TT TEXTBOOK TESTBANKS

Solutions Manual for Trigonometry 5th Young

SOLUTIONS MANUAL SAMPLE PREVIEW

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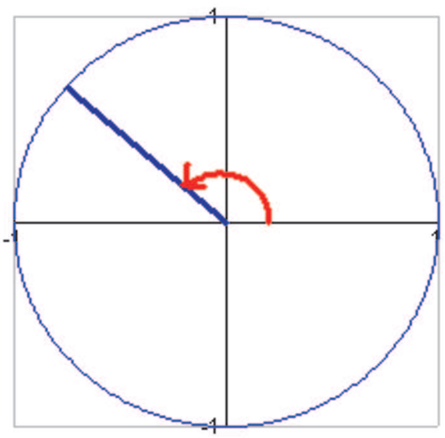
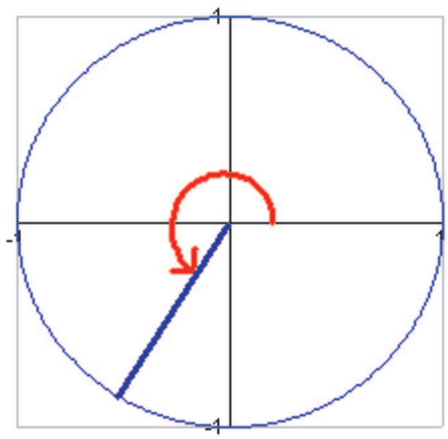
CHAPTER 2

Section 2.1 Solutions --

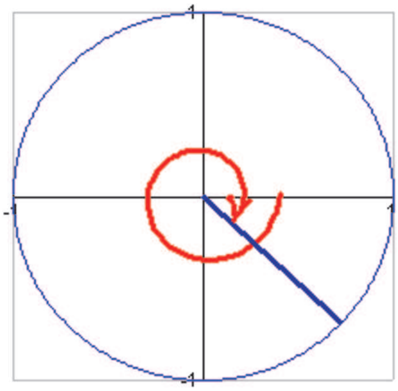
1. QI	**2.** QII	**3.** QII	**4.** QII	**5.** QIV	**6.** QIV

7. y-axis	**8.** x-axis
9. Since $-540° = -360° - 180°$, and $-360°$ corresponds to one full revolution clockwise about the circle, the terminal side ends up on the x-axis.	**10.** Since $-450° = -360° - 90°$, and $-360°$ corresponds to one full revolution clockwise about the circle, the terminal side ends up on the y-axis.

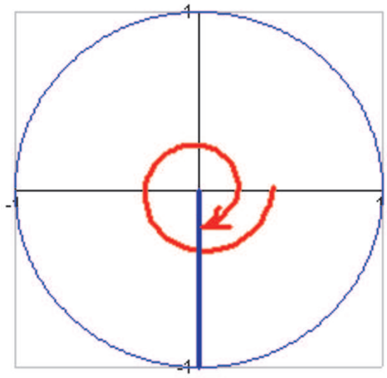
11. QIII	**12.** QIV	**13.** QI	**14.** QII

15. Since $595° = 360° + 235°$, and $360°$ corresponds to one full revolution counterclockwise about the circle, the terminal side ends up in QIII.	**16.** Since $620° = 360° + 260°$, and $360°$ corresponds to one full revolution counterclockwise about the circle, the terminal side ends up in QIII.
17. Since $525° = 360° + 165°$, and $360°$ corresponds to one full revolution counterclockwise about the circle, the terminal side ends up in QII.	**18.** Since $1085° = 3(360°) + 5°$, and $3(360°)$ corresponds to three full revolutions counterclockwise about the circle, the terminal side ends up in QI.
19. Since $-905° = 2(-360°) - 185°$, and $2(-360°)$ corresponds to two full revolutions clockwise about the circle, the terminal side ends up in QII.	**20.** Since $-640° = -360° - 280°$, and $-360°$ corresponds to one full revolution clockwise about the circle, the terminal side ends up in QI.
21. Sketch of $135°$: 	**22.** Sketch of $225°$: 

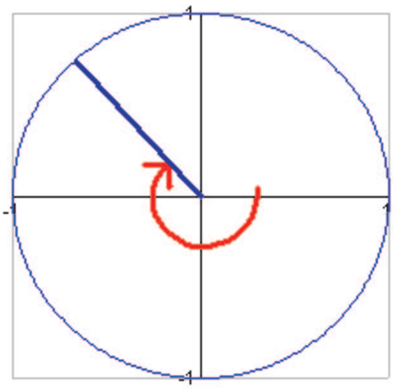
23. Sketch of –405°:



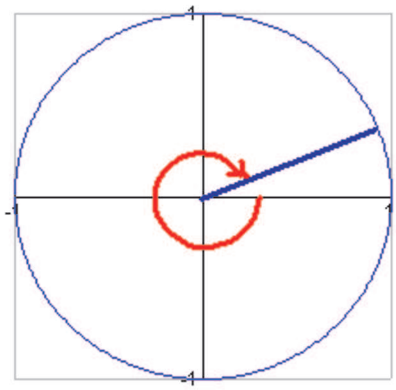
24. Sketch of –450°:



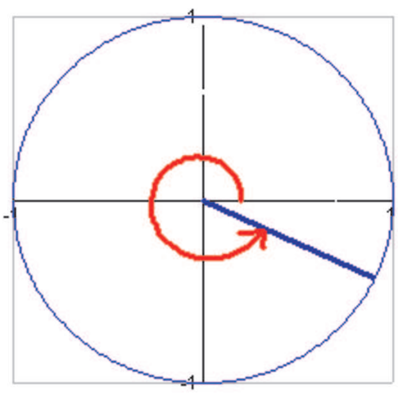
25. Sketch of –225°:



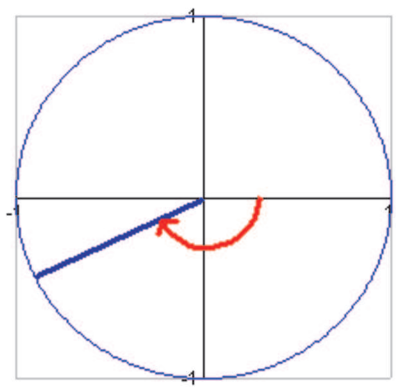
26. Sketch of –330°:



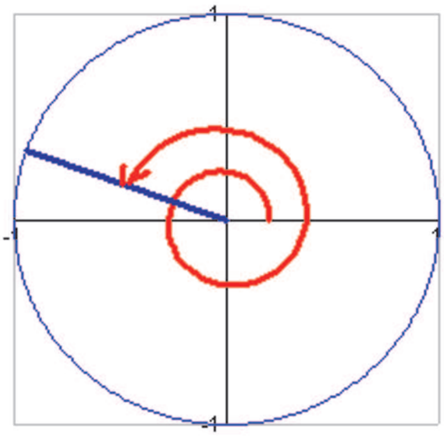
27. Sketch of 330°:



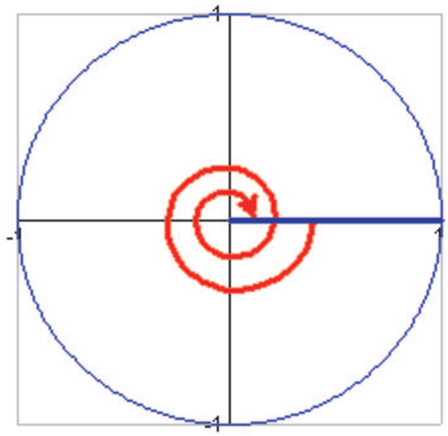
28. Sketch of –150°:



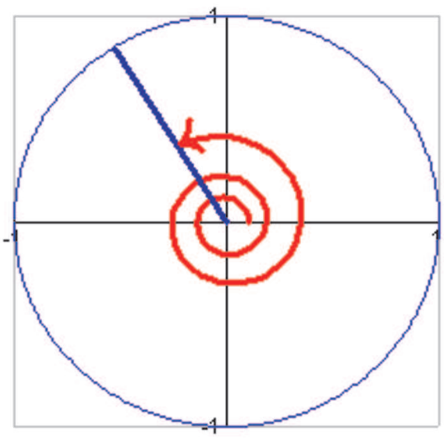
29. Sketch of $510°$:



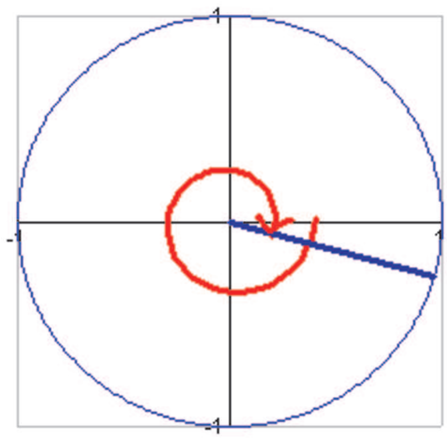
30. Sketch of $-720°$:



31. Sketch of $840°$:



32. Sketch of $-380°$:



33. Observe that $-535° = -360° - 175°$. Since $|-175°| + |185°| = 360°$, the coterminal angle corresponding to $-535°$ is $\boxed{\textbf{c}}$.

34. Observe that $-690° = -360° - 330°$. Since $|-330°| + |30°| = 360°$, the coterminal angle corresponding to $-690°$ is $\boxed{\textbf{a}}$.

35. Observe that $780° = 2(360°) + 60°$. So, $780°$ and $60°$ are coterminal angles Thus, the answer is $\boxed{\textbf{e}}$.

36. Observe that $|265°| + |-95°| = 360°$. So, they are coterminal angles Thus, the answer is $\boxed{\textbf{b}}$.

37. Observe that $-645° = -360° - 285°$. Since $|-285°| + |75°| = 360°$, we know that $-645°$ and $75°$ are coterminal angles Thus, the answer is $\boxed{\textbf{f}}$.

38. Observe that $-560° = -360° - 200°$. Since $\lvert -200° \rvert + \lvert 160° \rvert = 360°$, we know that$-560°$ and $160°$ are coterminal angles Thus, the answer is $\boxed{\mathbf{d}}$.	
39. Observe that $412° = 360° + 52°$. So, the angle with the smallest positive measure that is coterminal with $412°$ is $\boxed{52°}$.	
40. Observe that $379° = 360° + 19°$. So, the angle with the smallest positive measure that is coterminal with $379°$ is $\boxed{19°}$.	
41. Observe that $268° = 360° - 92°$. So, the angle with the smallest positive measure that is coterminal with $-92°$ is $\boxed{268°}$.	
42. Observe that $173° = 360° - 187°$. So, the angle with the smallest positive measure that is coterminal with $-187°$ is $\boxed{173°}$.	
43. Observe that $-390° = -360° - 30°$. Since $330°$ and $-30°$ are coterminal, the angle with the smallest positive measure that is coterminal with $-390°$ is $\boxed{330°}$.	
44. Observe that $945° = 2(360°) + 225°$. So, the angle with the smallest positive measure that is coterminal with $945°$ is $\boxed{225°}$.	
45. Observe that $510° = 360° + 150°$. So, the angle with the smallest positive measure that is coterminal with $510°$ is $\boxed{150°}$.	
46. Observe that $1395° = 3(360°) + 315°$. So, the angle with the smallest positive measure that is coterminal with $1395°$ is $\boxed{315°}$.	
47. Since the given angle is between $0°$ and $180°$, the smallest angle coterminal with it is itself, namely $154°$.	
48. $360° - 360° = 0°$	
49. The terminal side of $-1050°$ coincides with that of $-330°$. And, this is coterminal with $30°$.	
50. The terminal side of $2631°$ corresponds to that of $111°$. This is the angle with smallest positive measure with which it is coterminal.	
51. We seek the angle swept out by the second hand for a time duration of 3 min 20 sec. Note that 60 seconds corresponds to one complete revolution of the second-hand *clockwise*, and 20 seconds corresponds to $\frac{1}{3}$ of one complete revolution *clockwise*. As such, we have 3 minutes corresponds to the angle $3(-360°) = -1080°$, 20 seconds corresponds to the angle $\frac{1}{3}(-360°) = -120°$. Thus, we know that the result of such movement of the second-hand sweeps out the angle $\boxed{-1200°}$, regardless of its actual starting point on the clockface.	

52. Note that the time span from Wednesday 3pm to Thursday 5 pm constitutes 26 hours. For each hour, the hour hand moves $\frac{1}{12}$ of a complete revolution clockwise around the clockface. Hence, 26 hours corresponds to $\frac{26}{12} = 2\frac{1}{6}$ complete revolutions around the clockface. Since one such complete revolution corresponds to $-360°$, the angle swept out during the given time period is $\frac{26}{12}\left(-360°\right) = \boxed{-780°}$.

53. Alexandra's serve corresponds to $3.5 \times 360°$, while Joe's return is equivalent to $2 \times (-360°)$. The sum of these angles, namely 540° is the ball's position.

54. Alexandra's serve corresponds to $6 \times 360° = 2{,}160°$, while Joe's return is equivalent to $6 \times (-360°) = -2{,}160°$.

55. Observe that

$$\underline{\text{Don}}:\ 900° = 2(360°) + 180°, \qquad \underline{\text{Ron}}:\ -900° = 2(-360°) - 180°.$$

Since $180°$ and $-180°$ are coterminal, Don and Ron *do* end at the same place, namely $180°$ from their respective starting points.

56. Observe that

$$\underline{\text{Dan}}:\ 3640° = 10(360°) + 40°, \qquad \underline{\text{Stan}}:\ 1890° = 5(360°) + 90°.$$

Dan and Stan do not end at the same spot. Dan ends 40° counterclockwise from where he started, while Stan ends 90° counterclockwise from where he started.

57. Note that each multiple of 5 cuts off a sector that corresponds to 1/8 of the entire circular face. Follow these instructions to obtain the total angle sum that corresponds to the total turning to unlock the lock with combination **35 – 5 – 20 :**

<u>Step 1</u>: 2 full clockwise turns: $2(360°)$

<u>Step 2</u>: Going clockwise, stop at **35**: $\frac{1}{8}(360°)$

<u>Step 3</u>: 1 full counterclockwise turn: $(360°)$

<u>Step 4</u>: From **35**, go to **5** counterclockwise: $\frac{2}{8}(360°)$

<u>Step 5</u>: From **5**, go to **20** clockwise: $\frac{5}{8}(360°)$

Now, the resulting total angle sum is $360°\left[2 + \frac{1}{8} + 1 + \frac{2}{8} + \frac{5}{8}\right] = \boxed{1440°}$.

58. Note that each multiple of 5 cuts off a sector that corresponds to 1/8 of the entire circular face. Follow these instructions to obtain the total angle sum that corresponds to the total turning to unlock the lock with combination **20 – 15 – 5 :**

<u>Step 1</u>: 2 full clockwise turns: $2(360°)$

<u>Step 2</u>: Going clockwise, stop at **20**: $\frac{4}{8}(360°)$

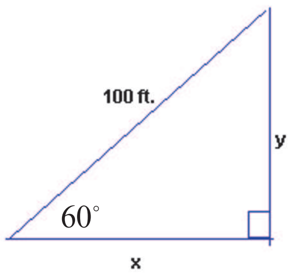
<u>Step 3</u>: 1 full counterclockwise turn: $(360°)$

<u>Step 4</u>: From **20**, go to **15** counterclockwise: $\frac{7}{8}(360°)$

<u>Step 5</u>: From **15**, go to **5** clockwise: $\frac{3}{8}(360°)$

Now, the resulting total angle sum is $360°\left[2+\frac{4}{8}+1+\frac{7}{8}+\frac{3}{8}\right]=\boxed{1665°}$.

59. Consider the following triangle:



Since $\sin 60° = \frac{y}{100 \text{ ft.}}$, we have

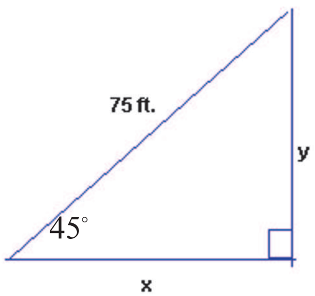
$$y = (100 \text{ ft.})\sin 60° \approx 87 \text{ ft.}$$

Similarly, since $\cos 60° = \frac{x}{100 \text{ ft.}}$, we have

$$x = (100 \text{ ft.})\cos 60° \approx 50 \text{ ft.}$$

So, the horizontal distance is 50 ft. and the vertical distance is 87 ft.

60. Consider the following triangle:



Since $\sin 45° = \frac{y}{75 \text{ ft.}}$, we have

$$y = (75 \text{ ft.})\sin 45° \approx 53 \text{ ft.}$$

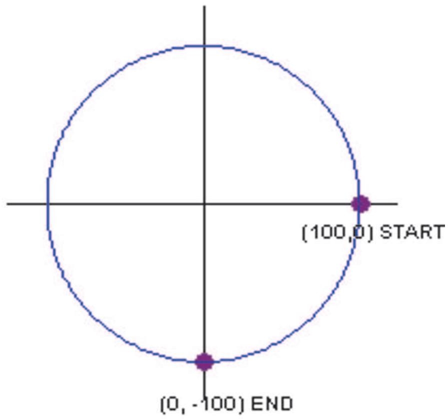
Since the triangle is isosceles, x and y have the same length.

So, the horizontal and vertical distances are both 53 ft.

61. QII

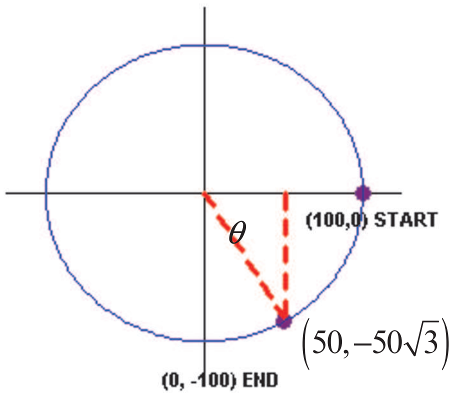
62. On the y-axis

63. Consider the following diagram:



Note that 5 revolutions counter-clockwise corresponds to $5\times360^\circ = 1800^\circ$, and that 3/4 revolution is equivalent to $3/4(360^\circ) = 270^\circ$. So, the angle made is the sum of these two, namely 2070°.

64. Consider the following diagram:



Note that

$$\cos\theta = \frac{50}{100} = \frac{1}{2}$$

$$\sin\theta = \frac{-50\sqrt{3}}{100} = -\frac{\sqrt{3}}{2}$$

Thus, $\theta = \frac{5\pi}{3}$, or equivalently 300°.

Hence, the resulting angle is 300°.

65. Coterminal angles are NOT supplementary angles. For instance, 300° and -60° are coterminal angles, but not supplementary (their sum does not add to 180°).

66. All work <u>except</u> the last line is correct. Indeed, the terminal side of -150° is in QIII, not QII.

67. True, by definition.

68. True. If α is an acute angle in <u>standard</u> position, then its terminal side is in QI if positive, and QIV if negative. However, an obtuse angle must have terminal side in QII if positive, and QIII if negative.

69. False. For example, when $n = 30$, the angles 30° and -30° are not coterminal."

70. Adding multiples of 360° yields coterminal angles. So, the desired angles are $30^\circ + 360^\circ k$, where k is an integer.

71. All angles with positive measure coterminal with 30° are of the form

$30° + n(360°)$, where $n = 0, 1, 2, \ldots$

All angles with negative measure coterminal with 30° are of the form

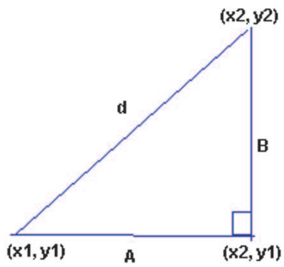
$-330° - n(360°)$, where $n = 0, 1, 2, \ldots$

72. Let x = measure of the desired angle.
We are given that $-360° < x < 0°$. Since the supplement of 130° is 50°, the coterminal angle satisfying the given inequality is $\boxed{-310°}$.

73. $30° - 360°k,\ k = 0, 1, 2, 3, \ldots$

74. These angles are given by $-60° + 360°k$, where $k = 0, \pm 1, \ldots, \pm 5$. So, there are 10 other angles coterminal with α. Including angle α itself, there are 11.

75. Consider the following diagram:



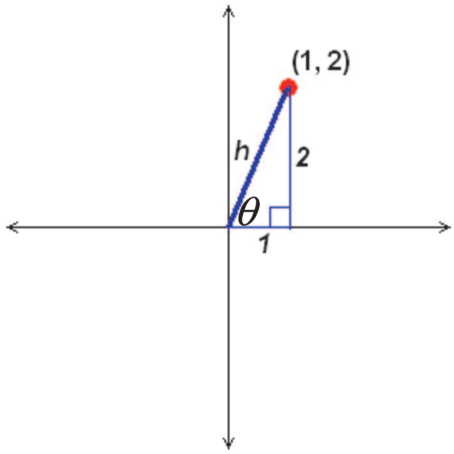
We are given (x_1, y_1) and (x_2, y_2). Consider the new point (x_2, y_1). Observe that $A^2 + B^2 = d^2$ by the Pythagorean theorem. This is the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

76. Since $\theta = 4.5 \times 360° + 45° = \underbrace{4 \times 360°}_{\text{Back to positive } x\text{-axis}} + \underbrace{\left(\frac{1}{2} \times 360° + 45°\right)}_{= 225°}$, the terminal side of θ is in QIII.

