

# Test Bank for Elementary Differential Equations 12th Boyce

TEST BANK SAMPLE PREVIEW

PUBLISHER

**John Wiley & Sons**

COPYRIGHT

**2021**

RESOURCE TYPE

**Test Bank**

ISBN

**9781119777755**

**Elementary Differential Equations, 12e (Boyce)**  
**Chapter 2 First-Order Differential Equations**

1) Find the general solution to the differential equation  $\tan(\frac{\pi}{4}x)y' + y = -5 \sin(\frac{\pi}{4}x)$ .

A)  $y = -\frac{1}{\pi} \cdot \frac{\cos(\frac{\pi}{2}x)}{\sin(\frac{\pi}{4}x)} + C \csc(\frac{\pi}{4}x)$

B)  $y = -\frac{1}{\pi} \cdot \cot(\frac{\pi}{4}x) + C \csc(\frac{\pi}{4}x)$

C)  $y = \frac{2}{\pi} \cdot \frac{\cos(\frac{\pi}{2}x)}{\sin(\frac{\pi}{4}x)} + C \csc(\frac{\pi}{4}x)$

D)  $y = \frac{2}{\pi} \cdot \cot(\frac{\pi}{4}x) + C \csc(\frac{\pi}{4}x)$

Answer: A

Type: MC Var: 1

2) Find the general solution of the differential equation  $xy' - 2y = x^7, x > 0$ .

A)  $y = 4x^5 + Cx^2$

B)  $y = 4x^6 + Cx^2$

C)  $y = \frac{x^7}{4} + Cx^{-2}$

D)  $y = \frac{x^7}{5} + Cx^2$

Answer: D

Type: MC Var: 1

3) Which of the following first-order differential equations are linear in  $y$ ? Select all that apply.

A)  $(x + 7)\frac{dy}{dx} = -3y + 3$

B)  $\frac{dy}{dx} = \frac{12}{5y - 2x}$

C)  $\frac{dy}{dx} = 6x + \frac{19y}{x} + 14$

D)  $\frac{dy}{dx} = \frac{y^3 - 2y}{x^3 - 2x}$

E)  $\frac{dy}{dx} = 2y + 7y^2$

Answer: A, C

Type: MC Var: 1

4) Consider the differential equation  $\frac{dy}{dx} + \frac{6x^5}{x^6 + 4} = \frac{1}{x^6 + 4}$ .

Which of the following is the general solution of this equation?

A)  $y = \frac{x}{x^6 + 4} + C$

B)  $y = \frac{x}{x^5 + 4} + C$

C)  $y = \frac{x + C}{x^6 + 4}$

D)  $y = \frac{x + C}{x^5 + 4}$

Answer: C

Type: MC Var: 1

5) Consider the differential equation  $\frac{dy}{dx} + \frac{7x^6}{x^7 + 7} = \frac{1}{x^7 + 7}$ .

What choice of the arbitrary constant in the general solution ensures that the solution curve passes through the point  $(1, 4)$ ?

Answer: 31

Type: SA Var: 1

6) Consider the differential equation  $\frac{dy}{dx} - 2y = e^{-2x}$ .

Which of these is the general solution to the equation?

A)  $y = -\frac{1}{4}e^{-2x} + Ce^{2x}$

B)  $y = \frac{Ce^{0x}}{-4}$

C)  $y = -\frac{1}{4}e^{-2x} + C$

D)  $y = e^{-4x} + C$

Answer: A

Type: MC Var: 1

7) Consider the differential equation  $\frac{dy}{dx} - 3y = e^{6x}$ .

What choice of the arbitrary constant in the general solution ensures that the solution satisfies the initial condition  $y(0) = 7$ ?

Answer:  $\frac{20}{3}$

Type: SA Var: 1

8) What is the general solution of this first-order differential equation?

$$x\frac{dy}{dx} + 2y = 4x^2$$

A)  $y = 4x + Cx^{-2}$

B)  $y = x^2 + Cx^{-2}$

C)  $y = Cx + 4x^{-2}$

D)  $y = Cx^2 + x^{-2}$

Answer: B

Type: MC Var: 1

9) Identify the integrating factor for this linear differential equation:

$$\frac{dy}{dx} - 2y = e^{-4x}$$

Answer:  $e^{-2x}$

Type: SA Var: 1

10) Identify the integrating factor for this linear differential equation:

$$\frac{dy}{dx} + \frac{5}{x \ln x} y = 4x^5$$

Answer:  $(\ln x)^5$

Type: SA Var: 1

11) Identify the integrating factor for this linear differential equation:

$$\cos(3x) \frac{dy}{dx} + y \sin(3x) = 2 \sin(3x) \cos(3x)$$

Answer:  $|\cos(3x)|^{-\frac{1}{3}}$

Type: SA Var: 1

12) Identify the integrating factor for this linear differential equation:

$$2x^7 \frac{dy}{dx} - 4y = -4x^6$$

Answer:  $e^{\frac{1}{3}x^{-6}}$

Type: SA Var: 1

13) Which of these is the general solution of this homogeneous differential equation?

$$x^2 \frac{dy}{dx} = xy + x^2 e^{\frac{y}{x}}$$

A)  $y = -\ln(C - \ln x)$

B)  $y = -x \ln(C - \ln x)$

C)  $y = \frac{\ln(C - \ln x)}{x}$

D)  $y = \frac{C - \ln(\ln x)}{x}$

Answer: B

Type: MC Var: 1

14) Consider the differential equation  $\frac{dy}{dx} = \frac{y}{x}(\ln y - \ln x + 1)$ .

