-3} = \frac{6}{-3} = -2 < 0.$$

In region B, let $x = -1$:

$$\frac{(-1)^2-(-1)-6}{-1} = \frac{1+1-6}{-1} = \frac{-4}{-1} = 4 > 0$$

In region C, let $x = 1$:

$$\frac{1^2-1-6}{1} = -6 < 0.$$

In region D, let $x = 4$:

$$\frac{4^2-4-6}{4} = \frac{16-10}{4} = \frac{6}{4} = \frac{3}{2} > 0.$$

The solution is $(-\infty, -2)$ or $(0, 3)$.

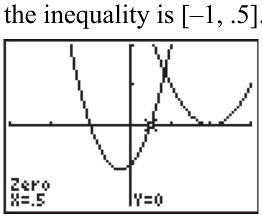
38. **a.** When $x > -4 \Rightarrow x+4 > 0$, let $x = 0$. $0+4 = 4 > 0$ is positive. Therefore $x+4$ is positive when $x > -4$.

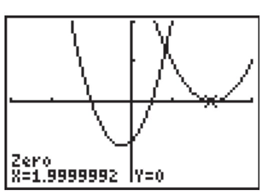
b. When $x < -4 \Rightarrow x+4 < 0$, $x+4$ is negative. Let $x = -5$. $-5+4 = -1 < 0$.

c. When $x > -4$, the quantity $x+4$ is positive, so you don't change the direction of the inequality. When $x < -4$, $x + 4$ is negative, so you must change the direction of the inequality sign.

d. Answers vary, but you must consider two separate cases ($x > -4$ and $x < -4$) and solve the inequality in each case.

39. To solve $\dfrac{2x^2+x-1}{x^2-4x+4} \le 0$, break the inequality into two inequalities $2x^2+x-1 \le 0$ and $x^2-4x+4 \le 0$. Graph the equations $y = 2x^2+x-1$ and $y = x^2-4x+4$. Enter these equations as y_1 and y_2, and use $-3 < x < 3$ and $-2 < y < 2$. On the CALC menu, use "zero" to find the x-values where the graphs cross the x-axis. These values for y_1 are $x = -1$ and $x = .5$. The graph of y_1 is below the x-axis to the right of -1 and to the left of .5. The graph of y_2 is never below the x-axis. The solution of the inequality is $[-1, .5]$.





40. To solve $\dfrac{x^3-3x^2+5x-29}{x^2-7} > 3$, rewrite the inequality with 0 on one side.

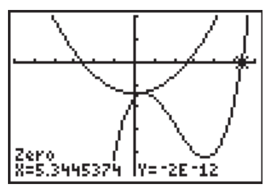
$$\frac{x^3-3x^2+5x-29}{x^2-7} - 3 > 0$$

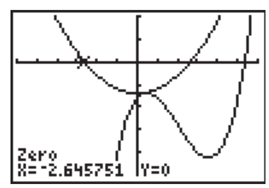
$$\frac{x^3-3x^2+5x-29-3\left(x^2-7\right)}{x^2-7} > 0$$

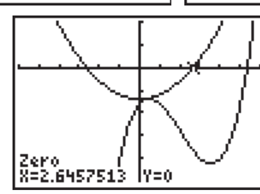
$$\frac{x^3-3x^2+5x-29-3x^2+21}{x^2-7} > 0$$

$$\frac{x^3-6x^2+5x-8}{x^2-7} > 0$$

Now break the inequality into two inequalities: $x^3-6x^2+5x-8 > 0$ and $x^2-7 > 0$. Graph the equations $y = x^3-6x^2+5x-8$ and $y = x^2-7$. Enter these equations as y_1 and y_2, and use $-6 < x < 6$ and $-25 < y < 10$. On the CALC menu, use "zero" or "root" to find the x-values where the graphs cross the x-axis. The value for y_1 is $x = 5.3445$. The graph of y_1 is above the x-axis to the right of 5.3445. The values for y_2 are $x = -2.6458$ and $x = 2.6458$. The graph of y_2 is above the x-axis to the left of -2.6458 and to the right of 2.6458. The solution of the inequality is $\left(-\sqrt{7}, \sqrt{7}\right)$ or $(5.3445, \infty)$.







41. $P = 2x^2 - 12x - 32$
The company makes a profit when
$2x^2 - 12x - 32 > 0$.
Solve the corresponding equation.

$$2x^2 - 12x - 32 = 0$$
$$2(x^2 - 6x - 16) = 0$$
$$(x+2)(x-8) = 0 \Rightarrow x = -2 \text{ or } x = 8$$

The test regions are $A(-\infty, -2)$, $B(-2, 8)$, and $C(8, \infty)$. Region A makes no sense in this context, so we ignore this. Test a number from regions B and C in the original inequality.
For region B, let $x = 0$.
$2(0)^2 - 12(0) - 32 = -32 < 0$
For region C, let $x = 10$.
$2(10)^2 - 12(10) - 32 = 48 > 0$
The numbers in region C satisfy the inequality. The company makes a profit when the amount spent on advertising in hundreds of thousands of dollars is in the interval $(8, \infty)$.

42. $P = 4t^2 - 30t + 14$
We want to find the values of t for which $P > 0$, that is, we must solve the inequality
$4t^2 - 30t + 14 > 0$.
Solve the corresponding equation.

$$4t^2 - 30t + 14 = 0$$
$$2(2t^2 - 15t + 7) = 0$$
$$(2t-1)(t-7) = 0 \Rightarrow t = \frac{1}{2} \text{ or } t = 7$$

We only consider positive values of t because t represents time (in months). The test regions are $A\left(0, \frac{1}{2}\right)$, $B\left(\frac{1}{2}, 7\right)$, and $C(7, \infty)$.
In region A, let $t = \frac{1}{4}$.:

$$4\left(\frac{1}{4}\right)^2 - 30\left(\frac{1}{4}\right) + 14 = \frac{27}{4} > 0.$$

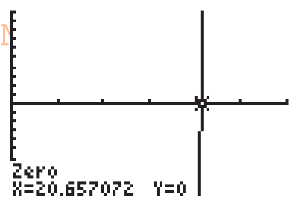
In region B, let $t = 3$:
$4(3)^2 - 30(3) + 14 = -40 < 0.$
In region C, let $t = 10$:
$4(10)^2 - 30(10) + 14 = 114 > 0.$.
The solution is $\left(0, \frac{1}{2}\right)$ or $(7, \infty)$.
The investor makes a profit between $t = 0$ and $t = \frac{1}{2}$ month and after 7 months.

43. $P = x^2 + 300x - 18{,}000$
The complex makes a profit when
$x^2 + 300x - 18{,}000 > 0$.
Solve the corresponding equation.

$$0 = x^2 + 300x - 18{,}000$$
$$x = \frac{-300 \pm \sqrt{(300)^2 - 4(1)(-18{,}000)}}{2(1)}$$
$$x \approx 51.25 \text{ or } x \approx -351.25$$

We only consider positive values of x because x represents the number of apartments rented.
The test regions are $A(0, 52)$ and $B(52, 200)$.
In region A, let $x = 1$:
$(1)^2 + 300(1) - 18{,}000 = -17{,}699 < 0$.
In region B, let $x = 100$:
$(100)^2 + 300(100) - 18{,}000 = 22{,}000 > 0$.
The complex makes a profit when the number of units rented is between 52 and 200, inclusive, or when x is in the interval [52, 200].

44. $x^2 + 5x - 530 > 0$
Use a graphing calculator to solve
$x^2 + 5x - 530 = 0$



[0, 30] by [–10, 10]

The graph lies above the x-axis for $x > 20.657$. Thus, the salesperson needs to make 21 pitches or more to earn a profit.

Chapter 2 Review Exercises

1. $y = x^2 - 2x - 5$

$(-2, 3)$:

$(-2)^2 - 2(-2) - 5 = 4 + 4 - 5 = 3$

$(0, -5)$

$(0)^2 - 2(0) - 5 = 0 - 0 - 5 = -5$

$(2, -3)$:

$(2)^2 - 2(2) - 5 = 4 - 4 - 5 = -5 \neq -3$

$(3, -2)$:

$(3)^2 - 2(3) - 5 = 9 - 6 - 5 = -2$

$(4, 3)$:

$(4)^2 - 2(4) - 5 = 16 - 8 - 5 = 3$

$(7, 2)$:

$(7)^2 - 2(7) - 5 = 49 - 14 - 5 = 30 \neq 2$

Solutions are (–2, 3), (0, –5), (3, –2), (4, 3).

2. $x - y = 5$

$(-2, 3): -2 - 3 = -5 \neq 5$

$(0, -5): 0 - (-5) = 0 + 5 = 5$

$(2, -3): 2 - (-3) = 2 + 3 = 5$

$(3, -2): 3 - (-2) = 3 + 2 = 5$

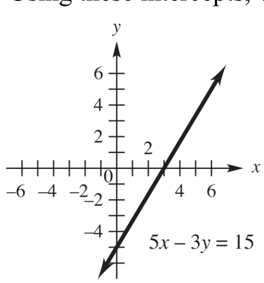
$(4, 3): 4 - 3 = 1 \neq 5$

$(7, 2): 7 - 2 = 5$

Solutions are (0, –5), (2, –3), (3, –2), (7, 2).

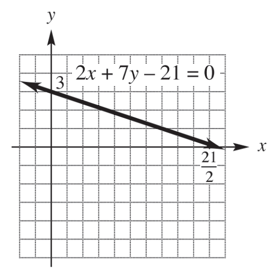
3. $5x - 3y = 15$

First, we find the y-intercept. If $x = 0$, $y = -5$, so the y-intercept is –5. Next we find the x-intercept. If $y = 0$, $x = 3$, so the x-intercept is 3. Using these intercepts, we graph the line.



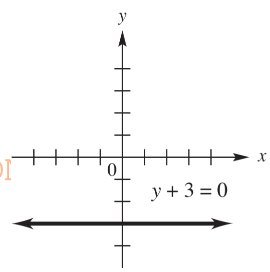
4. $2x + 7y - 21 = 0$

First we find the y-intercept. If $x = 0$, $y = 3$, so the y-intercept is 3. Next we find the x-intercept. If $y = 0$, $x = \frac{21}{2}$, so the x-intercept is $\frac{21}{2}$. Using these intercepts, we graph the line.



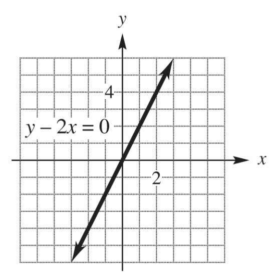
5. $y + 3 = 0$

The equation may be rewritten as $y = -3$. The graph of $y = -3$ is a horizontal line with y-intercept of –3.



6. $y - 2x = 0$

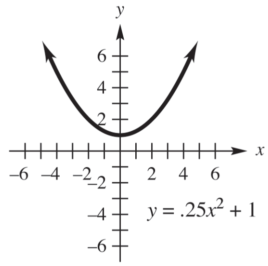
First, we find the y-intercept. If $x = 0$, $y = 0$, so the y-intercept is 0. Since the line passes through the origin, the x-intercept is also 0. We find another point on the line by arbitrarily choosing a value for x. Let $x = 2$. Then $y - 2(2) = 0$, or $y = 4$. The point with coordinates (2, 4) is on the line. Using this point and the origin, we graph the line.



7. $y = .25x^2 + 1$

First we find the y-intercept. If $x = 0$, $y = .25(0)^2 + 1 = 1$, so the y-intercept is 1. Next we find the x-intercepts. If $y = 0$, $0 = .25x^2 + 1 \Rightarrow .25x^2 = -1 \Rightarrow x = \sqrt{-4}$, not a real number. There are no x-intercepts.
Make a table of points and plot them.

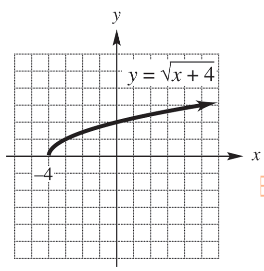
x	$.25x^2 + 1$
–4	5
–2	2
0	1
2	2
4	5



8. $y = \sqrt{x+4}$

Make a table of points and plot them.

x	$\sqrt{x+4}$
–4	0
–3	1
0	2
5	3



9. **a.** The temperature was over 55° from about 11:30 a.m. to about 7:30 p.m.

b. The temperature was below 40° from midnight until about 5 a.m., and after about 10:30 p.m.

10. At noon in Bratenahl the temperature was about 57°. The temperature in Greenville is 57° when the temperature in Bratenahl is 50°, or at about 10:30 a.m. and 8:30 p.m.

11. Answers vary. A possible answer is “rise over run”.

12. Through (–1, 3) and (2, 6)

$$\text{slope } = \frac{\Delta y}{\Delta x} = \frac{6-3}{2-(-1)} = \frac{3}{3} = 1$$

13. Through (4, –5) and (1, 4)

$$\text{slope } = \frac{-5-4}{4-1} = \frac{-9}{3} = -3$$

14. Through (8, –3) and the origin
The coordinates of the origin are (0, 0).

$$\text{slope} = \frac{-3-0}{8-0} = -\frac{3}{8}$$

15. Through (8, 2) and (0, 4)

$$\text{slope } = \frac{4-2}{0-8} = \frac{2}{-8} = -\frac{1}{4}$$

In exercises 16 and 17, we give the solution by rewriting the equation in slope-intercept form. Alternatively, the solution can be obtained by determining two points on the line and then using the definition of slope.

16. $3x + 5y = 25$
First we solve for y.

$$5y = -3x + 25 \Rightarrow y = -\frac{3}{5}x + 5$$

When the equation is written in slope-intercept form, the coefficient of x gives the slope. The slope is $-\frac{3}{5}$.

17. $6x - 2y = 7$
First we solve for y.

$$6x - 2y = 7 \Rightarrow 6x - 7 = 2y \Rightarrow 3x - \frac{7}{2} = y$$

The coefficient of x gives the slope, so the slope is 3.

18. $x - 2 = 0$
The graph of $x - 2 = 0$ is a vertical line. Therefore, the slope is undefined.

19. $y = -4$
The graph of $y = -4$ is a horizontal line. Therefore, the slope is 0.

20. Parallel to $3x + 8y = 0$
First, find the slope of the given line by solving for y.

$$8y = -3x \Rightarrow y = -\frac{3}{8}x$$

The slope is the coefficient of x, $-\frac{3}{8}$. A line parallel to this line has the same slope, so the slope of the parallel line is also $-\frac{3}{8}$.

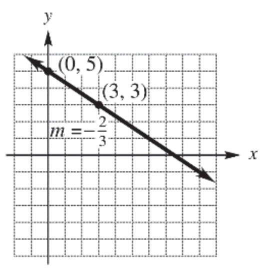
21. Perpendicular to $x = 3y$
First, find the slope of the given line by solving for y: $y = \frac{1}{3}x$

The slope of this line is the coefficient of x, $\frac{1}{3}$.

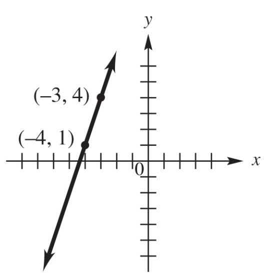
The slope of a line perpendicular to this line is the negative reciprocal of this slope, so the slope of the perpendicular line is –3.

22. Through (0, 5) with $m = -\frac{2}{3}$

Since $m = -\frac{2}{3} = \frac{-2}{3}$, we start at the point with coordinates (0, 5) and move 2 units down and 3 units to the right to obtain a second point on the line. Using these two points, we graph the line.



23. Through (–4, 1) with m = 3
Since $m = 3 = \frac{3}{1}$, we start at the point with coordinates (–4, 1) and move 3 units up and 1 unit to the right to obtain a second point on the line. Using these two points, we graph the line.