Section 2.2 Solutions ---

1. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{1^2+2^2}=\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

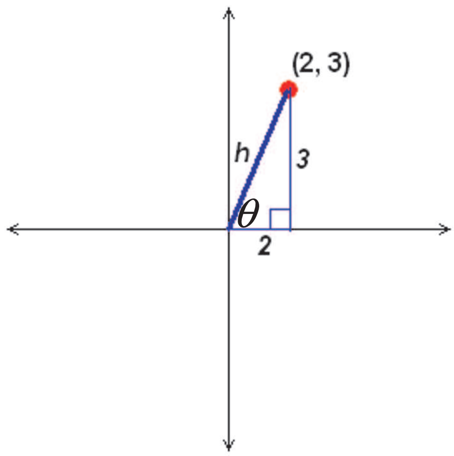
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{2}{1}=2$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{1}{2}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{1}{\sqrt{5}}}=\sqrt{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{2}{\sqrt{5}}}=\frac{\sqrt{5}}{2}$$

2. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{3^2+2^2}=\sqrt{13}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{3}{\sqrt{13}}=\frac{3\sqrt{13}}{13}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{2}{\sqrt{13}}=\frac{2\sqrt{13}}{13}$$

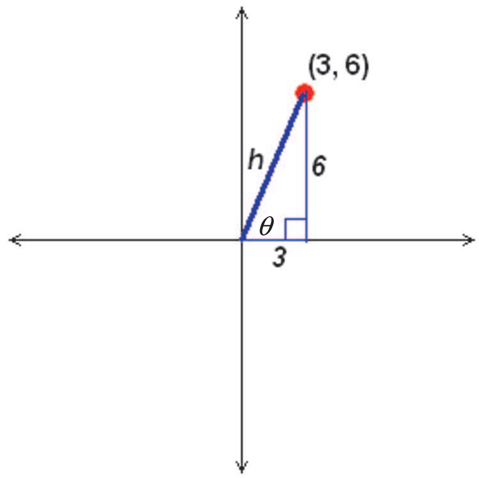
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{3}{2}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{2}{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{2}{\sqrt{13}}}=\frac{\sqrt{13}}{2}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{3}{\sqrt{13}}}=\frac{\sqrt{13}}{3}$$

3. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{3^2+6^2}=\sqrt{45}=3\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{6}{3\sqrt{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{3}{3\sqrt{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

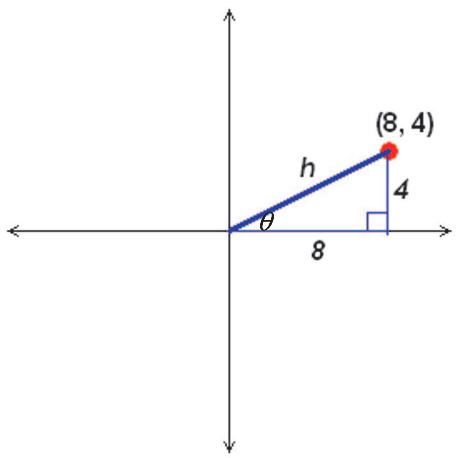
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{6}{3}=2$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{1}{2}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{1}{\sqrt{5}}}=\sqrt{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{2}{\sqrt{5}}}=\frac{\sqrt{5}}{2}$$

4. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{8^2+4^2}=\sqrt{80}=4\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{4}{4\sqrt{5}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{8}{4\sqrt{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

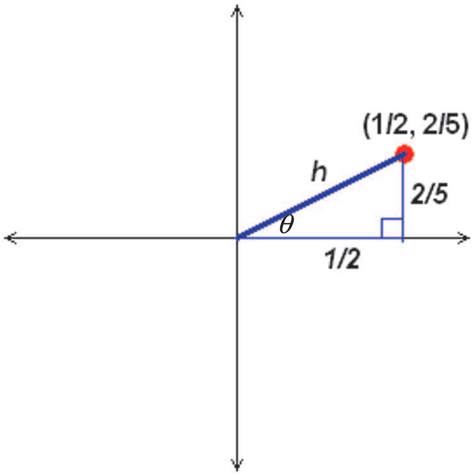
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{4}{8}=\frac{1}{2}$$

$$\cot\theta=\frac{1}{\tan\theta}=2$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{2}{\sqrt{5}}}=\frac{\sqrt{5}}{2}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{1}{\sqrt{5}}}=\sqrt{5}$$

5. Consider the following diagram:



Using the Pythagorean Theorem, we obtain

$$h=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{2}{5}\right)^2}=\sqrt{\frac{1}{4}+\frac{4}{25}}=\frac{\sqrt{41}}{10}.$$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{\frac{2}{5}}{\frac{\sqrt{41}}{10}}=\frac{2}{5}\times\frac{10}{\sqrt{41}}$$

$$=\frac{4}{\sqrt{41}}=\frac{4\sqrt{41}}{41}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{\frac{1}{2}}{\frac{\sqrt{41}}{10}}=\frac{1}{2}\times\frac{10}{\sqrt{41}}$$

$$=\frac{5}{\sqrt{41}}=\frac{5\sqrt{41}}{41}$$

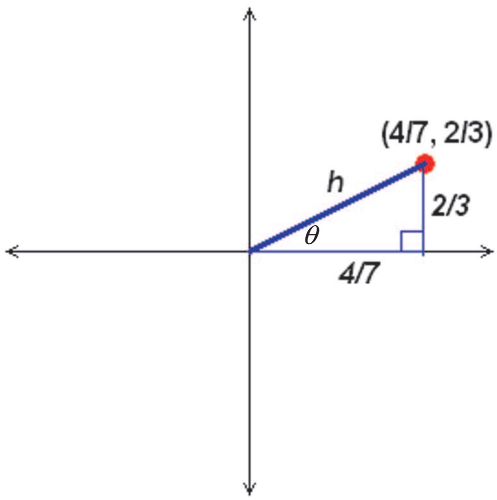
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{\frac{2}{5}}{\frac{1}{2}}=\frac{4}{5}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{5}{4}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{5}{\sqrt{41}}}=\frac{\sqrt{41}}{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{4}{\sqrt{41}}}=\frac{\sqrt{41}}{4}$$

6. Consider the following diagram:



Using the Pythagorean Theorem, we obtain

$$h=\sqrt{\left(\frac{4}{7}\right)^2+\left(\frac{2}{3}\right)^2}=\sqrt{\frac{16}{49}+\frac{4}{9}}=\frac{\sqrt{340}}{21}$$
$$=\frac{2\sqrt{85}}{21}.$$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{\frac{2}{3}}{\frac{2\sqrt{85}}{21}}=\frac{2}{3}\times\frac{21}{2\sqrt{85}}$$
$$=\frac{7}{\sqrt{85}}=\frac{7\sqrt{85}}{85}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{\frac{4}{7}}{\frac{2\sqrt{85}}{21}}=\frac{4}{7}\times\frac{21}{2\sqrt{85}}$$
$$=\frac{6}{\sqrt{85}}=\frac{6\sqrt{85}}{85}$$

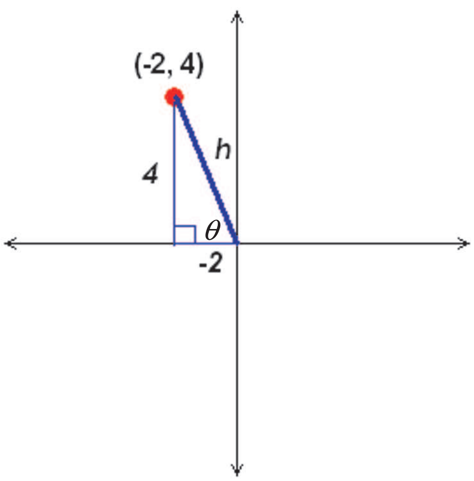
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{\frac{2}{3}}{\frac{4}{7}}=\frac{7}{6}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{6}{7}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{6}{\sqrt{85}}}=\frac{\sqrt{85}}{6}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{7}{\sqrt{85}}}=\frac{\sqrt{85}}{7}$$

7. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{(-2)^2+4^2}=\sqrt{20}=2\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{4}{2\sqrt{5}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-2}{2\sqrt{5}}=\frac{-1}{\sqrt{5}}=-\frac{\sqrt{5}}{5}$$

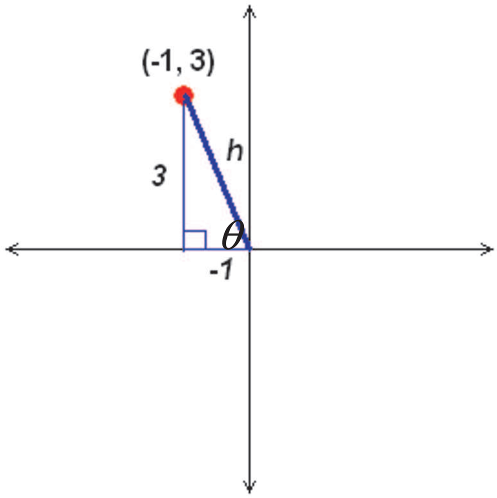
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{4}{-2}=-2$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{1}{2}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-1}{\sqrt{5}}}=-\sqrt{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{2}{\sqrt{5}}}=\frac{\sqrt{5}}{2}$$

8. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

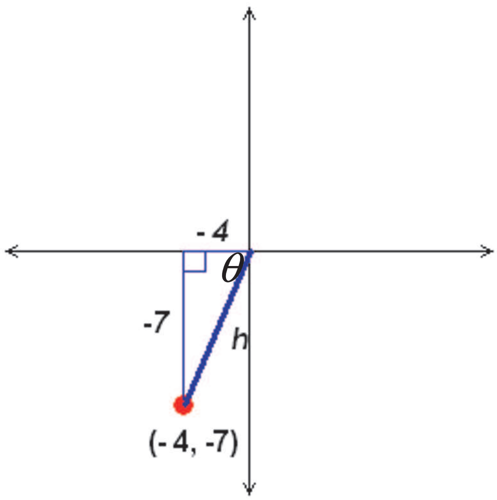
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3}{-1} = -3$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{1}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-1}{\sqrt{10}}} = -\sqrt{10}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{3}{\sqrt{10}}} = \frac{\sqrt{10}}{3}$$

9. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = \sqrt{(-4)^2 + (-7)^2} = \sqrt{65}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-7}{\sqrt{65}} = -\frac{7\sqrt{65}}{65}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-4}{\sqrt{65}} = -\frac{4\sqrt{65}}{65}$$

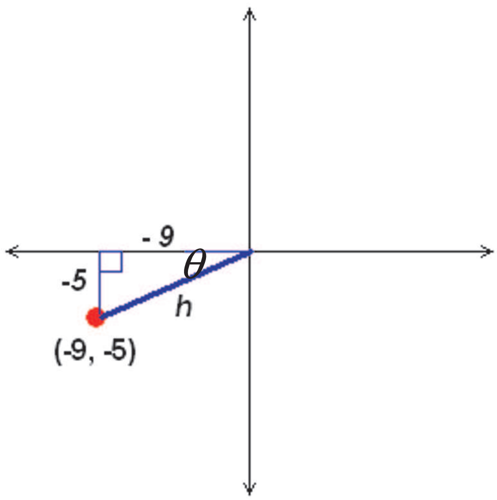
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{-7}{-4} = \frac{7}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{7}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-4}{\sqrt{65}}} = -\frac{\sqrt{65}}{4}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-7}{\sqrt{65}}} = -\frac{\sqrt{65}}{7}$$

10. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{(-5)^2+(-9)^2}=\sqrt{106}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-5}{\sqrt{106}}=-\frac{5\sqrt{106}}{106}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-9}{\sqrt{106}}=-\frac{9\sqrt{106}}{106}$$

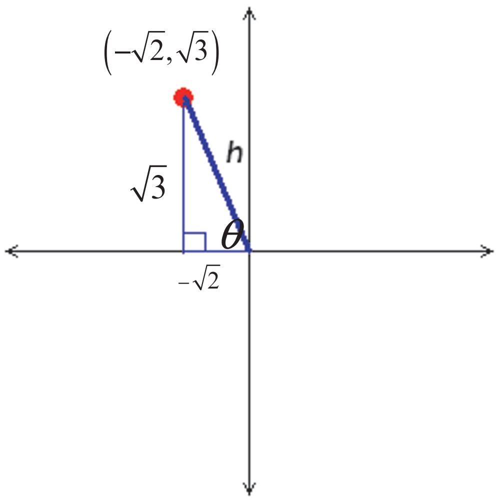
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-5}{-9}=\frac{5}{9}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{9}{5}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-9}{\sqrt{106}}}=-\frac{\sqrt{106}}{9}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{-5}{\sqrt{106}}}=-\frac{\sqrt{106}}{5}$$

11. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{\left(-\sqrt{2}\right)^2+\left(\sqrt{3}\right)^2}=\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{\sqrt{3}}{\sqrt{5}}=\frac{\sqrt{3}\sqrt{5}}{5}=\frac{\sqrt{15}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-\sqrt{2}}{\sqrt{5}}=\frac{-\sqrt{2}\sqrt{5}}{5}=-\frac{\sqrt{10}}{5}$$

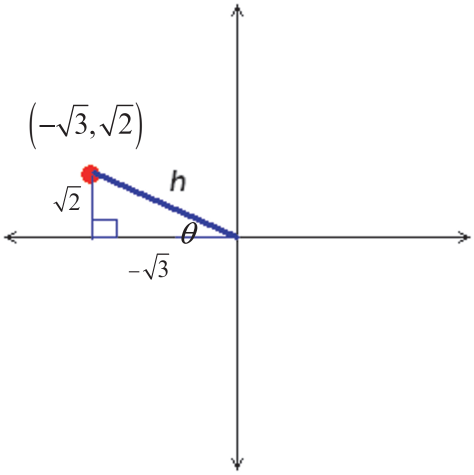
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{\sqrt{3}}{-\sqrt{2}}=\frac{-\sqrt{3}\sqrt{2}}{2}=-\frac{\sqrt{6}}{2}$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{2}{\sqrt{6}}=-\frac{2\sqrt{6}}{6}=-\frac{\sqrt{6}}{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-\sqrt{10}}{5}}=-\frac{5}{\sqrt{10}}=-\frac{\sqrt{10}}{2}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{\sqrt{15}}{5}}=\frac{5}{\sqrt{15}}=\frac{5\sqrt{15}}{15}=\frac{\sqrt{15}}{3}$$

12. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{\left(-\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2}=\sqrt{5}.$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{\sqrt{2}}{\sqrt{5}}=\frac{\sqrt{2}\sqrt{5}}{5}=\frac{\sqrt{10}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-\sqrt{3}}{\sqrt{5}}=\frac{-\sqrt{3}\sqrt{5}}{5}=-\frac{\sqrt{15}}{5}$$

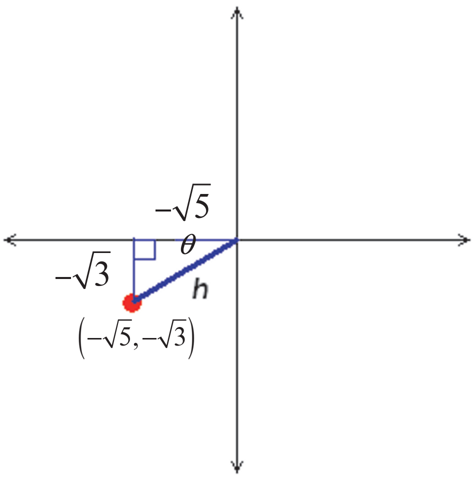
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{\sqrt{2}}{-\sqrt{3}}=\frac{-\sqrt{2}\sqrt{3}}{3}=-\frac{\sqrt{6}}{3}$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{3}{\sqrt{6}}=-\frac{3\sqrt{6}}{6}=-\frac{\sqrt{6}}{2}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-\sqrt{15}}{5}}=-\frac{5}{\sqrt{15}}=-\frac{\sqrt{15}}{3}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{\sqrt{10}}{5}}=\frac{5}{\sqrt{10}}=\frac{5\sqrt{10}}{10}=\frac{\sqrt{10}}{2}$$

13. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{\left(-\sqrt{5}\right)^2+\left(-\sqrt{3}\right)^2}=\sqrt{8}=2\sqrt{2}.$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-\sqrt{3}}{2\sqrt{2}}=\frac{-\sqrt{3}\sqrt{2}}{4}=-\frac{\sqrt{6}}{4}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-\sqrt{5}}{2\sqrt{2}}=\frac{-\sqrt{5}\sqrt{2}}{4}=-\frac{\sqrt{10}}{4}$$

$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-\sqrt{3}}{-\sqrt{5}}=\frac{\sqrt{3}\sqrt{5}}{5}=\frac{\sqrt{15}}{5}$$

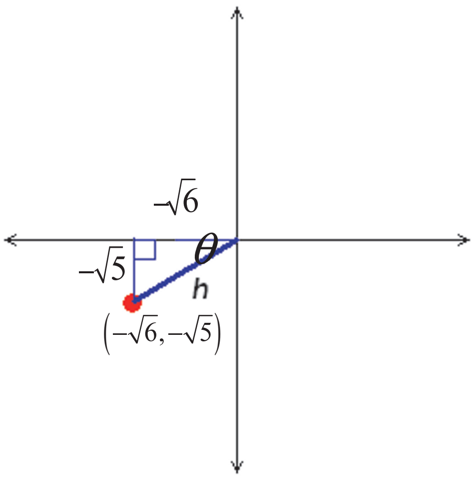
$$\cot\theta=\frac{1}{\tan\theta}=\frac{5}{\sqrt{15}}=\frac{5\sqrt{15}}{15}=\frac{\sqrt{15}}{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-\sqrt{10}}{4}}=-\frac{4}{\sqrt{10}}=-\frac{2\sqrt{10}}{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{-\sqrt{6}}{4}}=-\frac{4}{\sqrt{6}}$$

$$=-\frac{4\sqrt{6}}{6}=-\frac{2\sqrt{6}}{3}$$

14. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{\left(-\sqrt{5}\right)^2+\left(-\sqrt{6}\right)^2}=\sqrt{11}.$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-\sqrt{5}}{\sqrt{11}}=\frac{-\sqrt{5}\sqrt{11}}{11}=-\frac{\sqrt{55}}{11}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-\sqrt{6}}{\sqrt{11}}=\frac{-\sqrt{6}\sqrt{11}}{11}=-\frac{\sqrt{66}}{11}$$

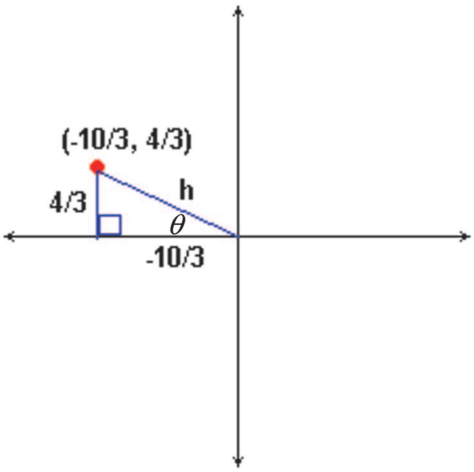
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-\sqrt{5}}{-\sqrt{6}}=\frac{\sqrt{5}\sqrt{6}}{6}=\frac{\sqrt{30}}{6}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{6}{\sqrt{30}}=\frac{6\sqrt{30}}{30}=\frac{\sqrt{30}}{5}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-\sqrt{66}}{11}}=-\frac{11}{\sqrt{66}}=-\frac{\sqrt{66}}{6}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{-\sqrt{55}}{11}}=-\frac{11}{\sqrt{55}}=-\frac{\sqrt{55}}{5}$$

15. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{\left(\frac{-10}{3}\right)^2+\left(\frac{4}{3}\right)^2}=\sqrt{\frac{100}{9}+\frac{16}{9}}$
$=\frac{\sqrt{116}}{3}=\frac{2\sqrt{29}}{3}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{\frac{4}{3}}{\frac{2\sqrt{29}}{3}}=\frac{4}{3}\times\frac{3}{2\sqrt{29}}$$
$$=\frac{2}{\sqrt{29}}=\frac{2\sqrt{29}}{29}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-\frac{10}{3}}{\frac{2\sqrt{29}}{3}}=\frac{-10}{3}\times\frac{3}{2\sqrt{29}}$$
$$=\frac{-5}{\sqrt{29}}=-\frac{5\sqrt{29}}{29}$$

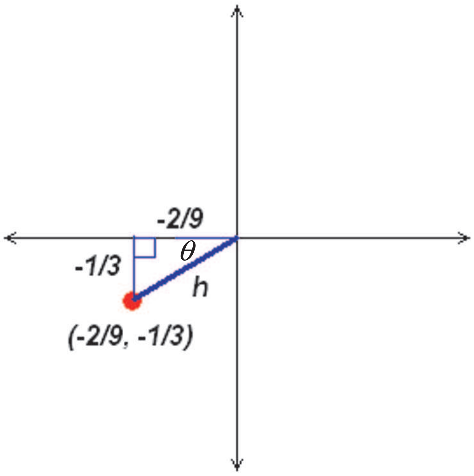
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{\frac{4}{3}}{-\frac{10}{3}}=-\frac{2}{5}$$

$$\cot\theta=\frac{1}{\tan\theta}=-\frac{5}{2}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-5}{\sqrt{29}}}=-\frac{\sqrt{29}}{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{2}{\sqrt{29}}}=\frac{\sqrt{29}}{2}$$

16. Consider the following diagram:



Using the Pythagorean Theorem, we obtain

$$h=\sqrt{\left(\frac{-2}{9}\right)^2+\left(\frac{-1}{3}\right)^2}=\sqrt{\frac{4}{81}+\frac{1}{9}}=\frac{\sqrt{13}}{9}.$$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-\frac{1}{3}}{\frac{\sqrt{13}}{9}}=\frac{-1}{3}\times\frac{9}{\sqrt{13}}=\frac{-3}{\sqrt{13}}=-\frac{3\sqrt{13}}{13}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-\frac{2}{9}}{\frac{\sqrt{13}}{9}}=\frac{-2}{9}\times\frac{9}{\sqrt{13}}=\frac{-2}{\sqrt{13}}=-\frac{2\sqrt{13}}{13}$$

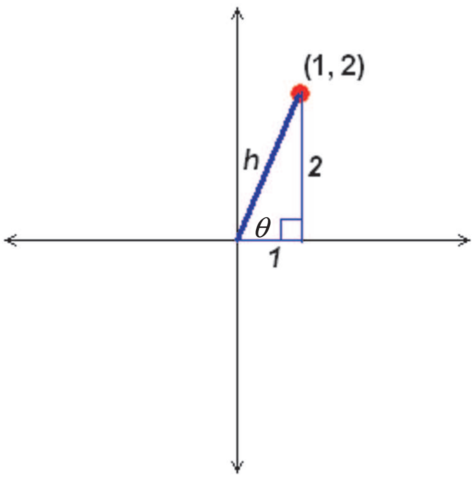
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-\frac{1}{3}}{-\frac{2}{9}}=\frac{3}{2}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{2}{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{-2}{\sqrt{13}}}=-\frac{\sqrt{13}}{2}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{-3}{\sqrt{13}}}=-\frac{\sqrt{13}}{3}$$

17. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{1^2+2^2}=\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

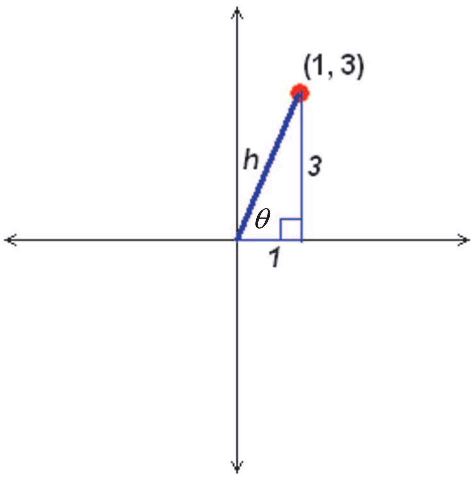
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{2}{1}=2$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{1}{2}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{1}{\sqrt{5}}}=\sqrt{5}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{2}{\sqrt{5}}}=\frac{\sqrt{5}}{2}$$

18. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{1^2+3^2}=\sqrt{10}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{1}{\sqrt{10}}=\frac{\sqrt{10}}{10}$$

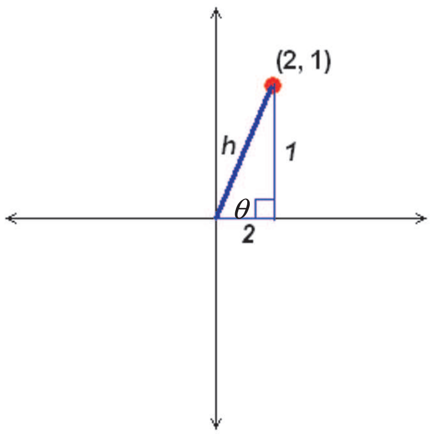
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{3}{1}=3$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{1}{3}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{1}{\sqrt{10}}}=\sqrt{10}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{3}{\sqrt{10}}}=\frac{\sqrt{10}}{3}$$

19. Consider the following diagram:



Note that we can use ANY point on the line $y=\frac{1}{2}x$ $(x\ge 0)$ to form the triangle. In order to avoid the use of fractions, we use the point (2, 1). Using the Pythagorean Theorem, we obtain $h=\sqrt{1^2+2^2}=\sqrt{5}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{1}{\sqrt{5}}=\frac{\sqrt{5}}{5}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$$

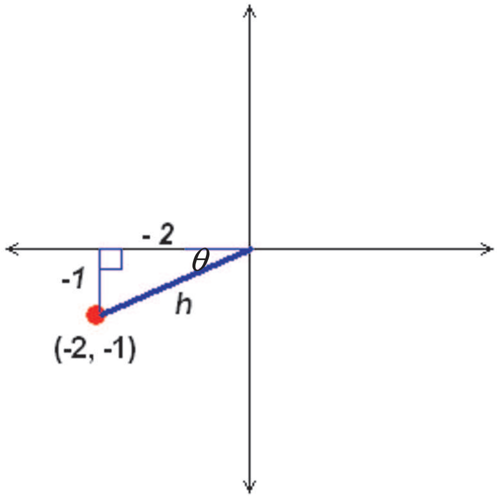
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{1}{2}$$

$$\cot\theta=\frac{1}{\tan\theta}=2$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{2}{\sqrt{5}}}=\frac{\sqrt{5}}{2}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{1}{\frac{1}{\sqrt{5}}}=\sqrt{5}$$

20. Consider the following diagram:



Note that we can use ANY point on the line $y = \frac{1}{2}x$ $(x \le 0)$ to form the triangle. In order to avoid the use of fractions, we use the point $(-2,-1)$. Using the Pythagorean Theorem yields $h = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

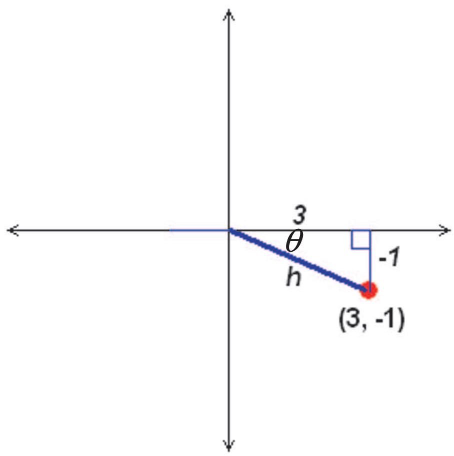
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{-1}{-2} = \frac{1}{2}$$

$$\cot\theta = \frac{1}{\tan\theta} = 2$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-2}{\sqrt{5}}} = -\frac{\sqrt{5}}{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-1}{\sqrt{5}}} = -\sqrt{5}$$

21. Consider the following diagram:



Note that we can use ANY point on the line $y = -\frac{1}{3}x$ $(x \ge 0)$ to form the triangle. In order to avoid the use of fractions, we use the point $(3,-1)$. Using the Pythagorean Theorem yields $h = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

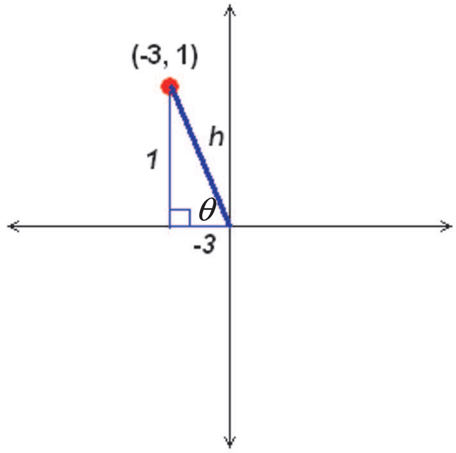
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -\frac{1}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = -3$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{3}{\sqrt{10}}} = \frac{\sqrt{10}}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-1}{\sqrt{10}}} = -\sqrt{10}$$