(i) Which of these is the general solution of this equation?

A.  $y = e^{Cx}$

B.  $y = Cxe^x$

C.  $y = \frac{e^x}{x} + \frac{C}{x}$

D.  $y = xe^{Cx}$

E.  $y = \frac{Ce^x}{x}$

(ii) What choice of the arbitrary constant in the general solution ensures that the solution curve passes through the point  $\left(\frac{1}{4}, 4\right)$ ?

Answer: (i) D

(ii)  $4\ln(16)$

Type: ES      Var: 1

15) What is the general solution of the differential equation  $\frac{dy}{dx} + 7x^3y^4 = 0$ ?

A)  $y = \sqrt[3]{\frac{21}{4}x^4 + C}$

B)  $y = \frac{1}{\sqrt[3]{\frac{21}{4}x^4 + C}}$

C)  $y = \sqrt[4]{\frac{21}{4}x^4 + C}$

D)  $y = \frac{1}{\sqrt[4]{\frac{21}{4}x^4 + C}}$

Answer: B

Type: MC      Var: 1

16) What is the general solution of the differential equation  $(1 + x^2)\frac{dy}{dx} = 1 + y^2$ ?

- A)  $y = x + C$
- B)  $y = (1 + x)\tan^{-1}x + C$
- C)  $y = \tan(C + \tan^{-1}x)$
- D)  $y = \frac{C \tan^{-1}x}{1 + x}$

Answer: C

Type: MC Var: 1

17) What is the general solution of the differential equation  $\frac{dy}{dx} = y \sin x + 6 \sin x$ ?

- A)  $y = Ce^{\cos x} - 6$
- B)  $y = Ce^{-\cos x} - 6$
- C)  $y = Ce^{\cos x} + 6$
- D)  $y = Ce^{-\cos x} + 6$

Answer: B

Type: MC Var: 1

18) What is the solution of this initial value problem?

- $$\begin{cases} \frac{dy}{dx} = \frac{x^2}{e^y} \\ y(3) = 0 \end{cases}$$
- A)  $y = -\ln\left[\frac{x^3}{3} + 8\right]$
  - B)  $y = -\ln\left[\frac{x^3}{3} - 8\right]$
  - C)  $y = \ln\left[\frac{x^3}{3} - 8\right]$
  - D)  $y = \ln\left[\frac{x^3}{3} + 8\right]$

Answer: C

Type: MC Var: 1

19) What is the solution of this initial value problem?

$$\begin{cases} y^4 y' = e^{3x}(y^5 + 1) \\ y(0) = 3 \end{cases}$$

A)  $y = 244e^{15x} + 1^{\frac{1}{5}}$

B)  $y = 244e^{15x} - 1^{\frac{1}{5}}$

C)  $y = 244e^{5(3-x)} + 1^{\frac{1}{5}}$

D)  $y = 244e^{\frac{5}{3}(3-x)} - 1^{\frac{1}{5}}$

Answer: D

Type: MC Var: 1

20) Consider this initial value problem:

$$\begin{cases} \frac{dy}{dx} = 2(1+x)(1+y^2) \\ y(0) = 0 \end{cases}$$

What is the solution of this initial value problem?

A)  $y = \tan^{-1}(x^2 + 2x)$

B)  $y = x^2 \tan x$

C)  $y = \tan(x^2 + 2x)$

D)  $y = x^2 \tan^{-1} x$

Answer: C

Type: MC Var: 1

21) Consider this initial value problem:

$$\begin{cases} \frac{dy}{dx} = 2(1+x)(1+y^2) \\ y(0) = 0 \end{cases}$$

What is approximately the interval where the solution applies? Round each of the endpoints of the interval to the nearest hundredth.

Answer: (-2.89, 0.90)

Type: SA Var: 1



22) What is the general solution of the differential equation  $\frac{dy}{dx} = \frac{3y^2 + 2xy}{2xy + x^2}$ ?

- A)  $\ln |x| + C = \ln \left| \frac{y}{y+x} \right|$   
B)  $\ln |x| + C = \ln \left| \frac{y+x}{y} \right|$   
C)  $\ln |y| + C = \ln \left| \frac{x}{y+x} \right|$   
D)  $\ln |y| + C = \ln \left| \frac{y+x}{x} \right|$

Answer: A

Type: MC Var: 1

23) Which of the following first-order differential equations are separable? Select all that apply.