24. Answers vary. One example is:
You need two points; one point and the slope; the y-intercept and the slope.

25. Through (5, –1), slope $\frac{2}{3}$
Use the point slope form with $x_1 = 5,\ y_1 = -1$, and $m = \frac{2}{3}$.

$$y - y_1 = m(x - x_1)$$
$$y - (-1) = \frac{2}{3}(x - 5)$$
$$y + 1 = \frac{2}{3}x - \frac{10}{3}$$

Multiplying by 3 gives

$$3y + 3 = 2x - 10$$
$$3y = 2x - 13$$

26. Through (8, 0), $m = -\frac{1}{4}$

$$y - 0 = -\frac{1}{4}(x - 8)$$
$$4y = -1(x - 8)$$
$$4y = -x + 8$$

27. Through (5, –2) and (1, 3)

$$m = \frac{3 - (-2)}{1 - 5} = \frac{5}{-4} = -\frac{5}{4}$$

$$y - 3 = -\frac{5}{4}(x - 1)$$
$$4(y - 3) = -5(x - 1)$$
$$4y - 12 = -5x + 5$$
$$4y = -5x + 17$$

28. (2, –3) and (–3, 4)

$$m = \frac{-3 - 4}{2 - (-3)} = -\frac{7}{5}$$
$$y - (-3) = -\frac{7}{5}(x - 2)$$
$$5(y + 3) = -7(x - 2)$$
$$5y + 15 = -7x + 14$$
$$5y = -7x - 1$$

29. Undefined slope, through (–1, 4)
This is a vertical line. Its equation is $x = –1$.

30. Slope 0, (–2, 5)
This is a horizontal line. Its equation is $y = 5$.

31. x-intercept –3, y-intercept 5
Use the points (–3, 0) and (0, 5).

$$m = \frac{5-0}{0-(-3)} = \frac{5}{3}$$

$$y = \frac{5}{3}x + 5$$

$$3y = 3(\tfrac{5}{3}x + 5)$$

$$3y = 5x + 15$$

32. x-intercept 3, y-intercept 2.
Use the points $(3, 0)$ and $(0, 2)$.

$$m = \frac{2-0}{0-3} = -\frac{2}{3}$$

$$y = -\frac{2}{3}x + 2$$

$$3y = 3\left(-\frac{2}{3}x + 2\right)$$

$$3y = -2x + 6$$

$$2x + 3y = 6$$

The answer is (d).

33. a. Let (x_1, y_1) be (15, 1.2) and (x_2, y_2) be (20, 1.7). Find the slope.

$$m = \frac{1.7-1.2}{20-15} = \frac{.5}{5} = .1$$

$$y - 1.2 = .1(x - 15)$$

$$y - 1.2 = .1x - 1.5$$

$$y = .1x - .3$$

b. The slope is positive because the total owed on federal student loans is increasing.

c. The year 2025 corresponds to $x = 25$.

$$y = .1(25) - .3 = 2.2$$

If the linear trend continued, there will be \$2.2 trillion owed on federal student loans in 2025.

34. a. Let (x_1, y_1) be (60, 7.5) and (x_2, y_2) be (120, 36). Find the slope.

$$m = \frac{36-7.5}{120-60} = \frac{28.5}{60} = 0.475$$

$$y - 7.5 = .475(x - 60)$$

$$y - 7.5 = .475x - 28.5$$

$$y = .475x - 21$$

b. The slope is positive because the number of adults living alone is increasing.

c. The year 2025 corresponds to $x = 125$.

$$y = .475(125) - 21 = 38.375$$

If the linear trend continued, there will be 38.375 million adults living alone in 2025.

35. a. Let (x_1, y_1) be (5, 46326) and (x_2, y_2) be (15, 56516).
Find the slope.

$$m = \frac{56,516 - 46,326}{15-5} = \frac{10,190}{10} = 1019$$

$$y - 46,326 = 1019(x - 5)$$

$$y - 46,326 = 1019x - 5,095$$

$$y = 1019x + 41,231$$

b. Using a graphing calculator, the least squares regression line is $y = 1557x + 36,133$.

c. The year 2019 corresponds to $x = 19$. Using the two-point model, we have

$$y = 1019(19) + 41,231 = 60,592.$$

Using the regression model, we have

$$y = 1557(19) + 36,133 = 65,716.$$

The two-point model is off by \$8,111, while the regression model is off by \$2,987, therefore the regression model is a better estimate.

d. The year 2026 corresponds to $x = 26$.
Regression model:

$$y = 1557(26) + 36,133 \approx 76,615$$

Thus, the median household income in the year 2026 will be about \$76,615.

36. a. Let (x_1, y_1) be (13, 330) and (x_2, y_2) be (17, 430).
Find the slope.

$$m = \frac{430-330}{17-13} = \frac{100}{4} = 25$$

$$y - 330 = 25(x - 13)$$

$$y - 330 = 25x - 325$$

$$y = 25x + 5$$

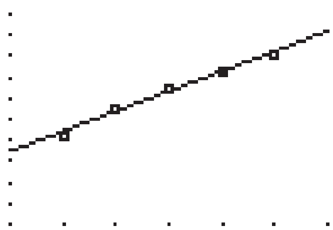
b. Using a graphing calculator, the least squares regression line is $y = 22x + 48$.

c. The year 2019 corresponds to $x = 19$. Using the two-point model, we have
$y = 25(19) + 5 \approx \$480$ billion.
Using the regression model, we have
$y = 22(19) + 48 \approx \$466$ billion.
The two-point model is off by \$20 billion, while the regression model is off by \$6 billion, therefore the least-squares approximation is a better estimate.

d. The year 2024 corresponds to $x = 24$.
Regression model:
$y = 22(24) + 48 \approx 576$
Thus, the total amount in charitable giving in the year 2024 will be about \$576 billion.

37. **a.** The least-squares regression line is $y = 95.2x + 2015.6$.

b.



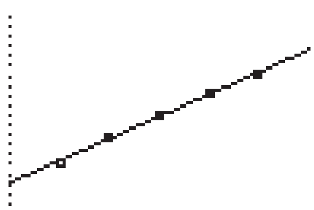
[14, 20] by [3000, 4000]

c. Yes; the line appears to fit.

d. The correlation coefficient is .995. This indicates that the line is a good fit.

38. **a.** The least-squares regression line is $y = 225x + 5165.2$.

b.



[14, 20] by [8000, 10000]

c. Yes; the line appears to fit.

d. The correlation coefficient is .999. This indicates that the line is a good fit.

39.
$$\begin{aligned} -6x+3 &< 2x \\ -6x+6x+3 &< 2x+6x \\ 3 &< 8x \\ \frac{3}{8} &< \frac{8x}{8} \\ \frac{3}{8} &< x \text{ or } x > \frac{3}{8} \end{aligned}$$

The solution is $\left(\frac{3}{8}, \infty\right)$.

40.
$$\begin{aligned} 12z &\ge 5z-7 \\ 12z-5z &\ge 5z-5z-7 \\ 7z &\ge -7 \\ \frac{7z}{7} &\ge \frac{-7}{7} \\ z &\ge -1 \end{aligned}$$

The solution is $[-1, \infty)$.

41.
$$\begin{aligned} 2(3-2m) &\ge 8m+3 \\ 6-4m &\ge 8m+3 \\ 6-4m-8m &\ge 8m-8m+3 \\ 6-12m &\ge 3 \\ 6-6-12m &\ge 3-6 \\ -12m &\ge -3 \\ \frac{-12m}{-12} &\le \frac{-3}{-12} \\ m &\le \frac{1}{4} \end{aligned}$$

The solution is $\left(-\infty, \frac{1}{4}\right]$.

42.
$$\begin{aligned} 6p-5 &> -(2p+3) \\ 6p-5 &> -2p-3 \\ 8p-5 &> -3 \\ 8p &> 2 \\ \frac{8p}{8} &> \frac{2}{8} \\ p &> \frac{1}{4} \end{aligned}$$

The solution is $\left(\frac{1}{4}, \infty\right)$.

43.
$$-3 \le 4x - 1 \le 7$$
$$-2 \le 4x \le 8$$
$$-\frac{1}{2} \le x \le 2$$
The solution is $\left[-\frac{1}{2}, 2\right]$.

44.
$$0 \le 3 - 2a \le 15$$
$$0 - 3 \le 3 - 3 - 2a \le 15 - 3$$
$$-3 \le -2a \le 12$$
$$\frac{-3}{-2} \ge \frac{-2a}{-2} \ge \frac{12}{-2}$$
$$\frac{3}{2} \ge a \ge -6$$
The solution is $\left[-6, \frac{3}{2}\right]$.

45. $|b| \le 8 \Rightarrow -8 \le b \le 8$
The solution is $[-8, 8]$.

46. $|a| > 7 \Rightarrow a < -7$ or $a > 7$
The solution is $(-\infty, -7)$ or $(7, \infty)$.

47. $|2x - 7| \ge 3$
$2x - 7 \le -3$ or $2x - 7 \ge 3$
$2x \le 4$ or $2x \ge 10$
$x \le 2$ or $x \ge 5$
The solution is $(-\infty, 2]$ or $[5, \infty)$.

48.
$$|4m + 9| \le 16$$
$$-16 \le 4m + 9 \le 16$$
$$-25 \le 4m \le 7$$
$$-\frac{25}{4} \le m \le \frac{7}{4}$$
The solution is $\left[-\frac{25}{4}, \frac{7}{4}\right]$.

49.
$$|5k + 2| - 3 \le 4$$
$$|5k + 2| \le 7$$
$$-7 \le 5k + 2 \le 7$$
$$-9 \le 5k \le 5$$
$$-\frac{9}{5} \le k \le 1$$
The solution is $\left[-\frac{9}{5}, 1\right]$.

50. $|3z - 5| + 2 \ge 10$
$|3x - 5| \ge 8$
$3z - 5 \le -8$ or $3z - 5 \ge 8$
$3z \le -3$ or $3z \ge 13$
$z \le -1$ or $z \ge \frac{13}{3}$
The solution is $(-\infty, -1]$ or $\left[\frac{13}{3}, \infty\right)$.

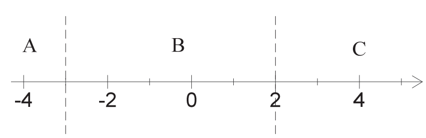
51. The inequalities that represent the weight of pumpkin that he will not use are $x < 2$ or $x > 10$. This is equivalent to the following inequalities:
$x - 6 < 2 - 6$ or $x - 6 > 10 - 6$
$x - 6 < -4$ or $x - 6 > 4$
$|x - 6| > 4$
Choose answer option (d).

52. Let x = the price of the snow thrower
$|x - 800| \le 55$

53.
$$45 + 8x \le 65$$
$$8x \le 20$$
$$x \le 2.5$$
Due to the fact that cell phone companies sell data by the whole gigabyte, the first plan is cheaper if you use less than or equal to 2 gigabytes.

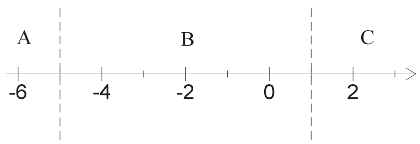
54. Let m = number of miles driven. The rate for the second rental company is $95 + .2m$. We want to determine when the second company is cheaper than the first.
$125 > 95 + .2m \Rightarrow 30 > .2m \Rightarrow 150 > m$
The second company is cheaper than the first company when the number of miles driven is less than 150.

55. $r^2 + r - 6 < 0$
Solve the corresponding equation.
$r^2 + r - 6 = 0 \Rightarrow (r + 3)(r - 2) = 0 \Rightarrow$
$r = -3$ or $r = 2$



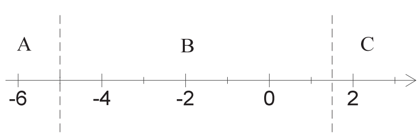
For region A, test –4:
$(-4)^2 + (-4) - 6 = 6 > 0$.
For region B, test 0:
$0^2 + 0 - 6 = -6 < 0$.
For region C, test 3:
$3^2 + 3 - 6 = 6 > 0$.
The solution is (–3, 2).

56. $y^2 + 4y - 5 \ge 0$
Solve the corresponding equation.
$y^2 + 4y - 5 = 0 \Rightarrow (y+5)(y-1) = 0$
$y = -5$ or $y = 1$



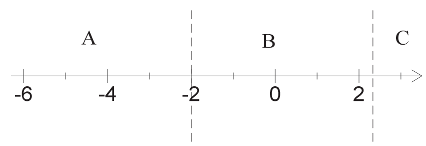
For region A, test –6:
$(-6)^2 + 4(-6) - 5 = 7 > 0$.
For region B, test 0:
$0^2 + 4(0) - 5 = -5 < 0$.
For region C, test 2:
$2^2 + 4(2) - 5 = 7 > 0$.
Both endpoints are included because the inequality symbol is "≥." The solution is (–∞, –5] or [1, ∞).

57. $2z^2 + 7z \ge 15$
Solve the corresponding equation.
$$2z^2 + 7z = 15$$
$$2z^2 + 7z - 15 = 0$$
$$(2z-3)(z+5) = 0$$
$$z = \frac{3}{2} \text{ or } z = -5$$
These numbers are solutions of the inequality because the inequality symbol is "≥."



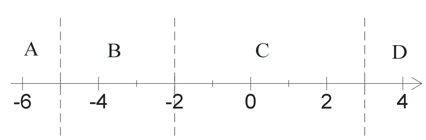
For region A, test –6:
$2(-6)^2 + 7(-6) = 30 > 15$.
For region B, test 0:
$2 \cdot 0^2 + 7 \cdot 0 = 0 < 15$.
For region C, test 2:
$2 \cdot 2^2 + 7 \cdot 2 = 22 > 15$.
The solution is (–∞, –5] or $\left[\frac{3}{2}, \infty\right)$.

58. $3k^2 \le k + 14$
Solve the corresponding equation.
$$3k^2 = k + 14$$
$$3k^2 - k - 14 = 0$$
$$(3k-7)(k+2) = 0$$
$$k = \frac{7}{3} \text{ or } k = -2$$



For region A, test –3:
$3(-3)^2 = 27, \ -3 + 14 = 11 \Rightarrow 27 > 11$
For region B, test 0:
$3(0)^2 = 0, \ 0 + 14 = 14 \Rightarrow 0 < 14$
For region C, test 3:
$3(3)^2 = 27, \ 3 + 14 = 17 \Rightarrow 27 > 17$
The given inequality is true in region B and at both endpoints, so the solution is $\left[-2, \frac{7}{3}\right]$.

59. $(x-3)\left(x^2 + 7x + 10\right) \le 0$
Solve the corresponding equation.
$$(x-3)\left(x^2 + 7x + 10\right) = 0$$
$$(x-3)(x+2)(x+5) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x + 2 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 3 \quad \text{or} \quad x = -2 \quad \text{or} \quad x = -5$$
Note that –5, –2, and 3 are solutions of the original inequality.



In region A, let $x = -6$:

$(-6-3)\left((-6)^2+7(-6)+10\right)$

$= -9(36 - 42 + 10) = -9(4) = -36 < 0$

In region B, let $x = -3$:

$(-3-3)\left((-3)^2+7(-3)+10\right)$

$= -6(9 - 21 + 10) = -6(-2) = 12 > 0.$

In region C, let $x = 0$:

$(0-3)\left(0^2+7(0)+10\right) = -3(10) = -30 < 0$.

In region D, let $x = 4$:

$(4-3)\left(4^2+7(4)+10\right)$

$= 1(16+28+10) = 54 > 0$.

The solution is $(-\infty, -5]$ or $[-2, 3]$.

60. $(x+4)\left(x^2-1\right) \geq 0$

Solve the corresponding equation.

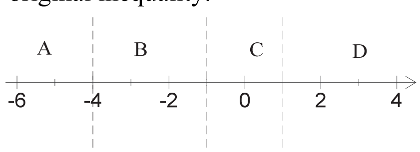
$$(x+4)\left(x^2-1\right) = 0$$

$(x+4)(x+1)(x-1) = 0$

$x+4=0$ or $x+1=0$ or $x-1=0$

$x=-4$ or $x=-1$ or $x=1$

Note that –4, –1, and 1 are solutions of the original inequality.



In region A, let $x = -5$:

$(-5+4)\left((-5)^2-1\right) = -1(24) = -24 \leq 0$.

In region B, let $x = -2$:

$(-2+4)\left((-2)^2-1\right) = 2(3) = 6 > 0$.

In region C, let $x = 0$:

$(0+4)\left(0^2-1\right) = 4(-1) = -4 < 0$.

In region D, let $x = 2$:

$(2+4)\left(2^2-1\right) = 6(3) = 18 > 0$.