22. Consider the following diagram:



Note that we can use ANY point on the line $y = -\frac{1}{3}x$ $(x \le 0)$ to form the triangle. In order to avoid the use of fractions, we use the point $(-3,1)$. Using the Pythagorean Theorem, we obtain $h = \sqrt{1^2 + (-3)^2} = \sqrt{10}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-3}{\sqrt{10}} = -\frac{3\sqrt{10}}{10}$$

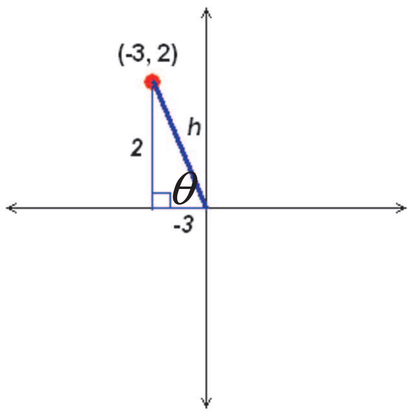
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -\frac{1}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = -3$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-3}{\sqrt{10}}} = -\frac{\sqrt{10}}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{1}{\sqrt{10}}} = \sqrt{10}$$

23. Consider the following diagram:



The equation of the line is $y = -\frac{2}{3}x$ $(x \le 0)$. Note that we can use ANY point on the line to form the triangle. In order to avoid the use of fractions, we use the point $(-3,2)$. Using the Pythagorean Theorem, we obtain $h = \sqrt{(-3)^2 + 2^2} = \sqrt{13}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

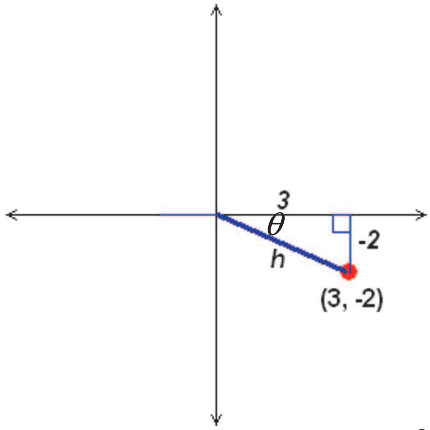
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -\frac{2}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{-3}{\sqrt{13}}} = -\frac{\sqrt{13}}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{2}{\sqrt{13}}} = \frac{\sqrt{13}}{2}$$

24. Consider the following diagram:



The equation of the line is $y = -\frac{2}{3}x$ $(x \geq 0)$. Note that we can use ANY point on the line to form the triangle. In order to avoid the use of fractions, we use the point $(3,-2)$. Using the Pythagorean Theorem, we obtain $h = \sqrt{3^2 + (-2)^2} = \sqrt{13}$.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -\frac{2}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{3}{\sqrt{13}}} = \frac{\sqrt{13}}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{-2}{\sqrt{13}}} = -\frac{\sqrt{13}}{2}$$

25. Since $\theta = 450° = 360° + 90°$, the standard position of θ is $90°$. So,

$\sin\theta = 1$
$\cos\theta = 0$
$\tan\theta$ is undefined
$\cot\theta = 0$
$\sec\theta$ is undefined
$\csc\theta = 1$

26. Since $\theta = 540° = 360° + 180°$, the standard position of θ is $180°$. So,

$\sin\theta = 0$
$\cos\theta = -1$
$\tan\theta = 0$
$\cot\theta$ is undefined
$\sec\theta = -1$
$\csc\theta$ is undefined

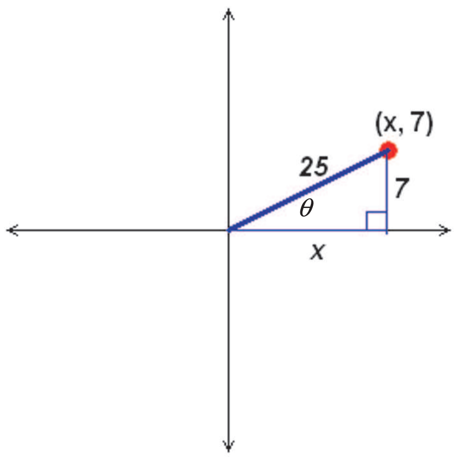
27. Since $\theta = 630° = 360° + 270°$, the standard position of θ is $270°$. So,

$\sin\theta = -1$
$\cos\theta = 0$
$\tan\theta$ is undefined
$\cot\theta = 0$
$\sec\theta$ is undefined
$\csc\theta = -1$

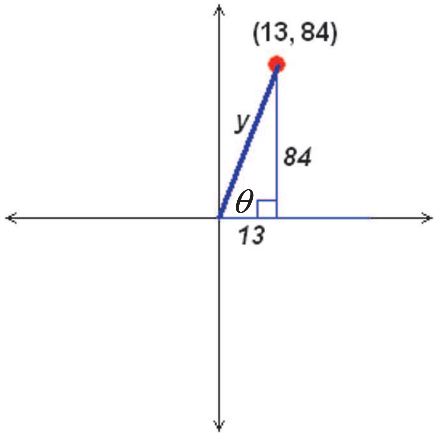
28. Since $\theta = 720° = 2(360°)$, the standard position of θ is $0°$. So,

$\sin\theta = 0$
$\cos\theta = 1$
$\tan\theta = 0$
$\cot\theta$ is undefined
$\sec\theta = 1$
$\csc\theta$ is undefined

29. Since $\theta = -270° = -360° + 90°$, the standard position of θ is $90°$. So, $\sin\theta = 1$ $\cos\theta = 0$ $\tan\theta$ is undefined $\cot\theta = 0$ $\sec\theta$ is undefined $\csc\theta = 1$	**30.** Since $\theta = -180° = -360° + 180°$, the standard position of θ is $180°$. So, $\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$ $\cot\theta$ is undefined $\sec\theta = -1$ $\csc\theta$ is undefined.
31. Since $\theta = -90° = -360° + 270°$, the standard position of θ is $270°$. So, $\sin\theta = -1$ $\cos\theta = 0$ $\tan\theta$ is undefined $\cot\theta = 0$ $\sec\theta$ is undefined $\csc\theta = -1$	**32.** Since the standard position of $\theta = -360°$ is $0°$, we have $\sin\theta = 0$ $\cos\theta = 1$ $\tan\theta = 0$ $\cot\theta$ is undefined $\sec\theta = 1$ $\csc\theta$ is undefined.
33. Since $\theta = -450° = -2(360°) + 270°$, the standard position of θ is $270°$. So, $\sin\theta = -1$ $\cos\theta = 0$ $\tan\theta$ is undefined $\cot\theta = 0$ $\sec\theta$ is undefined $\csc\theta = -1$	**34.** Since $\theta = -540° = -2(360°) + 180°$, the standard position of θ is $180°$. So, $\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$ $\cot\theta$ is undefined $\sec\theta = -1$ $\csc\theta$ is undefined
35. Since $\theta = -630° = -2(360°) + 90°$, the standard position of θ is $90°$. So, $\sin\theta = 1$ $\cos\theta = 0$ $\tan\theta$ is undefined $\cot\theta = 0$ $\sec\theta$ is undefined $\csc\theta = 1$	**36.** Since the standard position of $\theta = -720° = -2(360°)$ is $0°$, we have $\sin\theta = 0$ $\cos\theta = 1$ $\tan\theta = 0$ $\cot\theta$ is undefined $\sec\theta = 1$ $\csc\theta$ is undefined

37. Since $\theta = 810° = 2(360°) + 90°$, the standard position of θ is $90°$. So, $\sin 810° = 1$ $\cos 810° = 0$ $\tan 810°$ is undefined $\csc 810° = 1$ $\sec 810°$ is undefined $\cot 810° = 0$	**38.** Since $\theta = 900° = 2(360°) + 180°$, the standard position of θ is $180°$. So, $\sin 900° = 0$ $\cos 900° = -1$ $\tan 900° = 0$ $\csc 900°$ is undefined $\sec 900° = -1$ $\cot 900°$ is undefined
39. Since $\theta = -810° = -2(360°) - 90°$, the standard position of θ is $-90°$, which is coterminal with $270°$. So, $\sin(-810°) = -1$ $\cos(-810°) = 0$ $\tan(-810°)$ is undefined $\csc(-810°) = -1$ $\sec(-810°)$ is undefined $\cot(-810°) = 0$	**40.** Since $\theta = -900° = -2(360°) - 180°$, the standard position of θ is $-180°$, which is coterminal with $180°$. So, $\sin(-900°) = 0$ $\cos(-900°) = -1$ $\tan(-900°) = 0$ $\csc(-900°)$ = undefined $\sec(-900°) = -1$ $\cot(-900°)$ = undefined
41. Consider the following diagram:  	First, find x using the Pythagorean Theorem: $x^2 + 7^2 = 25^2$ $x = \sqrt{576} = 24$ Hence, $\tan\theta = \frac{\text{opp}}{\text{adj}} = \boxed{\frac{7}{24}}$.

42. Consider the following diagram:

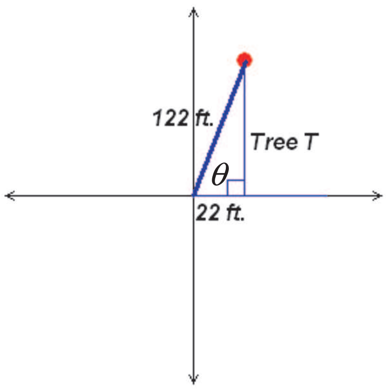


First, find y using the Pythagorean Theorem:

$$y^2 = 84^2 + 13^2$$
$$y = \sqrt{7225} = 85$$

Hence, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \boxed{\dfrac{13}{85}}$.

43. Consider the following diagram:



We need to determine T.

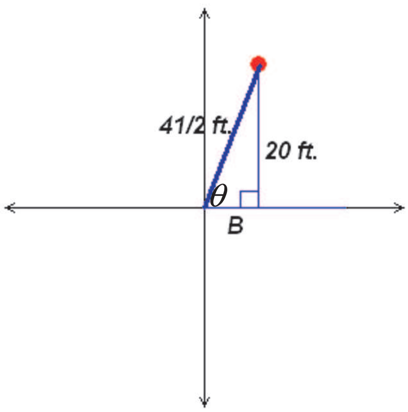
Observe that $\cos\theta = \dfrac{11}{61} = \dfrac{22}{122}$.

Hence, using the Pythagorean Theorem yields:

$$(22\text{ ft.})^2 + T^2 = (122\text{ ft.})^2$$
$$T = \sqrt{14{,}400\text{ ft.}^2} = 120\text{ ft.}$$

So, the tree is $\boxed{120\text{ ft.}}$ high.

44. Consider the following diagram:



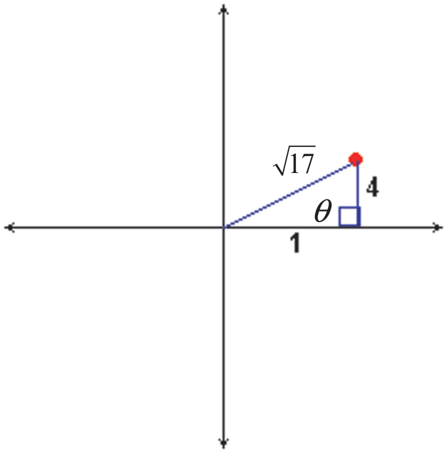
We need to determine B, the distance between the base of the tree and point on the ground.

Observe that $\sin\theta = \dfrac{40}{41} = \dfrac{20}{\frac{41}{2}}$.

Hence, using the Pythagorean Theorem yields:

$$(20\text{ ft.})^2 + B^2 = (\tfrac{41}{2}\text{ ft.})^2$$
$$T = \sqrt{\frac{81}{4}}\text{ ft.} = \boxed{4.5\text{ ft.}}$$

45. Consider this triangle:

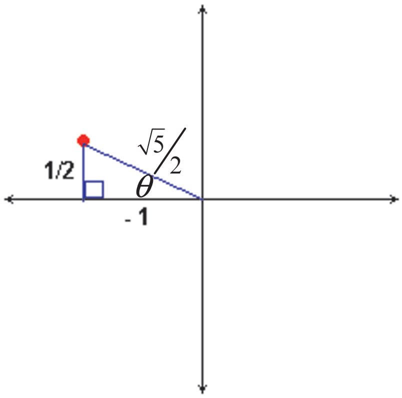


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = 4$$

46. Consider this triangle:

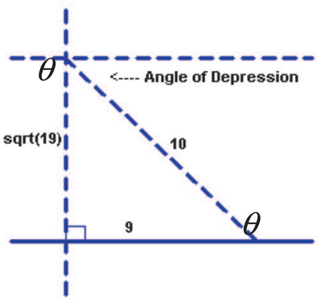


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1/2}{\sqrt{5}/2} = \frac{\sqrt{5}}{5}$$

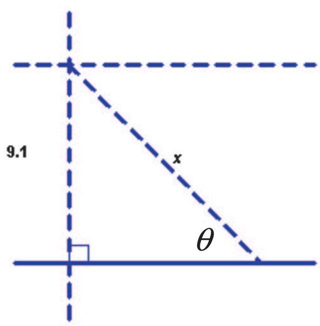
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = -\frac{1}{\sqrt{5}/2} = -\frac{2\sqrt{5}}{5}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -\frac{1}{2}$$

47. Since we are given that $\cos\theta = 0.9$, we have the triangle



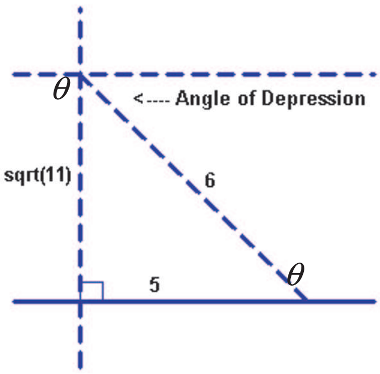
The picture for our current situation is



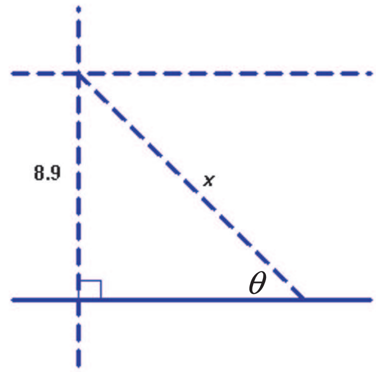
By similar triangles, we see that

$$\frac{\sqrt{19}}{9.1} = \frac{10}{x} \Rightarrow x \approx 20.87 \text{ ft.}$$

48. Since we are given that $\sec\theta = \frac{6}{5}$, and so $\cos\theta = \frac{5}{6}$, we have the triangle:



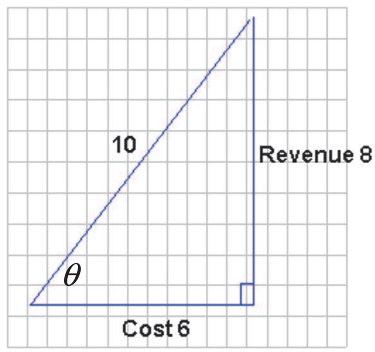
The picture for our current situation is



By similar triangles, we see that

$$\frac{\sqrt{11}}{8.9} = \frac{6}{x} \Rightarrow x \approx 16.1 \text{ ft.}$$

49. Since $\sin\theta = \frac{8}{10}$, we have the following triangle (where the length of the remaining leg was computed using the Pythagorean theorem):



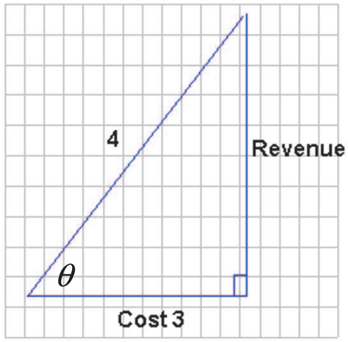
Hence,

$$\tan\theta = \frac{\text{Revenue}}{\text{Cost}} = \frac{8}{6} > 1.$$

Since $\tan\theta > 1$, this represents a profit.

50. Using the Pythagorean theorem in the following triangle yields

$$\text{Revenue}^2 + 3^2 = 4^2 \;\Rightarrow\; \text{Revenue} = \sqrt{7}.$$

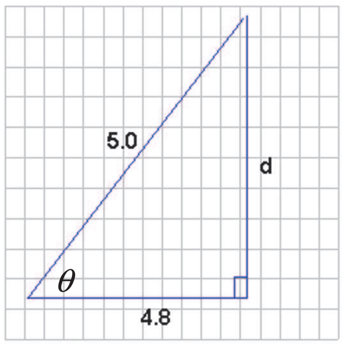


Hence,

$$\tan q = \frac{\text{Revenue}}{\text{Cost}} = \frac{\sqrt{7}}{3} < 1.$$

Since $\tan\theta < 1$, this represents a loss.

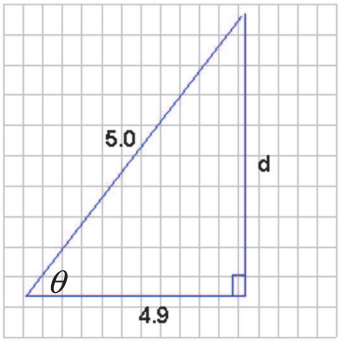
51. Since $\cos\theta = \dfrac{4.8}{5.0}$, we have the following triangle:



Now, using the Pythagorean theorem yields

$$4.8^2 + d^2 = 5.0^2 \;\Rightarrow\; d = \sqrt{1.96} = 1.4\,\text{cm}.$$

52. Since $\sec\theta = \frac{5.0}{4.9}$, we know that $\cos\theta = \frac{4.9}{5.0}$ and so, we have the following triangle:



Now, using the Pythagorean theorem yields

$$4.9^2 + d^2 = 5.0^2 \Rightarrow d = \sqrt{0.99} \approx 1.0\,\text{cm}.$$

53. Here, $r = \sqrt{1^2 + 2^2} = \sqrt{5}$, not 5.

54. $\frac{1}{0}$ does NOT equal 0, it is undefined.

55. False. Observe that

$$\sin 30^\circ = \frac{1}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \sin 90^\circ = 1.$$

But, $\frac{1}{2} + \frac{\sqrt{3}}{2} \neq 1$.

56. False. Even though $\tan 0^\circ = 0$, thereby resulting in the statement $\tan 90^\circ = \tan 90^\circ$, this statement is not a truth since $\tan 90^\circ$ is undefined.

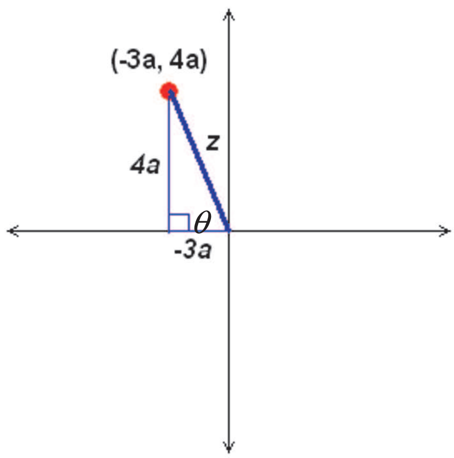
57. True. The angles θ and $\theta + 360^\circ n$ (where n is an integer) are coterminal, so that $\cos\theta = \cos(\theta + 360^\circ n)$.

58. True. This is true since sine is periodic with period 2π (which corresponds to 360°).

59. First, note that it is not possible for $a = 0$ since in such case there would be no triangle. Thus, there are only two cases to consider.

Case 1: $a > 0$

Consider the following diagram:

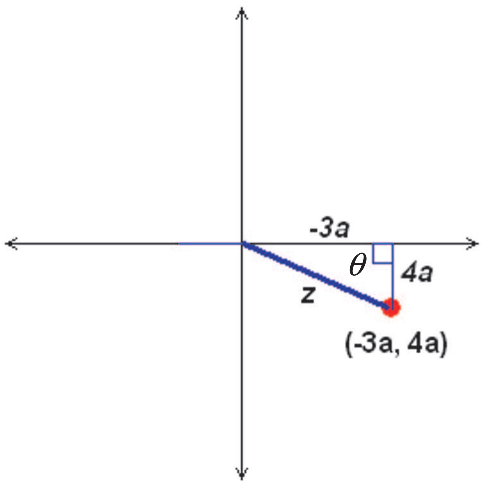


From the Pythagorean Theorem, it follows that $z = \sqrt{(4a)^2 + (-3a)^2} = 5a$.

So, $\cos\theta = \frac{-3a}{5a} = -\frac{3}{5}$.

Case 2: $a < 0$

Consider the following diagram:



From the Pythagorean Theorem, it follows that $z = \sqrt{(4a)^2 + (-3a)^2} = 5a$.

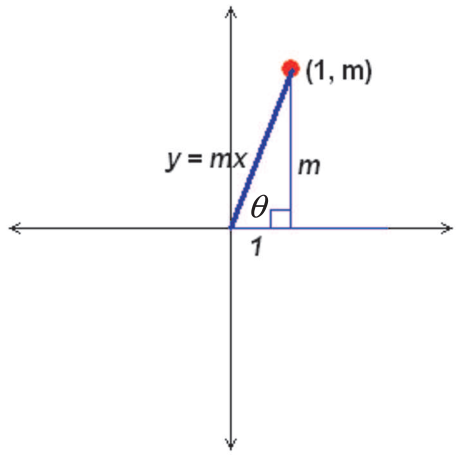
So, $\cos\theta = \frac{-3a}{5a} = -\frac{3}{5}$, which is the same conclusion of Case 1.

60. Referring to the diagram in Exercise 59, we observe that in both cases, $\sin\theta = \frac{4a}{5a} = \frac{4}{5}$.

61. **Case 1:** $m = 0$

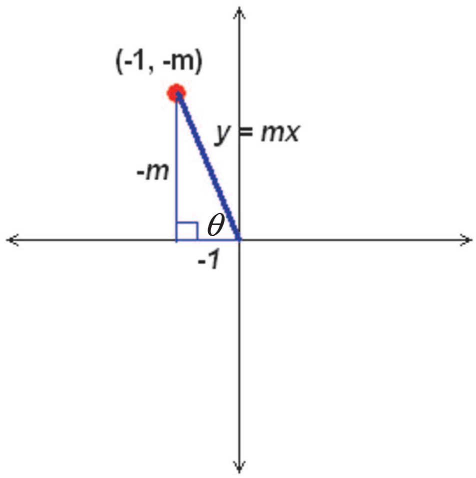
In such case, $\tan 0 = 0 = m$.

Case 2: $m > 0$ Consider the following diagram:



Observe that $\tan\theta = \frac{m}{1} = m$.

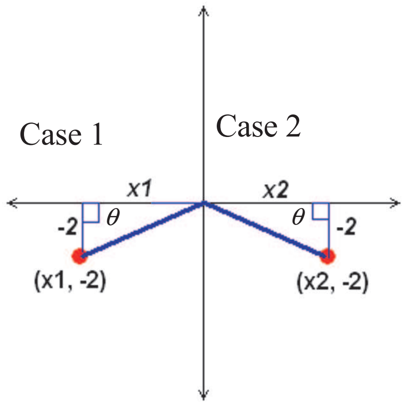
Case 3: $m < 0$ Consider the following diagram:



Similarly, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-m}{-1} = m$.

62. First, note that $\csc\theta = -\dfrac{\sqrt{29}}{2}$, so that $\sin\theta = -\dfrac{2}{\sqrt{29}}$.

Consider the following diagram:

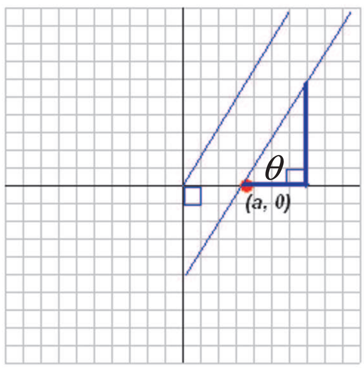


Using the Pythagorean Theorem, we see that

$$x_1^2 + (-2)^2 = \left(\sqrt{29}\right)^2 \quad \text{and} \quad x_2^2 + (-2)^2 = \left(\sqrt{29}\right)^2.$$

Hence, we have $x_1 = x_2 = \pm 5$. Thus, the two possible x values are $\boxed{\pm 5}$.

63. Consider the following diagram:



Suppose the form of the equation of the line is given by $y = mx + b$. We need to determine m and b to find the equation of this line.

First, substitute the point $(a, 0)$ into the equation: $0 = ma + b$ so that $b = -ma$. So, the equation of the line thus far is $y = mx - ma = m(x - a)$.