- A)  $\frac{dy}{dx} = e^{-7x} + 6y$   
B)  $\frac{dy}{dx} = -2x + 6y$   
C)  $\frac{dy}{dx} = x^4 \cdot \sqrt[4]{4+7x} \cdot \sqrt[5]{y+6}$   
D)  $\frac{dy}{dx} = \sin\left(\frac{1}{2}\pi x\right) + \cos(5\pi y)$   
E)  $\frac{dy}{dx} = \ln(4x + 9y)$

Answer: A, C

Type: MC Var: 1

24) A city's water reservoir contains 7 billion cubic meters (bcm) of water. The purification system ensures that the concentration of pollutants remains constant at 0.5 kilograms per bcm, and sensors will trigger an alarm if the concentration of pollutants rises above 1 kilogram per bcm. Water flows in and out of the reservoir at the same rate of 0.20 bcm per day, and the concentration of pollutants in the inflow is 1.9 kilograms per bcm. At all times, the reservoir is well mixed.

Set up a differential equation whose solution  $x(t)$  is the amount of pollutant in the reservoir at time  $t$ .

$$\frac{dx}{dt} = \underline{\hspace{2cm}}$$

Answer:  $0.38 - \frac{1}{35}x$

Type: SA Var: 1

25) A city's water reservoir contains 7 billion cubic meters (bcm) of water. The purification system ensures that the concentration of pollutants remains constant at 0.3 kilograms per bcm, and sensors will trigger an alarm if the concentration of pollutants rises above 1 kilogram per bcm. Water flows in and out of the reservoir at the same rate of 0.20 bcm per day, and the concentration of pollutants in the inflow is 1.9 kilograms per bcm. At all times, the reservoir is well mixed.

Set up a differential equation whose solution  $x(t)$  is the amount of pollutant in the reservoir at time  $t$  and find the general solution of the equation.

$$x(t) = \underline{\hspace{2cm}}$$

Answer:  $13.3 + Ce^{\frac{1}{35}t}$

Type: SA Var: 1

26) A city's water reservoir contains 7 billion cubic meters (bcm) of water. The purification system ensures that the concentration of pollutants remains constant at 0.6 kilograms per bcm, and sensors will trigger an alarm if the concentration of pollutants rises above 1 kilogram per bcm. Water flows in and out of the reservoir at the same rate of 0.25 bcm per day, and the concentration of pollutants in the inflow is 2 kilograms per bcm. At all times, the reservoir is well mixed.

Set up a differential equation whose solution  $x(t)$  is the amount of pollutant in the reservoir at time  $t$ . Let  $t = 0$  be the time when the purification system fails. What is  $x(0)$ ?

Answer: 4.2

Type: SA Var: 1

27) A city's water reservoir contains 6 billion cubic meters (bcm) of water. The purification system ensures that the concentration of pollutants remains constant at 0.2 kilograms per bcm, and sensors will trigger an alarm if the concentration of pollutants rises above 1 kilogram per bcm. Water flows in and out of the reservoir at the same rate of 0.20 bcm per day, and the concentration of pollutants in the inflow is 1.9 kilograms per bcm. At all times, the reservoir is well mixed.

Set up a differential equation whose solution  $x(t)$  is the amount of pollutant in the reservoir at time  $t$  and find the particular solution of the differential equation satisfying the initial condition  $x(0)$  at time  $t = 0$  when the purification system fails.

$x(t) =$  \_\_\_\_\_

Answer:  $11.4 - 10.2e^{\frac{1}{30}t}$

Type: SA Var: 1

28) A city's water reservoir contains 6 billion cubic meters (bcm) of water. The purification system ensures that the concentration of pollutants remains constant at 0.5 kilograms per bcm, and sensors will trigger an alarm if the concentration of pollutants rises above 1 kilogram per bcm. Water flows in and out of the reservoir at the same rate of 0.25 bcm per day, and the concentration of pollutants in the inflow is 1.9 kilograms per bcm. At all times, the reservoir is well mixed.

If the purification system fails, how much time (in days) elapses before the alarm is triggered? Round your answer to the nearest hundredth of a day.

Answer: 10.60 days

Type: SA Var: 1

29) A ball is thrown from the top of a tall building with a speed of 12 meters per second. Suppose the ball hits the ground with a speed of 69 meters per second. (Recall that the acceleration due to gravity is  $9.8 \text{ m/s}^2$ .)

What is the time  $t_*$  (in seconds) of impact? Round your answer to the nearest hundredth of a second.

Answer: 8.27 seconds

Type: SA Var: 1

30) A ball is thrown from the top of a tall building with a speed of 14 meters per second. Suppose the ball hits the ground with a speed of 69 meters per second. (Recall that the acceleration due to gravity is  $9.8 \text{ m/s}^2$ .)

How tall (in meters) is the building? Round your answer to the nearest hundredth of a meter.

Answer: 232.91 meters

Type: SA Var: 1

31) Which of these differential equations models the following situation?

In a town with 4 million people, the rate at which the inhabitants hear a rumor is proportional to the number of people who have not heard the rumor. Use  $N(t)$  for the number of people (in millions) who have heard the rumor at time  $t$ .

A)  $\frac{dN}{dt} = k(4 - N)$ , where  $k$  is a positive constant.

B)  $\frac{dN}{dt} = k(N - 4)$ , where  $k$  is a positive constant.

C)  $\frac{dN}{dt} = kN - 4$ , where  $k$  is a nonzero constant.