The solution is $[-4, -1]$ or $[1, \infty)$.

61. $\dfrac{m+2}{m} \leq 0$

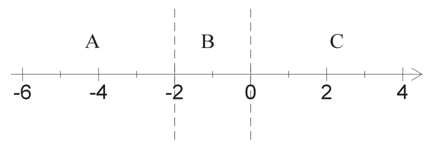
Solve the corresponding equation $\dfrac{m+2}{m} = 0$.

The quotient changes sign when

$m+2=0$ or $m=0$

$m=-2$ or $m=0$

–2 is a solution of the inequality, but the inequality is undefined when $m = 0$, so the endpoint 0 must be excluded.



For region A, test –3:

$\dfrac{-3+2}{-3} = \dfrac{1}{3} > 0$.

For region B, test –1:

$\dfrac{-1+2}{-1} = -1 < 0$.

For region C, test 1:

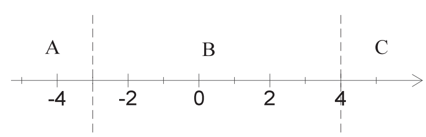
$\dfrac{1+2}{1} = 3 > 0$.

The solution is $[-2, 0)$.

62. $\dfrac{q-4}{q+3} > 0$

Solve the corresponding equation $\dfrac{q-4}{q+3} = 0$.

The numerator is 0 when $q = 4$. The denominator is 0 when $q = -3$.



For region A, test –4:

$\dfrac{-4-4}{-4+3} = \dfrac{-8}{-1} = 8 > 0$.

For region B, test 0:

$\dfrac{0-4}{0+3} = -\dfrac{4}{3} < 0$.

For region C, test 5:

$\dfrac{5-4}{5+3} = \dfrac{1}{8} > 0$.

The inequality is true in regions A and C, and both endpoints are excluded. Therefore, the solution is $(-\infty, -3)$ or $(4, \infty)$.

63. $\frac{5}{p+1} > 2$

Solve the corresponding equation.

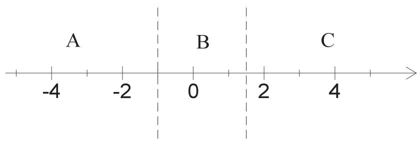
$$\frac{5}{p+1} = 2$$

$$\frac{5}{p+1} - 2 = 0$$

$$\frac{5 - 2(p+1)}{p+1} = 0$$

$$\frac{3 - 2p}{p+1} = 0$$

The numerator is 0 when $p = \frac{3}{2}$. The denominator is 0 when $p = -1$.
Neither of these numbers is a solution of the inequality.



In region A, test –2:

$\frac{5}{-2+1} = -5 < 2$.

In region B, test 0:

$\frac{5}{0+1} = 5 > 2$.

In region C, test 2:

$\frac{5}{2+1} = \frac{5}{3} < 2$.

The solution is $\left(-1, \frac{3}{2}\right)$.

64. $\frac{6}{a-2} \le -3$

Solve the corresponding equation.

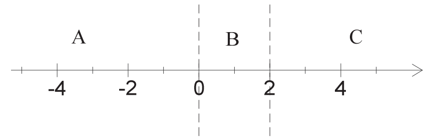
$$\frac{6}{a-2} = -3$$

$$\frac{6}{a-2} + 3 = 0$$

$$\frac{6 + 3(a-2)}{a-2} = 0$$

$$\frac{3a}{a-2} = 0$$

The numerator is 0 when $a = 0$. The denominator is 0 when $a = 2$.



For region A, test –1:

$\frac{6}{-1-2} = -2 \ge -3$.

For region B, test 1:

$\frac{6}{1-2} = -6 \le -3$.

For region C, test 3:

$\frac{6}{3-2} = 6 \ge -3$.

The given inequality is true in region B. The endpoint 0 is included because the inequality symbol is "≤." However, the endpoint 2 must be excluded because it makes the denominator 0. The solution is [0, 2).

65. $\frac{2}{r+5} \le \frac{3}{r-2}$

Write the corresponding equation and then set one side equal to zero.

$$\frac{2}{r+5} = \frac{3}{r-2}$$

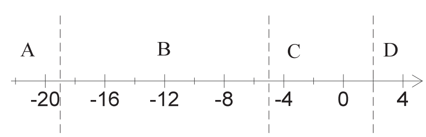
$$\frac{2}{r+5} - \frac{3}{r-2} = 0$$

$$\frac{2(r-2) - 3(r+5)}{(r+5)(r-2)} = 0$$

$$\frac{2r - 4 - 3r - 15}{(r+5)(r-2)} = 0$$

$$\frac{-r - 19}{(r+5)(r-2)} = 0$$

The numerator is 0 when $r = -19$. The denominator is 0 when $r = -5$ or $r = 2$.
–19 is a solution of the inequality, but the inequality is undefined when r = –5 or r = 2.



For region A, test –20:

$\frac{2}{-20+5} = -\frac{2}{15} \approx -.13$ and

$\frac{3}{-20-2} = -\frac{3}{22} \approx -.14$

Since –.13 > –.14, –20 is not a solution of the inequality.
For region B, test –6:

$$\frac{2}{-6+5} = -2 \text{ and } \frac{3}{-6-2} = -\frac{3}{8}.$$

Since $-2 < -\frac{3}{8}$, –6 is a solution.

For region C, test 0: $\frac{2}{0+5} = \frac{2}{5}$ and $\frac{3}{0-2} = -\frac{3}{2}$.

Since $\frac{2}{5} > -\frac{3}{2}$, 0 is not a solution.

For region D, test 3: $\frac{2}{3+5} = \frac{1}{4}$ and $\frac{3}{3-2} = 3$.

Since $\frac{1}{4} < 3$, 3 is a solution. The solution is [–19, –5) or (2, ∞).

66. $\frac{1}{z-1} > \frac{2}{z+1}$

Write the corresponding equation and then set one side equal to zero.

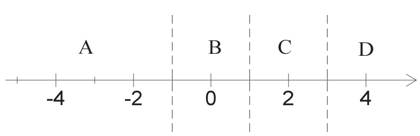
$$\frac{1}{z-1} = \frac{2}{z+1}$$

$$\frac{1}{z-1} - \frac{2}{z+1} = 0$$

$$\frac{(z+1)-2(z-1)}{(z-1)(z+1)} = 0$$

$$\frac{3-z}{(z-1)(z+1)} = 0$$

The numerator is 0 when $z = 3$. The denominator is 0 when $z = 1$ and when $z = -1$. These three numbers, –1, 1, and 3, separate the number line into four regions.



For region A, test –3.

$$\frac{1}{-3-1} > \frac{2}{-3+1} \Rightarrow -\frac{1}{4} > -1, \text{ which is true.}$$

For region B, test 0.

$$\frac{1}{0-1} > \frac{2}{0+1} \Rightarrow -1 > 2, \text{ which is false.}$$

For region C, test 2.

$$\frac{1}{2-1} > \frac{2}{2+1} \Rightarrow 1 > \frac{2}{3}, \text{ which is true.}$$

For region D, test 4.

$$\frac{1}{4-1} > \frac{2}{4+1} \Rightarrow \frac{1}{3} > \frac{2}{5}, \text{ which is false.}$$

Thus, the solution is (–∞, –1) or (1, 3).

Case 2 Using Extrapolation for Prediction

1.

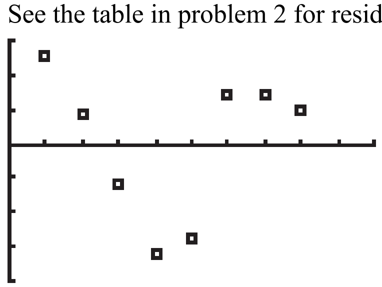
```
LinReg
 y=ax+b
 a=120
 b=-130
 r²=.8836937463
 r=.9400498638
```

This verifies the regression equation.

2.

Quarter	Table value	Predicted value	Residual
1	120	–10	130
2	150	110	40
3	170	230	–60
4	190	350	–160
5	330	470	–140
6	660	590	70
7	780	710	70
8	880	830	50

3. See the table in problem 2 for residual values.



4. No; because the residuals have a nonlinear U-shape.

5. Determine the least squares regression line.

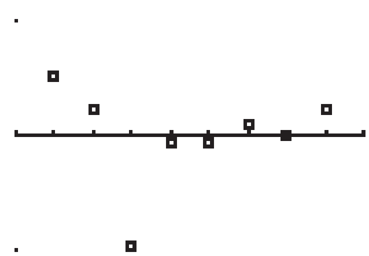
```
LinReg
 y=ax+b
 a=.639047619
 b=15.3472619
 r²=.9288467745
 r=.963766971
```

The model is verified.

6. The year 2020 corresponds to $x = 20$.
$y = .639(20) + 15.35 \approx 28.13.$
According to the model, the hourly wage in 2020 was about \$28.13, about \$0.43 too low.

7.

Year ($x = 12$ is 2012)	Table value	Predicted value	Residual
12	23.55	23.02	.53
13	23.87	23.66	.21
14	23.34	24.30	–.96
15	24.87	24.94	–.07
16	25.50	25.57	–.07
17	26.31	26.21	.10
18	26.86	26.85	.01
19	27.72	27.49	.23



8. You'll get 0 slope and 0 intercept, because the residual represents the vertical distance from the data point to the regression line. Since r is very close to 1, the data points lie very close to the regression line.

```
LinReg
 y=ax+b
 a=9.5238095E-4
 b=-.0172619048
 r²=2.887425E-5
 r=.0053734763
```

Chapter 3 Functions and Graphs

Section 3.1 Functions

1.

x	3	2	1	0	–1	–2	–3
y	9	4	1	0	1	4	9

This rule defines y as a function of x because each value of x determines one and only one value of y.

2.

x	10	20	30	40	50	60
y	–1	–1	0	0	1	1

This rule defines y as a function of x because each value of x determines one and only one value of y.

3.

x	9	4	1	0	1	4	9
y	3	2	1	0	–1	–2	–3

The rule does not define y as a function of x because some values of x determine two values for y. For example, when $x = 9$, $y = 3$ and $y = -3$.

4.

x	–1	–1	0	0	1	1
y	10	20	30	40	50	60

The rule does not define y as a function of x because some values of x determine two values for y. For example, when $x = 1$, $y = 50$ and $y = 60$.

5. The rule $y = x^3$ defines y as a function of x because each value of x determines one and only one value of y.

6. The rule $y = 21x^6$ defines y as a function of x because each value of x determines one and only one value of y.

7. The rule of $y = \sqrt{x-1}$ defines y as a function of x because each value of x determines one and only one value of y (or none).

8. The rule of $y = \sqrt{17-x}$ defines y as a function of x because each value of x determines one and only one value of y (or none).

9. The rule $x = |y+2|$ does not define y as a function of x because some values of x determine two values for y. For example, if $x = 4,\ 4 = |y+2| \Rightarrow$

$$\begin{aligned} y+2 &= 4 \quad \text{or} \quad y+2 = -4 \\ y &= 2 \quad \text{or} \quad\quad\ \ y = -6 \end{aligned}$$

10. The rule $x = 18 - 2y^2$ does not define y as a function of x because some values of x determine two values for y. For example, when $x = 0$, $y = 3$ and $y = -3$.

11. The rule $x = y^2 + 3$ does not define y as a function of x because some values of x determine two values for y. For example, when $x = 7$, $y = 2$ and $y = -2$.

12. The rule $x = |5-y|$ does not define y as a function of x because some values of x determine two values for y. For example, if $x = 4,\ 4 = |5-y| \Rightarrow$

$$\begin{aligned} 5-y &= 4 \quad \text{or} \quad 5-y = -4 \\ y &= 1 \quad \text{or} \quad\quad\ \ y = 9 \end{aligned}$$

13. The rule $y = \dfrac{-1}{x-1}$ defines y as a function of x because each value of x determines one and only one value of y (or none).

14. The rule $y = \dfrac{4}{2x+3}$ defines y as a function of x because each value of x determines one and only one value of y (or none).

15. $f(x) = 4x - 1$
The domain of f is all real numbers since x may take on any real-number value. Therefore, the domain is $(-\infty, \infty)$.

16. $f(x) = 2x + 7$
The domain of f is all real numbers since x may take on any real-number value. Therefore, the domain is $(-\infty, \infty)$.

17. $f(x) = x^4 - 1$
The domain of *f* is all real numbers since *x* may take on any real-number value. Therefore, the domain is $(-\infty, \infty)$.

18. $f(x) = (2x + 5)^2$
The domain of *f* is all real numbers since *x* may take any real-number value. Therefore, the domain is $(-\infty, \infty)$.

19. $f(x) = \sqrt{5 - x}$
In order for $\sqrt{5 - x}$ to be a real number, $5 - x \geq 0$ or $x \leq 5$. The domain is all real numbers less than or equal to 5, or $(-\infty, 5]$.

20. $f(x) = \sqrt{16 - x}$
In order to have $\sqrt{16 - x}$ be a real number, we must have $x \leq 16$. Thus, the domain is all nonpositive real numbers, or $(-\infty, 16]$.

21. $f(x) = \sqrt{-x} + 3$
In order to have $\sqrt{-x}$ be a real number, we must have $-x \geq 0$ or $x \leq 0$. Thus, the domain is all nonpositive real numbers, or $(-\infty, 0]$.

22. $f(x) = 13 - \sqrt{-x}$
In order to have $\sqrt{-x}$ be a real number, we must have $-x \geq 0$ or $x \leq 0$. Thus, the domain is all nonpositive real numbers, or $(-\infty, 0]$.

23. $g(x) = \dfrac{1}{x - 2}$
Since the denominator cannot be zero, $x \neq 2$. Thus, the domain is all real numbers except 2, which in interval notation is written $(-\infty, 2)$ or $(2, \infty)$.

24. $g(x) = \dfrac{9}{x + 9}$
Since the denominator cannot be zero, $x \neq -9$. Thus, the domain is all real numbers except -9, which in interval notation is written $(-\infty, -9)$ or $(-9, \infty)$.

25. $g(x) = \dfrac{x - 1}{x^2 + 7x + 10}$
Since the denominator cannot be zero, $x^2 + 7x + 10 \neq 0$. Solve $x^2 + 7x + 10 = 0$ and exclude the solutions from the domain.
$x^2 + 7x + 10 = 0$
$(x + 2)(x + 5) = 0$
$x + 2 = 0$ or $x + 5 = 0$
$x = -2$ or $x = -5$
Thus, the domain is all real numbers except -5 and -2, which in interval notation is written $(-\infty, -5)$ or $(-5, -2)$ or $(-2, \infty)$.

26. $g(x) = \dfrac{x}{x^2 + x - 2}$
Since the denominator cannot be zero, $x^2 + x - 2 \neq 0$. Solve $x^2 + x - 2 = 0$ and exclude the solutions from the domain.
$x^2 + x - 2 = 0$
$(x - 1)(x + 2) = 0$
$x - 1 = 0$ or $x + 2 = 0$
$x = 1$ or $x = -2$
Thus, the domain is all real numbers except -2 and 1, which in interval notation is written $(-\infty, -2)$ or $(-2, 1)$ or $(1, \infty)$.

27. $g(x) = \dfrac{x + 1}{x^2 - 25}$
Since the denominator cannot be zero, $x^2 - 25 \neq 0$. Solve $x^2 - 25 = 0$ and exclude the solutions from the domain.
$x^2 - 25 = 0$
$(x - 5)(x + 5) = 0$
$x - 5 = 0$ or $x + 5 = 0$
$x = 5$ or $x = -5$
Thus, the domain is all real numbers except -5 and 5, which in interval notation is written $(-\infty, -5)$ or $(-5, 5)$ or $(5, \infty)$.

28. $g(x) = \dfrac{x^2 + 4}{x^2 - 4}$

Solve $x^2 - 4 = 0$ and exclude the solutions from the domain because the solutions make the denominator equal to 0.

$x^2 - 4 = 0$
$(x-2)(x+2) = 0$
$x - 2 = 0$ or $x + 2 = 0$
$x = 2$ or $x = -2$

The domain of g is all real numbers except $x = \pm 2$ since x cannot take on $x = \pm 2$. Therefore, the domain is $(-\infty, -2)$ or $(-2, 2)$ or $(2, \infty)$.