Next, note that this line is parallel to the corresponding line translate to pass through the origin—the slope of that line is $\tan\theta$ (by Exercise 61). Since parallel lines have the same slope, we know that $m = \tan\theta$.

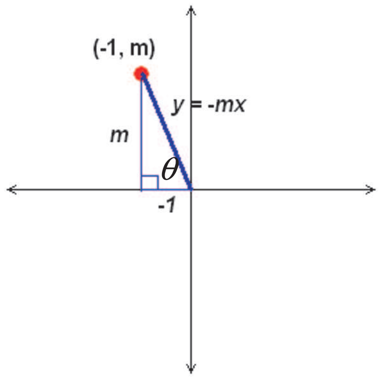
Thus, the equation of the line is $\boxed{y = (\tan\theta)(x - a)}$.

64. First, note that we need to find a relationship between the slope the line and θ (the angle the line makes with the negative x-axis). This is done in cases, as in Exercise 61.

Case 1: $m = 0$

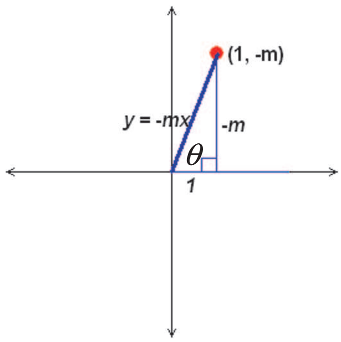
In such case, $\tan 0 = 0 = m$.

Case 2: $m > 0$ Consider the following diagram:



Observe that $\tan\theta = \frac{m}{-1} = -m$.

Case 3: $m < 0$ Consider the following diagram:



Similarly, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-m}{1} = -m.$

Now, to find the equation of a line with negative slope passing through the point $(a,0)$, we proceed as in Exercise 63. Suppose the form of the equation of the line is given by $y = mx + b$. We need to determine m and b to find the equation of this line.

First, substitute the point $(a,0)$ into the equation: $0 = ma + b$ so that $b = -ma$. So, the equation of the line thus far is $y = mx - ma = m(x-a)$.

Next, note that this line is parallel to the corresponding line translate to pass through the origin—the slope of that line is $-\tan\theta$ (by the above discussion). Since parallel lines have the same slope, we know that $m = -\tan\theta$.

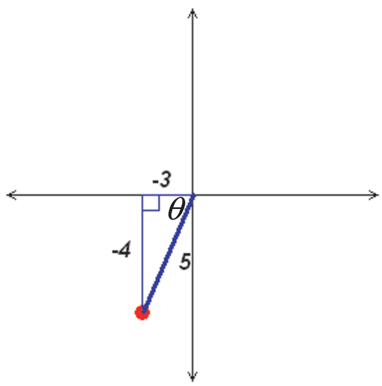
Thus, the equation of the line is $y = (-\tan\theta)(x-a) = \boxed{(\tan\theta)(a-x)}$.

65. $\sec x = \dfrac{1}{\cos x}$ is undefined when $\cos x = 0$. This occurs when $x = 90° + (180k)°$ for any integer k.	
66. If $\csc x = \dfrac{1}{\sin x}$ is undefined, then $\sin x = 0$.	
67. $\sin 270° = -1$	**68.** $\cos 270° = 0$
69. $\tan 270° = \dfrac{\sin 270°}{\cos 270°}$ is undefined.	**70.** $\cot 270° = \dfrac{\cos 270°}{\sin 270°} = 0$
71. $\cos(-270°) = 0$	**72.** $\sin(-270°) = 1$
73. $\sec(-270°)$ is undefined.	**74.** $\sec(270°)$ is undefined.
75. $\csc(270°) = -1$	**76.** $\csc(-270°) = 1$

Section 2.3 Solutions ---

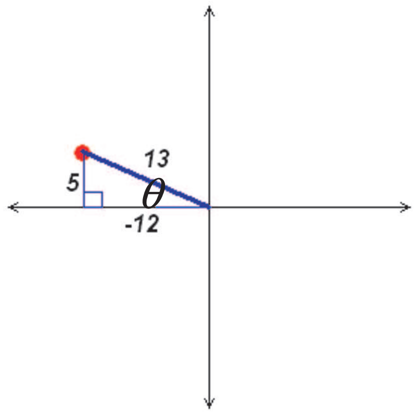
1. QIV	**2.** QII
3. QII (Need $\cos\theta < 0$ and $\sin\theta > 0$.)	**4.** QIII (Need $\cos\theta < 0$ and $\sin\theta < 0$.)
5. QI (Need $\cos\theta > 0$ and $\sin\theta > 0$.)	**6.** QIII (Need $\cos\theta < 0$ and $\sin\theta < 0$.)

7. Consider the following diagram:



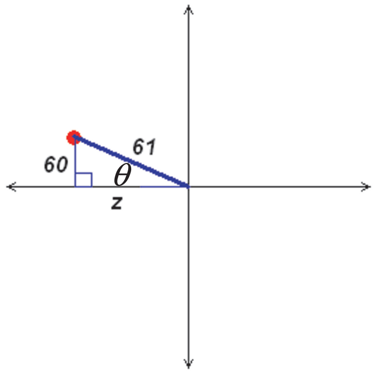
Observe that $\sin\theta = \boxed{-\dfrac{4}{5}}$.

8. Consider the following diagram:



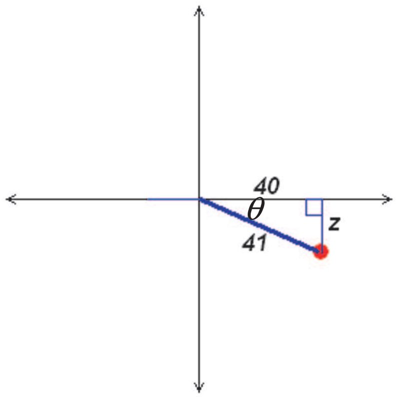
Observe that $\cos\theta = \boxed{-\dfrac{12}{13}}$.

9. Consider the following diagram:



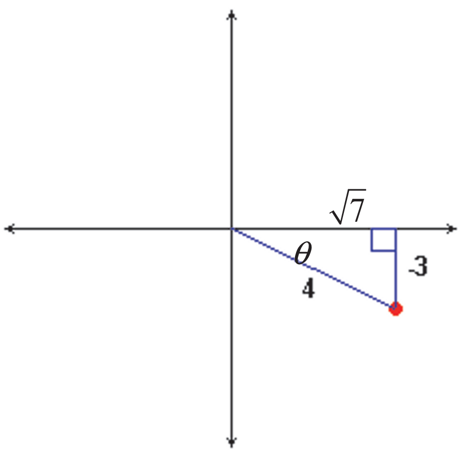
Using the Pythagorean Theorem, we obtain $z = -\sqrt{61^2 - 60^2} = -11$. So, $\tan\theta = \boxed{-\frac{60}{11}}$.

10. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $z = -\sqrt{41^2 - 40^2} = -9$. So, $\tan\theta = \boxed{-\frac{9}{40}}$.

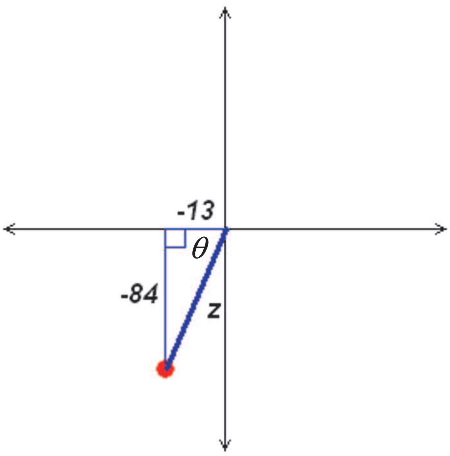
11. Consider the following diagram:



Observe that $\sec\theta = \frac{1}{\cos\theta} = \frac{4}{\sqrt{7}} = \frac{4\sqrt{7}}{7}$.

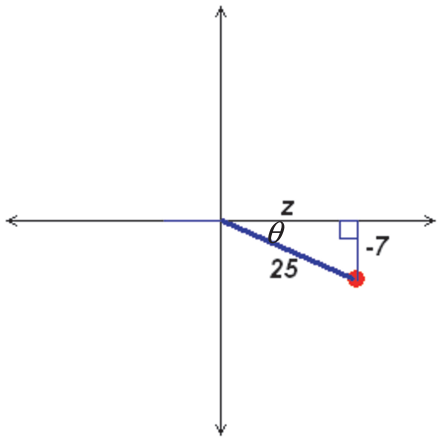
12. There is no solution here since there does not exist a value of θ for which $\cos\theta > 1$.

13. Consider the following diagram:



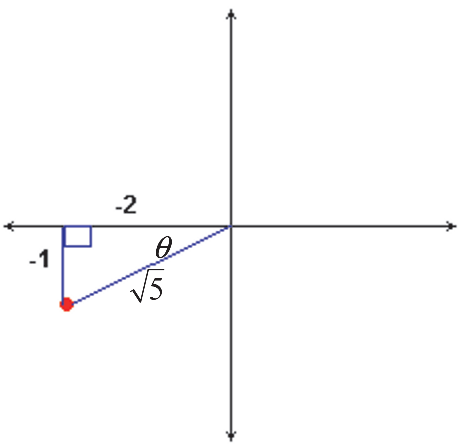
Using the Pythagorean Theorem, we obtain $z=\sqrt{(-13)^2+(-84)^2}=85$. So, $\sin\theta=\boxed{-\frac{84}{85}}$.

14. Consider the following diagram:



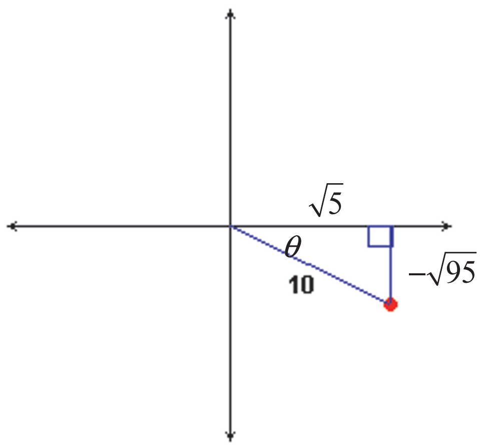
Using the Pythagorean Theorem, we obtain $z=\sqrt{25^2-(-7)^2}=24$. So, $\cos\theta=\boxed{\frac{24}{25}}$.

15. Consider the following diagram:



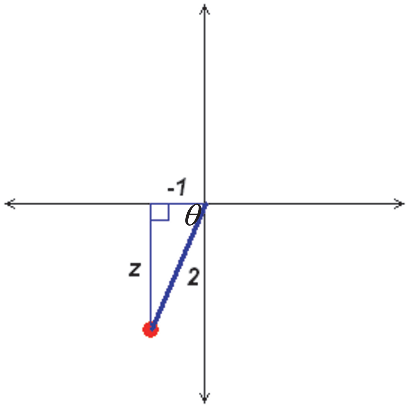
Observe that $\cot\theta=\frac{1}{\tan\theta}=\frac{1}{1/2}=2$.

16. Consider the following diagram:



Observe that $\sin\theta=-\frac{\sqrt{95}}{10}$.

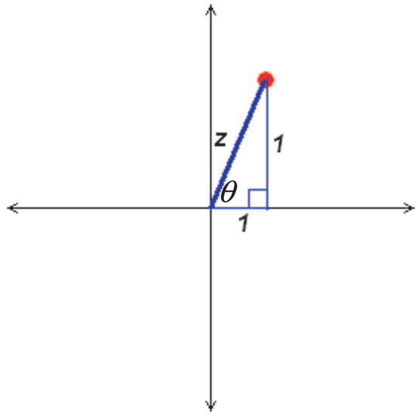
17. Since $\sec\theta = \dfrac{1}{\cos\theta} = -2$, we see that $\cos\theta = -\dfrac{1}{2}$. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $z = -\sqrt{2^2-(-1)^2} = -\sqrt{3}$. So,

$$\tan\theta = \frac{-\sqrt{3}}{-1} = \boxed{\sqrt{3}}.$$

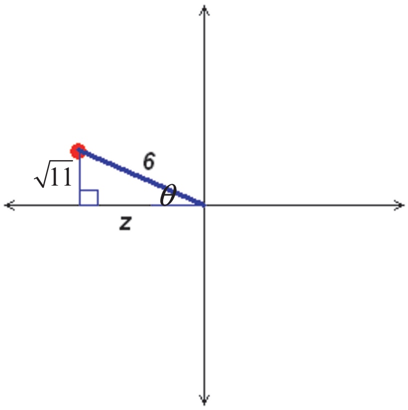
18. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $z = \sqrt{1^2+1^2} = \sqrt{2}$. So,

$$\sin\theta = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}.$$

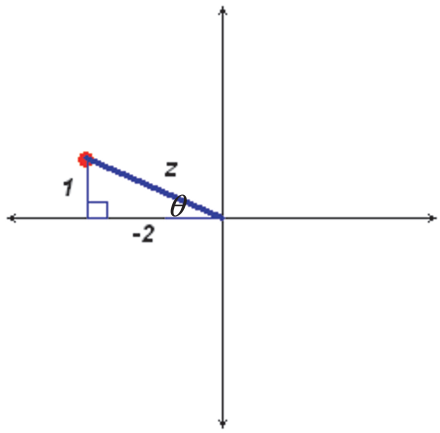
19. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $z = -\sqrt{6^2-\left(\sqrt{11}\right)^2} = -5$. So,

$$\tan\theta = \frac{\sqrt{11}}{-5} = \boxed{-\frac{\sqrt{11}}{5}}.$$

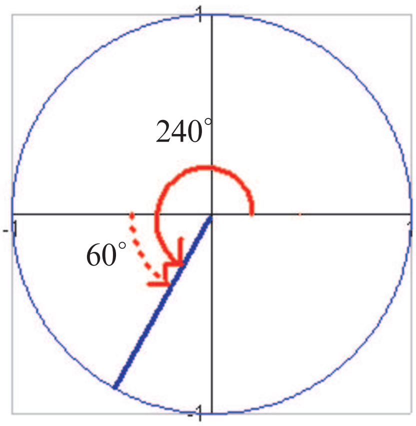
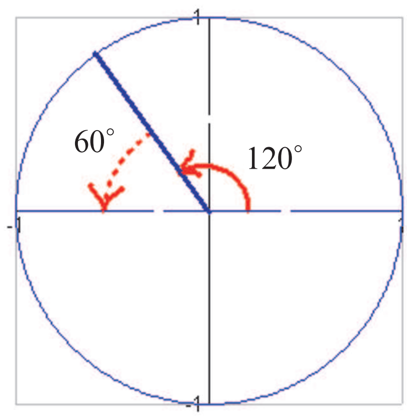
20. Consider the following diagram:



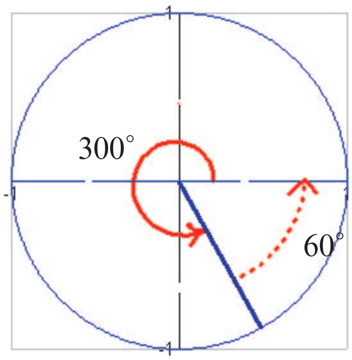
Using the Pythagorean Theorem, we obtain $z = \sqrt{1^2+(-2)^2} = \sqrt{5}$. So,

$$\cos\theta = \frac{-2}{\sqrt{5}} = \boxed{-\frac{2\sqrt{5}}{5}}.$$

21. First, $\cos(-270°)=\cos(90°)=0$. Also, since $450°=360°+90°$, we have $\sin(450°)=\sin(90°)=1$. Therefore, $\cos(-270°)+\sin(450°)=\boxed{1}$.	**22.** First, $\sin(-270°)=\sin(90°)=1$. Also, since $450°=360°+90°$, we have $\cos(450°)=\cos(90°)=0$. Therefore, $\sin(-270°)+\cos(450°)=\boxed{1}$.
23. Since $630°=360°+270°$, we have $\sin(630°)=\sin(270°)=-1$. Further, since $-540°=-360°-180°$, we have $\tan(-540°)=\tan(-180°)=\dfrac{\sin(-180°)}{\cos(-180°)}=0$. So, $\sin(630°)+\tan(-540°)=\boxed{-1}$.	**24.** Since $-720°=2(-360°)$, we have $\cos(-720°)=\cos(0°)=1$. Further, since $720°=2(360°)$, we have $\tan(720°)=\tan(0°)=\dfrac{\sin(0°)}{\cos(0°)}=0$. So, $\cos(-720°)+\tan(720°)=\boxed{1}$.
25. Since $540°=360°+180°$, we have $\cos(540°)=\cos(180°)=-1$. Further, since $-540°=-360°-180°$, we have $\cos(-540°)=\cos(-180°)=-1$, so that $\sec(-540°)=\dfrac{1}{\cos(-540°)}=\dfrac{1}{\cos(-180°)}=-1$. So, $\cos(540°)-\sec(-540°)=-1-(-1)=\boxed{0}$.	**26.** Since $-450°=-360°-90°$, we have $\sin(-450°)=\sin(-90°)=\sin(270°)=-1$. Further, $\csc(270°)=\dfrac{1}{\sin(270°)}=-1$. So, $\sin(-450°)+\csc(270°)=-1+(-1)=\boxed{-2}$.
27. Since $-630°=-360°-270°$, we have $\csc(-630°)=\csc(-270°)=\csc(90°)=1$. Further, since $630°=360°+270°$, we have $\cot(630°)=\dfrac{\cos(630°)}{\sin(630°)}=\dfrac{\cos(270°)}{\sin(270°)}=0$. So, $\csc(-630°)-\cot(630°)=\boxed{1}$.	**28.** Since $-540°=-360°-180°$, we have $\cos(-540°)=\cos(-180°)=-1$, so that $\sec(-540°)=\dfrac{1}{\cos(-540°)}=\dfrac{1}{\cos(-180°)}=-1$. Further, $\tan(540°)=\tan(180°)=0$. So, $\sec(-540°)+\tan(540°)=\boxed{-1}$.
29. Since $\tan(720°)=\tan(0°)=0$ and $\sec(720°)=\dfrac{1}{\cos(720°)}=\dfrac{1}{\cos(0°)}=1$, we have $\tan(720°)+\sec(720°)=\boxed{1}$.	**30.** Since $450°=360°+90°$, we have $\cos(450°)=\cos(90°)=0$. Further, since $-450°=-360°-90°$, we have $\cos(-450°)=\cos(-90°)=0$. So, $\cos(450°)-\cos(-450°)=\boxed{0}$.
31. $\boxed{\text{Possible}}$ since the range of sine is $[-1,1]$.	**32.** $\boxed{\text{Not possible}}$ since the range of cosine is $[-1,1]$.

33. $\boxed{\text{Not possible}}$ since $\frac{2\sqrt{6}}{3} > 1$, and hence out of the range of cosine (which is $[-1,1]$).	**34.** $\boxed{\text{Possible}}$ since $-1 < \frac{\sqrt{2}}{10} < 1$.
35. $\boxed{\text{Possible}}$ since the range of tangent is $\mathbb{R}$.	**36.** $\boxed{\text{Possible}}$ since the range of cotangent is $\mathbb{R}$.
37. $\boxed{\text{Possible}}$ since $\frac{-4}{\sqrt{7}} < -1$, and hence in the range of secant (which is $(-\infty,-1] \cup [1,\infty)$).	**38.** $\boxed{\text{Possible}}$ since the range of cosecant is $(-\infty,-1] \cup [1,\infty)$.
39.  The given angle is shown in **bold**, while the reference angle is *dotted*. $\cos(240°) = -\cos(60°) = \boxed{-\frac{1}{2}}$	**40.**  The given angle is shown in **bold**, while the reference angle is *dotted*. $\cos(120°) = -\cos(60°) = \boxed{-\frac{1}{2}}$

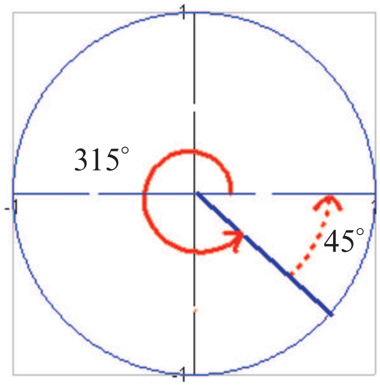
41.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sin(300°) = -\sin(60°) = \boxed{-\frac{\sqrt{3}}{2}}$$

42.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sin(315°) = -\sin(45°) = \boxed{-\frac{\sqrt{2}}{2}}$$

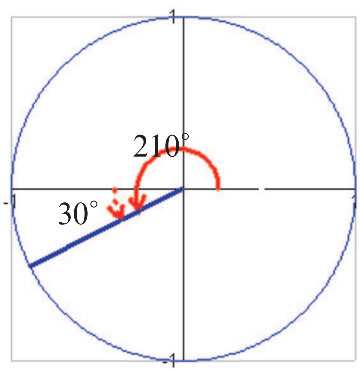
43. $\cot(-150°) = \dfrac{\cos(-150°)}{\sin(-150°)} = \dfrac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$

44. $\sin(-135°) = -\dfrac{\sqrt{2}}{2}$

45. $\cos(-30°) = \dfrac{\sqrt{3}}{2}$

46. $\sec(-135°) = \dfrac{1}{\cos(-135°)} = \dfrac{1}{-\sqrt{2}/2} = -\sqrt{2}$

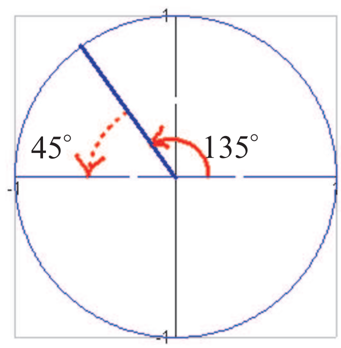
47.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\tan(210°) = \tan(30°) = \frac{\sin 30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{\sqrt{3}}{3}}$$

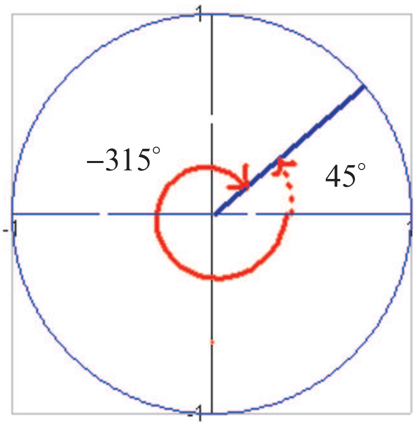
48.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sec(135°) = -\sec(45°) = \frac{-1}{\cos(45°)} = -\frac{2}{\sqrt{2}} = \boxed{-\sqrt{2}}$$

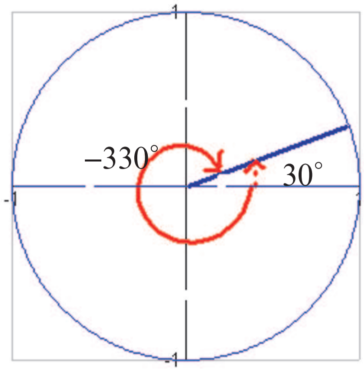
49.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\tan(135°) = \tan(45°) = \boxed{1}$$

50.

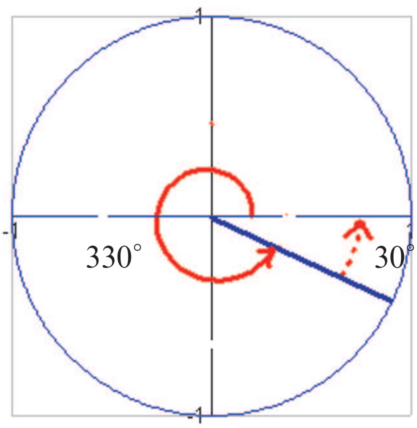


The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sec(-330°) = \sec(30°) = \frac{1}{\cos(30°)}$$

$$= \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$$

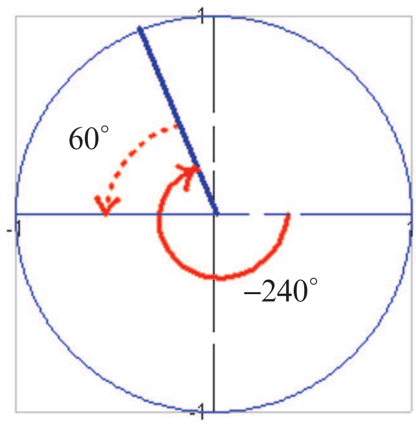
51.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\csc(330°) = -\csc(30°) = \frac{-1}{\sin(30°)} = \boxed{-2}$$

52.