D)  $\frac{dN}{dt} = 4 - kN$ , where  $k$  is a nonzero constant.

Answer: A

Type: MC Var: 1

32) On another planet, a ball dropped from a height of 5 meters takes 3 seconds to hit the ground.

Let  $g$  be the acceleration due to gravity on this planet,  $v_0$  the initial velocity of the ball, and  $x_0$  the initial height of the ball above the ground. Write a formula for the height of the ball,  $x(t)$ , above the ground at time  $t$ .

Answer:  $x(t) = -\frac{g}{2}t^2 + v_0t + x_0$

Type: SA Var: 1

33) On another planet, a ball dropped from a height of 5 meters takes 4 seconds to hit the ground.

Determine the value of  $g$ .

Answer:  $\frac{5}{8} \text{ m/s}^2$

Type: SA Var: 1

34) On another planet, a ball dropped from a height of 5 meters takes 4.5 seconds to hit the ground.

Find the amount of time it takes the ball to hit the ground if it is dropped from a height of 122 meters. Round your answer to the nearest tenth of a second.

Answer: 22.2 seconds

Type: SA Var: 1

35) A pie is moved from the oven at 475 degrees Fahrenheit to a freezer at 25 degrees Fahrenheit. After 10 minutes, the pie has cooled to 425 degrees Fahrenheit.

Let  $T(t)$  be the temperature of the pie  $t$  minutes after it has been moved to the freezer. Formulate an initial-value problem whose solution is  $T(t)$ .

Answer:  $\begin{cases} \frac{dT}{dt} = k(25 - T) \\ T(10) = 425 \end{cases}$ , where  $k$  is a negative constant of proportionality  
Type: SA Var: 1

36) A pie is moved from the oven at 425 degrees Fahrenheit to a freezer at 18 degrees Fahrenheit. After 10 minutes, the pie has cooled to 375 degrees Fahrenheit.

Let  $T(t)$  be the temperature of the pie  $t$  minutes after it has been moved to the freezer. Formulate and solve an initial-value problem for  $T(t)$ .

Answer:  $T(t) = 18 + 407 \frac{357}{407} \frac{t}{10}$

Type: SA Var: 1

37) The amount of medicine in the bloodstream decays exponentially with a half-life of 7 hours. There must be at least 30 milligrams of medicine per pound of body weight to keep the patient safe during a one-hour procedure.

Set up an initial value problem whose solution is the amount of medicine,  $x(t)$  (measured in milligrams), in the bloodstream at time  $t$ . Let  $t = 0$  correspond to the time when the procedure begins, and assume that  $x(0) = x_0$ .

Answer:  $\begin{cases} \frac{dx}{dt} = kx \\ x(0) = x_0 \end{cases}$ , where  $k$  is the constant of proportionality.  
Type: SA Var: 1

38) The amount of medicine in the bloodstream decays exponentially with a half-life of 4 hours. There must be at least 25 milligrams of medicine per pound of body weight to keep the patient safe during a one-hour procedure.

Set up an initial value problem whose solution is the amount of medicine,  $x(t)$  (measured in milligrams), in the bloodstream at time  $t$ . Let  $t = 0$  correspond to the time when the procedure begins, and assume that  $x(0) = x_0$ . Determine the *exact* value of proportionality constant in the differential equation from.

Answer:  $-\frac{\ln 2}{4}$

Type: SA Var: 1

39) The amount of medicine in the bloodstream decays exponentially with a half-life of 5 hours. There must be at least 25 milligrams of medicine per pound of body weight to keep the patient safe during a one-hour procedure.

If a patient weighs 130 pounds, how much medicine should be administered at the beginning of the procedure? Round your answer to the nearest milligram.

Answer: 3733 milligrams

Type: SA Var: 1

40) What is the general solution of the differential equation  $xy' + 4y = 24x^3y^{\frac{1}{2}}$ ?

A)  $y = (12x^3 + C)^{\frac{3}{2}}$

B)  $y = 12x^{-2} + \frac{C}{x^4}$

C)  $y = 12x^{-2} + \frac{C}{x^4}$

D)  $y = 12x^6 + \frac{C}{x^4}$

Answer: B

Type: MC Var: 1

41) Which of the following statements are true for this initial-value problem? Select all that apply.

$$(y - 9)\frac{dy}{dx} = x - 10 \text{ with } y(10) = 9$$

A) A locally unique solution is not guaranteed to exist by the local existence and uniqueness theorem for first-order differential equations because  $\frac{x - 10}{y - 9}$  is not continuous at the point (10, 9).

B)  $y = x - 19$  is the only solution of this initial value problem.

C)  $y = x - 19$  and  $y = 1 - x$  are both solutions of this initial value problem.

D) This initial value problem cannot have a solution because the conditions of the existence and uniqueness theorem for first-order linear equations are not satisfied.

E) The existence and uniqueness theorem for first-order linear equations ensures the existence of a unique local solution of this initial value problem because  $x - 10$  is continuous at the point (10, 9).