29. $g(x) = \dfrac{x+1}{x^2 + 25}$

$x^2 + 25$ is always positive when x is a real number. The domain of g is all real numbers since x can take on any real-number value. Therefore, the domain is $(-\infty, \infty)$.

30. $g(x) = \dfrac{x^2 - 1}{x^2 + 1}$

$x^2 + 1$ is always positive when x is a real number. The domain of g is all real numbers since x can take on any real-number value. Therefore, the domain is $(-\infty, \infty)$.

31. $h(x) = \dfrac{\sqrt{x+4}}{x^2 + x - 12}$

Solve $x^2 + x - 12 = 0$ and exclude the solutions from the domain since x cannot take on these numbers.

$x^2 + x - 12 = 0$
$(x-3)(x+4) = 0$
$x - 3 = 0$ or $x + 4 = 0$
$x = 3$ or $x = -4$

For $\sqrt{x+4}$, $x + 4$ must be positive or 0. That is $x + 4 \geq 0$ or $x \geq -4$. Therefore, the domain is all real numbers x such that $x \neq 3$ and $x \geq -4$.

32. $h(x) = \dfrac{\sqrt{x+1}}{x^2 - x - 6}$

Solve $x^2 - x - 6 = 0$ and exclude the solutions from the domain since x cannot take on these numbers.

$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x - 3 = 0$ or $x + 2 = 0$
$x = 3$ or $x = -2$

For $\sqrt{x+1}$, $x + 1$ must be positive or 0. That is $x + 1 \geq 0$ or $x \geq -1$. Therefore, the domain is all real numbers x such that $x \neq 3$ and $x \geq -1$.

33. $f(x) = 8$

For any value of x, the value of $f(x)$ will always be 8. (This is a constant function).

a. $f(4) = 8$

b. $f(-3) = 8$

c. $f(2.7) = 8$

d. $f(-4.9) = 8$

34. $f(x) = 0$

For any value of x, the value of $f(x)$ will always be 0. (This is a constant function).

a. $f(4) = 0$

b. $f(-3) = 0$

c. $f(2.7) = 0$

d. $f(-4.9) = 0$

35. $f(x) = 2x^2 + 4x$

a. $$\begin{aligned} f(4) &= 2(4^2) + 4(4) \\ &= 2(16) + 16 \\ &= 32 + 16 = 48 \end{aligned}$$

b. $$\begin{aligned} f(-3) &= 2(-3)^2 + 4(-3) \\ &= 2(9) + (-12) \\ &= 18 - 12 = 6 \end{aligned}$$

c. $$\begin{aligned} f(2.7) &= 2(2.7)^2 + 4(2.7) \\ &= 2(7.29) + 10.8 \\ &= 14.58 + 10.8 = 25.38 \end{aligned}$$

d. $f(-4.9) = 2(-4.9)^2 + 4(-4.9)$
$= 2(24.01) + (-19.6)$
$= 48.02 - 19.6 = 28.42$

36. $f(x) = x^2 - 2x$

a. $f(4) = (4)^2 - 2(4) = 16 - 8 = 8$

b. $f(-3) = (-3)^2 - 2(-3) = 9 + 6 = 15$

c. $f(2.7) = (2.7)^2 - 2(2.7) = 7.29 - 5.4 = 1.89$

d. $f(-4.9) = (-4.9)^2 - 2(-4.9)$
$= 24.01 + 9.8$
$= 33.81$

37. $f(x) = \sqrt{5 - x}$

a. $f(4) = \sqrt{5-4} = \sqrt{1} = 1$

b. $f(-3) = \sqrt{5-(-3)} = \sqrt{8}$

c. $f(2.7) = \sqrt{5-2.7} = \sqrt{2.3}$

d. $f(-4.9) = \sqrt{5-(-4.9)} = \sqrt{9.9}$

38. $f(x) = \sqrt{x+3}$

a. $f(4) = \sqrt{4+3} = \sqrt{7}$

b. $f(-3) = \sqrt{-3+3} = \sqrt{0} = 0$

c. $f(2.7) = \sqrt{2.7+3} = \sqrt{5.7}$

d. $f(-4.9) = \sqrt{-4.9+3} = \sqrt{-1.9}$ not defined

39. $f(x) = |x^2 - 6x - 4|$

a. $f(4) = |4^2 - 6(4) - 4|$
$f(4) = |16 - 24 - 4| = |-12| = 12$

b. $f(-3) = |(-3)^2 - 6(-3) - 4|$
$= |9 + 18 - 4| = 23$

c. $f(2.7) = |(2.7)^2 - 6(2.7) - 4|$
$= |7.29 - 16.2 - 4| = 12.91$

d. $f(-4.9) = |(-4.9)^2 - 6(-4.9) - 4|$
$= |24.01 + 29.4 - 4|$
$= |49.41| = 49.41$

40. $f(x) = |x^3 - x^2 + x - 1|$

a. $f(4) = |4^3 - 4^2 + 4 - 1|$
$= |64 - 16 + 4 - 1| = |51| = 51$

b. $f(-3) = |(-3)^3 - (-3)^2 + (-3) - 1|$
$= |-27 - 9 - 3 - 1| = |-40| = 40$

c. $f(2.7) = |2.7^3 - 2.7^2 + 2.7 - 1|$
$= |19.683 - 7.29 + 2.7 - 1|$
$= |14.093| = 14.093$

d. $f(-4.9) = |(-4.9)^3 - (-4.9)^2 + (-4.9) - 1|$
$= |-117.649 - 24.01 - 4.9 - 1|$
$= |-147.559| = 147.559$

41. $f(x) = \dfrac{\sqrt{x-1}}{x^2 - 1}$

a. $f(4) = \dfrac{\sqrt{4-1}}{4^2 - 1} = \dfrac{\sqrt{3}}{15}$

b. $f(-3) = \dfrac{\sqrt{-3-1}}{(-3)^2 - 1} = \dfrac{\sqrt{-4}}{8}$ not defined

c. $f(2.7) = \dfrac{\sqrt{2.7-1}}{2.7^2 - 1} = \dfrac{\sqrt{1.7}}{6.29}$

d. $f(-4.9) = \dfrac{\sqrt{-4.9-1}}{(-4.9)^2 - 1}$
$= \dfrac{\sqrt{-5.9}}{23.01}$ not defined

42. $f(x) = \sqrt{-x} + \dfrac{2}{x+1}$

a. $f(4) = \sqrt{-4} + \dfrac{2}{4+1}$, not defined

b. $f(-3) = \sqrt{-(-3)} + \dfrac{2}{(-3)+1}$
$= \sqrt{3} + \dfrac{2}{-2} = \sqrt{3} - 1$

c. $f(2.7) = \sqrt{-2.7} + \dfrac{2}{2.7+1}$, not defined

d. $f(-4.9) = \sqrt{-(-4.9)} + \dfrac{2}{(-4.9)+1}$
$= \sqrt{4.9} - \dfrac{2}{3.9}$

43. $f(x) = \begin{cases} -2x+4 & \text{if } x \le 1 \\ 3 & \text{if } 1 < x < 4 \\ x+1 & \text{if } x \ge 4 \end{cases}$

a. $f(4) = 4 + 1 = 5$

b. $f(-3) = -2(-3) + 4 = 10$

c. $f(2.7) = 3$

d. $f(-4.9) = -2(-4.9) + 4 = 13.8$

44. $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 5x-7 & \text{if } x \ge 2 \end{cases}$

a. $f(4) = 5(4) - 7 = 13$

b. $f(-3) = (-3)^2 = 9$

c. $f(2.7) = 5(2.7) - 7 = 6.5$

d. $f(-4.9) = (-4.9)^2 = 24.01$

45. $f(x) = 6 - x$

a. $f(p) = 6 - p$

b. $f(-r) = 6 - (-r)$
$= 6 + r$

c. $f(m+3) = 6 - (m+3)$
$= 6 - m - 3$
$= 3 - m$

46. $f(x) = 3x + 5$

a. $f(p) = 3p + 5$

b. $f(-r) = 3(-r) + 5 = -3r + 5$

c. $f(m+3) = 3(m+3) + 5$
$= 3m + 9 + 5 = 3m + 14$

47. $f(x) = \sqrt{4 - x}$

a. $f(p) = \sqrt{4 - p}\ \ (p \le 4)$

b. $f(-r) = \sqrt{4 - (-r)}$
$= \sqrt{4 + r}\ \ (r \ge -4)$

c. $f(m+3) = \sqrt{4 - (m+3)}$
$= \sqrt{4 - m - 3}$
$= \sqrt{1 - m}\ \ (m \le 1)$

48. $f(x) = \sqrt{-2x}$

a. $f(p) = \sqrt{-2p}\ \ (p \le 0)$

b. $f(-r) = \sqrt{-2(-r)} = \sqrt{2r}\ \ (r \ge 0)$

c. $f(m+3) = \sqrt{-2(m+3)}$
$= \sqrt{-2m - 6}\ \ (m \le -3)$

49. $f(x) = x^2 - 1$

a. $f(p) = p^2 - 1$

b. $f(-r) = (-r)^2 - 1 = r^2 - 1$

c. $f(m+3) = (m+3)^2 - 1$
$= m^2 + 6m + 9 - 1$
$= m^2 + 6m + 8$

50. $f(x) = 1 - x^2$

a. $f(p) = 1 - p^2$

b. $f(-r) = 1 - (-r)^2 = 1 - (r^2) = 1 - r^2$

c. $f(m+3) = 1 - (m+3)^2$
$= 1 - \left(m^2 + 6m + 9\right)$
$= -m^2 - 6m - 8$

51. $f(x)=\dfrac{3}{x-1}$

a. $f(p)=\dfrac{3}{p-1}\ (p\neq 1)$

b. $f(-r)=\dfrac{3}{-r-1}\ (r\neq -1)$

c. $f(m+3)=\dfrac{3}{(m+3)-1}=\dfrac{3}{m+2}\ (m\neq -2)$

52. $f(x)=\dfrac{-1}{5+x}$

a. $f(p)=\dfrac{-1}{5+p}\ (p\neq -5)$

b. $f(-r)=\dfrac{-1}{5+(-r)}=\dfrac{-1}{5-r}\ (r\neq 5)$

c. $f(m+3)=\dfrac{-1}{5+(m+3)}=\dfrac{-1}{8+m}\ (m\neq -8)$

53. $f(x)=2x-4$

$$\frac{f(x+h)-f(x)}{h}=\frac{[2(x+h)-4]-(2x-4)}{h}=\frac{2x+2h-4-2x+4}{h}=\frac{2h}{h}=2$$

54. $f(x)=2+4x$

$$\frac{f(x+h)-f(x)}{h}=\frac{2+4(x+h)-(2+4x)}{h}=\frac{2+4x+4h-2-4x}{h}=\frac{4h}{h}=4$$

55. $f(x)=x^2+1$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2+1-\left(x^2+1\right)}{h}=\frac{x^2+2hx+h^2+1-x^2-1}{h}=\frac{2hx+h^2}{h}=2x+h$$

56. $f(x)=x^2-x$

$$\frac{f(x+h)-f(x)}{h}=\frac{(x+h)^2-(x+h)-\left(x^2-x\right)}{h}=\frac{x^2+2hx+h^2-x-h-x^2+x}{h}=\frac{2hx+h^2-h}{h}=2x+h-1$$

57. $f(x)=707x+7700$

a. $f(10)=707(10)+7700=14,770$
In 2010, the GDP was about \$14,770 billion.

b. $f(18)=707(18)+7700=20,426$
In 2015, the GDP was about \$20,426 billion.

c. $f(19)=707(19)+7700=21,133$
In 2019, the GDP was about \$21,133 billion.

58. $f(x)=480x+5232$

a. $f(10)=480(10)+5232=10,032$
In 2010, personal consumption expendatures was about \$10,032 billion.

b. $f(18)=480(18)+5232=13,872$
In 2018, personal consumption expendatures was about \$13,872 billion.

c. $f(19)=480(19)+5232=14,352$
In 2019, personal consumption expendatures was about \$14,352 billion.

59. $f(x)=4x^2-115x+863$

a. $f(14)=4(14)^2-115(14)+863=37$
At the start of 2014, the closing price of Microsoft was \$37.

b. $f(17)=4(17)^2-115(17)+863=64$
At the start of 2017, the closing price of Microsoft was \$64.

c. $f(21) = 4(21)^2 - 115(21) + 863$
$= 212$
At the start of 2021, the closing price of Microsoft was \$212.

60. $f(x) = 48x^2 - 1296x + 9000$

a. $f(13) = 48(13)^2 - 1296(13) + 9000$
$= 264$
At the start of 2013, the closing price of Amazon was \$264.

b. $f(15) = 48(15)^2 - 1296(15) + 9000$
$= 360$
At the start of 2015, the closing price of Amazon was \$360.

c. $f(19) = 48(19)^2 - 1296(19) + 9000$
$= 1704$
At the start of 2019, the closing price of Amazon was \$1704.

61. $T(x) =$

$$\begin{cases} .02x & \text{if } 0 \le x \le 500 \\ 10 + .04(x - 500) & \text{if } 500 < x \le 3000 \\ 110 + .05(x - 3000) & \text{if } x > 3000 \end{cases}$$

a. $T(100) = .02(100) = \$2.00$
A person with a taxable income of \$100 pays \$2 in state income tax.

b. $T(2500) = 10 + .04(2500 - 500) = \$90.00$
A person with a taxable income of \$2500 pays \$90 in state income tax.

c. $T(103{,}000) = 110 + .05(103{,}000 - 3000)$
$= \$5110.00$
A person with a taxable income of \$103,000 pays \$5110 in state income tax.

62. $T(x) =$

$$\begin{cases} .02x & \text{if } 0 \le x \le 25{,}000 \\ 500 + .04(x - 25{,}000) & \text{if } 25{,}000 < x < 100{,}000 \\ 3500 + .06(x - 100{,}000) & \text{if } x \ge 100{,}000 \end{cases}$$

a. $T(20{,}000) = .02(20{,}000) = \$400.00$
A married person with a taxable income of \$20,000 pays \$400 in state income tax.

b. $T(100{,}000)$
$= 3500 + .06(100{,}000 - 100{,}000)$
$= \$3500$
A married person with a taxable income of \$100,000 pays \$3,500 in state income tax.

c. $T(150{,}000)$
$= 3500 + .06(150{,}000 - 100{,}000)$
$= \$6500$
A married person with a taxable income of \$150,000 pays \$6,500 in state income tax.

63. $f(x) = 27x^{.046}$

a. $f(10) = 27(10)^{.046} \approx 30.0$
In 2010, the mean age of a woman in a certain country when her first child is born was about 30.0 years.

b. $f(15) = 27(15)^{.046} \approx 30.6$
In 2015, the mean age of a woman in a certain country when her first child is born was about 30.6 years.

c. $f(20) = 27(20)^{.046} \approx 31.0$
In 2020, the mean age of a woman in a certain country when her first child is born was about 31.0 years

64. $f(x) = \left(\dfrac{x}{1960}\right)^{-.833}$

a. $f(110) = \left(\dfrac{110}{1960}\right)^{-.833} \approx 11$ min

b. $f(525) = \left(\dfrac{525}{1960}\right)^{-.833} \approx 3$ min

c. $f(1960) = \left(\dfrac{1960}{1960}\right)^{-.833} = 1$ min

d. $f(4500) = \left(\dfrac{4500}{1960}\right)^{-.833} \approx .5$ min

65. $f(t) = 2050 - 500t$

66. $f(t) = 1200 - 550t$

67. Fixed costs = \$1800
Each bag costs \$.50 to produce, and sells for \$1.20.

a. $c(x) = 1800 + .5x$

b. $r(x) = 1.2x$

c. $p(x) = r(x) - c(x)$
$p(x) = 1.2x - (1800 + .5x)$
$p(x) = .7x - 1800$

68. Fixed costs = \$120,000
Each can costs \$.03 to produce, and sells for \$.05.

a. $c(x) = 120{,}000 + .03x$

b. $r(x) = .05x$

c. $p(x) = r(x) - c(x)$
$p(x) = .05x - (120{,}000 + .03x)$
$p(x) = .02x - 120{,}000$

69. $f(x) = .0275x^2 - 1.96x + 35.8$

The corresponding years would be x = 35, 45, 55, and 65.