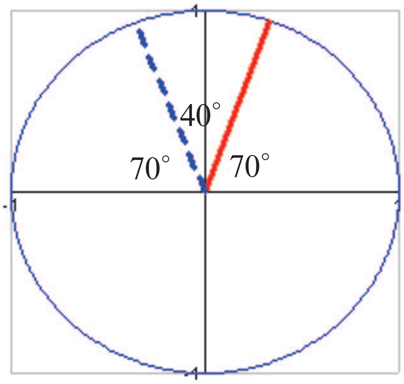
The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\csc(-240°) = \csc(60°) = \frac{1}{\sin(60°)}$$

$$= \frac{2}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$$

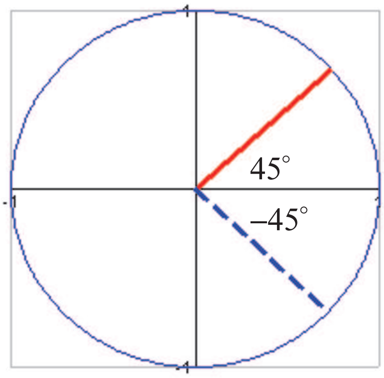
53. $\cos\theta = \frac{\sqrt{3}}{2}$ when $\boxed{\theta = 30° \text{ or } 330°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)	**54.** $\sin\theta = \frac{\sqrt{3}}{2}$ when $\boxed{\theta = 60° \text{ or } 120°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)
55. $\sin\theta = -\frac{1}{2}$ when $\boxed{\theta = 210° \text{ or } 330°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)	**56.** $\cos\theta = -\frac{1}{2}$ when $\boxed{\theta = 120° \text{ or } 240°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)
57. $\cos\theta = 0$ when $\boxed{\theta = 90° \text{ or } 270°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)	**58.** $\sin\theta = 0$ when $\boxed{\theta = 0°, 180°, \text{ or } 360°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)
59. $\sin\theta = -1$ when $\boxed{\theta = 270°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)	**60.** $\cos\theta = -1$ when $\boxed{\theta = 180°}$. (Note: Since the right side of the equation corresponds to standard angles, we refer the reader to the diagram in the text for a visual of the solutions to this equation.)
61. – 0.8387	**62.** 0.7314
63. – 11.4301	**64.** 7.1154
65. $\sec(421°) = \frac{1}{\cos(421°)} \approx \boxed{2.0627}$	**66.** $\sec(-222°) = \frac{1}{\cos(-222°)} \approx \boxed{-1.3456}$
67. $\csc(-111°) = \frac{1}{\sin(-111°)} \approx \boxed{-1.0711}$	**68.** $\csc(211°) = \frac{1}{\sin(211°)} \approx \boxed{-1.9416}$
69. $\cot(159°) = \frac{1}{\tan(159°)} \approx \boxed{-2.6051}$	**70.** $\cot(-82°) = \frac{1}{\tan(-82°)} \approx \boxed{-0.1405}$

71. Since $\sin\theta = 0.9397$, it follows that $\theta = \sin^{-1}(0.9397) \approx 70°$. This angle has terminal side in QI. Since the desired angle has terminal side in QII, we have the following diagram.
<u>**Note**</u>**:** The desired terminal side is the dashed segment.



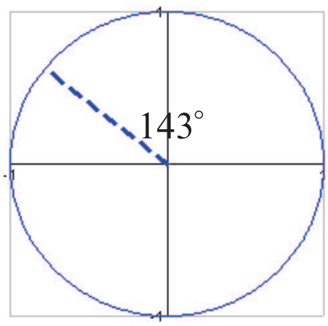
Hence, $\theta \approx 70° + 40° = \boxed{110°}$.

72. Since $\cos\theta = 0.7071$, it follows that $\theta = \cos^{-1}(0.7071) \approx 45°$. This angle has terminal side in QI. Since the desired angle has terminal side in QIV, we have the following diagram.
<u>**Note**</u>**:** The desired terminal side is the dashed segment.

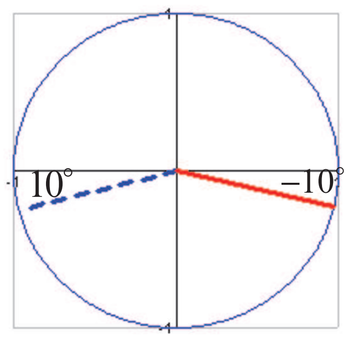


Hence, $\theta \approx 360° - 45° = \boxed{315°}$.

73. Since $\cos\theta = -0.7986$, it follows that $\theta = \cos^{-1}(-0.7986) \approx \boxed{143°}$. This angle has terminal side in QII. Since the desired angle has terminal side also in QII, no adjustment is necessary—this is pictured in the following diagram.
<u>**Note**</u>**:** The desired terminal side is the dashed segment.

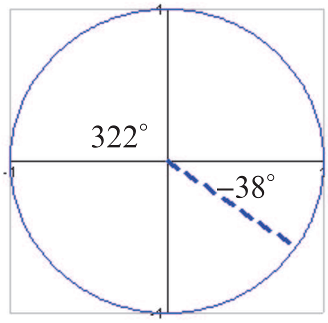


74. Since $\sin\theta = -0.1746$, it follows that $\theta = \sin^{-1}(-0.1746) \approx -10°$. This angle has terminal side in QIV. Since the desired angle has terminal side in QIII, we have the following diagram.
<u>**Note**</u>**:** The desired terminal side is the dashed segment.



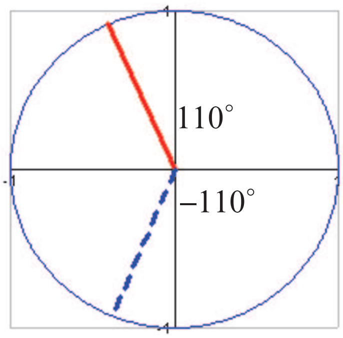
Hence, $\theta \approx 180° + 10° = \boxed{190°}$.

75. Since $\tan\theta = -0.7813$, it follows that $\theta = \tan^{-1}(-0.7813) \approx -38°$. This angle has terminal side in QIV. Since the desired angle has terminal side also in QIV, we need only a corresponding angle with positive measure – this is pictured in the following diagram.
Note: The desired terminal side is the dashed segment.



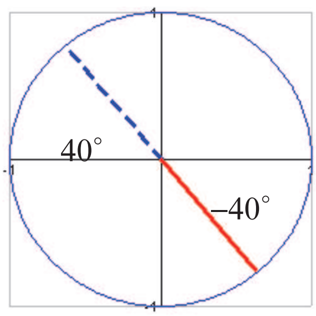
Hence, $\theta \approx 360° - 38° = \boxed{322°}$.

76. Since $\cos\theta = -0.3420$, it follows that $\theta = \cos^{-1}(-0.3420) \approx \boxed{110°}$. This angle has terminal side in QII. Since the desired angle has terminal side in QIII, we have the following diagram.
Note: The desired terminal side is the dashed segment.



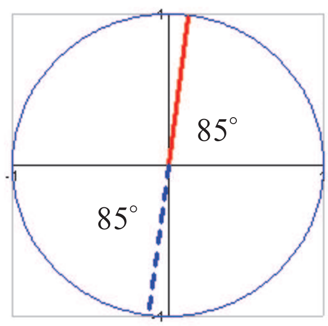
Hence, $\theta \approx 360° - 110° = \boxed{250°}$.

77. Since $\tan\theta = -0.8391$, it follows that $\theta = \tan^{-1}(-0.8391) \approx -40°$. This angle has terminal side in QIV. Since the desired angle has terminal side also in QII, we have the following diagram.
Note: The desired terminal side is the dashed segment.



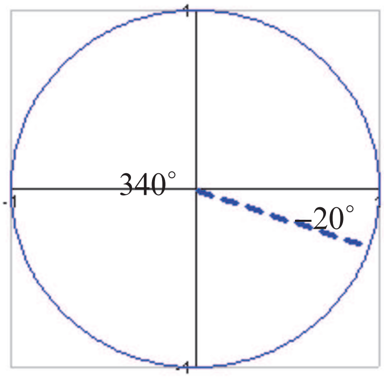
Hence, $\theta \approx 180° - 40° = \boxed{140°}$.

78. Since $\tan\theta = 11.4301$, it follows that $\theta = \tan^{-1}(11.4301) \approx 85°$. This angle has terminal side in QI. Since the desired angle has terminal side also in QIII, we have the following diagram.
Note: The desired terminal side is the dashed segment.



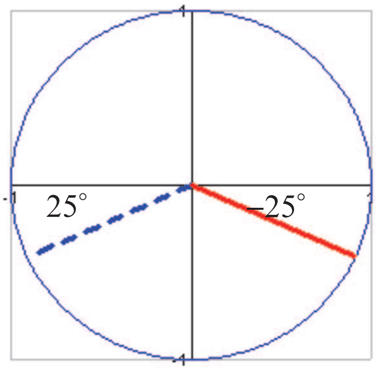
Hence, $\theta \approx 360° - 85° = \boxed{265°}$.

79. Since $\sin\theta = -0.3420$, it follows that $\theta = \sin^{-1}(-0.3420) \approx -20°$. This angle has terminal side in QIV. Since the desired angle has terminal side also in QIV, we need only a corresponding angle with positive measure—this is pictured in the following diagram.
<u>**Note:**</u> The desired terminal side is the dashed segment.



Hence, $\theta \approx 360° - 20° = \boxed{340°}$.

80. Since $\sin\theta = -0.4226$, it follows that $\theta = \sin^{-1}(-0.4226) \approx -25°$. This angle has terminal side in QIV. Since the desired angle has terminal side in QIII, we have the following diagram.
<u>**Note:**</u> The desired terminal side is the dashed segment.



Hence, $\theta \approx 180° + 25° = \boxed{205°}$.

81. Use $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ with $n_1 = 1,\ \theta_1 = 30°,\ \theta_2 = 22°$ to obtain

$$(1)\sin(30°) = n_2 \sin(22°)$$
$$n_2 = \frac{\sin(30°)}{\sin(22°)} \approx \boxed{1.3}.$$

82. Use $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ with $n_1 = 1,\ \theta_1 = 30°,\ \theta_2 = 18°$ to obtain

$$(1)\sin(30°) = n_2 \sin(18°)$$
$$n_2 = \frac{\sin(30°)}{\sin(18°)} \approx \boxed{1.6}.$$

83. Use $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ with $n_1 = 1,\ n_2 = 2.4,\ \theta_1 = 30$ to obtain

$$(1)\sin(30°) = (2.4)\sin(\theta_2)$$
$$\sin(\theta_2) = \frac{\sin(30°)}{2.4}$$
$$\theta_2 = \sin^{-1}\left[\frac{\sin(30°)}{2.4}\right] \approx \boxed{12°}.$$

84. Use $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$ with $n_1 = 1,\ n_2 = 1.9,\ \theta_1 = 30$ to obtain

$$(1)\sin(30°) = (1.9)\sin(\theta_2)$$
$$\sin(\theta_2) = \frac{\sin(30°)}{1.9}$$
$$\theta_2 = \sin^{-1}\left[\frac{\sin(30°)}{1.9}\right] \approx \boxed{15°}.$$

85. Observe that $\theta = 30° + 3.25(360°) = 30° + 3(360°) + 0.25(360°) = 3(360°) + 120°$.

Thus, $\sin\theta = \sin(120°) = \frac{\sqrt{3}}{2}$.

86. Observe that $\theta = 120° + \frac{2}{3}(-360°) + \frac{3}{4}(360°) = 150°$. Thus, $\cos\theta = \cos(150°) = -\frac{\sqrt{3}}{2}$.

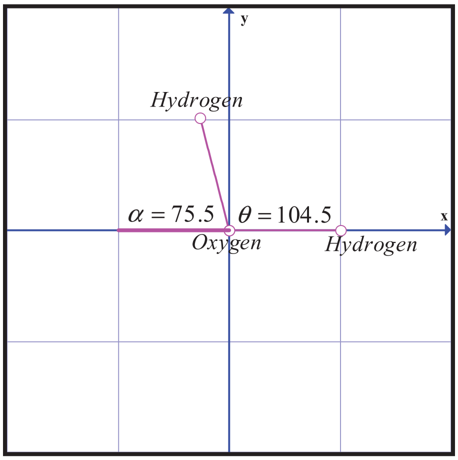
87. Since $\tan\alpha = 0 \Rightarrow \alpha = 0°$ or $180°$, the minute hand is on the 3 or the 9.

88. First, note that $\sin\alpha = \frac{1}{2} \Rightarrow \alpha = 30°$ or $150°$. Since the angle between consecutive hours on the face of the clock is 30°, the minute hand is on either the 2 or the 10.

89. The measure of the reference angle is $180° - 165° = 15°$. Physically, this means that the lower leg is bent at the knee in a backward direction at an angle of 15 degrees.

90. The measure of the reference angle is $180° - 160° = 20°$. Physically, an angle whose measure is greater than 180 degrees represents a hyperextended knee.

91. The diagram is as follows:

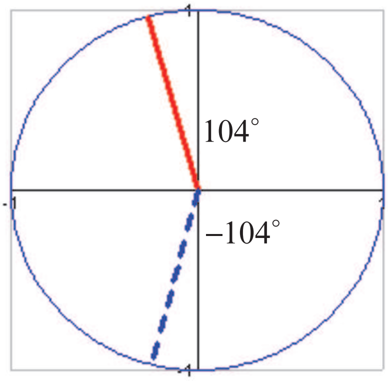


The reference angle is $180° - 104.5° = 75.5°$

92. Observe that $\cos(104.5°) = -\cos(75.5°) \approx -0.2504$

93. The reference angle is measured between the terminal side and the x-axis, not the y-axis. So, in this case, the reference angle is 60°. Since $\cos 60° = \frac{1}{2}$ and $\cos 120° = -\frac{1}{2}$, we see that $\sec 120° = \frac{1}{-\frac{1}{2}} = -2$.

94. Consider the following diagram:



The cosine of angles with terminal sides in QII and QIII is negative. The calculator correctly produces the angle 104°, but since we want the terminal side to lie in QIII, we need to use the correct reference angle. In this case, we angle would be $360° - 104° = 256°$.

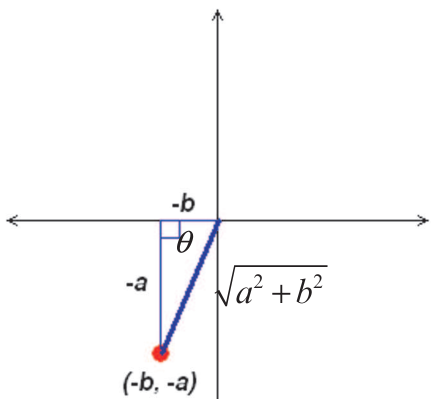
95. True. For any $0° < \theta < 90°$, $\sin\theta$ and $\cos\theta$ are both positive. Since all of the trigonometric functions are defined as quotients of sine, cosine, and 1, they are also positive for such values of θ.

96. False. This requires both $\sin\theta$ and $\cos\theta$ to be negative. But, in such case, $\tan\theta$ and $\cot\theta$ are both positive.

97. False. For instance, $\cos(-45°) = \frac{\sqrt{2}}{2}$.

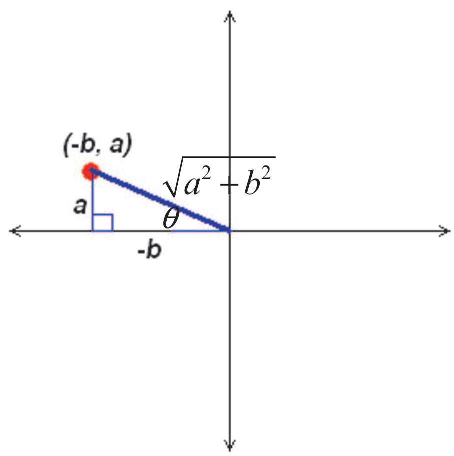
98. False. For instance, $\cos(120°) = -\frac{1}{2}$.

99. Assume that $a > 0,\ b > 0$. Consider the following diagram:



Observe that $\sin\theta = \boxed{-\frac{a}{\sqrt{a^2+b^2}}}$.

100. Assume that $a > 0,\ b > 0$. Consider the following diagram:



Observe that $\cos\theta = \boxed{-\frac{b}{\sqrt{a^2+b^2}}}$.

101. If $\cos\theta = -\frac{\sqrt{3}}{2}$, then $\theta = 150°$ or $210°$. The reference angle for both is $30°$.	**102.** $\tan(45° + 180°k) = \tan(45°) = 1$, for any integer k.
103. If the reference angle for β is $60°$, then β could be any of $60°, 120°, 240°, 300°$. The cosine of these angles is either $-\frac{1}{2}$ or $\frac{1}{2}$.	
104. Observe that $\csc(3(-210°)+30°) - \frac{1}{2} = \csc(-600°) - \frac{1}{2} = \csc(-240°) - \frac{1}{2} = \frac{1}{\sin(-240°)} - \frac{1}{2} = \frac{2\sqrt{3}}{3} - \frac{1}{2} = \frac{4\sqrt{3}-3}{6}$	
105. Observe that $\sin 80° \approx 0.9848 \approx \sin 100°.$ Also, $\sin^{-1}(0.9848) \approx 80°$. So, the calculator is assuming that the terminal side of the angle lies in QI.	**106.** Observe that $\sin 260° \approx -0.9848 \approx \sin 280°.$ Also, $\sin^{-1}(-0.9848) \approx -80°$, which corresponds to the angle $280°$. So, the calculator is assuming that the terminal side of the angle lies in QIV.
107. Observe that $\cos 75° \approx 0.2588 \approx \cos(-75°).$ Also, $\cos^{-1}(0.2588) \approx 75°$. So, the calculator is assuming that the terminal side of the angle lies in QI.	**108.** Observe that $\cos 105° \approx -0.2588 \approx \cos(255°).$ Also, $\cos^{-1}(-0.2588) \approx 105°$. So, the calculator is assuming that the terminal side of the angle lies in QII.
109. Observe that $\tan 65° \approx 2.1445 \approx \tan 245°.$ Also, $\tan^{-1}(2.1445) \approx 65°$. So, the calculator is assuming that the terminal side of the angle lies in QI.	**110.** Using a calculator yields: $\tan(136°) \approx -0.9657 \approx \tan(316°)$ $\tan^{-1}(-0.9657) \approx -44°$ So, the calculator assumes the terminal side is in QIV when calculating inverse tangent of a negative quantity.

Section 2.4 Solutions --

1. $\sec\theta = \frac{1}{\cos\theta} = \boxed{\frac{8}{7}}$	**2.** $\csc\theta = \frac{1}{\sin\theta} = \boxed{-\frac{7}{3}}$
3. $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-0.6} = \frac{1}{-\frac{3}{5}} = \boxed{-\frac{5}{3}}$	**4.** $\sec\theta = \frac{1}{\cos\theta} = \frac{1}{0.8} = \frac{1}{\frac{4}{5}} = \boxed{\frac{5}{4}}$
5. $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-5} = \boxed{-\frac{1}{5}}$	**6.** $\cot\theta = \frac{1}{\tan\theta} = \frac{1}{0.5} = \frac{1}{\frac{1}{2}} = \boxed{2}$

7. Since $\frac{\sqrt{5}}{2} = \csc\theta = \frac{1}{\sin\theta}$, we see that $\sin\theta = \frac{2}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{5}}$.

8. Since $\frac{\sqrt{11}}{2} = \sec\theta = \frac{1}{\cos\theta}$, we see that $\cos\theta = \frac{2}{\sqrt{11}} = \boxed{\frac{2\sqrt{11}}{11}}$.

9. $\tan\theta = \frac{1}{\cot\theta} = \frac{1}{-\frac{\sqrt{7}}{5}} = -\frac{5}{\sqrt{7}} = \boxed{-\frac{5\sqrt{7}}{7}}$

10. $\tan\theta = \frac{1}{\cot\theta} = \frac{1}{3.5} = \frac{1}{\frac{7}{2}} = \boxed{\frac{2}{7}}$

11. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = \boxed{-\frac{\sqrt{3}}{3}}$

12. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \boxed{-\sqrt{3}}$

13. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \boxed{-\frac{4}{3}}$

14. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \boxed{-\frac{3}{4}}$

15. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{-0.8}{-0.6} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \boxed{\frac{4}{3}}$

16. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-0.6}{-0.8} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \boxed{\frac{3}{4}}$

17.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{\sqrt{11}}{6}}{-\frac{5}{6}} = -\frac{\sqrt{11}}{6} \times -\frac{6}{5} = \boxed{\frac{\sqrt{11}}{5}}$$

18. Using Exercise 17, observe that

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sqrt{11}}{5}} = \frac{5}{\sqrt{11}} = \boxed{\frac{5\sqrt{11}}{11}}$$

19. $\sin^2\theta = (\sin\theta)^2 = \left(\frac{\sqrt{5}}{8}\right)^2 = \boxed{\frac{5}{64}}$

20. $\cos^3\theta = (\cos\theta)^3 = (0.1)^3 = \boxed{0.001}$

21. $\csc^3\theta = (\csc\theta)^3 = (-2)^3 = \boxed{-8}$

22. $\tan^2\theta = (\tan\theta)^2 = (-5)^2 = \boxed{25}$

23. Solving the equation $\sin^2\theta + \cos^2\theta = 1$ for $\cos\theta$ yields $\cos\theta = \pm\sqrt{1-\sin^2\theta}$. Since the terminal side of θ is in QIII, we use $\cos\theta = -\sqrt{1-\sin^2\theta}$. Hence,

$$\cos\theta = -\sqrt{1-\left(-\frac{1}{2}\right)^2} = -\sqrt{\frac{3}{4}} = \boxed{-\frac{\sqrt{3}}{2}}.$$

24. Solving the equation $\sin^2\theta + \cos^2\theta = 1$ for $\cos\theta$ yields $\cos\theta = \pm\sqrt{1-\sin^2\theta}$. Since the terminal side of θ is in QIII, we use $\cos\theta = -\sqrt{1-\sin^2\theta}$. Hence,

$$\cos\theta = -\sqrt{1-\left(-\frac{3}{5}\right)^2} = -\sqrt{\frac{16}{25}} = \boxed{-\frac{4}{5}}.$$

25. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QIV, we use $\sin\theta=-\sqrt{1-\cos^2\theta}$. Hence,

$$\sin\theta=-\sqrt{1-\left(\frac{2}{5}\right)^2}=-\sqrt{\frac{21}{25}}=\boxed{-\frac{\sqrt{21}}{5}}.$$

26. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QIV, we use $\sin\theta=-\sqrt{1-\cos^2\theta}$. Hence,

$$\sin\theta=-\sqrt{1-\left(\frac{2}{7}\right)^2}=-\sqrt{\frac{45}{49}}=\boxed{-\frac{3\sqrt{5}}{7}}.$$

27. Solving the equation $1+\tan^2\theta=\sec^2\theta$ for $\sec\theta$ yields $\sec\theta=\pm\sqrt{1+\tan^2\theta}$. Since the terminal side of θ is in QIII, $\cos\theta<0$ and so $\sec\theta<0$. So, we use $\sec\theta=-\sqrt{1+\tan^2\theta}$. Hence, $\sec\theta=-\sqrt{1+\tan^2\theta}=-\sqrt{1+(4)^2}=\boxed{-\sqrt{17}}$.

28. Solving the equation $1+\tan^2\theta=\sec^2\theta$ for $\sec\theta$ yields $\sec\theta=\pm\sqrt{1+\tan^2\theta}$. Since the terminal side of θ is in QII, $\cos\theta<0$ and so $\sec\theta<0$. So, we use $\sec\theta=-\sqrt{1+\tan^2\theta}$. Hence, $\sec\theta=-\sqrt{1+\tan^2\theta}=-\sqrt{1+(-5)^2}=\boxed{-\sqrt{26}}$.

29. Solving the equation $1+\cot^2\theta=\csc^2\theta$ for $\csc\theta$ yields $\csc\theta=\pm\sqrt{1+\cot^2\theta}$. Since the terminal side of θ is in QIII, $\sin\theta<0$ and so $\csc\theta<0$. So, we use $\csc\theta=-\sqrt{1+\cot^2\theta}$. Hence, $\csc\theta=-\sqrt{1+\cot^2\theta}=-\sqrt{1+(2)^2}=\boxed{-\sqrt{5}}$.

30. Solving the equation $1+\cot^2\theta=\csc^2\theta$ for $\csc\theta$ yields $\csc\theta=\pm\sqrt{1+\cot^2\theta}$. Since the terminal side of θ is in QII, $\sin\theta>0$ and so $\csc\theta>0$. So, we use $\csc\theta=\sqrt{1+\cot^2\theta}$. Hence, $\csc\theta=\sqrt{1+\cot^2\theta}=\sqrt{1+(-3)^2}=\boxed{\sqrt{10}}$.

31. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\cos\theta$ yields $\cos\theta=\pm\sqrt{1-\sin^2\theta}$. Since the terminal side of θ is in QII, we use $\cos\theta=-\sqrt{1-\sin^2\theta}$. Hence,

$$\cos\theta=-\sqrt{1-\left(\frac{8}{15}\right)^2}=-\sqrt{\frac{161}{225}}=-\frac{\sqrt{161}}{15}.$$

Thus, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{\frac{8}{15}}{-\frac{\sqrt{161}}{15}}=-\dfrac{8}{\sqrt{161}}=\boxed{\dfrac{-8\sqrt{161}}{161}}$.

32. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\cos\theta$ yields $\cos\theta=\pm\sqrt{1-\sin^2\theta}$. Since the terminal side of θ is in QIII, we use $\cos\theta=-\sqrt{1-\sin^2\theta}$. Hence,

$$\cos\theta=-\sqrt{1-\left(-\frac{7}{15}\right)^2}=-\sqrt{\frac{176}{225}}=-\frac{\sqrt{176}}{15}.$$

Thus, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{-\frac{7}{15}}{-\frac{\sqrt{176}}{15}}=\dfrac{7}{\sqrt{176}}=\dfrac{7}{4\sqrt{11}}=\boxed{\dfrac{7\sqrt{11}}{44}}.$

33. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QIII, we use $\sin\theta=-\sqrt{1-\cos^2\theta}$. Hence,

$$\sin\theta=-\sqrt{1-\left(-\frac{7}{15}\right)^2}=-\sqrt{\frac{176}{225}}=-\frac{\sqrt{176}}{15}.$$

Thus, $\csc\theta=\dfrac{1}{\sin\theta}=\dfrac{-15}{\sqrt{176}}=\dfrac{-15}{4\sqrt{11}}=\boxed{\dfrac{-15\sqrt{11}}{44}}.$

34. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QII, we use $\sin\theta=\sqrt{1-\cos^2\theta}$. Hence,

$$\sin\theta=\sqrt{1-\left(-\frac{8}{15}\right)^2}=\sqrt{\frac{161}{225}}=\frac{\sqrt{161}}{15}.$$

Thus, $\csc\theta=\dfrac{1}{\sin\theta}=\dfrac{15}{\sqrt{161}}=\boxed{\dfrac{15\sqrt{161}}{161}}.$

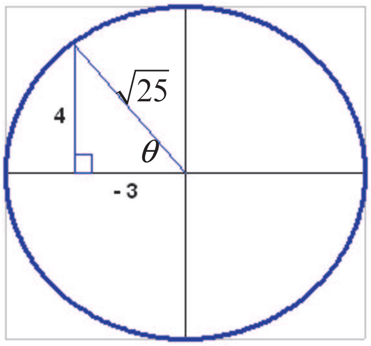
35. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QIV, we use $\sin\theta=-\sqrt{1-\cos^2\theta}$. From the given information, observe that $\cos\theta=\dfrac{1}{\sec\theta}=\dfrac{3}{\sqrt{13}}$. Hence, we obtain

$$\sin\theta=-\sqrt{1-\left(\frac{3}{\sqrt{13}}\right)^2}=-\sqrt{\frac{4}{13}}=-\frac{2}{\sqrt{13}}=\boxed{-\frac{2\sqrt{13}}{13}}.$$

36. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QII, we use $\sin\theta=\sqrt{1-\cos^2\theta}$. From the given information, observe that $\cos\theta=\dfrac{1}{\sec\theta}=-\dfrac{5}{\sqrt{61}}$. Hence, we obtain

$$\sin\theta=\sqrt{1-\left(-\frac{5}{\sqrt{61}}\right)^2}=\sqrt{\frac{36}{61}}=\frac{6}{\sqrt{61}}=\boxed{\frac{6\sqrt{61}}{61}}.$$

37. Consider the following diagram:

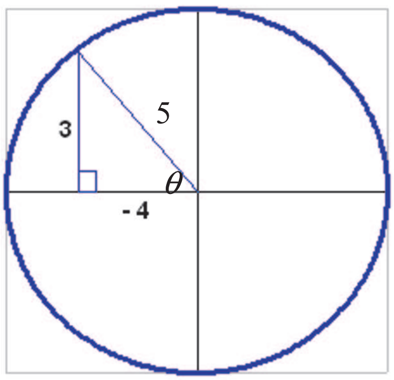


Since $\tan\theta = -\frac{4}{3}$ and the terminal side of θ lies in QII, we conclude that

$$\sin\theta = \frac{4}{\sqrt{25}} = \frac{4}{5}$$

$$\cos\theta = -\frac{3}{\sqrt{25}} = -\frac{3}{5}.$$

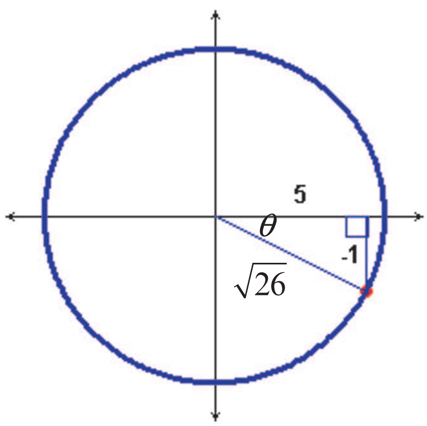
38. Consider the following diagram:



Since $\tan\theta = -\frac{3}{4}$ and the terminal side of θ lies in QII, we conclude that

$$\sin\theta = \frac{3}{5}, \quad \cos\theta = -\frac{4}{5}.$$

39. Consider the following diagram:

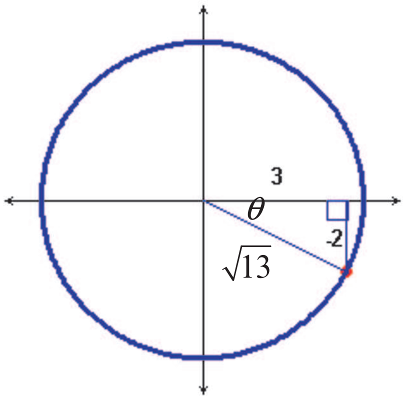


Since the terminal side of θ lies in QIV, we conclude that

$$\sin\theta = -\frac{1}{\sqrt{26}} = -\frac{\sqrt{26}}{26}$$

$$\cos\theta = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$$

40. Consider the following diagram:

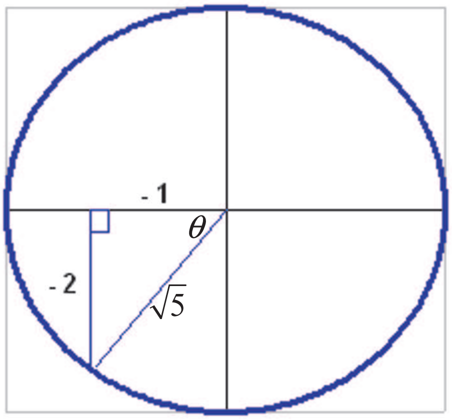


Since the terminal side of θ lies in QIV, we conclude that

$$\sin\theta = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

41. Consider the following diagram:

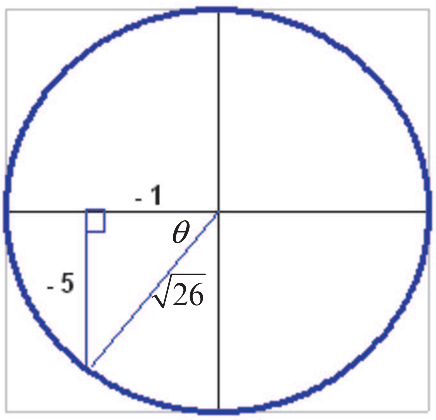


Since $\tan\theta = 2$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}.$$

42. Consider the following diagram:

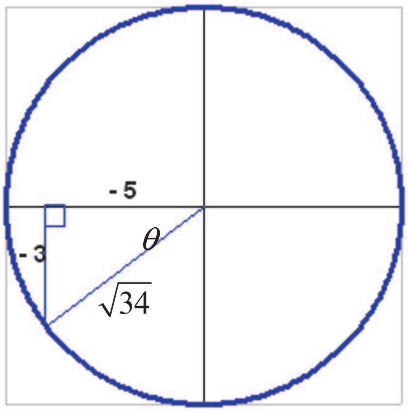


Since $\tan\theta = 5$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta = -\frac{5}{\sqrt{26}} = -\frac{5\sqrt{26}}{26}$$

$$\cos\theta = -\frac{1}{\sqrt{26}} = -\frac{\sqrt{26}}{26}.$$

43. Consider the following diagram:

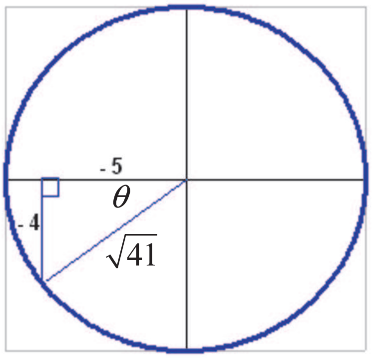


Since $\tan\theta = 0.6 = \frac{-3}{-5}$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta = -\frac{3}{\sqrt{34}} = -\frac{3\sqrt{34}}{34}$$

$$\cos\theta = -\frac{5}{\sqrt{34}} = -\frac{5\sqrt{34}}{34}$$

44. Consider the following diagram:



Since $\tan\theta = 0.8 = \frac{-4}{-5}$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta = -\frac{4}{\sqrt{41}} = -\frac{4\sqrt{41}}{41}$$

$$\cos\theta = -\frac{5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$$

45. $\sec\theta\cot\theta = \frac{1}{\cancel{\cos\theta}} \times \frac{\cancel{\cos\theta}}{\sin\theta} = \boxed{\frac{1}{\sin\theta}}$

46. $\csc\theta\tan\theta = \frac{1}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{\cos\theta} = \boxed{\frac{1}{\cos\theta}}$

47. $\tan^2\theta - \sec^2\theta = \frac{\sin^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta} = \frac{\sin^2\theta - 1}{\cos^2\theta} = \frac{-\cos^2\theta}{\cos^2\theta} = \boxed{-1}$

Equivalently, you can compute as follows:

$\tan^2\theta - \left(1 + \tan^2\theta\right) = \cancel{\tan^2\theta} - 1 \cancel{- \tan^2\theta} = -1$

48. $\cot^2\theta - \csc^2\theta = \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta - 1}{\sin^2\theta} = \frac{-\sin^2\theta}{\sin^2\theta} = \boxed{-1}$

Equivalently, you can compute as follows: $\cot^2\theta - \left(1 + \cot^2\theta\right) = \cancel{\cot^2\theta} - 1 \cancel{- \cot^2\theta} = -1$

49. $\csc\theta - \sin\theta = \dfrac{1}{\sin\theta} - \sin\theta = \dfrac{1-\sin^2\theta}{\sin\theta} = \boxed{\dfrac{\cos^2\theta}{\sin\theta}}$

50. $\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta} = \boxed{\dfrac{\sin^2\theta}{\cos\theta}}$

51. $\dfrac{\sec\theta}{\tan\theta} = \dfrac{\dfrac{1}{\cancel{\cos\theta}}}{\dfrac{\sin\theta}{\cancel{\cos\theta}}} = \boxed{\dfrac{1}{\sin\theta}}$

52. $\dfrac{\csc\theta}{\cot\theta} = \dfrac{\dfrac{1}{\cancel{\sin\theta}}}{\dfrac{\cos\theta}{\cancel{\sin\theta}}} = \boxed{\dfrac{1}{\cos\theta}}$

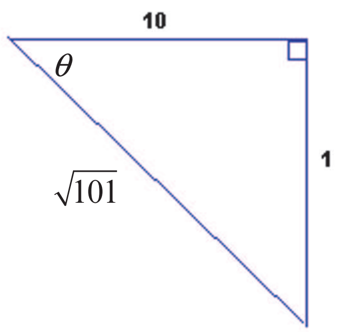
53.

$$\begin{aligned}(\sin\theta+\cos\theta)^2 &= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta \\ &= (\sin^2\theta+\cos^2\theta) + 2\sin\theta\cos\theta \\ &= \boxed{1+2\sin\theta\cos\theta}\end{aligned}$$

54.

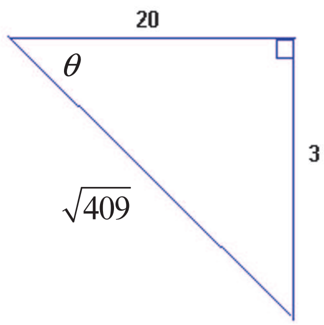
$$\begin{aligned}(\sin\theta-\cos\theta)^2 &= \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta \\ &= (\sin^2\theta+\cos^2\theta) - 2\sin\theta\cos\theta \\ &= \boxed{1-2\sin\theta\cos\theta}\end{aligned}$$

55. Consider the following diagram:



Observe that $\tan\theta = \dfrac{1}{10} = 0.1$. So, 10%.

56. Consider the following diagram:

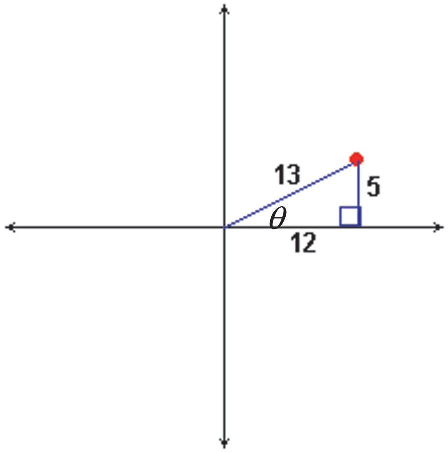


Observe that

$\sec\theta = \dfrac{\sqrt{409}}{20}$, so that $\cos\theta = \dfrac{20}{\sqrt{409}}$.

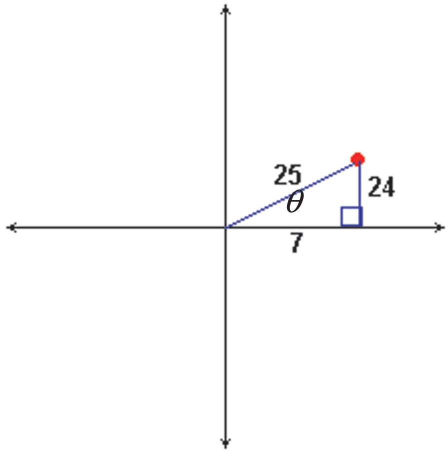
Hence, $\tan\theta = \dfrac{3}{20} = 0.15$. So, 15%.

57. Consider the following triangle:



Observe that $\tan\theta = \frac{5}{12}$.

58. Consider the following triangle:



Observe that

$\sec\theta = \frac{25}{7}$, so that $\cos\theta = \frac{7}{25}$.

Hence, $\tan\theta = \frac{24}{7}$.

59. Observe that $r = 8\sin\theta\cos^2\theta = 8\sin\theta\left(1-\sin^2\theta\right) = 8\sin\theta - 8\sin^3\theta$.

θ	r
30°	$r = 8\sin(30°) - 8\sin^3(30°) = 8\left(\frac{1}{2}\right) - 8\left(\frac{1}{2}\right)^3 = 4-1=3$
60°	$r = 8\sin(60°) - 8\sin^3(60°) = 8\left(\frac{\sqrt{3}}{2}\right) - 8\left(\frac{\sqrt{3}}{2}\right)^3 = 4\sqrt{3}-3\sqrt{3}=\sqrt{3}$
90°	$r = 8\sin(90°) - 8\sin^3(90°) = 8(1) - 8(1)^3 = 8-8=0$

60. Observe that $r = 8\sin\theta\cos^2\theta = 8\sin\theta\left(1-\sin^2\theta\right) = 8\sin\theta - 8\sin^3\theta$.

θ	r
-30°	$r = 8\sin(-30°) - 8\sin^3(-30°) = 8\left(-\frac{1}{2}\right) - 8\left(-\frac{1}{2}\right)^3 = -4+1=-3$
-60°	$r = 8\sin(-60°) - 8\sin^3(-60°) = 8\left(-\frac{\sqrt{3}}{2}\right) - 8\left(-\frac{\sqrt{3}}{2}\right)^3 = -4\sqrt{3}+3\sqrt{3}=-\sqrt{3}$
-90°	$r = 8\sin(-90°) - 8\sin^3(-90°) = 8(-1) - 8(-1)^3 = -8+8=0$

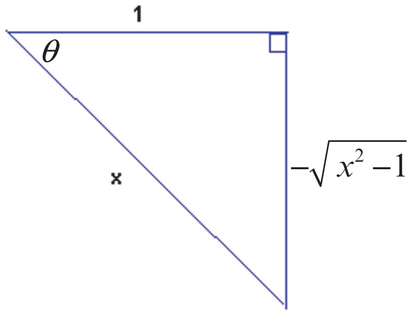
61. Note that $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{4/5}{3/5} = \frac{4}{3} \approx 1.33$. This means that each dollar spent on costs produces \$1.33 in revenue.	
62. Note that $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}/5}{2\sqrt{5}/5} = \frac{1}{2} = 0.5$. This means that each dollar spent on costs produces \$0.50 in revenue.	
63. Cosine is *negative* in QIII, so that $\cos\theta = -\frac{2\sqrt{2}}{3}$.	
64. Must use the Pythagorean identity here, not the ratio identities. Further, note that -4 is not a possible value for $\cos\theta$ (since the only tenable values are in the interval $[-1,1]$). In this case, the correct values would be: $$\sin\theta = \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17}, \quad \cos\theta = -\frac{4}{\sqrt{17}} = -\frac{4\sqrt{17}}{17}$$	
65. False. Since $\csc\theta = \frac{1}{\sin\theta}$, they have the same sign for a given value of θ.	
66. False. Since $\sec\theta = \frac{1}{\cos\theta}$, they have the same sign for a given value of θ.	
67. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{b}{a}$, so that $a \neq 0$.	
68. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$, so that $b \neq 0$.	
69. Observe that $\sin\theta = \csc\theta = \frac{1}{\sin\theta}$, so that $\sin^2\theta = 1$. Hence, we seek those values of θ for which either $\sin\theta = 1$ or $\sin\theta = -1$. The only value of θ such that $0° < \theta \leq 180°$ that satisfies either one of these equations is $\theta = 90°$. (<u>Note</u>: The equation $\sin\theta = -1$ has no solution when θ is restricted to this interval.)	
70. Observe that $\cos\theta = \sec\theta = \frac{1}{\cos\theta}$, so that $\cos^2\theta = 1$. Hence, we seek those values of θ for which either $\cos\theta = 1$ or $\cos\theta = -1$. The only value of θ such that $0° < \theta \leq 180°$ that satisfies either one of these equations is $\theta = 180°$. (<u>Note</u>: Even though $\cos(0°) = 1$, $0°$ is not included in the interval and so, the equation $\cos\theta = 1$ has no solution when θ is restricted to this interval.)	
71. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\pm\sqrt{1-\sin^2\theta}}{\sin\theta}$, where the sign is chosen appropriately depending on the angle θ.	

72. $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{\pm\sqrt{1-\cos^2\theta}}$, where the sign is chosen appropriately depending on the angle θ.

73. $\sqrt{64-(8\sin\theta)^2} = \sqrt{64(1-\sin^2\theta)} = \sqrt{64\cos^2\theta} = \boxed{8|\cos\theta|}$

74. $\sqrt{36-(6\cos\theta)^2} = \sqrt{36(1-\cos^2\theta)} = \sqrt{36\sin^2\theta} = \boxed{6|\sin\theta|}$

75. Consider the following diagram:



Observe that $\cos\theta = \dfrac{1}{x}$.

So, $\tan\theta = -\dfrac{\sqrt{x^2-1}}{1} = -\sqrt{x^2-1}$.

76. Observe that

$$\frac{\cos\theta}{\tan\theta(1-\sin\theta)} = \frac{\cos\theta}{\dfrac{\sin\theta}{\cos\theta}(1-\sin\theta)} = \frac{\cos^2\theta}{\sin\theta(1-\sin\theta)} = \frac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$$

$$= \frac{(1+\sin\theta)\cancel{(1-\sin\theta)}}{\sin\theta\cancel{(1-\sin\theta)}} = \frac{1+\sin\theta}{\sin\theta}$$

77. **a.** $\sin(22°) \approx 0.3746$ **b.** $\cos(22°) \approx 0.9272$ **c.** $\dfrac{\sin(22°)}{\cos(22°)} \approx 0.4040$

d. $\tan(22°) \approx 0.4040$ Yes, the values in parts **c.** and **d.** are the same.

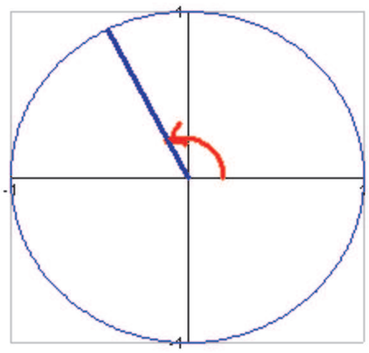
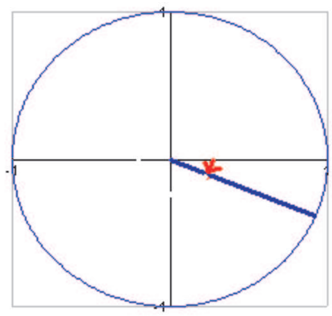
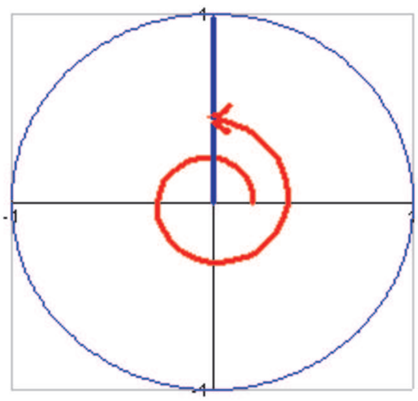
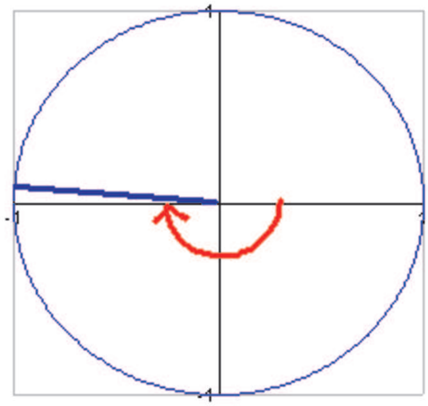
78. **a.** $\sin(160°) \approx 0.3420$ **b.** $\cos(16°) \approx 0.9613$ **c.** $\dfrac{\sin(160°)}{\cos(16°)} \approx 0.3558$

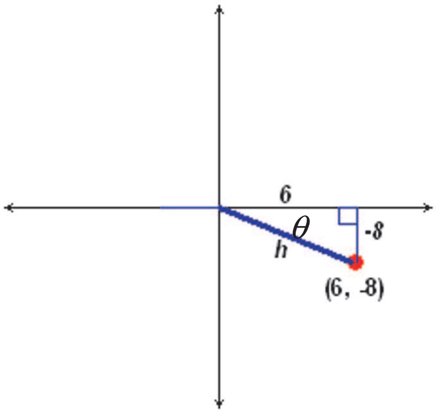
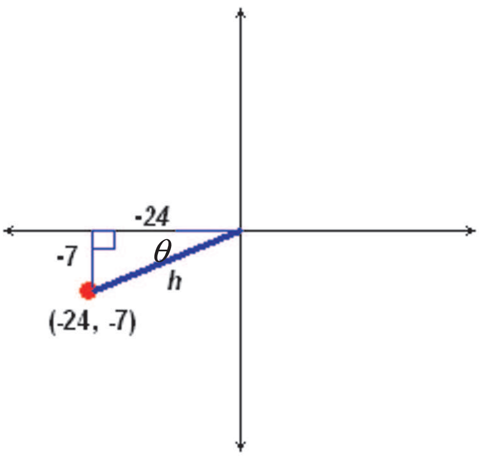
d. $\tan(10°) \approx 0.1763$ No, the values in parts **c.** and **d.** are not the same.

79. **a.** $\sin(35°) \approx 0.5736$ **b.** $1-\cos(35°) \approx 0.1808$ **c.** $\sqrt{1-\cos^2(35°)} \approx 0.5736$
The values in parts **a** and **c** are the same.

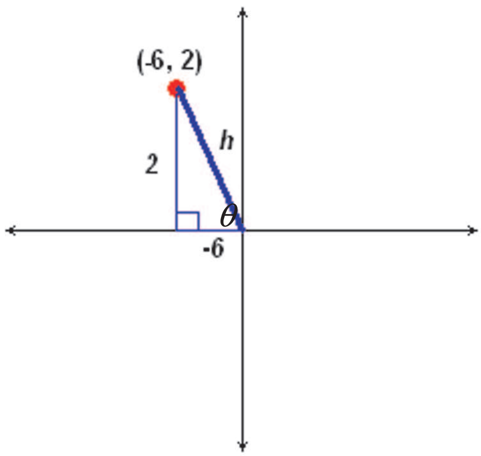
80. **a.** $\tan(68°) \approx 2.4751$ **b.** $1+\tan(68°) \approx 3.4751$ **c.** $\sqrt{1+\tan^2(68°)} \approx 2.6695$
None of the values are the same.