Answer: A, C

Type: MC Var: 1

42) Which of the following is an accurate conclusion that can be made using the existence and uniqueness theorem for first-order differential equations for this initial value problem?

$$\frac{dy}{dx} = \frac{x^6(y-6)}{(x+9)^4} \text{ with } y(10) = 6$$

Throughout,  $f(x, y) = \frac{x^6(y-6)}{(x+9)^4}$ .

A) The initial value problem has a unique solution because  $f(x, y)$  is continuous on a rectangle containing the point  $(10, 6)$ .

B) The initial value problem is not guaranteed to have a unique solution because  $f_x(x, y)$  is not continuous when  $x = -9$ .

C) The initial value problem has a unique solution because both  $f(x, y)$  and  $f_y(x, y)$  are continuous on a rectangle containing the point  $(10, 6)$ .

D) The initial value problem does not have a solution because  $f_x(x, y)$  and  $f_y(x, y)$  are not both continuous on a rectangle containing the point  $(10, 6)$ .

Answer: C

Type: MC Var: 1

43) Which of the following is an accurate conclusion that can be made using the existence and uniqueness theorem for first-order nonlinear equations for this initial value problem?

$$\frac{dy}{dx} = \frac{x^5\sqrt{y-8}}{(x+1)^4} \text{ with } y(2) = 8$$

Throughout,  $f(x, y) = \frac{x^5\sqrt{y-8}}{(x+1)^4}$ .

A) The initial value problem has a unique solution because  $f(x, y)$  is continuous on a rectangle containing the point  $(2, 8)$  on its boundary.

B) The initial value problem is not guaranteed to have a unique solution because  $f_x(x, y)$  is not continuous when  $x = -1$ .

C) The initial value problem has a unique solution because both  $f(x, y)$  and  $f_y(x, y)$  are continuous on a rectangle containing the point  $(2, 8)$ .

D) The initial value problem is not guaranteed to have a unique local solution because there is no rectangle surrounding the point  $(2, 8)$  on which both  $f(x, y)$  and  $f_y(x, y)$  are continuous.

Answer: D

Type: MC Var: 1

44) Which of these is the general solution of the differential equation  $3y^2y' + y^3 = e^{-x}$ ?

A)  $y = (x + C)^{\frac{1}{3}} e^{-\frac{x}{3}}$

B)  $y = (x + C)^3 e^{-3x}$

C)  $y = (x + C)^{\frac{1}{3}} e^{\frac{x}{3}}$

D)  $y = (x + C)^3 e^{3x}$

Answer: A

Type: MC Var: 1

45) Consider the initial value problem

$$(y - 6)\frac{dy}{dt} = t^3, y(0) = 6$$

What can be said about the applicability of the existence and uniqueness theorem to this initial value problem?

A) The theorem does not apply because the initial condition is prescribed at  $t = 0$  and the

function  $f(t, y) = \frac{t^3}{y - 6}$  equals 0 when evaluated at such a point.

B) The theorem does not apply because the function  $f(t, y) = \frac{t^3}{y - 6}$  is discontinuous at any point of the form  $(t, 6)$ .

C) The theorem applies and ensures the existence of a unique local solution of this initial value problem.

D) The theorem does not apply because the differential equation is nonlinear.

Answer: B

Type: MC Var: 1

46) Consider the initial value problem

$$(y - 4)\frac{dy}{dt} = t^8, y(0) = 4$$

Find all solutions of this initial value problem.

Answer:  $y = 8 \pm \sqrt{\frac{2}{9}} \cdot t^{\frac{9}{2}}$

Type: SA Var: 1



47) Consider the autonomous differential equation

$$\frac{dy}{dt} = 8y(y - 2)(y + 1)$$

Which of these is an equilibrium solution of this differential equation? Select all that apply.

A)  $y = 0$

B)  $y = 2$

C)  $y = 1$

D)  $y = -2$

E)  $y = -1$

Answer: A, B, E

Type: MC Var: 1

48) Consider the autonomous differential equation

$$\frac{dy}{dt} = 2y(y - 1)(y + 6)$$

Which of the following statements are true? Select all that apply.

A)  $y = 1$  is asymptotically stable.

B)  $y = -6$  is unstable.

C)  $y = 0$  is asymptotically stable.

D)  $y = 6$  is unstable.

E)  $y = 1$  is semi-stable.

Answer: B, C

Type: MC Var: 1

49) Consider the autonomous differential equation

$$\frac{dy}{dt} = 7y(y - 1)(y + 2)$$

Determine  $\lim_{t \rightarrow \infty} y(t)$  for the initial condition  $y(t_0) = (8, -4)$ .

Answer:  $-\infty$

Type: SA Var: 1

50) Consider the autonomous differential equation

$$\frac{dy}{dt} = \ln \left( \frac{6}{6 + y^2} \right) \cdot \sin^2 y$$

Which of these is a complete list of the equilibrium solutions of this differential equation?

A)  $y = 2k\pi$ , where  $k$  is an integer

B)  $y = 0$

C)  $y = k\pi$ , where  $k$  is an integer

D)  $y = \frac{(2k+1)\pi}{2}$ , where  $k$  is an integer

Answer: C

Type: MC Var: 1

51) Consider the autonomous differential equation

$$\frac{dy}{dt} = \ln \left( \frac{3}{3 + y^2} \right) \cdot \sin^2 y$$

Which of the following statements is true?