X	Y1
25	3.9875
35	.8875
45	3.2875
55	11.188
65	24.588
75	43.488
85	67.888

X=25

The corrsponding percentage of the U. S. population with hearing loss is 1%, 3%, 11%, and 25%.

70. $f(x) = .02x^2 - 1.24x + 26$

The years 2026–2030 correspond to $x = 26$ through $x = 30$.

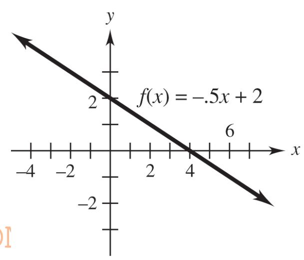
X	Y1
25	7.5
26	7.28
27	7.1
28	6.96
29	6.86
30	6.8
31	6.78

X=25

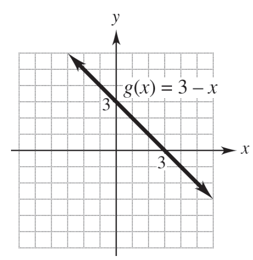
The corrsponding percentage of the population living in extreme poverty is 7.3%, 7.1%, 7.0%, 6.9%, and 6.8%.

Section 3.2 Graphs of Functions

1. $f(x) = -.5x + 2$
The graph is a straight line with slope –.5 and y-intercept 2.

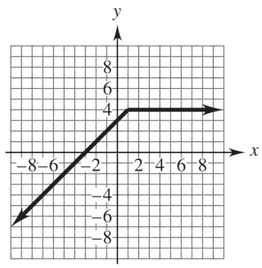


2. $g(x) = 3 - x$
The graph is a straight line with slope –1 and y-intercept 3.



3. $f(x) = \begin{cases} x + 3 & \text{if } x \le 1 \\ 4 & \text{if } x > 1 \end{cases}$

Graph the line $y = x + 3$ for $x \le 1$.
Graph the horizontal line $y = 4$ for $x > 1$.



$f(x) = \begin{cases} x+3 & \text{if } x \le 1 \\ 4 & \text{if } x > 1 \end{cases}$

4. $g(x) = \begin{cases} 2x-1 & \text{if} \quad x < 0 \\ -1 & \text{if} \quad x \ge 0 \end{cases}$

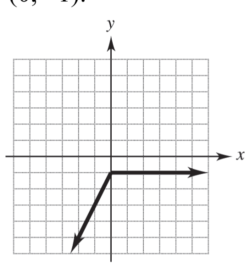
For $x < 0$, use the equation y = $2x - 1$ to find ordered pairs in the following table. Note that we include the endpoint with 0 as the x-value in one of the ordered pairs.

x	-2	-1	0
$y = 2x - 1$	-5	-3	-1

For $x \ge 0$, use the equation $y = -1$ to find ordered pairs in the following table. Again, include the endpoint at 0.

x	0	1	2
$y = -1$	-1	-1	-1

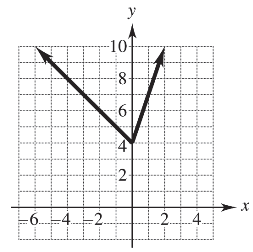
Note that this second part of the graph is a portion of a horizontal line. The graph of the given function is made up of the two partial lines, which are joined at the common endpoint $(0, -1)$.



$g(x) = \begin{cases} 2x-1 & \text{if } x < 0 \\ -1 & \text{if } x \ge 0 \end{cases}$

5. $y = \begin{cases} 4-x & \text{if} \quad x \le 0 \\ 3x+4 & \text{if} \quad x > 0 \end{cases}$

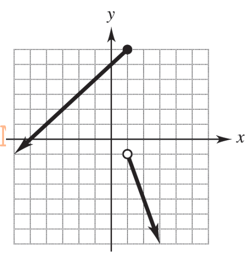
Graph the line $y = 4 - x$ for $x \le 0$.
Graph the line $y = 3x + 3$ for $x > 0$.



$y = \begin{cases} 4-x & \text{if } x \le 0 \\ 3x+4 & \text{if } x > 0 \end{cases}$

6. $y = \begin{cases} x+5 & \text{if} \quad x \le 1 \\ 2-3x & \text{if} \quad x > 1 \end{cases}$

For $x \le 1$, use the equation $y = x + 5$ to graph a partial line through $(-1, 4)$, $(0, 5)$, and $(1, 6)$. For $x > 1$, use the equation $y = 2 - 3x$ to graph a partial line through $(1, -1)$, $(2, -4)$, and $(3, -7)$. Notice that the two parts of the graphs do not have a common endpoint. To show that the endpoint $(1, 6)$ is part of the graph, put a closed circle at this point. To show that the endpoint. $(1, -1)$ is not part of the graph, put an open circle at this point.



$y = \begin{cases} x+5 & \text{if } x \le 0 \\ 2-3x & \text{if } x > 1 \end{cases}$

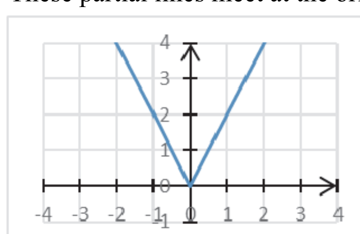
7. $g(x) = 2|x|$

The definition of absolute value gives

$g(x) = \begin{cases} 2x & \text{if} \quad x \ge 0 \\ 2(-x) & \text{if} \quad x < 0 \end{cases}$ or

$g(x) = \begin{cases} 2x & \text{if} \quad x \ge 0 \\ -2x & \text{if} \quad x < 0 \end{cases}$

We graph the line $y = 2x$ for $x \ge 0$.
We graph the line $y = -2x$ for $x < 0$.
These partial lines meet at the origin.



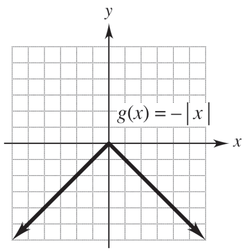
8. $g(x) = -|x|$

The definition of absolute value gives

$$g(x) = \begin{cases} -x & \text{if} \quad x \geq 0 \\ -(-x) & \text{if} \quad x < 0 \end{cases} \text{ or}$$

$$g(x) = \begin{cases} -x & \text{if} \quad x \geq 0 \\ x & \text{if} \quad x < 0 \end{cases}$$

We graph the line $y = -x$ for $x \geq 0$.
We graph the line $y = x$ for $x < 0$.
These partial lines meet at the origin.



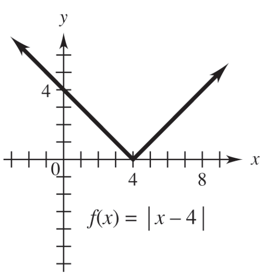
9. $f(x) = |x - 4|$

Using the definition of absolute values gives

$$f(x) = \begin{cases} x - 4 & \text{if} \quad x - 4 \geq 0 \\ -(x-4) & \text{if} \quad x - 4 < 0 \end{cases} \text{ or}$$

$$f(x) = \begin{cases} x - 4 & \text{if} \quad x \geq 4 \\ -x + 4 & \text{if} \quad x < 4 \end{cases}$$

We graph the line $y = x - 4$ with slope 1 and y-intercept –4 for $x \geq 4$. We graph the line $y = -x + 4$ with slope –1 and y-intercept 4 for $x < 4$. Note that these partial lines meet at the point (4, 0).

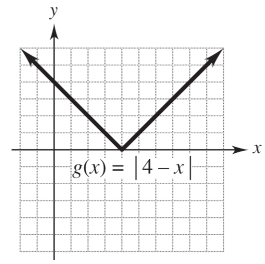


10. $g(x) = |4 - x|$

$$g(x) = \begin{cases} 4 - x & \text{if} \quad 4 - x \geq 0 \\ -(4 - x) & \text{if} \quad 4 - x < 0 \end{cases} \text{ or}$$

$$g(x) = \begin{cases} 4 - x & \text{if} \quad x \leq 4 \\ -4 + x & \text{if} \quad x > 4 \end{cases}$$

Graph the line $y = 4 - x$ for $x \leq 4$.
Graph the line $y = x - 4$ for $x > 4$.
The graph consists of two lines that meet at (4, 0).

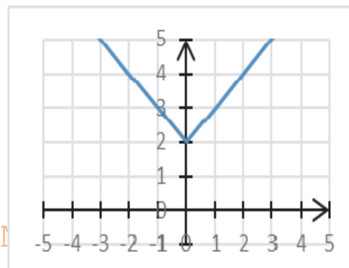


11. $f(x) = |x| + 2$

$$f(x) = \begin{cases} x + 2 & \text{if} \quad x \geq 0 \\ -(x) + 2 & \text{if} \quad x < 0 \end{cases} \text{ or}$$

$$f(x) = \begin{cases} x + 2 & \text{if} \quad x \geq 0 \\ -x + 2 & \text{if} \quad x > 0 \end{cases}$$

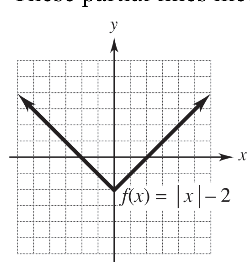
Graph the line $y = x + 2$ for $x \geq 0$.
Graph the line $y = -x + 2$ for $x < 0$.
The graph consists of two lines that meet at (0, 2).



12. $f(x) = |x| - 2$

$$f(x) = \begin{cases} x - 2 & \text{if} \quad x \geq 0 \\ -x - 2 & \text{if} \quad x < 0 \end{cases}$$

We graph the line $y = x - 2$ for $x \geq 0$.
We graph the line $y = -x - 2$ for $x < 0$.
These partial lines meet at the point (0, –2).

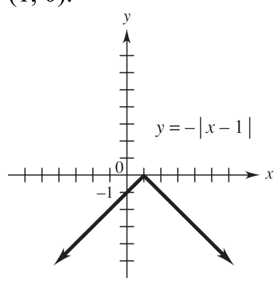


13. $y = -|x-1|$

$$y = \begin{cases} -(x-1) & \text{if} \quad x-1 \geq 0 \\ -[-(x-1)] & \text{if} \quad x-1 < 0 \end{cases} \text{ or}$$

$$y = \begin{cases} -x+1 & \text{if} \quad x \geq 1 \\ x-1 & \text{if} \quad x < 1 \end{cases}$$

Graph the line $y = -x + 1$ for $x \geq 1$.
Graph the line $y = x - 1$ for $x < 1$.
The graph consists of two lines that meet at (1, 0).

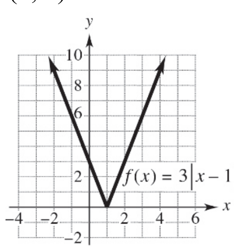


14. $f(x) = |3 - 3x|$

$$f(x) = \begin{cases} 3-3x & \text{if} \quad 3-3x \geq 0 \\ -(3-3x) & \text{if} \quad 3-3x < 0 \end{cases} \text{ or}$$

$$f(x) = \begin{cases} 3-3x & \text{if} \quad x \leq 1 \\ -3+3x & \text{if} \quad x > 1 \end{cases}$$

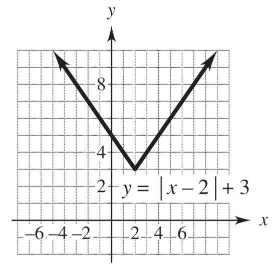
Graph the line $y = 3 - 3x$ for $x \leq 1$.
Graph the line $y = 3x - 3$ for $x > 1$.
The graph consists of two lines that meet at (1, 0).



15. $y = |x-2| + 3$

$$y = \begin{cases} x+1 & \text{if} \quad x \geq 2 \\ -x+5 & \text{if} \quad x < 2 \end{cases}$$

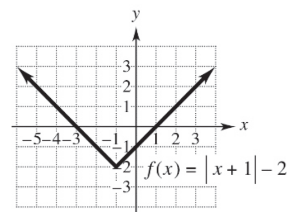
Graph the line $y = x + 1$ for $x \geq 2$.
Graph the line $y = -x + 5$ for $x < 2$.
The graph consists of two lines that meet at (2, 3).



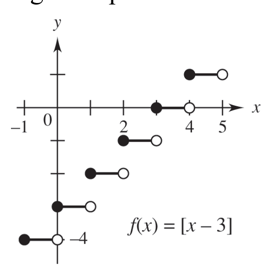
16. $y = |x+1| - 2$

$$y = \begin{cases} x-1 & \text{if} \quad x \geq -1 \\ -x-3 & \text{if} \quad x < -1 \end{cases}$$

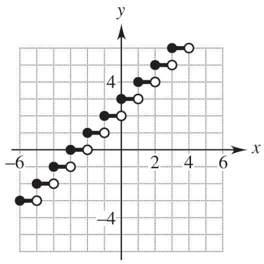
Graph the line $y = x - 1$ for $x \geq -1$.
Graph the line $y = -x - 3$ for $x < -1$.
The graph consists of two lines that meet at (−1, −2).

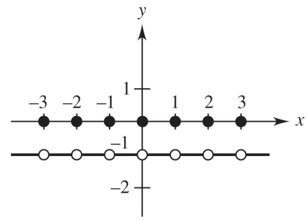


17. $f(x) = [x - 3]$
For x in the interval [0, 1), the value of $[x - 3] = -3$. For x in the interval [1, 2), the value of $[x - 3] = -2$. For x in the interval [2, 3), the value of $[x - 3] = -1$, and so on. The graph consists of a series of line segments. In each case, the left endpoint is included, and the right endpoint is excluded.



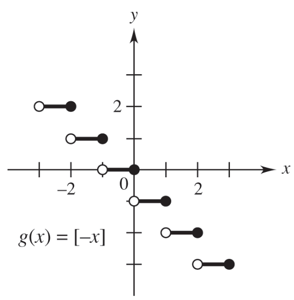
18. $g(x) = [x + 3]$
For x in the interval $0 \leq x < 1$, the value of $[x + 3] = 3$.
For x in the interval $1 \leq x < 3$, the value of $[x + 3] = 4$.
For x in the interval $2 \leq x < 3$, the value of $[x + 3] = 5$.
Continue the graph in this pattern. The graph consists of a series of line segments. In each case, the left endpoint is included, and the right endpoint is excluded.




$f(x) = [x] + [-x]$

19. $g(x) = [-x]$

For x in the interval $(0, 1]$, the value of $[-x] = -1$. For x in the interval $(1, 2]$, the value of $[-x] = -2$. For x in the interval $(2, 3]$, the value of $[-x] = -3$, and so on. The graph consists of a series of line segments. In each case, the left endpoint is excluded, and the right endpoint is included.



20. $f(x) = [x] + [-x]$

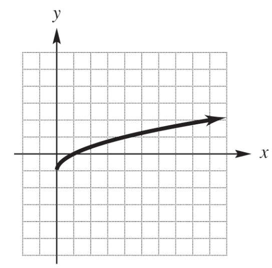
Make a table of intervals and their endpoints:

x	$[x]$	$[-x]$	$f(x)$
–3	–3	3	0
(–3, –2)	–3	2	–1
–2	–2	2	0
(–2, –1)	–2	1	–1
–1	–1	1	0
(–1, 0)	–1	0	–1
0	0	0	0
(0, 1)	0	–1	–1
1	1	–1	0
(1, 2)	1	–2	–1
2	2	–2	0
(2, 3)	2	–3	–1
3	3	–3	0
(3, 4)	3	–4	–1

21. $h(x) = \sqrt{x} - 1$

Make a table of values and plot the corresponding points. The function is not defined for $x < 0$.

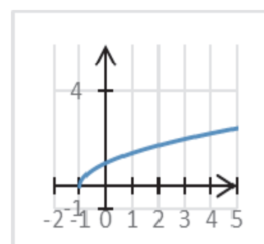
x	$h(x) = \sqrt{x} - 1$
0	–1
1	0
4	1
9	2


$h(x) = \sqrt{x} - 1$

22. $h(x) = \sqrt{x+1}$

Make a table of values and plot the corresponding points. The function is not defined for $x < -1$.

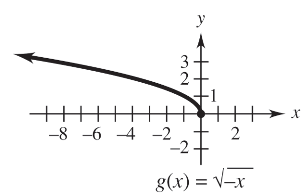
x	$h(x) = \sqrt{x+1}$
–1	0
0	1
3	2
8	3



23. $g(x) = \sqrt{-x}$

Make a table of values and plot the corresponding points. The function is not defined for $x > 0$.