Chapter 2 Review Solutions --

1. QIV	**2.** QIV
3. QII	**4.** x-axis $(x < 0)$
5. Sketch of $120°$: 	**6.** Sketch of $-30°$: 
7. Sketch of $450°$: 	**8.** Sketch of $-185°$: 
9. Observe that $1000° = 2(360°) + 280°$. So, angle with smallest positive measure that is coterminal with $1000°$ is $\boxed{280°}$.	**10.** Observe that $-800° = 2(-360°) - 80°$. Since $-80°$ and $280°$ are coterminal, the angle with smallest positive measure that is coterminal with $-800°$ is $\boxed{280°}$.
11. Observe that $480° = 360° + 120°$. So, the angle with smallest positive measure that is coterminal with $480°$ is $\boxed{120°}$.	**12.** Observe that $-10° = -360° + 350°$. So, angle with smallest positive measure that is coterminal with $-10°$ is $\boxed{350°}$.

13. Consider the following diagram:  Using the Pythagorean Theorem, we obtain $h=\sqrt{6^2+(-8)^2}=10.$	$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-8}{10}=\frac{-4}{5}$ $\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{6}{10}=\frac{3}{5}$ $\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-8}{6}=\frac{-4}{3}$ $\cot\theta=\frac{1}{\tan\theta}=\frac{-3}{4}$ $\sec\theta=\frac{1}{\cos\theta}=\frac{5}{3}$ $\csc\theta=\frac{1}{\sin\theta}=\frac{-5}{4}$
14. Consider the following diagram:  Using the Pythagorean Theorem, we obtain $h=\sqrt{(-7)^2+(-24)^2}=25.$	$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-7}{25}$ $\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-24}{25}$ $\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-7}{-24}=\frac{7}{24}$ $\cot\theta=\frac{1}{\tan\theta}=\frac{24}{7}$ $\sec\theta=\frac{1}{\cos\theta}=\frac{-25}{24}$ $\csc\theta=\frac{1}{\sin\theta}=\frac{-25}{7}$

15. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{(-6)^2+2^2}=\sqrt{40}=2\sqrt{10}$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{2}{2\sqrt{10}}=\frac{\sqrt{10}}{10}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-6}{2\sqrt{10}}=\frac{-3\sqrt{10}}{10}$$

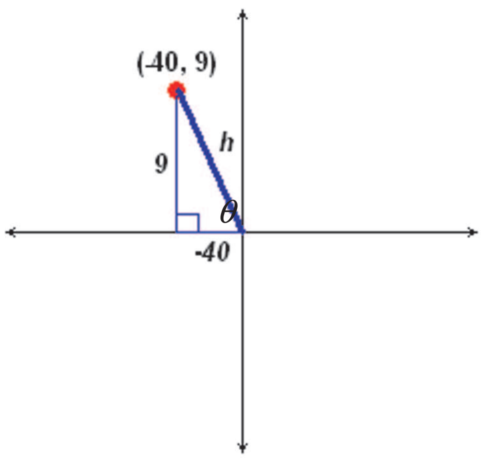
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{2}{-6}=\frac{-1}{3}$$

$$\cot\theta=\frac{1}{\tan\theta}=-3$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{-\sqrt{10}}{3}$$

$$\csc\theta=\frac{1}{\sin\theta}=\sqrt{10}$$

16. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h=\sqrt{(-40)^2+9^2}=41$.

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{9}{41}$$

$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{-40}{41}$$

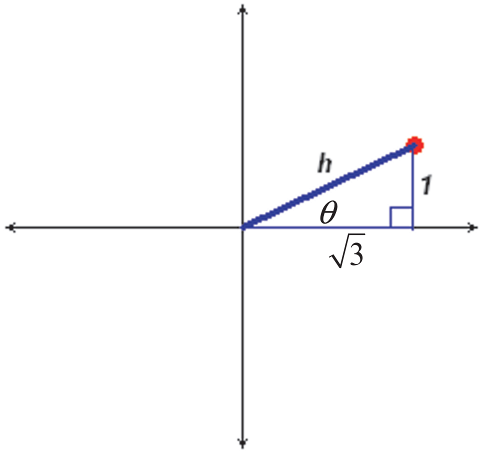
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{9}{-40}=\frac{-9}{40}$$

$$\cot\theta=\frac{1}{\tan\theta}=\frac{-40}{9}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{-41}{40}$$

$$\csc\theta=\frac{1}{\sin\theta}=\frac{41}{9}$$

17. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2.$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2}$$

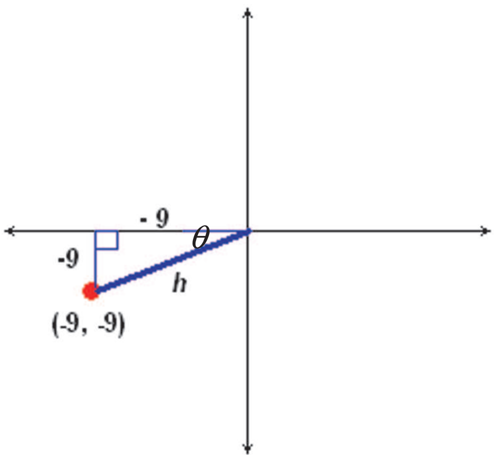
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \sqrt{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\csc\theta = \frac{1}{\sin\theta} = 2$$

18. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = \sqrt{(-9)^2 + (-9)^2} = \sqrt{162} = 9\sqrt{2}.$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{-9}{9\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{-9}{9\sqrt{2}} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{-9}{-9} = 1$$

$$\cot\theta = \frac{1}{\tan\theta} = 1$$

$$\sec\theta = \frac{1}{\cos\theta} = -\sqrt{2}$$

$$\csc\theta = \frac{1}{\sin\theta} = -\sqrt{2}$$

<table>
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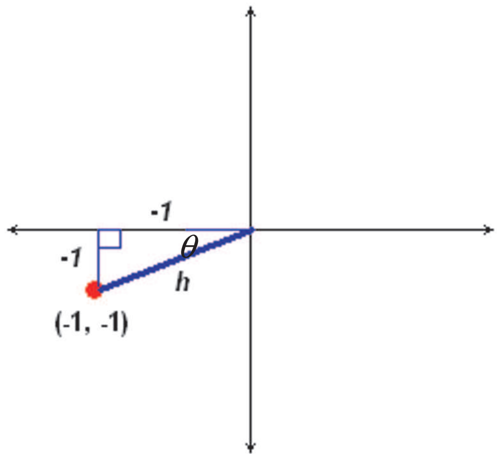
19. Consider the following diagram:



Note that we can use ANY point on the line $y = x$ $(x \le 0)$ to form the triangle. For convenience, we use $(-1,-1)$. Using the Pythagorean Theorem, we obtain $h = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$.

</td>
<td>

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{-1}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{-1}{\sqrt{2}} = \dfrac{-\sqrt{2}}{2}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{-1}{-1} = 1$

$\cot\theta = \dfrac{1}{\tan\theta} = 1$

$\sec\theta = \dfrac{1}{\cos\theta} = -\sqrt{2}$

$\csc\theta = \dfrac{1}{\sin\theta} = -\sqrt{2}$

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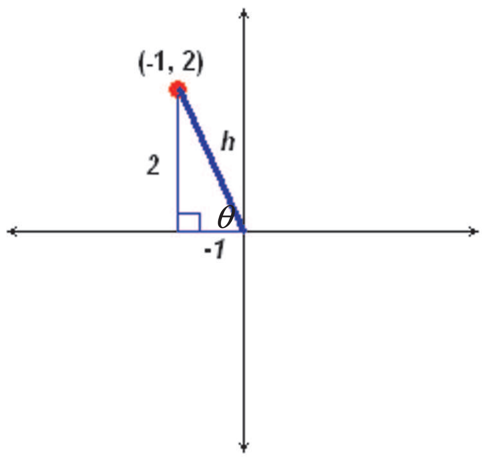
20. Consider the following diagram:



Note that we can use ANY point on the line $y = -2x$ $(x \le 0)$ to form the triangle. For convenience, we use the point $(-1,2)$. Using the Pythagorean Theorem, we obtain $h = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$.

</td>
<td>

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{2}{\sqrt{5}} = \dfrac{2\sqrt{5}}{5}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{-1}{\sqrt{5}} = \dfrac{-\sqrt{5}}{5}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{2}{-1} = -2$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{-1}{2}$

$\sec\theta = \dfrac{1}{\cos\theta} = -\sqrt{5}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{5}}{2}$

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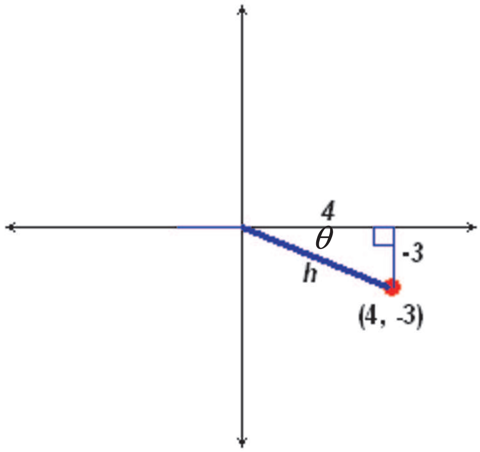
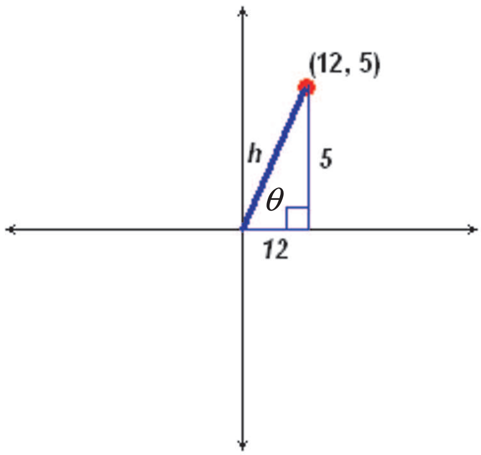
21. Consider the following diagram:



Note that we can use ANY point on the line $y=-\frac{3}{4}x$ $(x \geq 0)$ to form the triangle. In order to avoid the use of fractions, we use $(4,-3)$. Using the Pythagorean Theorem, we obtain $h=\sqrt{4^2+(-3)^2}=5.$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{-3}{5}$$
$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{4}{5}$$
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{-3}{4}$$
$$\cot\theta=\frac{1}{\tan\theta}=\frac{-4}{3}$$
$$\sec\theta=\frac{1}{\cos\theta}=\frac{5}{4}$$
$$\csc\theta=\frac{1}{\sin\theta}=\frac{-5}{3}$$

22. Consider the following diagram:



Note that we can use ANY point on the line $y=\frac{5}{12}x$ $(x \geq 0)$ to form the triangle. In order to avoid the use of fractions, we use the point (12,5). Using the Pythagorean Theorem, we obtain $h=\sqrt{12^2+5^2}=13.$

$$\sin\theta=\frac{\text{opp}}{\text{hyp}}=\frac{5}{13}$$
$$\cos\theta=\frac{\text{adj}}{\text{hyp}}=\frac{12}{13}$$
$$\tan\theta=\frac{\text{opp}}{\text{adj}}=\frac{5}{12}$$
$$\cot\theta=\frac{1}{\tan\theta}=\frac{12}{5}$$
$$\sec\theta=\frac{1}{\cos\theta}=\frac{13}{12}$$
$$\csc\theta=\frac{1}{\sin\theta}=\frac{13}{5}$$

23. For $\theta = 270°$, we have:

$$\sin\theta = -1$$
$$\cos\theta = 0$$
$$\tan\theta \text{ is undefined}$$
$$\cot\theta = 0$$
$$\sec\theta \text{ is undefined}$$
$$\csc\theta = -1$$

24. Since $\theta = 1080° = 3(360°)$, the standard position of θ is $0°$. So,

$$\sin\theta = 0$$
$$\cos\theta = 1$$
$$\tan\theta = 0$$
$$\cot\theta \text{ is undefined}$$
$$\sec\theta = 1$$
$$\csc\theta \text{ is undefined}$$

25. Since $\theta = -1080° = 3(-360°) + 0°$, we know that $-1080°$ is coterminal with $0°$. So,

$$\sin\theta = 0$$
$$\cos\theta = 1$$
$$\tan\theta = 0$$
$$\cot\theta \text{ is undefined}$$
$$\sec\theta = 1$$
$$\csc\theta \text{ is undefined}$$

26. Let n be an integer. Observe that since $\theta = (-360n°)$, the standard position of θ is $0°$. So,

$$\sin\theta = 0$$
$$\cos\theta = 1$$
$$\tan\theta = 0$$
$$\cot\theta \text{ is undefined}$$
$$\sec\theta = 1$$
$$\csc\theta \text{ is undefined}$$

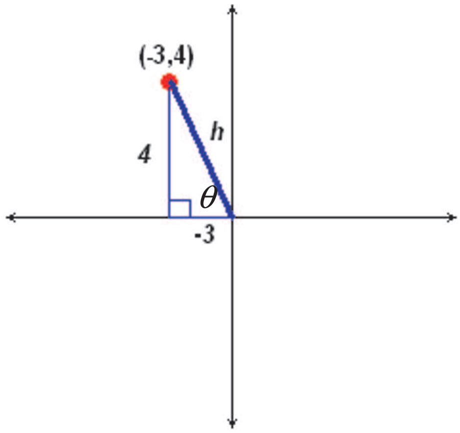
27. Recall that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. So, if $\sin\theta < 0$ and $\tan\theta > 0$, it follows that $\cos\theta < 0$. Hence, the terminal side of θ must be in QIII.

28. Since $\csc\theta = \dfrac{1}{\sin\theta} > 0$, it follows that $\sin\theta > 0$. Since also we have $\cos\theta < 0$, the terminal side of θ is in QII.

29. Since $\sec\theta = \dfrac{1}{\cos\theta} > 0$, it follows that $\cos\theta > 0$. Since also we have $\tan\theta = \dfrac{\sin\theta}{\cos\theta} < 0$, it follows that $\sin\theta < 0$, so that the terminal side of θ is in QIV.

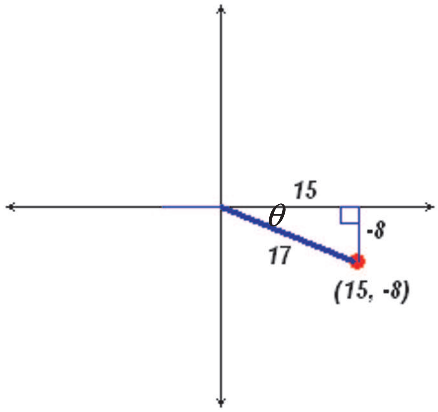
30. Recall that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. So, if $\sin\theta < 0$ and $\tan\theta < 0$, it follows that $\cos\theta > 0$. Hence, the terminal side of θ must be in QIV.

31. Consider the following diagram:



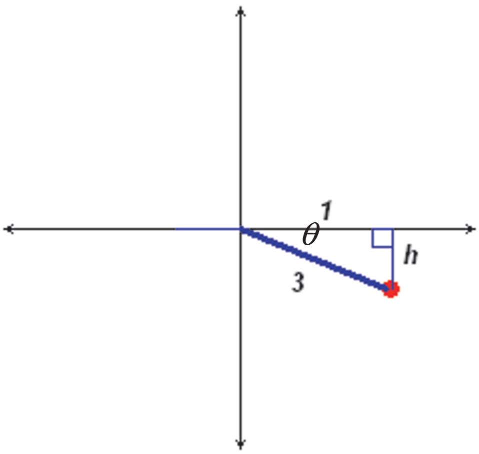
Using the Pythagorean Theorem, we obtain $h = \sqrt{(-3)^2 + 4^2} = 5$. So, we have that $\sin\theta = \boxed{\frac{4}{5}}$.

32. Consider the following diagram:



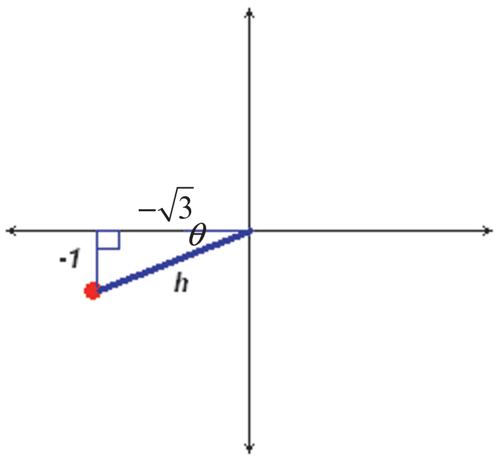
Using the Pythagorean Theorem, we obtain $h = \sqrt{17^2 - (-8)^2} = 15$. So, we have that $\cos\theta = \boxed{\frac{15}{17}}$.

33. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = -\sqrt{3^2 - 1^2} = -2\sqrt{2}$. So, we have that $\tan\theta = \frac{-2\sqrt{2}}{1} = \boxed{-2\sqrt{2}}$.

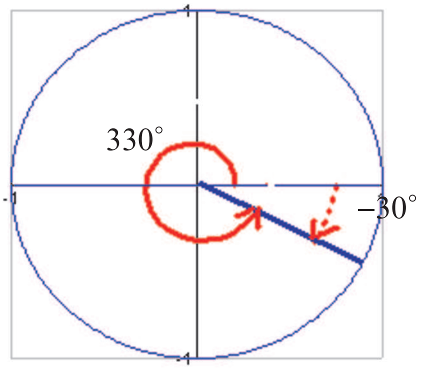
34. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = \sqrt{\left(-\sqrt{3}\right)^2 + (-1)^2} = 2$. So, we have that $\sin\theta = \boxed{\frac{-1}{2}}$.

35. First, $\cos(270°)=0$. Also, $\sin(-270°)=\sin(90°)=1$. Therefore, $\cos(270°)+\sin(-270°)=\boxed{1}$.	**36.** Since $450°=360°+90°$, we have $\cos(450°)=\cos(90°)=0$. Therefore, $\tan(450°)$ is undefined and hence, one cannot compute $\tan(450°)-\cot(540°)$.
37. First, since $-720°=2(-360°)$, we have $\cos(-720°)=\cos(0°)=1$, so that $\sec(-720°)=\frac{1}{1}=1$. Also, since $-450°=-360°-90°$, we have $\sin(-450°)=\sin(-90°)=\sin(270°)=-1$, so that $\csc(-450°)=\dfrac{1}{\sin(270°)}=-1$. So, $\sec(-720°)+\csc(-450°)=1-1=\boxed{0}$.	**38.** Since $\theta=1080°=3(360°)$, we have $$\tan(1080°)=\frac{\sin(1080°)}{\cos(1080°)} =\frac{\sin(0°)}{\cos(0°)}=0$$ Further, $\sin(-1080°)=-\sin(1080°)=0$ So, $\tan(1080°)-\sin(-1080°)=\boxed{0}$.
39. $\boxed{\text{Not possible}}$ because the equation $\sin\theta=-\sqrt{2}$ has no solution since the range of sine is $[-1,1]$.	**40.** $\boxed{\text{Possible}}$ since $-1\le 0.0004\le 1$.
41. $\boxed{\text{Possible}}$ since the tangent function can attain any real value for the appropriate choice of θ.	**42.** Observe that solutions to the equation $\csc\theta=\dfrac{1}{\sin\theta}=\dfrac{15}{17}$ need to satisfy $\sin\theta=\dfrac{17}{15}$. However, the latter equation has no solutions since the range of sine is $[-1,1]$, and $\dfrac{17}{15}>1$.

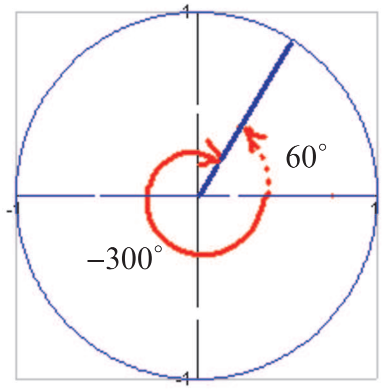
43.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sin(330°) = -\sin(30°) = \boxed{-\frac{1}{2}}$$

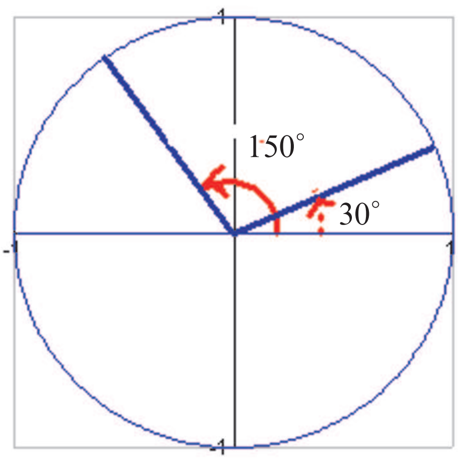
44.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\cos(-300°) = \cos(60°) = \boxed{\frac{1}{2}}$$

45.

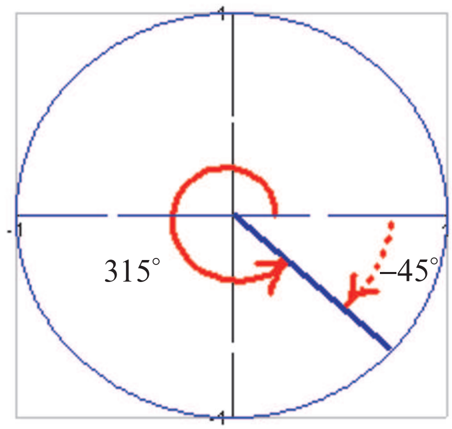


The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\tan(150°) = \frac{\sin(150°)}{\cos(150°)} = \frac{\sin 30°}{-\cos 30°}$$

$$= \frac{1/2}{-\sqrt{3}/2} = \boxed{-\frac{\sqrt{3}}{3}}$$

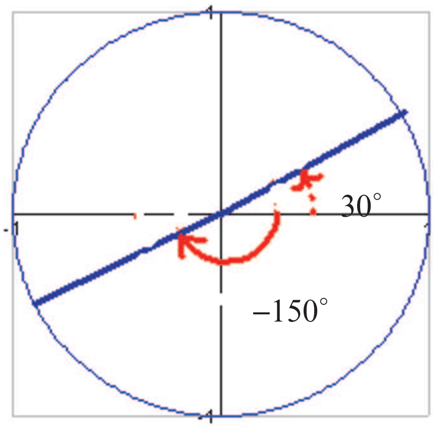
46.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\cot(315°) = \frac{\cos(315°)}{\sin(315°)} = \frac{\cos 45°}{-\sin 45°} = \boxed{-1}$$

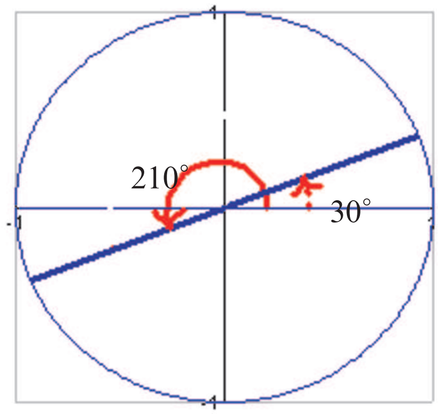
47.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sec(-150°) = \frac{1}{\cos(-150°)} = \frac{1}{-\cos(30°)} = \frac{-2}{\sqrt{3}} = \boxed{\frac{-2\sqrt{3}}{3}}$$

48.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\csc(210°) = \frac{1}{\sin(210°)} = \frac{-1}{\sin(30°)} = \boxed{-2}$$

49. – 0.2419

50. 0.0872

51. 1.0355

52. $\cot(500°) = \dfrac{1}{\tan(500°)} \approx \boxed{-1.1918}$

53. $\sec(111°) = \dfrac{1}{\cos(111°)} \approx \boxed{-2.7904}$

54. $\csc(215°) = \dfrac{1}{\sin(215°)} \approx \boxed{-1.7434}$

55. $\cot(-57°) = \dfrac{1}{\tan(-57°)} \approx \boxed{-0.6494}$

56. $\csc(-59°) = \dfrac{1}{\sin(-59°)} \approx \boxed{-1.1666}$

57. $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{-\frac{7}{11}} = \boxed{-\dfrac{11}{7}}$

58. Since $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{3\sqrt{5}}{5}$, we see that $\tan\theta = \dfrac{5}{3\sqrt{5}} = \dfrac{5\sqrt{5}}{15} = \boxed{\dfrac{\sqrt{5}}{3}}$.

59. $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{8}{17}}{\frac{-15}{17}} = \boxed{-\dfrac{8}{15}}$

60. $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\frac{-5}{13}}{\frac{-12}{13}} = \boxed{\dfrac{5}{12}}$

61. $\sin^2\theta = (\sin\theta)^2 = \left(\dfrac{2\sqrt{3}}{9}\right)^2 = \boxed{\dfrac{4}{27}}$

62.
$\cos^2\theta = (\cos\theta)^2 = (-0.45)^2 = \boxed{0.2025}$

63. $\tan^3\theta = (\tan\theta)^3 = \left(\sqrt{3}\right)^3 = \boxed{3\sqrt{3}}$

64. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\cos\theta$ yields $\cos\theta=\pm\sqrt{1-\sin^2\theta}$. Since the terminal side of θ is in QII, we use $\cos\theta=-\sqrt{1-\sin^2\theta}$. Hence,

$$\cos\theta=-\sqrt{1-\left(\frac{11}{61}\right)^2}=-\sqrt{\frac{3600}{61^2}}=-\frac{60}{61}.$$

Thus, $\tan\theta=\dfrac{\sin\theta}{\cos\theta}=\dfrac{\frac{11}{61}}{-\frac{60}{61}}=\boxed{\dfrac{-11}{60}}$.