A) All nonequilibrium solutions tend toward  $-\infty$  as  $t \rightarrow \infty$ .

B)  $y = 2k\pi$  is stable and  $y = (2k+1)\pi$  is unstable, for any integer  $k$ .

C)  $y = 2k\pi$  is unstable and  $y = (2k+1)\pi$  is stable, for any integer  $k$ .

D) All equilibrium solutions are semi-stable.

Answer: D

Type: MC Var: 1

52) Consider the autonomous differential equation

$$\frac{dy}{dt} = \ln \left( \frac{2}{2 + y^2} \right) \cdot \sin^8 y$$

Determine  $\lim_{t \rightarrow \infty} y(t)$  for the initial condition  $y(t_0) = (4, \frac{11\pi}{5})$ . Enter the *exact* answer.

Answer:  $2\pi$

Type: SA Var: 1

53) Consider the autonomous differential equation

$$\frac{dy}{dt} = y^3 + 10y^2 + 25y$$

Which of these is an equilibrium solution of this differential equation? Select all that apply.

A)  $y = -5$

B)  $y = 5$

C)  $y = 0$

D)  $y = -10$

E)  $y = 10$

Answer: A, C

Type: MC Var: 1

54) Consider the autonomous differential equation

$$\frac{dy}{dt} = y^3 + 8y^2 + 16y$$

Identify the following statement as TRUE or FALSE:

A solution curve passing through the point  $(0, -2)$  tends to 0 as  $t \rightarrow \infty$ .

Answer: False

Type: SA Var: 1

55) A model of a fishery which grows logistically and is harvested at a constant rate is given by

$$\begin{cases} \frac{dy}{dx} = -6 + 7x - x^2 \\ x(0) = x_0 \end{cases}$$

Which of these is an equilibrium solution of this differential equation? Select all that apply.

A)  $y = 0$

B)  $y = -1$

C)  $y = 6$

D)  $y = 1$

E)  $y = -6$

Answer: C, D

Type: MC Var: 1

56) A model of a fishery which grows logistically and is harvested at a constant rate is given by

$$\begin{cases} \frac{dy}{dx} = -5 + 6x - x^2 \\ x(0) = x_0 \end{cases}$$

For what values of  $x_0$  does the fish population become extinct?

A) For all  $x_0$  in  $(0, 1]$ .

B) For all  $x_0$  in  $(0, 1)$ .

C) For all  $x_0$  in  $(0, 5)$ .

D) For all  $x_0$  in  $(0, 5]$ .

E) For all  $x_0$  in  $(1, 5)$ .

F) For all  $x_0$  in  $[1, 5]$ .

Answer: B

Type: MC Var: 1

57) Consider the differential equation  $(-3x^{-4}y^5 + e^{6x}) + (5x^{-3}y^4 + \sin(\frac{1}{4}y))y' = 0$ .

Let  $M(x, y) = -3x^{-4}y^5 + e^{6x}$  and  $N(x, y) = 5x^{-3}y^4 + \sin(\frac{1}{4}y)$ . Which of the following statements

regarding this differential equation is true?

A) It is not exact because  $M_x \neq N_y$ .

B) It is exact because  $M_y = N_x$ .

C) It is exact because  $M_y = N_y$ .

D) It is not exact because  $M_x \neq N_x$ .

Answer: B

Type: MC Var: 1

58) Consider the differential equation  $(4x^3y^8 + e^{2x}) + (8x^4y^7 + \sin(\frac{1}{3}y))y' = 0$ .

What is the general solution of this differential equation?

A)  $x^4y^8 + \frac{e^{2x}}{2} + \frac{\cos(\frac{1}{3}y)}{\frac{1}{3}} = C$

B)  $x^4y^8 + \frac{e^{2x}}{2} - \frac{\cos(\frac{1}{3}y)}{\frac{1}{3}} = C$

C)  $x^4y^8 + 2e^{2x} - \frac{1}{3}\cos(\frac{1}{3}y) = C$

D)  $x^4y^8 + 2e^{2x} + \frac{1}{3}\cos(\frac{1}{3}y) = C$

Answer: B

Type: MC Var: 1

59) Solve this initial value problem:

$$\begin{cases} (6x - y) + (8y - x)y' = 0 \\ y(0) = 0 \end{cases}$$

A)  $6x^2 - xy + 8y^2 = 0$

B)  $6x^2 + xy + 8y^2 = 0$

C)  $3x^2 - xy + 4y^2 = 0$

D)  $3x^2 + xy + 4y^2 = 0$

Answer: C

Type: MC Var: 1

60) Which of the following first-order differential equations are exact? Select all that apply.