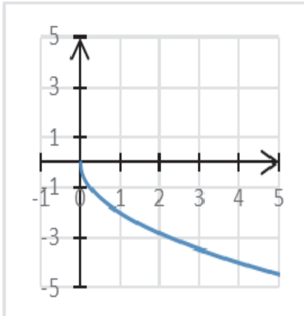
x	$g(x) = \sqrt{-x}$
–9	3
–4	2
–1	1
0	0



24. $f(x) = -2\sqrt{x}$

Make a table of values and plot the corresponding points. The function is not defined for $x < 0$.

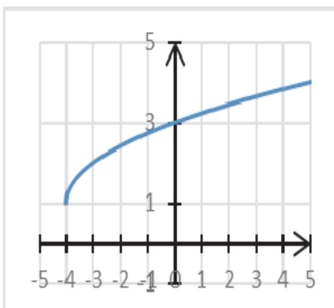
x	$f(x) = -2\sqrt{x}$
0	0
1	–2
4	–4
9	–6



25. $f(x) = \sqrt{x+4} + 1$

Make a table of values and plot the corresponding points. The function is not defined for $x > -4$.

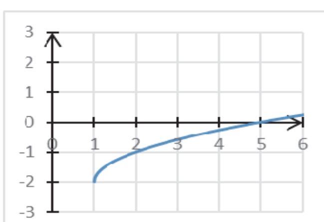
x	$f(x) = \sqrt{x+4} + 1$
–4	1
–3	2
0	3
5	4



26. $f(x) = \sqrt{x-1} - 2$

Make a table of values and plot the corresponding points. The function is not defined for $x > 1$.

x	$f(x) = \sqrt{x-1} - 2$
1	–2
2	–1
5	0
10	1



27. Every vertical line intersects this graph in at most one point, so this is the graph of a function.

28. Every vertical line intersects this graph in at most one point, so this is the graph of a function.

29. A vertical line intersects the graph in more than one point, so this is not the graph of a function.

30. A vertical line intersects the graph in more than one point, so this is not the graph of a function.

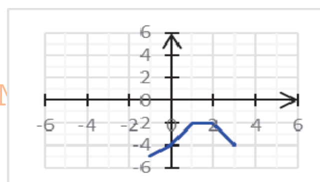
31. Every vertical line intersects this graph in at most one point, so this is the graph of a function.

32. A vertical line intersects the graph in more than one point, so this is not the graph of a function.

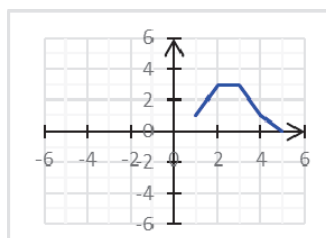
33. **a.** No Match
b. $k(x)$
c. $g(x)$
d. $h(x)$

34. **a.** No Match
b. $f(x)$
c. $k(x)$
d. $h(x)$

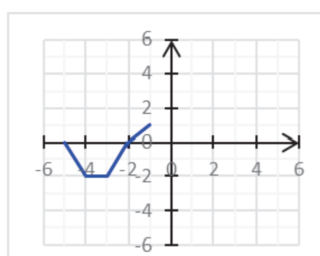
35.



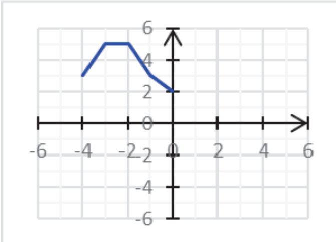
36.



37.



38.



39. Domain: $[-3,1]$

Range: $[-1,2]$

40. Domain: $[-7,-3]$

Range: $[-4,-1]$

41. Domain: $[-3,1]$

Range: $[-5,-2]$

42. Domain: $[-1,3]$

Range: $[3,6]$

43. Domain: $(-\infty,\infty)$

Range: $(-\infty,3]$

44. Domain: $(-\infty,\infty)$

Range: $(-\infty,-5)$

45. Domain: $[7,\infty)$

Range: $[3,\infty)$

46. Domain: $[-5,\infty)$

Range: $[-2,\infty)$

47. Domain: $(-\infty,0]$

Range: $[0,\infty)$

48. Domain: $[0,\infty)$

Range: $(-\infty,0]$

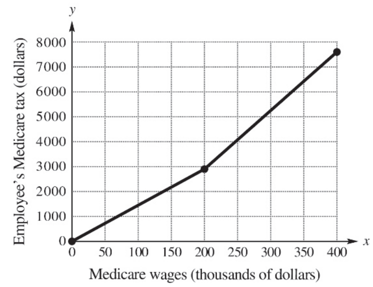
49. $f(x)=\begin{cases}14.5x & \text{if } 0\le x\le 200\\ 2900+23.5(x-200) & \text{if } x>200\end{cases}$

a. $f(0)=14.5(0)=0$

$f(200)=14.5(200)=2900$

$f(400)=2900+23.5(400)=7600$

b. The graph would look as follows:



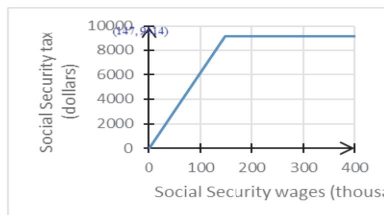
50. $f(x)=\begin{cases}62x & \text{if } 0\le x\le 147\\ 9114 & \text{if } x>147\end{cases}$

a. $f(0)=62(0)=0$

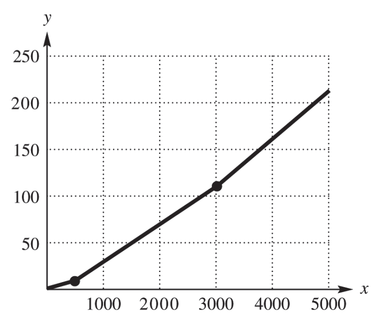
$f(147)=62(147)=9114$

$f(400)=9114$

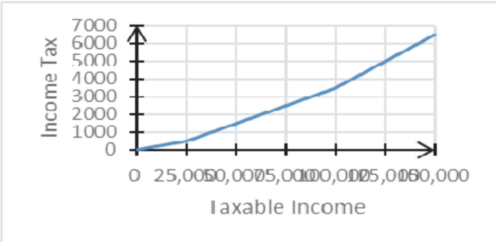
b. The graph would look as follows:



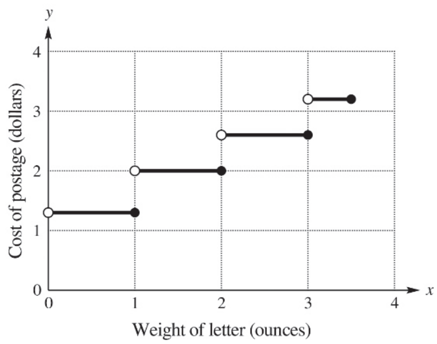
51. $T(x)=\begin{cases}.02x & \text{if } 0\le x\le 500\\ 10+.04(x-500) & \text{if } 500<x\le 3000\\ 110+.05(x-3000) & \text{if } x>3000\end{cases}$



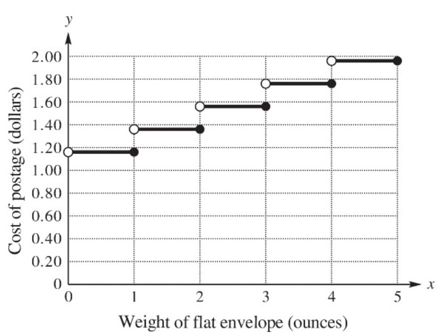
52. $T(x)=\begin{cases}.02x & \text{if } 0\le x\le 25{,}000\\ 500+.04(x-25{,}000) & \text{if } 25{,}000<x\le 100{,}000\\ 3500+.06(x-100{,}000) & \text{if } x>100{,}000\end{cases}$



53. $f(x)=\begin{cases}1.30x & \text{if } 0 \le x \le 1\\ 1.96x & \text{if } 1<x \le 2\\ 2.60x & \text{if } 2<x \le 3\\ 3.25x & \text{if } 3<x \le 3.5\end{cases}$



54. $f(x)=\begin{cases}1.16x & \text{if } 0 \le x \le 1\\ 1.36x & \text{if } 1<x \le 2\\ 1.56x & \text{if } 2<x \le 3\\ 1.76x & \text{if } 3<x \le 4\\ 1.96x & \text{if } 4<x \le 5\end{cases}$



55. a. The function values in the year 2020 was 55 cents.

b. The figure has vertical line segments, which can't be part of the graph of a function. To make the figure into the graph of *f*, delete the vertical line segments, then for each horizontal segment of the graph, put a closed dot on the left end and an open-circle dot on the right end (or vice versa, depending on when the cost change occurred).

56. To calculate $S(x)$ when x is not an integer, take 20 times the next integer larger than x and add 7. If x is an integer, $S(x) = 20x + 7$.

a. $S\left(\frac{1}{2}\right) = 20 \cdot 1 + 7 = \$27$

b. $S(1) = 20 \cdot 1 + 7 = \$27$

c. $S\left(1\frac{2}{3}\right) = 20 \cdot 2 + 7 = \$47$

d. $S\left(4\frac{3}{4}\right) = 20 \cdot 5 + 7 = 107$

e. $S\left(5\frac{7}{8}\right) = 20 \cdot 6 + 7 = 127$

It costs \$127 to rent a saw for $5\frac{7}{8}$ days.

f. Insert one-unit long horizontal segments similar to the two now on the graph; segments should start at these points: (2, .67), (3, .87).

g. The domain variable is x.

h. The range variable is $S(x)$.

i. Answers vary. Sample answer: This represents the cost to rent a saw for $1\frac{2}{3}$ days or 1 day 16 hours.

j. Answers vary. Sample answer: You wouldn't rent a saw for 0 days, so $S(0)$ makes no sense. If you rent a saw for a part of a day, we use $x = 1$ in the function to determine the cost. (See part a.)

57. First convert 75 minutes to 1.25 hours, and 15 minutes to .25 hours. Let x represent the number of hours to rent the van. To calculate $C(x)$, subtract 1.25 from x, then divide this number by .25 and round to the next integer. Finally, multiply this result by 5 and add \$19.99.

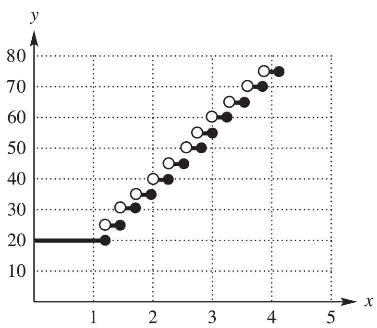
a. 2 hours
$$\frac{2-1.25}{.25}=3$$
$$5(3)+19.99=34.99$$
It costs \$34.99 to rent a van for 2 hours.

b. 1.5 hours
$$\frac{1.5-1.25}{.25}=1$$
$$5(1)+19.99=24.99$$
It costs \$24.99 to rent a van for 1.5 hours.

c. 3.5 hours
$$\frac{3.5-1.25}{.25}=9$$
$$5(9)+19.99=64.99$$
It costs \$64.99 to rent a van for 3.5 hours.

d. 4 hours
$$\frac{4-1.25}{.25}=11$$
$$5(11)+19.99=74.99$$
It costs \$74.99 to rent a van for 4 hours.

e.



Note that for each step, the left endpoint is not included and the right point is included.

58. Let x be the number of miles and $D(x)$ be the delivery charge. If x is an integer, then $D(x) = .6x + 25$.
If x is not an integer, $D(x)$ is the sum of .6 times the next integer larger than x added to 3.

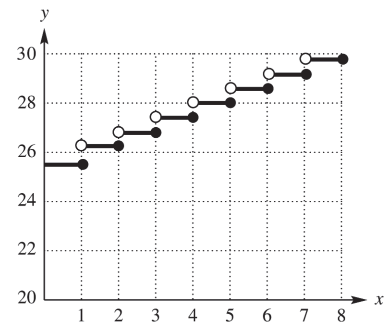
a. Let $x = 3$.
$D(3) = .6(3) + 25 = 26.80$
The delivery charge for a trip of 3 miles is \$26.80.

b. Since $x = 4.2$, the next integer larger than 4.2 is 5.
$D(4.2) = .6(5) + 25 = 28$
The delivery charge for a trip of 4.2 miles is \$28.00.

c. $D(5.9) = .6(6) + 25 = 28.60$
The delivery charge for a trip of 5.9 miles is \$28.60.

d. $D(8) = .6(8) + 25 = 29.80$
The delivery charge for a trip of 8 miles is \$29.80.

e.



f. Yes, this is a function because every trip has a unique cost.

59. a. The percent change in CPI for food away from home was the largest in 2020.

b. The percent change in CPI for apparel was the largest in 2012.

c. The percent change in CPI for all items was approximately zero in 2015.

60. a. The P/E ratio in 2018 was approximately 7.

b. The P/E ratio was the largest in 2012.

c. The P/E ratio was the smallest in 2018.

61. a. The shareholders' equity for Amazon in the year 2020 was approximately \$100 billion.

b. The domain of the assets function is $[2015, 2020]$.

c. The ramge of the liabilities function is $[50, 225]$.

62. a. The shareholders' equity for Comcast in the year 2018 was approximately \$75 billion.

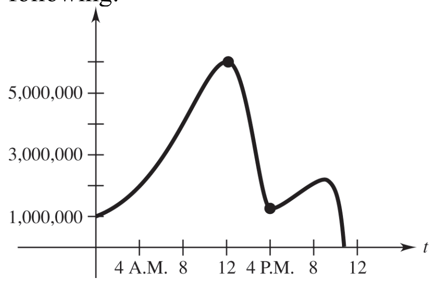
b. The domain of the liabilities function is $[2016, 2020]$.

c. The ramge of the assets function is $[175, 275]$.

63. Make table of values:

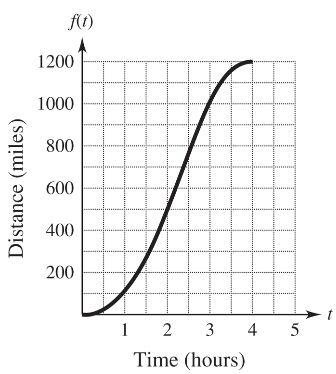
Time	t	$g(t)$	Slope
midnight	0	1,000,000	increasing
noon	12	?	decreasing
4:00 P.M.	16	?	increasing
9:00 P.M.	21	?	vertical

There are many correct answers, including the following:

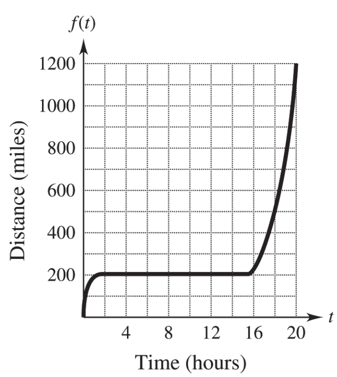


64. There are many correct answers, including the following:Section 3.3 Applications of Linear Functions

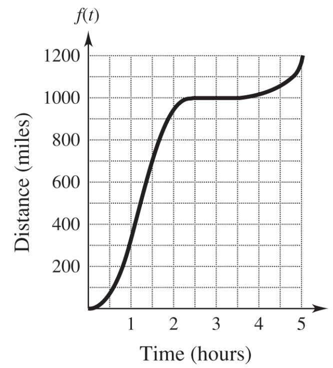
a.



b.



c.



Section 3.3 Applications of Linear Functions

1. The marginal cost is \$5, while the fixed cost is \$25. Let $C(x)$ be the cost of renting a saw for x hours. Then $C(x) = 5x + 25$.

2. Let x = the number of miles and $C(x)$ = the total charge for x miles. Then $C(x) = 8x + 95$.

3. Let x = the number of half-hours and $C(x)$ = the total cost in dollar for x hours. Then $C(x) = 2.5x + 8$

4. Let x = the number of miles and $C(x)$ = the total charge for x miles. Then $C(x) = .45x + 65$.

5. Fixed cost, \$200, 50 items cost \$2000 to produce. Since the fixed cost is \$200,

$$C(x) = mx + 200$$
$$C(50) = 2000.$$

Therefore,

$$2000 = m(50) + 200 \Rightarrow 50m = 1800 \Rightarrow m = 36$$
$$C(x) = 36x + 200$$

6. Since the fixed cost is \$2000, $C(x) = mx + 2000$. Since 40 items cost \$5000 to produce, $C(40) = 2000$. Therefore,

$$5000 = m(40) + 2000 \Rightarrow 3000 = 40m \Rightarrow 75 = m$$

and $C(x) = 75x + 2000$.