65. Solving the equation $\sin^2\theta+\cos^2\theta=1$ for $\sin\theta$ yields $\sin\theta=\pm\sqrt{1-\cos^2\theta}$. Since the terminal side of θ is in QIV, we use $\sin\theta=-\sqrt{1-\cos^2\theta}$. Hence,

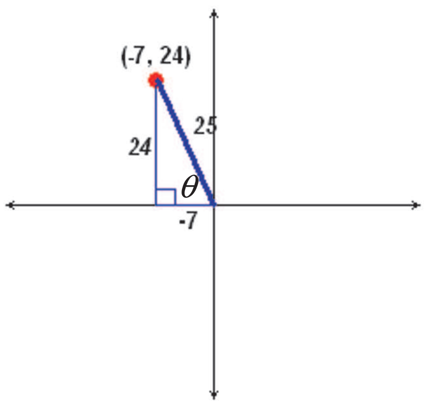
$$\sin\theta=-\sqrt{1-\left(\frac{7}{25}\right)^2}=-\sqrt{\frac{576}{25^2}}=-\frac{24}{25}.$$

Thus, $\cot\theta=\dfrac{\cos\theta}{\sin\theta}=\dfrac{\frac{7}{25}}{-\frac{24}{25}}=\boxed{\dfrac{-7}{24}}$.

66. Solving the equation $1+\tan^2\theta=\sec^2\theta$ for $\sec\theta$ yields $\sec\theta=\pm\sqrt{1+\tan^2\theta}$. Since the terminal side of θ is in QIV, $\cos\theta>0$ and so $\sec\theta>0$. So, we use $\sec\theta=\sqrt{1+\tan^2\theta}$. Hence, $\sec\theta=\sqrt{1+\tan^2\theta}=\sqrt{1+(-1)^2}=\boxed{\sqrt{2}}$.

67. Solving the equation $1+\cot^2\theta=\csc^2\theta$ for $\cot\theta$ yields $\cot\theta=\pm\sqrt{\csc^2\theta-1}$. Since the terminal side of θ is in QII, $\sin\theta>0$ and $\cos\theta<0$, so $\cot\theta<0$. So, we use $\cot\theta=-\sqrt{\csc^2\theta-1}$. Hence, $\cot\theta=-\sqrt{\csc^2\theta-1}=-\sqrt{\left(\frac{13}{12}\right)^2-1}=\boxed{-\dfrac{5}{12}}$.

68. Consider the following diagram:

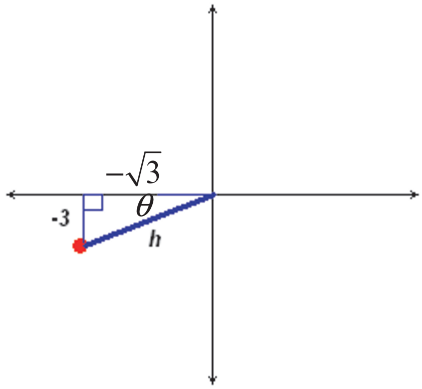


Since $\tan\theta=-\dfrac{24}{7}$ and the terminal side of θ lies in QII, we conclude that

$$\sin\theta=\frac{24}{25}$$

$$\cos\theta=-\frac{7}{25}$$

69. Consider the following diagram:



Using the Pythagorean Theorem, we see that

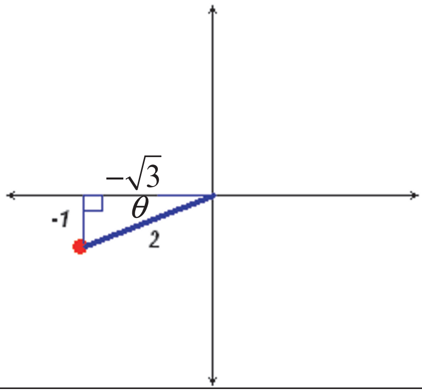
$h=\sqrt{(-3)^2+\left(-\sqrt{3}\right)^2}=\sqrt{12}=2\sqrt{3}$.

Since $\cot\theta=\frac{\sqrt{3}}{3}$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta=-\frac{3}{2\sqrt{3}}=-\frac{3\sqrt{3}}{6}=-\frac{\sqrt{3}}{2}$$

$$\cos\theta=-\frac{\sqrt{3}}{2\sqrt{3}}=-\frac{3}{6}=-\frac{1}{2}$$

70. Consider the following diagram:



Since $\cot\theta=\sqrt{3}$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta=-\frac{1}{2}$$

$$\cos\theta=-\frac{\sqrt{3}}{2}$$

71.

$$\sec\theta\cot^2\theta=\frac{1}{\cancel{\cos\theta}}\times\frac{\cancel{\cos^2}\theta}{\sin^2\theta}=\boxed{\frac{\cos\theta}{\sin^2\theta}}$$

72. $\sin\theta\cot\theta=\cancel{\sin\theta}\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right)=\boxed{\cos\theta}$

73. $\dfrac{\tan\theta}{\sec\theta}=\dfrac{\frac{\sin\theta}{\cancel{\cos\theta}}}{\frac{1}{\cancel{\cos\theta}}}=\boxed{\sin\theta}$

74.

$$\left(\cos\theta+\sec\theta\right)^2=\left(\cos\theta+\frac{1}{\cos\theta}\right)^2$$

$$=\frac{\left(\cos^2\theta+1\right)^2}{\cos^2\theta}$$

$$=\frac{\cos^4\theta+2\cos^2\theta+1}{\cos^2\theta}$$

$$=\cos^2\theta+2+\frac{1}{\cos^2\theta}$$

75.

$$\cot\theta(\sec\theta+\sin\theta)=\frac{\cos\theta}{\sin\theta}\left(\frac{1}{\cos\theta}+\sin\theta\right)=\frac{\cancel{\cos\theta}}{\sin\theta}\left(\frac{1}{\cancel{\cos\theta}}\right)+\frac{\cos\theta}{\cancel{\sin\theta}}\left(\cancel{\sin\theta}\right)=\frac{1}{\sin\theta}+\cos\theta$$

76. $\dfrac{\sin^2\theta}{\csc\theta}=\dfrac{\sin^2\theta}{\frac{1}{\sin\theta}}=\sin^3\theta.$

77. $\sec 180°=-1$

78. $\csc 180°$ is undefined.

79. Using a calculator yields:

$$\sin(218°)\approx-0.6157$$
$$\sin^{-1}(-0.6157)\approx-38°$$

So, the calculator assumes the terminal side is in QIV.

80. Using a calculator yields:

$$\cos(28°)\approx 0.8829$$
$$\cos^{-1}(0.8829)\approx 152°$$

So, the calculator assumes the terminal side is in QII.

81. **a.** $\cos(73°)\approx 0.2924$ **b.** $1-\sin(73°)\approx 0.4370$ **c.** $\sqrt{1-\sin^2(73°)}\approx 0.2924$

The values in parts **a** and **c** are the same.

82. **a.** $\cot(28°)\approx 1.8807$ **b.** $1+\cot(28°)\approx 2.8807$ **c.** $\sqrt{1+\cot^2(28°)}\approx 2.1301$

None of the values are the same.

Chapter 2 Practice Test Solutions

1.

	0°	**QI**	90°	**QII**	180°	**QIII**	270°	**QIV**	360°
$\sin\theta$	0	+	1	+	0	–	–1	–	0
$\cos\theta$	1	+	0	–	–1	–	0	+	1

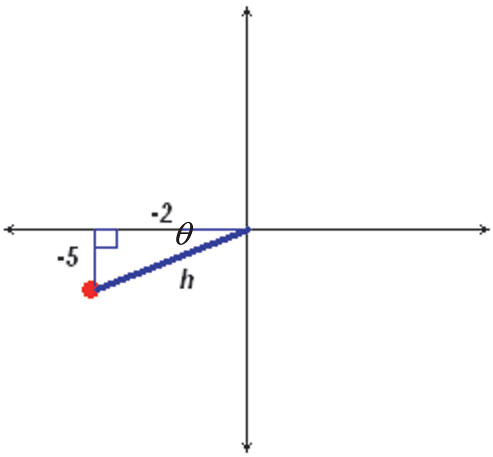
2. Since $\sec\theta=\dfrac{1}{\cos\theta}>0$, it follows that $\cos\theta>0$. Since also we have $\cot\theta=\dfrac{\cos\theta}{\sin\theta}<0$, it follows that $\sin\theta<0$, so that the terminal side of θ is in $\boxed{\text{QIV}}$.

3. Note that $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ is undefined when $\cos\theta=0$, which occurs when $\theta=\boxed{90°,\ 270°}$.

4. Observe that $890°=2(360°)+170°$. This angle is not coterminal with $-170°$. The angle with the smallest positive measure that is coterminal with 890° is $\boxed{170°}$.

5. Observe that $-500°=-360°-140°$. So, the angle with the smallest positive measure that is coterminal with $-500°$ is $\boxed{220°}$.

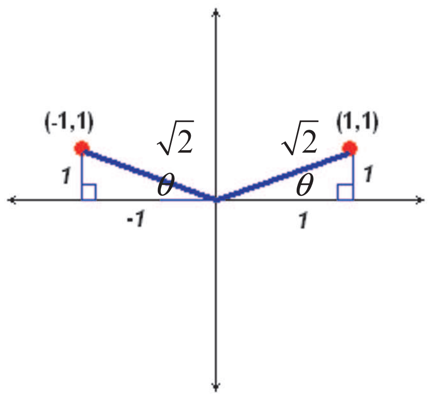
6. Consider the following diagram:



Using the Pythagorean Theorem, we obtain $h = \sqrt{(-5)^2 + (-2)^2} = \sqrt{29}$.
So, we have that

$$\cos\theta = \frac{-2}{\sqrt{29}} = \boxed{\frac{-2\sqrt{29}}{29}}.$$

7. Consider the following diagram:

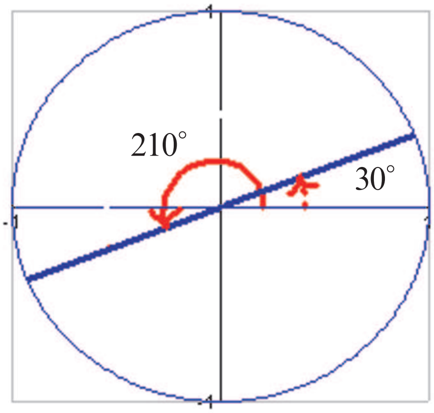


<u>Case 1</u>: $x \geq 0$ Consider the point (1,1) on $y = x$. Then, $\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

<u>Case 2</u>: $x < 0$ Consider the point $(-1,1)$ on $y = x$. Then, $\sin\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$.

Thus, we conclude that $\sin\theta = \frac{\sqrt{2}}{2}$ if the terminal side of θ lies along the graph of $y = |x|$.

8. Consider the following diagram:



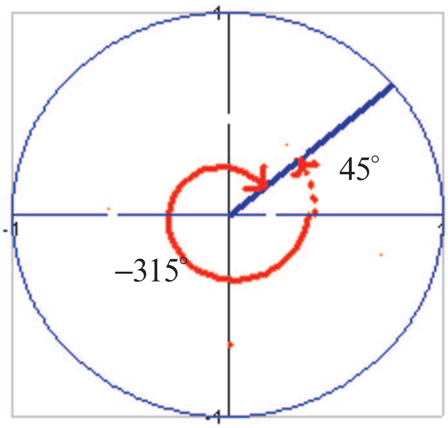
The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sin(210°) = -\sin(30°) = \boxed{-\frac{1}{2}}$$

9. $\frac{\sqrt{2}}{2}$

10. $-\frac{\sqrt{3}}{3}$

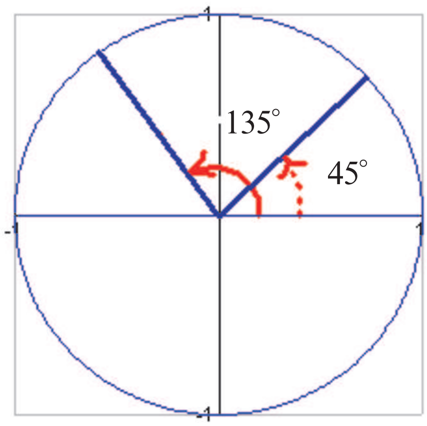
11.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\tan(-315°) = \frac{\sin(-315°)}{\cos(-315°)} = \frac{\sin 45°}{\cos 45°} = \boxed{1}$$

12.



The given angle is shown in **bold**, while the reference angle is *dotted*.

$$\sec(135°) = \frac{1}{\cos(135°)} = \frac{1}{-\cos(45°)}$$

$$= \frac{-2}{\sqrt{2}} = \boxed{-\sqrt{2}}$$

13. $\cot(222°) = \frac{1}{\tan(222°)} \approx \boxed{1.1106}$

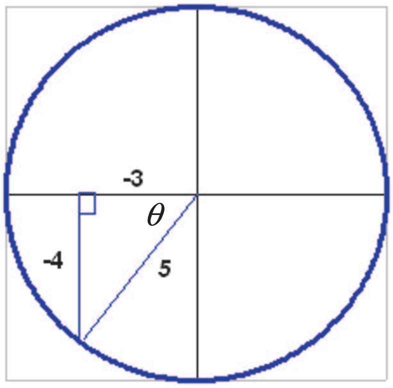
14. $\sec(-165°) = \frac{1}{\cos(-165°)} \approx -1.0353$

15. First, we convert the angle to DD:

$$110°11' = 110° + 11'\left(\frac{1°}{60'}\right) \approx 110.1833°$$

Hence, $\cos(110°11') \approx -0.3450$.

16. Consider the following diagram:

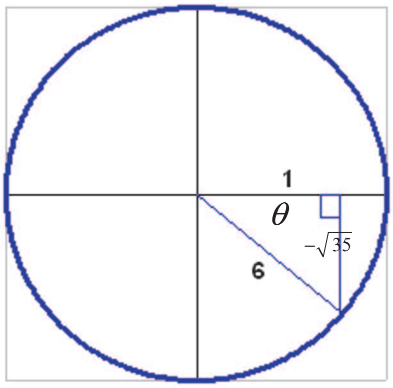


Since $\tan\theta = \frac{4}{3}$ and the terminal side of θ lies in QIII, we conclude that

$$\sin\theta = \frac{-4}{5}$$

$$\cos\theta = \frac{-3}{5}$$

17. Consider the following diagram:



Since $\cos\theta = \frac{1}{6}$ and the terminal side of θ lies in QIV, we see that $\sin\theta = \frac{-\sqrt{35}}{6}$.

So,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{-\sqrt{35}}{6}}{\frac{1}{6}} = \boxed{-\sqrt{35}}.$$

18. $$\frac{(1-\cos\theta)(1+\cos\theta)}{\tan^2\theta} = \frac{\overbrace{1-\cos^2\theta}^{=\sin^2\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} = \cancel{\sin^2\theta}\left(\frac{\cos^2\theta}{\cancel{\sin^2\theta}}\right) = \cos^2\theta$$

19. a. $\boxed{\text{Not possible}}$ since the range of cosine is $[-1,1]$.

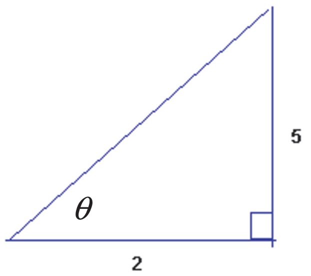
b. $\boxed{\text{Possible}}$ since the range of tangent is $\mathbb{R}$.

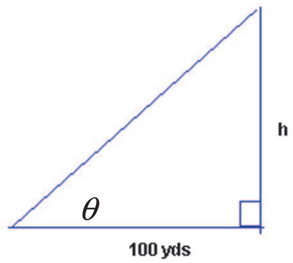
20. $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\sqrt{15}/4}{1/4} = \sqrt{15}$

21. $\cot^2\theta + 1 = \csc^2\theta \;\Rightarrow\; \cot\theta = \pm\sqrt{\csc^2\theta - 1} = \pm\sqrt{\frac{4}{3} - 1} = \pm\frac{\sqrt{3}}{3}$

22. $\cos^2\theta + \sin^2\theta = 1 \;\Rightarrow\; \cos\theta = \pm\sqrt{1 - \sin^2\theta} = \pm\sqrt{1 - \left(\frac{3}{7}\right)^2} = \pm\frac{\sqrt{40}}{7} = \pm\frac{2\sqrt{10}}{7}$

23. Since $\tan\theta = \frac{5}{2}$, we have the triangle, and the subsequent similar one:





Since sides are in proportion, we have $\frac{5}{2} = \frac{h}{100\text{ yd.}} \;\Rightarrow\; h = 250\text{ yd.}$

24. $\cos\theta = 0.1125 \;\Rightarrow\; \theta = \cos^{-1}(0.1125) \approx 83.54^\circ$. Since the terminal side is in QIV, we use $\theta = 360^\circ - 83.54^\circ \approx 276^\circ$.

25. $\sec(870^\circ) = \sec(720^\circ + 150^\circ) = \frac{1}{\cos(720^\circ + 150^\circ)} = \frac{1}{\cos(150^\circ)} = \frac{1}{-\sqrt{3}/2} = -\frac{2\sqrt{3}}{3}$

26. **a.** undefined since dividing by zero.
b. defined and equals 0
c. undefined since dividing by zero.

Chapter 2 Cumulative Review Solutions ---

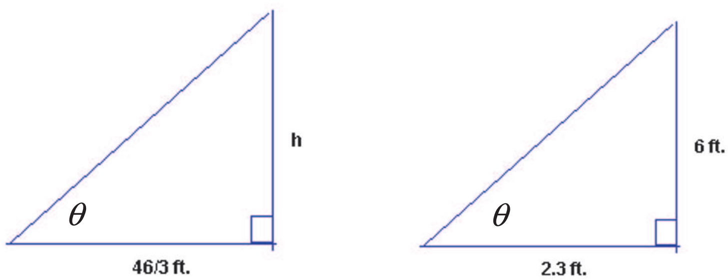
1. a. $90° - 37° = 53°$ **b.** $180° - 37° = 143°$

2. Using the Pythagorean theorem, we have $24^2 + b^2 = 25^2 \Rightarrow b = 7$.

3. The central angle of the sector formed between consecutive minutes on the face of a clock is $\frac{360°}{12} = 30°$. Since there are 7 such sectors used in forming 35 minutes, the corresponding central angle would be $7 \times 30° = 210°$.

4. If $x = 15$ ft. is a leg of a $45° - 45° - 90°$ triangle, then the hypotenuse is $15\sqrt{2}$ ft. ≈ 21.21 ft.

5. Consider the following two triangles:



Using similar triangles, we obtain

$$\frac{h}{46/3 \text{ ft.}} = \frac{6 \text{ ft.}}{2.3 \text{ ft.}} \Rightarrow h = 40 \text{ ft.}$$

6. Note that angles A and B are supplementary, so that $A = 95°$. Since angles A and E are corresponding angles, $E = 95°$.

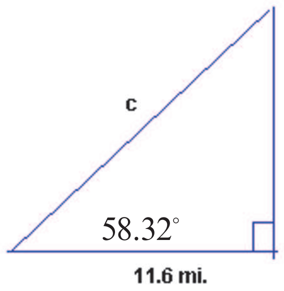
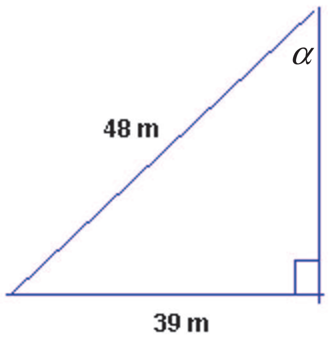
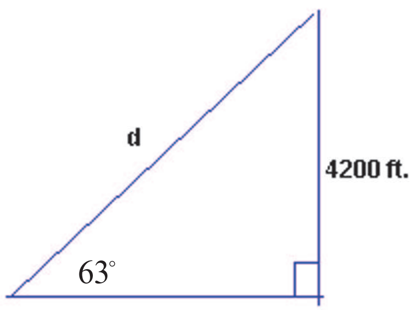
7. $\tan 60° = \cot(90° - 60°) = \cot(30°)$

8. $\cos(30° - x) = \sin(90° - (30° - x)) = \sin(60° + x)$

9. $\frac{\sqrt{3}}{2}$

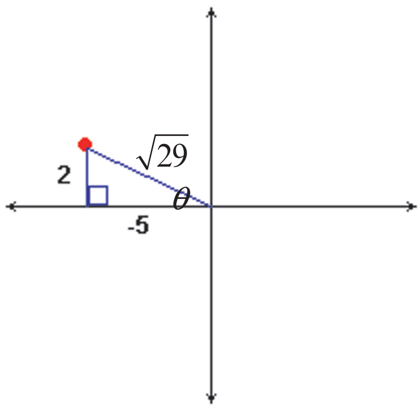
10. 0.4695

11. $17.525° = 17° + 0.525°\left(\frac{60'}{1°}\right) = 17° + 31.5' = 17° + 31' + 0.5'\left(\frac{60''}{1'}\right) = 17°31'30''$

12. Consider the following triangle: 	Observe that $\cos(58.32°) = \dfrac{11.6 \text{ mi.}}{c}$, and so $c = \dfrac{11.6 \text{ mi.}}{\cos(58.32°)} \approx 22.1 \text{ mi.}$
13. Consider the following triangle: 	Observe that $\sin\alpha = \dfrac{39}{48} \Rightarrow \alpha = \sin^{-1}\left(\dfrac{39}{48}\right) \approx 54°.$
14. Consider the following triangle: 	Observe that $\sin 63° = \dfrac{4,200 \; ft.}{d} \Rightarrow d = \dfrac{4,200 \; ft.}{\sin 63°} \approx 4,713 \; ft.$
15. QIII	**16.** $360° + (-170°) = 190°$

17. QIV

18. Consider the following triangle:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{29}}{29}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = -\frac{5\sqrt{29}}{29}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -\frac{2}{5}$$

$$\cot\theta = \frac{1}{\tan\theta} = -\frac{5}{2}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{\sqrt{29}}{5}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{29}}{2}$$

19. Since $\theta = -1170° = -3(360°) - 90°$, the standard position of θ is $270°$. So,

$\sin\theta = -1$

$\cos\theta = 0$

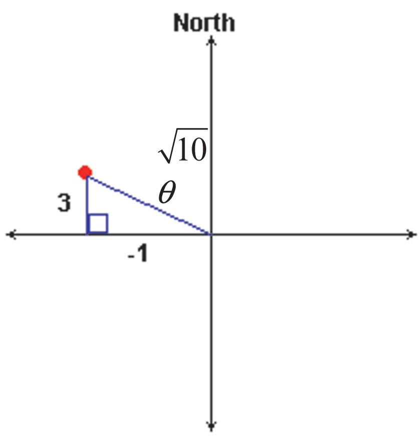
$\tan\theta$ is undefined

$\cot\theta = 0$

$\sec\theta$ is undefined

$\csc\theta = -1$

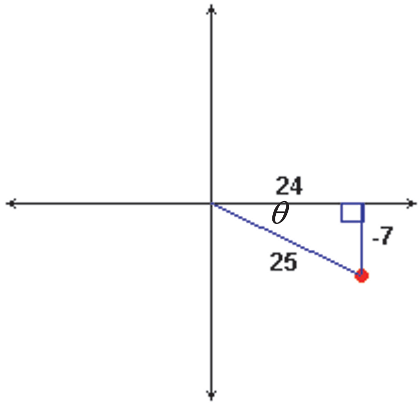
20. Consider the following triangle:



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{3\sqrt{10}}{10}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = -\frac{\sqrt{10}}{10}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = -3$$

21. Consider the following triangle: 	$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{1}{24/25} = \dfrac{25}{24}$
22. Not possible since $-1 \le \sin\theta \le 1$ and so, either $\dfrac{1}{\sin\theta} \ge 1$ or $\dfrac{1}{\sin\theta} \le -1$. Since $\dfrac{2}{\pi}$ is between -1 and 1, there is no value of θ for which $\dfrac{1}{\sin\theta} = \dfrac{2}{\pi}$.	
23. $-\dfrac{\sqrt{3}}{2}$	**24.** $(0.2)^3 = 0.008$
25. $\tan\theta(\csc\theta + \cos\theta) = \dfrac{\sin\theta}{\cos\theta}\left(\dfrac{1}{\sin\theta} + \cos\theta\right) = \dfrac{1}{\cos\theta} + \sin\theta$	