A)  $(8y - 4x)y' - 4y = 0$

B)  $(2y - 5x) + (5y + 2x)y' = 0$

C)  $4y^3 + (x^4 + y^4 - 7x)y' = 0$

D)  $(7x - \cos(\frac{5}{2}y)) - \sin(\frac{5}{2}y)y' = 0$

Answer: A, B

Type: MC Var: 1

61) Which of the following is the general solution of the differential equation

$$x\frac{dy}{dx} + y - 6x^2 - 2xe^x = 0?$$

A)  $2e^x + xy - 2xe^x - 2x^3 = C$

B)  $2e^x + xy + 2xe^x - \frac{1}{4}x^4 = C$

C)  $\frac{1}{2}e^x + xy + 2xe^x - 3x^4 = C$

D)  $2xe^x - xy - 2e^x - 3x^2 = C$

Answer: A

Type: MC Var: 1

62) For what value of **K** is this differential equation exact? Enter the exact answer, not a decimal approximation.

$$(10y + Kx^2)y' + (-8x^5 + 7xy) = 0$$

Answer:  $\frac{7}{2}$

Type: SA Var: 1

63) For what value of **K** is this differential equation exact? Enter the exact answer, not a decimal approximation.

$$\left(\frac{2 + Kx}{y^8}\right)dy + \left(\frac{2}{x^7} + \frac{1}{y^7}\right)dx = 0$$

Answer: -7

Type: SA Var: 1

64) Consider the differential equation  $(6x^2y + 6xy + 2y^3)dx + (3x^2 + 3y^2)dy = 0$ . Find an integrating factor  $\mu(x)$  so that the following differential equation is exact:

$$\mu(x)(6x^2y + 6xy + 2y^3)dx + \mu(x)(3x^2 + 3y^2)dy = 0$$

Answer:  $\mu(x) = e^{2x}$

Type: SA Var: 1

65) Consider the following initial value problem:

$$\begin{cases} \frac{dy}{dx} = x + 4 \\ y(0) = 3 \end{cases}$$

(i) Use Euler's method with two equal steps to find an approximation of  $y(1)$ . Enter the exact answer, not an approximation.

(ii) Solve the initial value problem and compute the solution at  $x = 1$ . Enter the exact answer, not an approximation.

(iii) What is the error in Euler's method in making this approximation?

Answer: (i)  $7\frac{1}{4}$

(ii)  $7\frac{1}{2}$

(iii)  $\frac{1}{4}$

Type: ES      Var: 1

66) Consider the following initial value problem:

$$\begin{cases} \frac{dy}{dt} = y^2 + t^2 \\ y(0) = 0 \end{cases}$$

Determine the first two iterations of Picard's iteration method.

$\varphi_0(t) = \underline{\hspace{2cm}}$

$\varphi_1(t) = \underline{\hspace{2cm}}$

$\varphi_2(t) = \underline{\hspace{2cm}}$

Answer:  $\varphi_0(t) = 0$

$\varphi_1(t) = \frac{t^3}{3}$

$\varphi_2(t) = \frac{t^3}{3} + \frac{t^7}{63}$

Type: ES      Var: 1

67) Consider the difference equation  $y_{n+1} = -0.55y_n$ ,  $n = 0, 1, 2, 3, \dots$

Find the explicit solution of this difference equation in terms of  $y_0$ .

- A)  $y_n = -0.55ny_0$ ,  $n = 0, 1, 2, 3, \dots$
- B)  $y_n = (-0.55)^n y_0$ ,  $n = 0, 1, 2, 3, \dots$
- C)  $y_n = -0.55^n y_0$ ,  $n = 0, 1, 2, 3, \dots$
- D)  $y_n = (-0.55y_0)^n$ ,  $n = 0, 1, 2, 3, \dots$

Answer: B

Type: MC Var: 1

68) Consider the difference equation  $y_{n+1} = -0.40y_n$ ,  $n = 0, 1, 2, 3, \dots$

Which of the following is an accurate description of the behavior of the solutions to this difference equation in terms of  $y_0$ ?

- A) If  $y_0 \neq 0$ , then the sequence  $\{y_n\}$  diverges in an oscillatory manner.
- B) If  $y_0 > 0$ , then the sequence  $\{y_n\}$  converges to 0 as  $n \rightarrow \infty$ ; for all other choices of  $y_0$ , the sequence  $\{y_n\}$  diverges.
- C) If  $y_0 \neq 0$ , then the sequence  $\{y_n\}$  diverges toward  $-\infty$  as  $n \rightarrow \infty$ .
- D) The sequence  $\{y_n\}$  converges to 0 as  $n \rightarrow \infty$  for all values of  $y_0$ .

Answer: D

Type: MC Var: 1

69) Consider the difference equation  $y_{n+1} = 0.50y_n + 6$ ,  $n = 0, 1, 2, 3, \dots$

Find the explicit solution of this difference equation in terms of  $y_0$ .

- A)  $y_n = 0.50^n y_0 + 12$ ,  $n = 0, 1, 2, 3, \dots$
- B)  $y_n = 0.50^n y_0 + 4$ ,  $n = 0, 1, 2, 3, \dots$
- C)  $y_n = 0.50^{n-1} y_0 + 6$ ,  $n = 0, 1, 2, 3, \dots$
- D)  $y_n = 0.50^{n-1} y_0 + 12$ ,  $n = 0, 1, 2, 3, \dots$

Answer: A

Type: MC Var: 1



70) Consider the difference equation  $y_{n+1} = 0.20y_n + 5$ ,  $n = 0, 1, 2, 3, \dots$

Which of the following is an accurate description of the behavior of the solutions to this difference equation in terms of  $y_0$ ?