7. Marginal cost, \$120; 100 items cost \$15,800 to produce.

$$C(x) = mx + b,\ m = 120$$
$$C(100) = 15,800 = 120(100) + b$$
$$b = 15,800 - 12,000 = 3800$$
$$C(x) = 120x + 3800$$

8. Since the marginal cost is \$90, $C(x) = 90x + b$. Since 150 items cost \$16,000 to produce, $C(150) = 16{,}000$. Therefore,
$$16{,}000 = 90(150) + b \Rightarrow$$
$$16{,}000 = 13{,}500 + b \Rightarrow 2500 = b$$
$$C(x) = 90x + 2500$$

9. $\overline{C}(50) = \dfrac{C(50)}{50} = \dfrac{12(50)+1800}{50} = 48$

The average cost per item when 50 items are produced is \$48.

$\overline{C}(500) = \dfrac{C(500)}{500} = \dfrac{12(500)+1800}{500} = 15.60$

The average cost per item when 500 items are produced is \$15.60.

$\overline{C}(1000) = \dfrac{C(1000)}{1000} = \dfrac{12(1000)+1800}{1000} = 13.80$

The average cost per item when 1000 items are produced is \$13.80.

10. $\overline{C}(100) = \dfrac{C(100)}{100} = \dfrac{80(100)+12{,}000}{100} = 200$

The average cost per item when 100 items are produced is \$200.

$\overline{C}(1000) = \dfrac{C(1000)}{1000} = \dfrac{80(1000)+12{,}000}{1000}$
$= 92$

The average cost per item when 1000 items are produced is \$92.

$\overline{C}(10{,}000) = \dfrac{C(10{,}000)}{10{,}000}$
$= \dfrac{80(10{,}000)+12{,}000}{10{,}000} = 81.20$

The average cost per item when 10,000 items are produced is \$81.20.

11. $\overline{C}(200) = \dfrac{C(200)}{200} = \dfrac{6.5(200)+9800}{200} = 55.50$

The average cost per item when 200 items are produced is \$55.50.

$\overline{C}(2000) = \dfrac{C(2000)}{2000} = \dfrac{6.5(2000)+9800}{2000}$
$= \$11.40$

The average cost per item when 2000 items are produced is \$11.40.

$\overline{C}(5000) = \dfrac{C(5000)}{5000} = \dfrac{6.5(5000)+9800}{5000}$
$= \$8.46$

The average cost per item when 5000 items are produced is \$8.46.

12. $\overline{C}(1000) = \dfrac{C(1000)}{1000} = \dfrac{8.75(1000)+16{,}500}{1000}$
$= 25.25$

The average cost per item when 1000 items are produced is \$25.25.

$\overline{C}(10{,}000) = \dfrac{C(10{,}000)}{10{,}000}$
$= \dfrac{8.75(10{,}000)+16{,}500}{10{,}000} = 10.40$

The average cost per item when 10,000 items are produced is \$10.40.

$\overline{C}(75{,}000) = \dfrac{C(75{,}000)}{75{,}000}$
$= \dfrac{8.75(75{,}000)+16{,}500}{75{,}000} = 8.97$

The average cost per item when 75,000 items are produced is \$8.97.

13. a. Let $(x_1, y_1) = (0,\ 16645)$ and let $(x_2, y_2) = (4,\ 12385)$.
$$m = \frac{16{,}645 - 12{,}385}{0-4} = -1065$$
The y-intercept is 16,645, so the equation is $f(x) = -1065x + 16{,}645$.

b. $f(5) = -1065(5) + 16{,}645 = 11{,}320$

The car will be worthe \$11,320.

c. Because the slope is –1065, the car is depreciating at a rate of \$1065 per year.

14. a. $f(x) = -250x + 1250$

b. When $x = 5$, $f(5) = -250(5) + 1250 = 0$.

The value of the computer will be 0, and so it cannot be depreciated further.

15. a. Let $(x_1, y_1) = (0,\ 120{,}000)$ and $(x_2, y_2) = (8,\ 25{,}000)$.
$$m = \frac{25{,}000 - 120{,}000}{8-0} = -11{,}875$$
$$y - y_1 = m(x - x_1)$$
$$y - 120{,}000 = -11{,}875(x - 0)$$
$$y - 120{,}000 = -11{,}875x$$
$$y = -11{,}875x + 120{,}000$$
$$f(x) = -11{,}875x + 120{,}000$$

b. The domain ranges from 0 to 8 years, that is [0, 8].

c. $f(6) = -11,875(6) + 120,000 = 48,750$
The machine will be worth \$48,750 in 6 years.

16. a. Let $(x_1, y_1) = (0,\ 222,000)$ and let $(x_2, y_2) = (6,\ 300,000)$

$$m = \frac{300,000 - 222,000}{6 - 0} = 13,000$$

Since the y-intercept is 222,000, the equation is $g(x) = 13,000x + 222,000.$

b. $g(12) = 13,000(12) + 222,000 = 378,000$
In 12 years, the house will be worth \$378,000.

17. a. The fixed costs are
$C(0) = \$42.5(0) + 80,000 = \$80,000$

b. The slope of $C(x) = 42.5x + 80,000$ is 42.5, so the marginal cost is \$42.50.

c. The cost of producing 1000 books is
$C(1000) = 42.5(1000) + 80,000 = \$122,500.$
The cost of producing 32,000 books is

$$C(32,000) = 42.5(32,000) + 80,000 = \$1,440,000.$$

d. The average cost per book when 1000 are produced is:

$$\overline{C}(1000) = \frac{C(1000)}{1000} = \frac{42.5(1000) + 80,000}{1000} = \$122.50$$

The average cost per book when 32,000 are produced is,

$$\overline{C}(32,000) = \frac{C(32,000)}{32,000} = \frac{42.5(32,000) + 80,000}{32,000} = \$45$$

18. a. The fixed costs are
$C(0) = 6.80(0) + 450,000 = \$450,000$

b. The slope of $C(x) = 6.80x + 450,000$ is 6.80, so the marginal cost is \$6.80.

c. The cost of producing 50,000 chargers is

$$C(50,000) = 6.80(50,000) + 450,000 = \$790,000.$$

The cost of producing 600,000 chargers is

$$C(600,000) = 6.80(600,000) + 450,000 = \$4,530,000.$$

d. The average cost per charger when 50,000 are produced is

$$\overline{C}(50,000) = \frac{C(50,000)}{50,000} = \frac{6.80(50,000) + 450,000}{50,000} = \$15.80.$$

The average cost per charger when 500,000 are produced is

$$\overline{C}(500,000) = \frac{C(500,000)}{500,000} = \frac{6.80(500,000) + 450,000}{500,000} = \$7.70.$$

19. a. Let (x_1, y_1) be (50, 58) and (x_2, y_2) be (250, 142).
Find the slope.

$$m = \frac{142 - 58}{250 - 50} = \frac{84}{200} = .42$$

Use the point-slope form with (50, 58).

$$y - 58 = .42(x - 50)$$
$$y = .42x + 37$$
$$C(x) = .42x + 37$$

b. The fixed costs are the constant added to the cost function or \$37.

c. The total cost of producing 1000 tacos is
$C(1000) = .42(1000) + 37 = \$457.$

d. The total cost of producing 1001 tacos is
$C(1001) = .42(1001) + 37 = \$457.42.$

e. The marginal cost of the 1001st cup is
$C(1001) - C(1000) = 457.42 - 457 = \$.42,$
which is about 42¢.

f. The slope of $C(x) = .42x + 37$ is .42, so the marginal cost is \$.42 or 42 cents.

20. **a.** Let (x_1, y_1) be (10,000, 547,500) and (x_2, y_2) be (50,000, 737,500).
Find the slope.

$$m = \frac{737,500 - 547,500}{50,000 - 10,000} = \frac{190,000}{40,000} = 4.75$$

Use the point-slope form with (10,000, 547,500)

$$(y - 547,500) = 4.75(x - 10,000)$$
$$y = 4.75x + 500,000$$
$$C(x) = 4.75x + 500,000$$

b. The total cost to produce 100,000 items is

$$C(100,000) = 4.75(100,000) + 500,000 = \$975,000.$$

c. The slope of $C(x) = 4.75x + 500,000$ is 4.75, so the marginal cost is \$4.75.

21. Each customer pays $5.46 + 1.90x$ per month, where x is the number of thousands of gallons of water used. Then,

$$R(x) = 200,000(5.46) + 1.90x = 1,092,000 + 1.90x.$$

22. Each customer pays $13.17 + .09433x$ per month, where x is the number of thousands of gallons of water used. Then,

$$R(x) = 10,000(13.17) + .09433x = 131,700 + .09433x.$$

23. **a.** $C(x) = 10x + 750$

b. $R(x) = 35x$

c. $P(x) = R(x) - C(x) = 35x - (10x + 750) = 25x - 750$

d. The profit on 100 items is,
$P(100) = 25(100) – 750 = \$1750$

24. **a.** $C(x) = 11x + 150$

b. $R(x) = 20x$

c. $P(x) = R(x) - C(x) = 20x - (11x + 150) = 9x - 150$

d. The profit on 100 items is,
$P(100) = 9(100) – 150 = \$750$

25. **a.** $C(x) = 18x + 300$

b. $R(x) = 28x$

c. $P(x) = R(x) - C(x) = 28x - (18x + 300) = 10x - 300$

d. The profit on 100 items is,
$P(100) = 10(100) – 300 = \$700$

26. **a.** $C(x) = 30x + 17,000$

b. $R(x) = 80x$

c.
$$P(x) = R(x) - C(x) = 80x - (30x + 17,000) = 50x - 17,000$$

d. The profit on 100 items is,
$P(100) = 50(100) – 17,000 = –\$12,000$
(a loss of \$12,000)

27. $y = 100 + 7x$ and $y = 11x$
Set the two equations equal and solve for x.
$100 + 7x = 11x \Rightarrow 4x = 100 \Rightarrow x = 25$
Substitute x into one equation and solve for y.
$y = 11(25) = 275$

The lines intersect at $(25, 275)$.

28. $y = 23x$ and $y = 1500 + 11x$
Set the two equations equal and solve for x.
$23x = 1500 + 11x \Rightarrow 12x = 1500 \Rightarrow x = 125$
Substitute x into one equation and solve for y.
$y = 23(125) = 2875$
The lines intersect at $(125, 2875)$.

29. $2x – y = 7$ and $y = 8 – 3x$
Solve the first equation for y:
$2x - y = 7 \Rightarrow y = 2x - 7.$
Set the two equations equal and solve for x.
$2x - 7 = 8 - 3x \Rightarrow 5x = 15 \Rightarrow x = 3$
Substitute x into one equation to find y.
$y = 8 – 3(3) = –1$
The lines intersect at (3, –1).

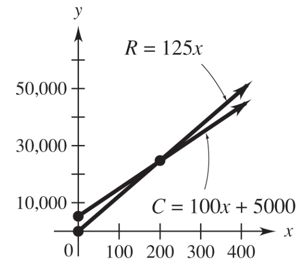
30. $6x - y = 2$ and $y = 4x + 7$
Solve the first equation for y:
$6x - y = 2 \Rightarrow 6x - 2 = y$
Set the two equations equal and solve for x.
$6x - 2 = 4x + 7 \Rightarrow 2x = 9 \Rightarrow x = \frac{9}{2}$
Substitute x into one equation to find y.
$y = 4\left(\frac{9}{2}\right) + 7 = 25$
The lines intersect at $\left(\frac{9}{2}, 25\right)$.

31. $y = 3x - 7$ and $y = 7x + 4$
Set the two equations equal and solve for x.
$3x - 7 = 7x + 4 \Rightarrow -4x = 11 \Rightarrow x = -\frac{11}{4}$
Substitute x into one equation to find y.
$y = 3\left(-\frac{11}{4}\right) - 7 = -\frac{33}{4} - \frac{28}{4} = -\frac{61}{4}$
The lines intersect at $\left(-\frac{11}{4}, -\frac{61}{4}\right)$.

32. $y = 3x + 5$ and $y = 12 - 2x$
Set the two equations equal and solve for x.
$3x + 5 = 12 - 2x \Rightarrow 5x = 7 \Rightarrow x = \frac{7}{5}$
Substitute x into one equation and solve for y.
$y = 3\left(\frac{7}{5}\right) + 5 = \frac{21}{5} + \frac{25}{5} = \frac{46}{5}$
The lines intersect at $\left(\frac{7}{5}, \frac{46}{5}\right)$.

33. a. The break-even point occurs when $R(x) = C(x)$.
$125x = 100x + 5000 \Rightarrow 25x = 5000 \Rightarrow$
$x = 200$
The break-even point is $x = 200$ or 200,000 policies.

b. To graph the revenue function, graph $y = 125x$. If $x = 0$, $y = 0$. If $x = 20$, $y = 2500$. Use the points (0, 0) and (20, 2500) to graph the line. To graph the cost function, graph $y = 100x + 5000$. If $x = 0$, $y = 5000$. If $x = 30$, $y = 8000$. Use the points (0, 5000) and (30, 8000) to graph the line.

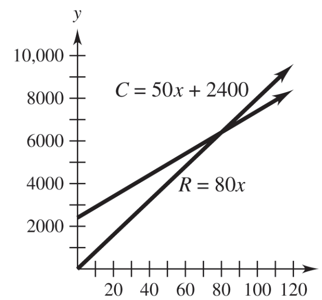


c. From the graph, when $x = 100$, cost is \$15,000 and revenue is \$12,500.

34. Given $R(x) = 80x$ and $(x)C = 50x + 2400$.

a. The break-even point occurs when $R = C$, so $80x = 50x + 2400 \Rightarrow 30x = 2400 \Rightarrow$
$x = 80$
The break-even point is 80.

b.



c. When $x = 60$, $R = 80(60) = 4800$ and $C = 50(60) + 2400 = 5400$.
Revenue is \$4800; cost is \$5400.

35. Given $R(x) = .21x$ and $P(x) = .084x - 1.5$

a. Cost equals revenue – profit, so
$C(x) = R(x) - P(x) = .21x - (.084x - 1.5)$
$= .21x - .084x + 1.5 = .126x + 1.5$

b. $C(7) = .126(7) + 1.5 = 2.382$
The cost of producing 7 units is \$2.382 million.

c. At the break-even point, profit $P(x) = 0$, so $C(x) = R(x)$
$.126x + 1.5 = .21x \Rightarrow 1.5 = .084x \Rightarrow$
$x \approx 17.857$
The break-even point occurs at about 17.857 units.

36. Given $P(x) = .7x - 25.5$ and $C(x) = .9x + 25.5$.

a. Profit equals revenue minus cost, so

$$P(x) = R(x) - C(x)$$
$$.7x - 25.5 = R(x) - (.9x + 25.5)$$
$$.7x - 25.5 = R(x) - .9x - 25.5$$
$$R(x) = 1.6x$$

b. $R(10) = 1.6(10) = 16$

The revenue from 10 million units is $16 million.

c. At the break-even point, profit $P(x) = 0$, so

$0 = .7x - 25.5 \Rightarrow 25.5 = .7x \Rightarrow x \approx 36.4$

The break-even point occurs at about 36.4 million units.

37. $C(x) = 80x + 7000$; $R(x) = 95x$

$95x = 80x + 7000 \Rightarrow 15x = 7000 \Rightarrow x \approx 467$

The break-even point is about 467 units. Do not produce the item since 467 > 400.

38. $C(x) = 37x + 9000$; $R(x) = 53x$

$53x = 37x + 9000 \Rightarrow 16x = 9000 \Rightarrow$

$x = 562.5$

Since the break-even point is about 563units, the product will never make a profit. Do not produce it.

39. $C(x) = 125x + 42,000$; $R(x) = 165.5x$

$125x + 42,000 = 165.5x \Rightarrow 42,000 = 40.5x \Rightarrow$

$x \approx 1037$

The break-even point is about 1037 units. Produce the item since 1037 < 2000.

40. $C(x) = 1750x + 95,000$; $R(x) = 1975x$

$1750x + 95,000 = 1975x \Rightarrow 95,000 = 225x \Rightarrow$

$x \approx 422$

The break-even point is about 422 units. Produce the item since 422 < 600.

41. **a.** The revenue function is $R(x) = 75x$.

b. Let (x_1, y_1) be (0, 1000) and (x_2, y_2) be (20, 2000).

Find the slope.

$$m = \frac{2,000 - 1,000}{20 - 0} = \frac{1,000}{20} = 50$$

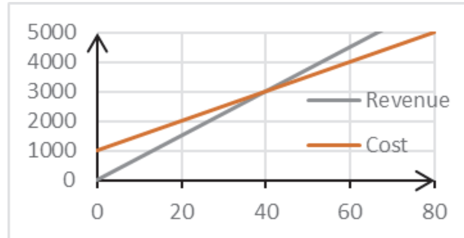
Since the y-intercept is 1,000, the cost equation is $C(x) = 50x + 1,000$.

c. $C(x) = 50x + 1000$; $R(x) = 75x$

$75x = 50x + 1000 \Rightarrow 25x = 1000 \Rightarrow x = 40$

The break-even point is 40 items.

d. The graph will look as follows:



42. **a.** The revenue function is $R(x) = 100x$.

b. Let (x_1, y_1) be (0, 600) and (x_2, y_2) be (80, 3800).

Find the slope.

$$m = \frac{3,800 - 600}{80 - 0} = \frac{3,200}{80} = 40$$

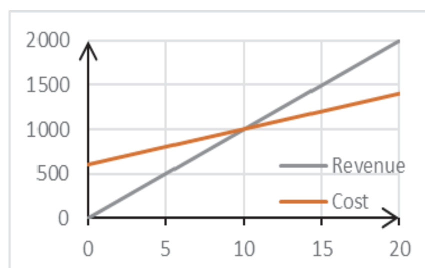
Since the y-intercept is 600, the cost equation is $C(x) = 40x + 600$.

c. $C(x) = 40x + 600$; $R(x) = 100x$

$100x = 40x + 600 \Rightarrow 60x = 600 \Rightarrow x = 10$

The break-even point is 10 items.

d. The graph will look as follows:



43. **a.** The revenue function is $R(x) = 15x$.

b. Let (x_1, y_1) be (50, 750) and (x_2, y_2) be (140, 1200).

Find the slope.

$$m = \frac{1,200 - 750}{140 - 50} = \frac{450}{90} = 5$$

Use the point-slope form with (50, 750)

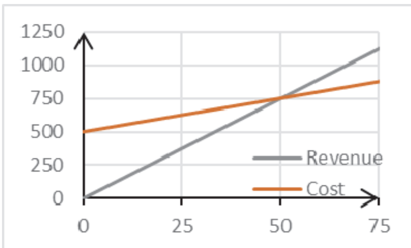
$$(y-750)=5(x-50)$$
$$y=5x+500$$
$$C(x)=5x+500$$

c. $C(x)=5x+500;\ R(x)=15x$

$15x=5x+500 \Rightarrow 10x=500 \Rightarrow x=50$

The break-even point is 10 items.

d. The graph will look as follows:



44. **a.** The revenue function is $R(x)=50x$.

b. Let (x_1, y_1) be (10, 2250) and (x_2, y_2) be (35, 2875).

Find the slope.

$$m=\frac{2,875-2,250}{35-10}=\frac{625}{25}=25$$

Use the point-slope form with (10, 2250)

$$(y-2,250)=25(x-10)$$
$$y=25x+2,000$$
$$C(x)=25x+2,000$$

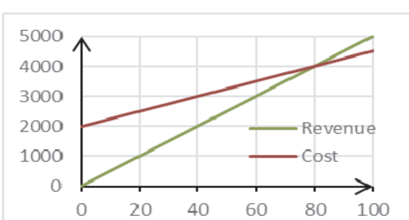
c. $C(x)=25x+2,000;\ R(x)=50x$

$50x=25x+2,000 \Rightarrow 25x=2,000$

$\Rightarrow x=80$

The break-even point is 80 items.

d. The graph will look as follows:



45. On the supply curve, when $q=20$, $p=140$. The point (20, 140) is on the graph. When 20 items are supplied, the price is \$140.

46. From the graph, the price is \$90 when the demand is 20.

47. The two curves intersect at the point (10, 120). The equilibrium supply and equilibrium demand are both $q=10$ or 10 items.

48. From the graph, the equilibrium price, when supply equals demand, is \$120.

49. $p=16-\frac{5}{4}q$

a. If $q=0$, $p=16$, so for a demand of 0 units, the price is \$16.

b. If $q=4$, $p=16-\frac{5}{4}(4)=16-5=11$, so for a demand of 4 units, the price is \$11.

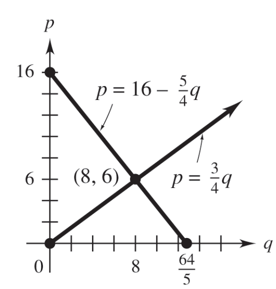
c. If $q=8$, $p=16-\frac{5}{4}(8)=16-10=6$, so for a demand of 8 units, the price is \$6.

d. From (c), if $p=6$, $q=8$, so at a price of \$6, the demand is 8 units.

e. From (b), if $p=11$, $q=4$, so at a price of \$11, the demand is 4 units.

f. From (a), if $p=16$, $q=0$, so at a price of \$16, the demand is 0 units.

g.



h. $p=\frac{3}{4}q$

If $p=0$, $q=0$, so at a price of \$0, the supply is 0 units.

i. If $p=10$,

$$10=\frac{3}{4}q \Rightarrow \frac{4}{3}(10)=\frac{4}{3}\left(\frac{3}{4}\right)q \Rightarrow \frac{40}{3}=q$$

When the price is \$10, the supply is $\frac{40}{3}$ units.

j. If $p = 20$,

$$20 = \frac{3}{4}q \Rightarrow \frac{4}{3}(20) = \frac{4}{3}\left(\frac{3}{4}q\right) \Rightarrow \frac{80}{3} = q$$

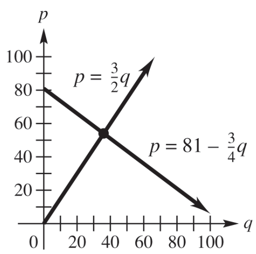
When the price is \$20, the supply is $\frac{80}{3}$ units.

k. See (g).

l. The two graphs intersect at the point (8, 6). The equilibrium supply is 8 units.

m. The equilibrium price is \$6.

50. a.



b. The equilibrium demand occurs when supply equals demand, so

$$\frac{3}{2}q = 81 - \frac{3}{4}q \Rightarrow \frac{3}{2}q + \frac{3}{4}q = 81 \Rightarrow$$
$$\frac{9}{4}q = 81 \Rightarrow q = 36$$

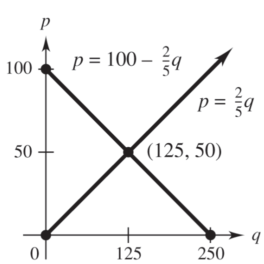
The equilibrium demand is 36 tires. This is confirmed by the above graph.

c. Substitute the equilibrium demand from part (b) in either equation.

$$p = \frac{3}{2}(36) = 54$$

The equilibrium price is \$54. This is also confirmed by the above graph.

51. a.



b. The two graphs intersect at the point (125, 50). The equilibrium demand is 125 units.

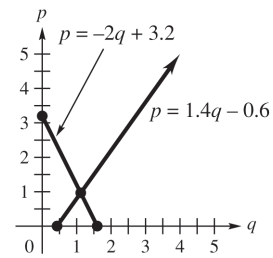
c. The equilibrium price is 50 cents.

d. $40 = \frac{2}{5}q \Rightarrow q = \frac{5}{2}(40) \Rightarrow$

$q = 100$

Since the value is less than the equilibrium demand value, there will be a shortage of supply.

52. a.



b. The equilibrium demand occurs when supply equals demand, so

$$1.4q - .6 = -2q + 3.2 \Rightarrow 3.4q = 3.8 \Rightarrow$$
$$q \approx 1.118$$

The equilibrium demand is about 1.118 pounds, as reflected in the above graph.

c. $p = 1.4(1.118) - .6 = .9652$

The equilibrium price is about \$.96.

d. $2 = 1.4q - .6 \Rightarrow$

$$q = \frac{2 + .6}{1.4} \Rightarrow$$
$$q \approx 1.857$$

Since the value is greater than the equilibrium demand value, there will be a surplus of supply.

53. Total cost increases when more items are made (because it includes the cost of all previously made items), so the graph cannot move downward. No; the average cost can decrease as more items are made, so its graph may move downward.

54. Profit is negative at the beginning (because of the fixed costs that apply even if no items are made), so the graph can be below the *x*-axis. Depending on revenue and cost, the profit may be positive, negative, or zero at various points, so its graph may rise and fall.

Section 3.4 Quadratic Functions and Applications

1. $f(x) = x^2 - 3x - 12$
This function is written in the form $y = ax^2 + bx + c$ with a = 1, b = –3 and the parabola opens upward since $a = 1 > 0$.

2. $g(x) = -x^2 + 5x + 15$
The parabola opens downward since $a = -1 < 0$.

3. $h(x) = -3x^2 + 14x + 1$
The parabola opens downward since $a = -3 < 0$.

4. $f(x) = 6.5x^2 - 7.2x + 4$
The parabola opens upward since $a = 6.5 > 0$.

5. $f(x) = 2(x-5)^2 + 7$
This function is written in the form $y = a(x-h)^2 + k$ with $a = 2$, $h = 5$, and $k = 7$. The vertex of the parabola, (h, k), is (5, 7). The parabola opens upward since $a = 2 > 0$.

6. $g(x) = 7(x-8)^2 - 3$
This function is written in the form $y = a(x-h)^2 + k$ with $a = 7$, $h = 8$, and $k = -3$. The vertex of the parabola, (h, k), is (8, –3). The parabola opens upward since $a = 7 > 0$.

7. $h(x) = -4(x+1)^2 - 9$
This function is written in the form $y = a(x-h)^2 + k$ with $a = -4$, $h = -1$, and $k = -9$. The vertex of the parabola, (h, k), is (–1, –9). The parabola opens downward since $a = -4 < 0$.

8. $f(x) = -8(x+12)^2 + 9$
This function is written in the form $y = a(x-h)^2 + k$ with $a = -8$, $h = -12$, and $k = 9$. The vertex of the parabola, (h, k), is (–12, 9). The parabola opens downward since $a = -8 < 0$.

9. i **10.** e **11.** k **12.** c

13. j **14.** l **15.** f **16.** d

17. vertex (1, 2); point (5, 6)
$f(x) = a(x-h)^2 + k$
$f(x) = a(x-1)^2 + 2$
Use (5, 6) to find a.
$6 = a(5-1)^2 + 2 \Rightarrow 4 = 16a \Rightarrow a = \frac{1}{4}$
$f(x) = \frac{1}{4}(x-1)^2 + 2$

18. vertex (–3, 2); point (2, 1)
$f(x) = a(x-h)^2 + k$
$f(x) = a(x+3)^2 + 2$
Use (2, 1) to find a:
$1 = a(2+3)^2 + 2 \Rightarrow -1 = 25a \Rightarrow a = \frac{-1}{25}$
$f(x) = -\frac{1}{25}(x+3)^2 + 2$

19. vertex (–1, –2); point (1, 2)
$f(x) = a(x-h)^2 + k$
$f(x) = a(x+1)^2 - 2$
Use (1, 2) to find a.
$2 = a(1+1)^2 - 2 \Rightarrow 4 = 4a \Rightarrow a = 1$
$f(x) = (x+1)^2 - 2$

20. vertex (2, –4); point (5, 2)
$f(x) = a(x-h)^2 + k$
$f(x) = a(x-2)^2 - 4$
Use (5, 2) to find a.
$2 = a(5-2)^2 - 4 \Rightarrow 6 = 9a \Rightarrow a = \frac{2}{3}$
$f(x) = \frac{2}{3}(x-2)^2 - 4$

21. $f(x) = x^2 + 6x - 3$
$a = 1, b = 6$
To find the vertex of the function, use the vertex formula $x = -\frac{b}{2a}$ and $y = f\left(-\frac{b}{2a}\right)$.
$x = -\frac{6}{2(1)} = -3$
$f(-3) = (-3)^2 + 6(-3) - 3 = -12$
The vertex is (–3, –12).

22. $g(x) = x^2 + 10x + 9$

$a = 1,\ b = 10$

To find the vertex of the function, use the vertex formula $x = -\frac{b}{2a}$ and $y = f\left(-\frac{b}{2a}\right)$.

$x = -\frac{10}{2(1)} = -5$

$f(-5) = (-5)^2 + 10(-5) + 9 = -16$

The vertex is (–5, –16).

23. $f(x) = 3x^2 - 12x + 5$

$a = 3,\ b = -12$

To find the vertex of the function, use the vertex formula $x = -\frac{b}{2a}$ and $y = f\left(-\frac{b}{2a}\right)$.

$x = -\frac{-12}{2(3)} = 2$

$f(2) = 3(2)^2 - 12(2) + 5 = -7$

The vertex is (2, –7).

24. $g(x) = -4x^2 - 16x + 9$

$a = -4,\ b = -16$

To find the vertex of the function, use the vertex formula $x = -\frac{b}{2a}$ and $y = f\left(-\frac{b}{2a}\right)$.

$x = -\frac{-16}{2(-4)} = -2$

$f(-2) = -4(-2)^2 - 16(-2) + 9 = 25$

The vertex is (–2, 25).

25. $f(x) = 3(x-2)^2 - 3$

To find the x-intercepts, set $f(x) = 0$ and then solve for x.

$0 = 3(x-2)^2 - 3 \Rightarrow 3(x-2)^2 = 3 \Rightarrow$

$(x-2)^2 = 1 \Rightarrow x - 2 = \pm 1 \Rightarrow x = 1$ or $x = 3$.

The x-intercepts are 1 and 3 or (1, 0) and (3, 0).

Let $x = 0$ to find the y-intercept.

$f(0) = 3(0-2)^2 - 3 = 9$

The y-intercept is 9.

26. $g(x) = 2x^2 + 8x + 6$

To find the intercepts, set each variable equal to 0. If $x = 0$, $g(0) = 2(0)^2 + 8(0) + 6 = 6$.

The y-intercept is 6.

If $y = 0$, $f(x) = y = 0$, so

$2x^2 + 8x + 6 = 0 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow$

$(x+3)(x+1) = 0 \Rightarrow x = -3$ or $x = -1$

The x-intercepts are –3 and –1.

27. $g(x) = x^2 - 10x + 20$

Set each variable equal to 0.

If $x = 0$,

$g(0) = 0^2 - 10(0) + 20 = 20$.

The y-intercept is 20.

If $y = 0$, $g(x) = y = 0$, so $0 = x^2 - 10x + 20$.

Use the quadratic formula.

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(20)}}{2(1)}$$

$$= \frac{10 \pm \sqrt{100 - 80}}{2} = \frac{10 \pm \sqrt{20}}{2} = \frac{10 \pm 2\sqrt{5}}{2}$$

$$= 5 \pm \sqrt{5}$$

The x-intercepts are $5 \pm \sqrt{5}$.

28. $f(x) = x^2 - 4x - 1$

To find the intercepts, set each variable equal to 0. If $x = 0$, $f(0) = 0^2 - 4(0) - 1 = -1$.

The y-intercept is -1.

If $y = 0$, $f(x) = y = 0$, so $0 = x^2 - 4x - 1$.

Use the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2(1)} = \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

The x-intercepts are $2 \pm \sqrt{5}$.

29. $f(x) = (x+2)^2 \Rightarrow y = 1(x+2)^2 + 0$

The vertex is (–2, 0). The axis is $x = -2$.

x	–1	0	1
y	1	4	9

Plot these points and use the axis of symmetry to find corresponding points on the other side of the axis. Connect the points with a smooth curve.