- A) If  $y_0 \neq 0$ , then the sequence  $\{y_n\}$  diverges to  $\infty$  as  $n \rightarrow \infty$ .
- B) The sequence  $\{y_n\}$  converges to 6.25 as  $n \rightarrow \infty$  for all values of  $y_0$ .
- C) If  $y_0 > 0$ , then the sequence  $\{y_n\}$  converges to 6.25 as  $n \rightarrow \infty$ ; for all other choices of  $y_0$ , the sequence  $\{y_n\}$  converges to 0 as  $n \rightarrow \infty$ .
- D) If  $y_0 \leq 0$ , then the sequence  $\{y_n\}$  converges to 6.25 as  $n \rightarrow \infty$ ; for all other choices of  $y_0$ , the sequence  $\{y_n\}$  converges to 0 as  $n \rightarrow \infty$ .

Answer: B

Type: MC Var: 1

71) What is the two-parameter family of solutions of the second-order differential equation  $yy'' + (y')^2 = yy'$ ?

- A)  $y^2 = C_1 e^x + C_2$
- B)  $\arctan(y) + C_1 = C_2 x$
- C)  $\frac{1}{4}y^2 + C_1 \ln|y| = x + C_2$
- D)  $\ln|y^2 + C_1| = x + C_2$

Answer: A

Type: MC Var: 1

72) What is the two-parameter family of solutions of the second-order differential equation  $\frac{1}{7}xy'' + y' = 3x$ ?

- A)  $y = \frac{21}{16}x^2 + C_1 x^{\frac{6}{7}} + C_2$
- B)  $y = C_1 x^2 + C_1 x^{-6}$
- C)  $y = C_1 x^2 + C_1 x^{\frac{6}{7}}$
- D)  $y = \frac{21}{16}x^2 + C_1 + C_2 x^{-6}$

Answer: D

Type: MC Var: 1

73) For each differential equation, select each category in which it falls.

| Differential Equation                             | Linear<br>in y | Separable | Bernoulli | Homogeneous | Exact |
|---------------------------------------------------|----------------|-----------|-----------|-------------|-------|
| $3y + e^{-8x} \frac{dy}{dx} = 0$                  |                |           |           |             |       |
| $x\sqrt{4+y^2} - y\sqrt{3+x^2} \frac{dy}{dx} = 0$ |                |           |           |             |       |
| $5x^4 + (x^5 - 7) \frac{dy}{dx} = 0$              |                |           |           |             |       |
| $\frac{dy}{dx} = -\frac{9x + 9y^2x}{5y + 8x^2y}$  |                |           |           |             |       |
| $(e^{-3x} + 5) \frac{dy}{dx} = y - ye^{-3x}$      |                |           |           |             |       |
| $\frac{dy}{dx} = (5\cos x + 1)y - y$              |                |           |           |             |       |
| $\frac{dy}{dx} = \frac{1 - 8x}{7y + 9}$           |                |           |           |             |       |
| $\frac{dy}{dx} = \frac{7y^2 + 8xy}{8xy + 2x^2}$   |                |           |           |             |       |
| $\frac{dy}{dx} = \frac{5x + 6y}{5x}$              |                |           |           |             |       |
| $\frac{dy}{dx} = \frac{y^2 + y}{x^9 + x}$         |                |           |           |             |       |

Answer:

| Differential Equation                             | Linear<br>in y | Separable | Bernoulli | Homogeneous | Exact |
|---------------------------------------------------|----------------|-----------|-----------|-------------|-------|
| $3y + e^{-8x} \frac{dy}{dx} = 0$                  | X              | X         |           |             |       |
| $x\sqrt{4+y^2} - y\sqrt{3+x^2} \frac{dy}{dx} = 0$ |                | X         |           |             |       |
| $5x^4 + (x^5 - 7) \frac{dy}{dx} = 0$              | X              | X         |           |             | X     |
| $\frac{dy}{dx} = -\frac{9x + 9y^2x}{5y + 8x^2y}$  |                | X         |           |             |       |
| $(e^{-3x} + 5) \frac{dy}{dx} = y - ye^{-3x}$      | X              | X         |           |             |       |
| $\frac{dy}{dx} = (5\cos x + 1)y - y$              |                | X         | X         |             |       |
| $\frac{dy}{dx} = \frac{1 - 8x}{7y + 9}$           |                | X         |           |             | X     |
| $\frac{dy}{dx} = \frac{7y^2 + 8xy}{8xy + 2x^2}$   |                |           |           | X           |       |
| $\frac{dy}{dx} = \frac{5x + 6y}{5x}$              | X              |           |           | X           |       |
| $\frac{dy}{dx} = \frac{y^2 + y}{x^9 + x}$         |                | X         | X         |             | X     |

Type: ES      Var: 1

© (2022) John Wiley & Sons, Inc. All rights reserved. Instructors who are authorized users of this course are permitted to download these materials and use them in connection with the course. Except as permitted herein or by law, no part of these materials should be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise.