TT TEXTBOOK TESTBANKS

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Solution and Answer Guide

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Yes. $|3-10|=|-7|=7$ and $|10-3|=|7|=7.$

P.1.4.

(a) iii

(b) iv

(c) ii

(d) v

(e) i

P.1.5.

$-9, -\frac{7}{2}, 5, \frac{2}{3}, \sqrt{3}, 0, 3, -4, 2, -11$

(a) Natural numbers: 5, 8, 2

(b) Whole numbers: 5, 0, 8, 2

(c) Integers: $-9, 5, 0, 8, -4, 2, -11$

(d) Rational numbers: $-9, -\frac{7}{2}, 5, \frac{2}{3}, 0, 8, -4, 2, -11$

(e) Irrational numbers: $\sqrt{3}$

P.1.6.

$\sqrt{5}, -7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5$

(a) Natural numbers: 12, 5

(b) Whole numbers: 0, 12, 5

(c) Integers: $-7, 0, -3, 12, 5$

(d) Rational numbers: $-7, -\frac{7}{3}, 0, 3.14, \frac{5}{4}, -3, 12, 5$

(e) Irrational numbers: $\sqrt{5}$

P.1.7.

$2.01, 0.\overline{6}, -13, 0.010110111\ldots, 1, -6$

(a) Natural numbers: 1

(b) Whole numbers: 1

(c) Integers: $-13,\ 1,\ -6$

(d) Rational numbers: $2.01, 0.\overline{6},\ -13, 1, -6$

(e) Irrational numbers: $0.010110111\ldots$

P.1.8.

$25,\ -17,\ -\frac{12}{5},\ \sqrt{9},\ 3.12,\ \frac{1}{2}\pi,\ 18,\ -11.1,\ 13$

(a) Natural numbers: $25,\ \sqrt{9}\ = 3,\ 18,\ 13$

(b) Whole numbers: $25,\ \sqrt{9}\ = 3,\ 18,\ 13$

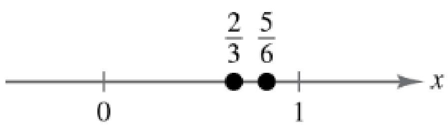
(c) Integers: $25,\ -17,\ \sqrt{9} = 3,\ 18,\ 13$

(d) Rational numbers: $25,\ -17,\ -\frac{12}{5},\ \sqrt{9}\ = 3,\ 3.12,\ 18,\ -11.1,\ 13$

(e) Irrational numbers: $\frac{1}{2}\pi$

P.1.9.

$\frac{5}{6} > \frac{2}{3}$

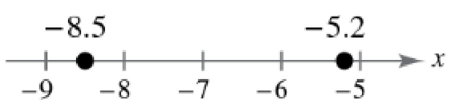


P.1.10.

$-4 > -8$

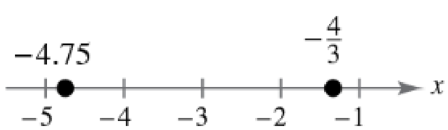
P.1.11.

$-5.2 > -8.5$



P.1.12.

$-\frac{4}{3} > -4.75$



P.1.13.

The inequality $x \le 5$ denotes the set of all real numbers less than or equal to 5.

P.1.14.

The inequality $x < 0$ denotes the set of all real numbers less than zero.

P.1.15.

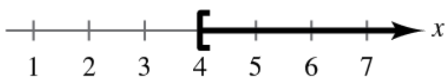
The inequality $-2 < x < 2$ denotes the set of all real numbers greater than -2 and less than 2.

P.1.16.

The inequality $0 < x \le 6$ denotes the set of all positive real numbers less than or equal to 6.

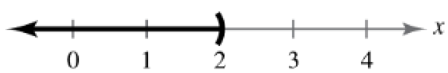
P.1.17.

The interval $\left[4, \infty\right)$ denotes the set of all real numbers greater than or equal to 4; $x \ge 4$.



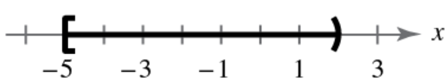
P.1.18.

The interval $\left(-\infty, 2\right)$ denotes the set of all real numbers less than 2; $x < 2$.



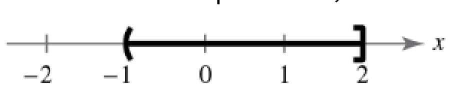
P.1.19.

The interval $\left[-5, 2\right)$ denotes the set of all real numbers greater than or equal to -5 and less than 2; $-5 \le x < 2$.



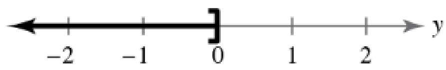
P.1.20.

The interval $\left(-1, 2\right]$ denotes the set of all real numbers greater than -1 and less than or equal to 2; $-1 < x \le 2$.



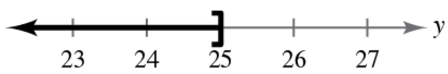
P.1.21.

$(-\infty, 0]$; $y \le 0$



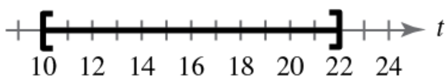
P.1.22.

$(-\infty, 25]$; $y \le 25$



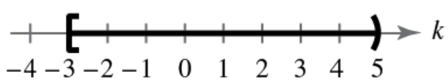
P.1.23.

$[10, 22]$; $10 \le t \le 22$



P.1.24.

$[-3, 5)$; $-3 \le k < 5$



P.1.25.

$|-10| = -(-10) = 10$

P.1.26.

$|0| = 0$

P.1.27.

$|-1| - |-2| = 1 - 2 = -1$

P.1.28.

$-3 - |-3| = -3 - (3) = -6$

P.1.29.

$5\ |-5| = 5(5) = 25$

P.1.30.

$-4\ |-4| = -4(4) = -16$

P.1.31.

If $x < -2$, then $x + 2$ is negative.

So, $\dfrac{|x+2|}{x+2} = \dfrac{-(x+2)}{x+2} = -1.$

P.1.32.

If $x > 1$, then $x - 1$ is positive.

So, $\dfrac{|x-1|}{x-1} = \dfrac{x-1}{x-1} = 1.$

P.1.33.

$|-4| = |4|$ because $|-4| = 4$ and $|4| = 4.$

P.1.34.

$-5 = -\ |5|$ because $-5 = -5.$

P.1.35.

$-|-6| < |-6|$ because $|-6| = 6$ and

$-|-6| = -(6) = -6.$

P.1.36.

$-|-2| = -\ |2|$ because $-2 = -2.$

P.1.37.

$d(126, 75) = |75 - 126| = 51$

P.1.38.

$d(-20, 30) = |30 - (-20)| = |50| = 50$

P.1.39.

$d\left(-\frac{5}{2}, 0\right) = \left|0 - \left(-\frac{5}{2}\right)\right| = \frac{5}{2}$

P.1.40.

$d\left(-\frac{1}{4}, -\frac{11}{4}\right) = \left|-\frac{11}{4} - \left(-\frac{1}{4}\right)\right| = \left|-\frac{5}{2}\right| = \frac{5}{2}$

P.1.41.

$7x+4$

Terms: $7x,\ 4$

Coefficient: 7

P.1.42.

$6x^3-5x$

Terms: $6x^3,\ -5x$

Coefficients: $6,\ -5$

P.1.43.

$4x^3+0.5x-5$

Terms: $4x^3,\ 0.5x,\ -5$

Coefficients: $4,\ 0.5$

P.1.44.

$3\sqrt{3}x^2+1$

Terms: $3\sqrt{3}x^2,\ 1$

Coefficient: $3\sqrt{3}$

P.1.45.

Receipts: R = \$3268.0 billion

Expenditures: E = \$3852.6 billion

$|R-E|=|3268.0-3852.6|=|-584.6|=\$584.6$ billion

P.1.46.

Receipts: R = \$3329.9 billion

Expenditures: E = \$4109.0 billion

$|R-E|=|3329.9-4109.0|=|-779.1|=\$779.1$ billion

P.1.47.

Receipts: R = \$3421.2 billion

Expenditures: E = \$6553.6 billion

$|R-E|=|3421.2-6553.6|=|-3132.4|=\$3132.4$ billion

P.1.48.

Receipts: R = \$4897.3 billion

Expenditures: E = \$6273.3 billion

$|R-E|=|4897.3-6273.3|=|-1376.0|=\$1376.0$ billion

P.1.49.

$x^2 - 3x + 2$

(a) $(0)^2 - 3(0) + 2 = 2$

(b) $(-1)^2 - 3(-1) + 2 = 1 + 3 + 2 = 6$

P.1.50.

$\dfrac{x-2}{x+2}$

(a) $\dfrac{2-2}{2+2} = \dfrac{0}{4} = 0$

(b) $\dfrac{-2-2}{-2+2} = \dfrac{-4}{0}$

Division by zero is undefined.

P.1.51.

$$\frac{2x}{3} - \frac{x}{4} = \frac{8x}{12} - \frac{3x}{12} = \frac{5x}{12}$$

P.1.52.

$$\frac{3x}{4} + \frac{x}{5} = \frac{15x}{20} + \frac{4x}{20} = \frac{19x}{20}$$

P.1.53.

$$\frac{3x}{10} \cdot \frac{5}{6} = \frac{x}{2} \cdot \frac{1}{2} = \frac{x}{4}$$

P.1.54.

$$\frac{2x}{3} \div \frac{6}{7} = \frac{2x}{3} \cdot \frac{7}{6} = \frac{x}{3} \cdot \frac{7}{3} = \frac{7x}{9}$$

P.1.55.

False. Because zero is nonnegative but not positive, not every nonnegative number is positive.

P.1.56.

True. The product of two negative numbers is positive.

P.1.57.

The Distributive Property was used incorrectly. The 5 was not distributed to the 3.

P.1.58.

(a) Because the price can only be a positive rational number with at most two decimal places, the description matches graph (ii).
A range of prices can only include zero and positive numbers with at most two decimal places. So, a range of prices can be represented by whole numbers and some noninteger positive fractions.

(b) Because the distance is a positive real number, the description matches graph (i).
A range of lengths can only include positive numbers. So, a range of lengths can be represented by positive real numbers.

P.1.59.

$5/n$ increases or decreases without bound. As n approaches 0, $5/n$ increases when n is positive and decreases when n is negative.

P.1.60.

$5/n$ approaches 0 as n increases without bound. As n increases, $5/n$ decreases but stays positive.

P.1.61.

$6 = 2 \cdot 3,\ 8 = 2 \cdot 2 \cdot 2$

(a) Greatest common factor: 2

(b) Least common multiple: $2 \cdot 2 \cdot 2 \cdot 3 = 24$

P.1.62.

$10 = 2 \cdot 5,\ 25 = 5 \cdot 5$

(a) Greatest common factor: 5

(b) Least common multiple: $2 \cdot 5 \cdot 5 = 50$

P.1.63.

$27 = 3 \cdot 3 \cdot 3,\ 36 = 2 \cdot 2 \cdot 3 \cdot 3,\ 54 = 2 \cdot 3 \cdot 3 \cdot 3$

(a) Greatest common factor: $3 \cdot 3 = 9$

(b) Least common multiple: $2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = 108$

P.1.64.

$49 = 7 \cdot 7,\ 98 = 2 \cdot 7 \cdot 7,\ 112 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$

(a) Greatest common factor: 7

(b) Least common multiple: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 7 \cdot 7 = 784$

P.1.65.

$-(3 \cdot 3) = -9$

P.1.66.

$$(-3)(-3) = 3 \cdot 3 = 9$$

P.1.67.

$$\frac{(-5)(-5)}{-5} = -5$$

P.1.68.

$$\frac{-(5 \cdot 5)}{(-5)(-5)(-5)} = \frac{5 \cdot 5}{5 \cdot 5 \cdot 5} = \frac{1}{5}$$

P.1.69.

$$48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 = 2^4 \cdot 3$$

P.1.70.

$$250 = 2 \cdot 5 \cdot 5 \cdot 5 = 2 \cdot 5^3$$

P.1.71.

$$792 = 8 \cdot 9 \cdot 11 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 11 = 2^3 \cdot 3^2 \cdot 11$$

P.1.72.

$$4802 = 2 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 2 \cdot 7^4$$

P.1.73.

$$3.785(10,000) = 37,850$$

P.1.74.

$$1.42(1,000,000) = 1,420,000$$

P.1.75.

$$\frac{6.09}{1000} = 0.00609$$

P.1.76.

$$\frac{8.603}{100,000} = 0.00008603$$

Section P.2 Exponents and Radicals

P.2.1.

exponent; base

P.2.2.

index; radicand

P.2.3.

An expression involving radicals is in simplest form when the following conditions are satisfied:

1. All possible factors have been removed from the radical.
2. All fractions have radical-free denominators.
3. The index of the radical is reduced.

P.2.4.

64 is both a perfect square $\left(8^2 = 64\right)$ and a perfect cube $\left(4^3 = 64\right)$.

P.2.5.

$$5 \cdot 5^3 = 5^4 = 625$$

P.2.6.

$$\begin{aligned}\left(2^3 \cdot 3^2\right)^2 &= 2^{3 \cdot 2} \cdot 3^{2 \cdot 2} \\ &= 2^6 \cdot 3^4 = 64 \cdot 81 = 5184\end{aligned}$$

P.2.7.

$$\frac{5^2}{5^4} = 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

P.2.8.

$$\begin{aligned}\left(-\frac{3}{5}\right)^3\left(\frac{5}{3}\right)^2 &= \left(-1\right)^3 \frac{3^3}{5^3} \cdot \frac{5^2}{3^2} = -1 \cdot 3^{3-2} \cdot 5^{2-3} \\ &= -3 \cdot 5^{-1} = -\frac{3}{5}\end{aligned}$$

P.2.9.

$$-3^2 = -9$$

P.2.10.

$$\left(-4\right)^{-3} = \frac{1}{\left(-4\right)^3} = -\frac{1}{64}$$

P.2.11.

$$\frac{4 \cdot 3^{-2}}{2^{-2} \cdot 3^{-1}} = 4 \cdot 2^2 \cdot 3^{-2-(-1)} = 4 \cdot 4 \cdot 3^{-1} = \frac{16}{3}$$

P.2.12.

$$\frac{3}{3^{-4}} = 3 \cdot 3^4 = 3^5 = 243$$

P.2.13.

When $x = 2$,

$$-3x^3 = -3(2)^3 = -24.$$

P.2.14.

When $x = 4$,

$$7x^{-2} = 7(4)^{-2} = \frac{7}{4^2} = \frac{7}{16}.$$

P.2.15.

When $x = 0.1$,

$$6x^2 = 6(0.1)^2 = 6(0.01) = 0.06.$$

P.2.16.

When $x = -\frac{1}{3}$,

$$12(-x)^3 = 12\left(\tfrac{1}{3}\right)^3 = 12\left(\tfrac{1}{27}\right) = \tfrac{4}{9}.$$

P.2.17.

$$6y^2\left(2y^0\right)^2 = 6y^2(2 \cdot 1)^2 = 6y^2(4) = 24y^2$$

P.2.18.

$$\begin{aligned}(-z)^3\left(3z^4\right) &= (-1)^3\left(z^3\right)3z^4 \\ &= -1 \cdot 3 \cdot z^{3+4} = -3z^7\end{aligned}$$

P.2.19.

$$\frac{7x^2}{x^3} = 7x^{2-3} = 7x^{-1} = \frac{7}{x}$$

P.2.20.

$$\frac{12(x+y)^3}{9(x+y)} = \frac{4}{3}(x+y)^{3-1} = \frac{4}{3}(x+y)^2$$

P.2.21.

$$\left(\frac{4}{y}\right)^3\left(\frac{3}{y}\right)^4 = \frac{4^3}{y^3} \cdot \frac{3^4}{y^4} = \frac{64 \cdot 81}{y^{3+4}} = \frac{5184}{y^7}$$

P.2.22.

$$\left(\frac{b^{-2}}{a^{-2}}\right)\left(\frac{b}{a}\right)^2 = \left(\frac{a^2}{b^2}\right)\left(\frac{b^2}{a^2}\right) = 1,\ a \neq 0, \quad b \neq 0$$

P.2.23.

$$\left[\left(x^2y^{-2}\right)^{-1}\right]^{-1} = \left(x^{-2}y^2\right)^{-1} = x^2y^{-2} = \frac{x^2}{y^2}$$

P.2.24.

$$\begin{aligned}\left(5x^2z^6\right)^3\left(5x^2z^6\right)^{-3} &= \left(5^3\right)\left(x^6\right)\left(z^{18}\right)\left(5^{-3}\right)\left(x^{-6}\right)\left(z^{-18}\right)\\ &= 5^0\left(x^0\right)\left(z^0\right)\\ &= (1)(1)(1)\\ &= 1\end{aligned}$$

P.2.25.

$$\left(2x^2\right)^{-2} = \frac{1}{\left(2x^2\right)^2} = \frac{1}{4x^4}$$

P.2.26.

$$\left(4y^{-2}\right)\left(8y^{-4}\right) = \left(\frac{4}{y^2}\right)\left(\frac{8}{y^4}\right) = \frac{32}{y^6}$$

P.2.27.

$$\left(\frac{x^{-3}y^4}{5}\right)^{-3} = \left(\frac{5x^3}{y^4}\right)^3 = \frac{125x^9}{y^{12}}$$

P.2.28.

$$\left(\frac{a^{-2}}{b^{-2}}\right)\left(\frac{b}{a}\right)^{-3} = \left(\frac{b^2}{a^2}\right)\left(\frac{a}{b}\right)^3 = \frac{b^2a^3}{a^2b^3} = \frac{a}{b}$$

P.2.29.

$$10,250.4 = 1.02504 \times 10^4$$

P.2.30.

$$-0.000125 = -1.25 \times 10^{-4}$$

P.2.31.

$$3.14 \times 10^{-4} = 0.000314$$

P.2.32.

$$-2.058 \times 10^6 = -2,058,000$$

P.2.33.

$$\left(2.0\times10^{9}\right)\left(3.4\times10^{-4}\right)=6.8\times10^{5}$$

P.2.34.

$$\left(1.2\times10^{7}\right)\left(5.0\times10^{-3}\right)=6.0\times10^{4}$$

P.2.35.

$$\frac{6.0\times10^{8}}{3.0\times10^{-3}}=2.0\times10^{11}$$

P.2.36.

$$\frac{2.5\times10^{-3}}{5.0\times10^{2}}=0.5\times10^{-5}=5.0\times10^{-6}$$

P.2.37.

(a) $\sqrt{9}=3$

(b) $\sqrt[3]{\frac{27}{8}}=\frac{\sqrt[3]{27}}{\sqrt[3]{8}}=\frac{3}{2}$

P.2.38.

(a) $\sqrt[3]{27}=3$

(b) $\left(\sqrt{36}\right)^{3}=(6)^{3}=216$

P.2.39.

(a) $\left(\sqrt[5]{2}\right)^{5}=2^{5/5}=2^{1}=2$

(b) $\sqrt[5]{32x^{5}}=\sqrt[5]{(2x)^{5}}=2x$

P.2.40.

(a) $\sqrt{12}\cdot\sqrt{3}=\sqrt{36}=6$

(b) $\sqrt[4]{\left(3x^{2}\right)^{4}}=\sqrt[4]{3^{4}x^{8}}$

$=3x^{2}$

P.2.41.

$$\begin{aligned}\sqrt{20}&=\sqrt{4\cdot5}\\&=\sqrt{4}\sqrt{5}=2\sqrt{5}\end{aligned}$$

P.2.42.

$$\begin{aligned}\sqrt[3]{128}&=\sqrt[3]{64\cdot2}\\&=\sqrt[3]{64}\sqrt[3]{2}=4\sqrt[3]{2}\end{aligned}$$

P.2.43.

$$\sqrt[3]{\frac{16}{27}} = \frac{\sqrt[3]{2^3 \cdot 2}}{\sqrt[3]{3^3}} = \frac{2\sqrt[3]{2}}{3}$$

P.2.44.

$$\sqrt{\frac{75}{4}} = \frac{\sqrt{5^2 \cdot 3}}{\sqrt{2^2}} = \frac{5\sqrt{3}}{2}$$

P.2.45.

$$\begin{aligned}\sqrt{72x^3} &= \sqrt{36x^2 \cdot 2x}\\ &= 6x\sqrt{2x}\end{aligned}$$

P.2.46.

$$\begin{aligned}\sqrt{54xy^4} &= \sqrt{6 \cdot 3^2 \cdot x \cdot \left(y^2\right)^2}\\ &= 3y^2\sqrt{6x}\end{aligned}$$

P.2.47.

$$\begin{aligned}8\sqrt{147x} - 3\sqrt{48x} &= 8\sqrt{49 \cdot 3x} - 3\sqrt{16 \cdot 3x}\\ &= 8\sqrt{7^2 \cdot 3x} - 3\sqrt{4^2 \cdot 3x}\\ &= 56\sqrt{3x} - 12\sqrt{3x}\\ &= 44\sqrt{3x}\end{aligned}$$

P.2.48.

$$\begin{aligned}3\sqrt[3]{54x^3} + \sqrt[3]{16x^3} &= 3\sqrt[3]{27x^3 \cdot 2} + \sqrt[3]{8x^3 \cdot 2}\\ &= 3\sqrt[3]{(3x)^3 \cdot 2} + \sqrt[3]{(2x)^3 \cdot 2}\\ &= 9x\sqrt[3]{2} + 2x\sqrt[3]{2}\\ &= 11x\sqrt[3]{2}\end{aligned}$$

P.2.49.

$$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

P.2.50.

$$\frac{8}{\sqrt[3]{2}} = \frac{8}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{8\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{8\sqrt[3]{4}}{2} = 4\sqrt[3]{4}$$

P.2.51.

$$\frac{5}{\sqrt{14}-2}=\frac{5}{\sqrt{14}-2}\cdot\frac{\sqrt{14}+2}{\sqrt{14}+2}=\frac{5\left(\sqrt{14}+2\right)}{\left(\sqrt{14}\right)^2-\left(2\right)^2}=\frac{5\left(\sqrt{14}+2\right)}{14-4}=\frac{5\left(\sqrt{14}+2\right)}{10}=\frac{\sqrt{14}+2}{2}$$

P.2.52.

$$\frac{3}{\sqrt{5}+\sqrt{6}}=\frac{3}{\sqrt{5}+\sqrt{6}}\cdot\frac{\sqrt{5}-\sqrt{6}}{\sqrt{5}-\sqrt{6}}=\frac{3\left(\sqrt{5}-\sqrt{6}\right)}{5-6}=\frac{3\left(\sqrt{5}-\sqrt{6}\right)}{-1}=-3\left(\sqrt{5}-\sqrt{6}\right)=3\left(\sqrt{6}-\sqrt{5}\right)$$

P.2.53.

$$\frac{\sqrt{5}+\sqrt{3}}{3}=\frac{\sqrt{5}+\sqrt{3}}{3}\cdot\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}=\frac{5-3}{3\left(\sqrt{5}-\sqrt{3}\right)}=\frac{2}{3\left(\sqrt{5}-\sqrt{3}\right)}$$

P.2.54.

$$\frac{\sqrt{7}-3}{4}=\frac{\sqrt{7}-3}{4}\cdot\frac{\sqrt{7}+3}{\sqrt{7}+3}=\frac{7-9}{4\left(\sqrt{7}+3\right)}=\frac{-2}{4\left(\sqrt{7}+3\right)}=-\frac{1}{2\left(\sqrt{7}+3\right)}$$

P.2.55.

$$\sqrt[3]{64}=64^{1/3}$$

P.2.56.

$$\begin{aligned}x^2\sqrt{x}&=x^2\cdot x^{1/2}\\&=x^{5/2}\end{aligned}$$

P.2.57.

$$\begin{aligned}3x^{-2/3}&=\frac{3}{x^{2/3}}\\&=\frac{3}{\sqrt[3]{x^2}},\quad x\neq 0\end{aligned}$$

P.2.58.

$$a^{0.4}=a^{2/5}$$

P.2.59.

$$32^{-3/5}=\frac{1}{32^{3/5}}=\frac{1}{\left(\sqrt[5]{32}\right)^3}=\frac{1}{(2)^3}=\frac{1}{8}$$

P.2.60.

$$\left(\frac{16}{81}\right)^{-3/4}=\left(\frac{81}{16}\right)^{3/4}=\left(\sqrt[4]{\frac{81}{16}}\right)^3=\left(\frac{3}{2}\right)^3=\frac{27}{8}$$

P.2.61.

$$\sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \sqrt{3}$$

P.2.62.

$$\sqrt[4]{\left(3x^2\right)^4} = 3x^2$$

P.2.63.

$$\begin{aligned}\sqrt{\sqrt{32}} &= \left(32^{1/2}\right)^{1/2} \\ &= 32^{1/4} = \sqrt[4]{32} = \sqrt[4]{16 \cdot 2} = 2\sqrt[4]{2}\end{aligned}$$

P.2.64.

$$\sqrt{\sqrt[4]{2x}} = \left((2x)^{1/4}\right)^{1/2} = (2x)^{1/8} = \sqrt[8]{2x}$$

P.2.65.

$$\begin{aligned}(x-1)^{1/3}(x-1)^{-4/3} &= (x-1)^{-3/3} \\ &= (x-1)^{-1} \\ &= \frac{1}{x-1}\end{aligned}$$

P.2.66.

$$\begin{aligned}(4x+3)^{-5/2}(4x+3)^{2/3} &= (4x+3)^{-15/6}(4x+3)^{4/6} \\ &= (4x+3)^{-11/6} \\ &= \frac{1}{(4x+3)^{11/6}}\end{aligned}$$

P.2.67.

(a) $t = 0.03\left[12^{5/2} - (12-h)^{5/2}\right], \quad 0 \le h \le 12$

h (in centimeters)	t (in seconds)
0	0
1	2.93
2	5.48
3	7.67
4	9.53
5	11.08
6	12.32
7	13.29
8	14.00
9	14.50

10	14.80
11	14.93
12	14.96

(b) From the table in part (a), you can see that the water height that corresponds to an emptying time of 10 seconds is between 4 and 5 centimeters. Create a table of values to find a better approximation.

h (in centimeters)	b (in seconds)
4.20	9.87
4.21	9.88
4.22	9.90
4.23	9.92
4.24	9.93
4.25	9.95
4.26	9.96
4.27	9.98
4.28	10.00

A water height of about 4.28 centimeters corresponds to an emptying time of 10 seconds.

P.2.68.

The error is in the third step.

$$\left(\frac{1}{a^2}\right)^{-3} \neq \frac{-1}{a^{-6}}$$

In fact, $\left(\frac{1}{a^2}\right)^{-3} = (a^2)^3 = a^6$, not $-a^6$.

P.2.69.

False. When $x = 0$, the expressions are not equal.

P.2.70.

False. Raising the numerator and denominator to the second power changes the expression. To rationalize the denominator, multiply the fraction by $\frac{\sqrt{b}}{\sqrt{b}} = 1$.

P.2.71.

For $a \neq 0$, $1 = \frac{a}{a} = \frac{a^1}{a^1} = a^{1-1} = a^0$.

Thus, $a^0 = 1$.

P.2.72.

Package A: $x^3 = 500$

$$x = \sqrt[3]{500} \approx 7.9 \text{ in.}$$

Package B: $x^3 = 250$

$$x = \sqrt[3]{250} \approx 6.3 \text{ in.}$$

So, $2x = 2(6.3) = 12.6$ in.

Because $7.9 < 12.6$, the length of a side of package A is *less than* twice the length of a side of package B.

P.2.73.

Consider $x^2 = n$, x a positive integer.

Unit digit of x	*Unit digit of* $n = x^2$
1	1
2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1
0	0

Therefore, the possible digits are 0, 1, 4, 5, 6, and 9 thus $\sqrt{5233}$ is *not* an integer because its unit digit is 3.

P.2.74.

(a) When $x = 2$,

$$(x+4)(x-4) = (2+4)(2-4) = 6(-2) = -12.$$

(b) When $x = -3$,

$$(x+4)(x-4) = (-3+4)(-3-4) = 1(-7) = -7.$$

P.2.75.

(a) When $x = 2$,

$$\begin{aligned}(9x+5)(3x-1) &= [9(2)+5][3(2)]-1 \\ &= (18+5)(6-1) \\ &= (23)(5) = 115.\end{aligned}$$

(b) When $x = -3$

$$\begin{aligned}(9x+5)(3x-1) &= [9(-3)+5][3(-3)-1] \\ &= (-27+5)(-9-1) \\ &= (-22)(-10) = 220.\end{aligned}$$

P.2.76.

$2x - 3$

Terms: $2x,\ -3$

Coefficient: 2

P.2.77.

$4x^3 - x^2 + 5x + 1$

Terms: $4x^3,\ -x^2,\ 5x,\ 1$

Coefficients: 4, −1, 5

Section P.3 Polynomials and Factoring

P.3.1.

n; a_n; a_0

P.3.2.

First terms; Outer terms; Inner terms; Last terms

P.3.3.

Yes, it is possible for a binomial and a trinomial to have the same degree. For example, $x^3 + 1$ and $x^3 + x + 5$ have the same degree, 3.

P.3.4.

(a) iv

(b) iii

(c) v

(d) ii

(e) i

P.3.5.

(a) Standard form: $7x$

(b) Degree: 1
Leading coefficient: 7

(c) Monomial

P.3.6.

(a) Standard form: 3

(b) Degree: 0
Leading coefficient: 3

(c) Monomial

P.3.7.

(a) Standard form: $-\frac{1}{2}x^5 + 14x$

(b) Degree: 5
Leading coefficient: $-\frac{1}{2}$

(c) Binomial

P.3.8.

(a) Standard form: $2x + 3$

(b) Degree: 1
Leading coefficient: 2

(c) Binomial

P.3.9.

(a) Standard form: $-4x^5 + 6x^4 + 1$

(b) Degree: 5
Leading coefficient: −4

(c) Trinomial

P.3.10.

(a) Standard form: $25y^2 - y + 1$

(b) Degree: 2
Leading coefficient: 25

(c) Trinomial

P.3.11.

$2x - 3x^3 + 8$ *is* a polynomial.

Standard form: $-3x^3 + 2x + 8$

P.3.12.

$5x^4 - 2x^2 + x^{-2}$ is *not* a polynomial because it includes a term with a negative exponent.

P.3.13.

$\dfrac{3x+4}{x} = 3 + \dfrac{4}{x} = 3 + 4x^{-1}$ is *not* a polynomial because it includes a term with a negative exponent.

P.3.14.

$\dfrac{x^2 + 2x - 3}{2}$ is a polynomial.

Standard form: $\dfrac{1}{2}x^2 + x - \dfrac{3}{2}$

P.3.15.

$y^2 - y^4 + y^3$ is polynomial.

Standard form: $-y^4 - y^3 + y^2$

P.3.16.

$y^4 - \sqrt{y}$ is *not* a polynomial because it includes a term with a square root.

P.3.17.

$$\begin{aligned}(6x+5)-(8x+15) &= 6x+5-8x-15\\ &= (6x-8x)+(5-15)\\ &= -2x-10\end{aligned}$$

P.3.18.

$$\begin{aligned}\left(t^3-1\right)+\left(6t^3-5t\right)&=t^3-1+6t^3-5t\\&=\left(t^3+6t^3\right)-5t-1\\&=7t^3-5t-1\end{aligned}$$

P.3.19.

$$\begin{aligned}\left(4y^2-3\right)+\left(-7y^2+9\right)&=4y^2-3-7y^2+9\\&=\left(4y^2-7y^2\right)+\left(-3+9\right)\\&=-3y^2+6\end{aligned}$$

P.3.20.

$$\begin{aligned}\left(2x^2+1\right)-\left(x^2-2x+1\right)&=2x^2+1-x^2+2x-1\\&=\left(2x^2-x^2\right)+2x+\left(1-1\right)\\&=x^2+2x\end{aligned}$$

P.3.21.

$$\begin{aligned}\left(15x^2-6\right)+\left(-8.3x^3-14.7x^2-17\right)&=15x^2-6-8.3x^3-14.7x^2-17\\&=-8.3x^3+\left(15x^2-14.7x^2\right)+\left(-6-17\right)\\&=-8.3x^3+0.3x^2-23\end{aligned}$$

P.3.22.

$$\begin{aligned}\left(15.6w^4-14w-17.4\right)+\left(16.9w^4-9.2w+13\right)&=15.6w^4-14w-17.4+16.9w^4-9.2w+13\\&=\left(15.6w^4+16.9w^4\right)+\left(-14w-9.2w\right)+\left(-17.4+13\right)\\&=32.5w^4-23.2w-4.4\end{aligned}$$

P.3.23.

$$\begin{aligned}5z-\left[3z-\left(10z+8\right)\right]&=5z-\left(3z-10z-8\right)\\&=5z-3z+10z+8\\&=\left(5z-3z+10z\right)+8\\&=12z+8\end{aligned}$$

P.3.24.

$$\begin{aligned}\left(y^3+1\right)-\left[\left(y^2+1\right)+\left(3y-7\right)\right]&=y^3+1-\left(y^2+1\right)-\left(3y-7\right)\\&=y^3+1-y^2-1-3y+7\\&=y^3-y^2-3y+\left(1-1+7\right)\\&=y^3-y^2-3y+7\end{aligned}$$

P.3.25.

$$\begin{aligned}3x\left(x^2-2x+1\right)&=3x\left(x^2\right)+3x\left(-2x\right)+3x\left(1\right)\\&=3x^3-6x^2+3x\end{aligned}$$

P.3.26.

$$\begin{aligned}y^2\left(4y^2+2y-3\right)&=y^2\left(4y^2\right)+y^2\left(2y\right)+y^2\left(-3\right)\\&=4y^4+2y^3-3y^2\end{aligned}$$

P.3.27.

$$\begin{aligned}-5z\left(3z-1\right)&=-5z\left(3z\right)+\left(-5z\right)\left(-1\right)\\&=-15z^2+5z\end{aligned}$$

P.3.28.

$$\begin{aligned}-3x\left(5x+2\right)&=-3x\left(5x\right)+\left(-3x\right)\left(2\right)\\&=-15x^2-6x\end{aligned}$$

P.3.29.

$$\begin{aligned}\left(1.5t^2+5\right)\left(-3t\right)&=\left(1.5t^2\right)\left(-3t\right)+\left(5\right)\left(-3t\right)\\&=-\,4.5t^3-15t\end{aligned}$$

P.3.30.

$$\begin{aligned}\left(2-3.5y\right)\left(2y^3\right)&=2\left(2y^3\right)+\left(-3.5y\right)\left(2y^3\right)\\&=4y^3-7y^4\\&=-7y^4+4y^3\end{aligned}$$

P.3.31.

$$\begin{aligned}\left(3x-5\right)\left(2x+1\right)&=6x^2+3x-10x-5\\&=6x^2-7x-5\end{aligned}$$

P.3.32.

$$\begin{aligned}\left(7x-2\right)\left(4x-3\right)&=28x^2-21x-8x+6\\&=28x^2-29x+6\end{aligned}$$

P.3.33.

$$\begin{aligned}\left(x+7\right)\left(x^2+2x+5\right)&=\left(x^3+2x^2+5x\right)+\left(7x^2+14x+35\right)\\&=x^3+9x^2+19x+35\end{aligned}$$

P.3.34.

$$\begin{aligned}\left(x-8\right)\left(2x^2+x+4\right)&=\left(2x^3+x^2+4x\right)-\left(16x^2+8x+32\right)\\&=2x^3-15x^2-4x-32\end{aligned}$$

P.3.35.

$$(x^2 - x + 2)(x^2 + x + 1)$$

$$\begin{array}{r} x^2 - x + 2 \\ \times\, x^2 + x + 1 \\ \hline x^4 - x^3 + 2x^2 \qquad\qquad \\ x^3 - x^2 + 2x \qquad \\ x^2 - x + 2 \\ \hline x^4 + 0x^3 + 2x^2 + x + 2 = x^4 + 2x^2 + x + 2 \end{array}$$

P.3.36.

$$(2x^2 - x + 4)(x^2 + 3x + 2)$$

$$\begin{array}{r} 2x^2 - x + 4 \\ \times\; x^2 + 3x + 2 \\ \hline 2x^4 - x^3 + 4x^2 \qquad\qquad \\ 6x^3 - 3x^2 + 12x \qquad \\ 4x^2 - 2x + 8 \\ \hline 2x^4 + 5x^3 + 5x^2 + 10x + 8 \end{array}$$

P.3.37.

$$\begin{aligned}(-3x^3 + x^2 + 9) - (4x^2 - 5) &= -3x^3 + x^2 + 9 - 4x^2 + 5 \\ &= -3x^3 + (x^2 - 4x^2) + (9 + 5) \\ &= -3x^3 - 3x^2 + 14\end{aligned}$$

P.3.38.

$$\begin{aligned}(2t^4 - 10t^3 - 4t) - (-7t^4 + 5t^2 - 1) &= 2t^4 - 10t^3 - 4t + 7t^4 - 5t^2 + 1 \\ &= (2t^4 + 7t^4) - 10t^3 - 5t^2 - 4t + 1 \\ &= 9t^4 - 10t^3 - 5t^2 - 4t + 1\end{aligned}$$

P.3.39.

$$(y^2 + 3y - 5)(y^2 - 6y + 4)$$

$$\begin{array}{r} y^2 + 3y - 5 \\ \times\; y^2 - 6y + 4 \\ \hline y^4 + 3y^3 - 5y^2 \qquad\qquad \\ 6y^3 - 18y^2 + 30y \qquad \\ + \; 4y^2 + 12y - 20 \\ \hline y^4 - 3y^3 - 19y^2 + 42y - 20 \end{array}$$

P.3.40.

$$\left(x^2+4x-1\right)\left(x^2-x+3\right)$$

$$\begin{array}{rrrrr}
 & & x^2 & +4x & -1 \\
\times & & x^2 & -\quad x & +3 \\
\hline
x^4 & +4x^3 & -x^2 & & \\
 & -x^3 & -4x^2 & +x & \\
 & + & 3x^2 & +12x & -3 \\
\hline
x^4 & +3x^3 & -2x^2 & +13x & -3
\end{array}$$

P.3.41.

$$\left(x+10\right)\left(x-10\right)=x^2-10^2=x^2-100$$

P.3.42.

$$\left(2x+3\right)\left(2x-3\right)=\left(2x\right)^2-3^2=4x^2-9$$

P.3.43.

$$\left(x+2y\right)\left(x-2y\right)=x^2-\left(2y\right)^2=x^2-4y^2$$

P.3.44.

$$\left(4a+5b\right)\left(4a-5b\right)=\left(4a\right)^2-\left(5b\right)^2=16a^2-25b^2$$

P.3.45.

$$\begin{aligned}\left(2x+3\right)^2&=\left(2x\right)^2+2\left(2x\right)\left(3\right)+3^2\\&=4x^2+12x+9\end{aligned}$$

P.3.46.

$$\begin{aligned}\left(5-8x\right)^2&=5^2+\left(2\right)\left(5\right)\left(-8x\right)+\left(-8x\right)^2\\&=25-80x+64x^2\\&=64x^2-80x+25\end{aligned}$$

P.3.47.

$$\begin{aligned}\left(4x^3-3\right)^2&=\left(4x^3\right)^2-2\left(4x^3\right)\left(3\right)+\left(3\right)^2\\&=16x^6-24x^3+9\end{aligned}$$

P.3.48.

$$\begin{aligned}\left(8x+3\right)^2&=\left(8x\right)^2+2\left(8x\right)\left(3\right)+3^2\\&=64x^2+48x+9\end{aligned}$$

P.3.49.

$$\begin{aligned}(x+3)^3 &= x^3+3(x)^2(3)+3(x)(3)^2+3^3\\ &= x^3+9x^2+27x+27\end{aligned}$$

P.3.50.

$$\begin{aligned}(x-2)^3 &= x^3-3x^2(2)+3x(2)^2-2^3\\ &= x^3-6x^2+12x-8\end{aligned}$$

P.3.51.

$$\begin{aligned}(2x-y)^3 &= (2x)^3-3(2x)^2y+3(2x)y^2-y^3\\ &= 8x^3-12x^2y+6xy^2-y^3\end{aligned}$$

P.3.52.

$$\begin{aligned}(3x+2y)^3 &= (3x)^3+3(3x)^2(2y)+3(3x)(2y)^2+(2y)^3\\ &= 27x^3+54x^2y+36xy^2+8y^3\end{aligned}$$

P.3.53.

$$\begin{aligned}\left[(x-3)+y\right]^2 &= (x-3)^2+2y(x-3)+y^2\\ &= x^2-6x+9+2xy-6y+y^2\\ &= x^2+2xy+y^2-6x-6y+9\end{aligned}$$

P.3.54.

$$\begin{aligned}\left[(x+1)-y\right]^2 &= (x+1)^2+2(x+1)(-y)+(-y)^2\\ &= x^2+2x+1-2xy-2y+y^2\\ &= x^2-2xy+y^2+2x-2y+1\end{aligned}$$

P.3.55.

$$\begin{aligned}(3y-6x)(-3y-6x) &= -(3y-6x)(3y+6x)\\ &= -\left[(3y)^2-(6x)^2\right]\\ &= -9y^2+36x^2\end{aligned}$$

P.3.56.

$$\begin{aligned}(3a^3-4b^2)(3a^3+4b^2) &= (3a^3)^2-(4b^2)^2\\ &= 9a^6-16b^4\end{aligned}$$

P.3.57.

$$\begin{aligned}\left[(m-3)+n\right]\left[(m-3)-n\right] &= (m-3)^2-(n)^2\\ &= m^2-6m+9-n^2\\ &= m^2-n^2-6m+9\end{aligned}$$

P.3.58.

$$\left[(x-3y)+z\right]\left[(x-3y)-z\right] = (x-3y)^2-(z)^2$$
$$= x^2-6xy+9y^2-z^2$$

P.3.59.

$$(u+2)(u-2)(u^2+4) = (u^2-4)(u^2+4)$$
$$= u^4-16$$

P.3.60.

$$(x+y)(x-y)(x^2+y^2) = (x^2-y^2)(x^2+y^2)$$
$$= (x^2)^2-(y^2)^2 = x^4-y^4$$

P.3.61.

$$(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2-(\sqrt{y})^2$$
$$= x-y$$

P.3.62.

$$(5+\sqrt{x})(5-\sqrt{x}) = (5)^2-(\sqrt{x})^2$$
$$= 25-x$$

P.3.63.

$$(x-\sqrt{y})^2 = x^2-2x\sqrt{y}+(\sqrt{y})^2$$
$$= x^2-2x\sqrt{y}+y$$

P.3.64.

$$(x+\sqrt{y})^2 = x^2+2x\sqrt{y}+(\sqrt{y})^2$$
$$= x^2+2x\sqrt{y}+y$$

P.3.65.

(a) The possible gene combinations of an offspring with albino coloring is $\frac{1}{4}$, or 25%.

(b) $$(0.5N+0.5a)^2 = (0.5N)^2+2(0.5N)(0.5a)+(0.5a)^2$$
$$= 0.25N^2+0.5Na+0.25a^2$$

(c) The probability of an offspring with albino coloring is represented by the coefficient of the a^2 term, which is 0.25, or 25%.

P.3.66.

(a) $\left(100-x\right)\left(100+x\right)=\left(100\right)^2-\left(x\right)^2$

$$=10{,}000-x^2$$

(b) As x increases, the area of the foundation decreases.

(c) When $x=21$ feet, the area of the new foundation is

$$A=10{,}000-\left(21\right)^2$$

$$=9559 \text{ square feet.}$$

P.3.67.

(a)
$$\begin{aligned} V &= l\cdot w\cdot h \\ &=\left(26-2x\right)\left(18-2x\right)\left(x\right) \\ &=2\left(13-x\right)\left(2\right)\left(9-x\right)\left(x\right) \\ &=4x\left(-1\right)\left(x-13\right)\left(-1\right)\left(x-9\right) \\ &=4x\left(x-13\right)\left(x-9\right) \\ &=4x^3-88x^2+468x \end{aligned}$$

(b) When $x=1$: $V=4\left(1\right)^3-88\left(1\right)^2+468\left(1\right)=384 \text{ cm}^3$

When $x=2$: $V=4\left(2\right)^3-88\left(2\right)^2+468\left(2\right)=616 \text{ cm}^3$

When $x=3$: $V=4\left(3\right)^3-88\left(3\right)^3+468\left(3\right)=720 \text{ cm}^3$

x (cm)	1	2	3
V (cm^3)	384	616	720

P.3.68.

(a)
$$\begin{aligned} V &= l\cdot w\cdot h \\ &=\tfrac{1}{2}\left(45-3x\right)\left(15-2x\right)x \\ &=\tfrac{1}{2}\left(45-3x\right)\left(15x-2x^2\right) \\ &=\tfrac{1}{2}\left(675x-90x^2-45x^2+6x^3\right) \\ &=\tfrac{1}{2}\left(6x^3-135x^2+675x\right) \end{aligned}$$

(b) When $x=3$: $V=\tfrac{1}{2}\left[6\left(3\right)^3-135\left(3\right)^3+675\left(3\right)\right]=\tfrac{1}{2}\left(6\cdot 27-135\cdot 9+2025\right)$

$$=\tfrac{1}{2}\left[162-1215+2025\right]=\tfrac{1}{2}\left(972\right)$$

$$=486 \text{ cm}^3$$

When $x=5$: $V=\tfrac{1}{2}\left[6\left(5\right)^3-135\left(5\right)^2+675\left(5\right)\right]=\tfrac{1}{2}\left[6\cdot 125-135\cdot 25+3375\right]$

$$=\tfrac{1}{2}\left[750-3375+3375\right]=\tfrac{1}{2}\left(750\right)$$

$$=375 \text{ cm}^3$$

When $x = 7$: $V = \frac{1}{2}\left[6(7)^3 - 135(7)^2 + 675(7)\right] = \frac{1}{2}\left[6 \cdot 343 - 135 \cdot 49 + 4725\right]$

$= \frac{1}{2}\left[2058 - 6615 + 4725\right] = \frac{1}{2}(168)$

$= 84 \text{ cm}^3$

x (cm)	3	5	7
Volume (cm^3)	486	375	84

P.3.69.

(a) Approximations will vary. Actual safe loads for $x = 12$:

$$S_6 = \left[0.06(12)^2 - 2.42(12) + 38.71\right]^2 = 335.2561 \quad \text{(using a calculator)}$$

$$S_8 = \left[0.08(12)^2 - 3.30(12) + 51.93\right]^2 = 568.8225 \quad \text{(using a calculator)}$$

(b) The difference in safe loads decreases in magnitude as the span increases.

P.3.70.

(a) $T = R + B = 1.1x + \left(0.0475x^2 - 0.001x + 0.23\right)$

$= 0.0475x^2 + 1.099x + 0.23$

(b)

x (mi/hr)	30	40	55
T (ft)	75.95	120.19	204.36

(c) Stopping distance increases at an accelerating rate as speed increases.

P.3.71.

False. $\left(4x^2 + 1\right)(3x + 1) = 12x^3 + 4x^2 + 3x + 1$

P.3.72.

False. $\left(x^2 + 1\right) + \left(-x^2 + 3\right) = 4$, which is not a second-degree polynomial.

P.3.73.

False. $(4x + 3) + (-4x + 6) = 4x + 3 - 4x + 6$

$= 3 + 6$

$= 9$

P.3.74.

True. The leading coefficient of

$\left(a_n x^n + \cdots + a_0\right)\left(b_m x^m + \cdots + b_0\right)$ is $a_n b_m$.

P.3.75.

Because $x^m x^n = x^{m+n}$, the degree of the product is $m + n$.

P.3.76.

If the degree of one polynomial is m *and* the degree of the second polynomial is n (and $n > m$), the degree of the sum of the polynomials is n.

P.3.77.

The middle term was omitted when squaring the binomial.

$$\begin{aligned}(x-3)^2 &= (x)^2 - 2(x)(3) + (3)^2 \\ &= x^2 - 6x + 9 \\ &\neq x^2 + 9\end{aligned}$$

P.3.78.

(a) The cardboard was 52 inches long and 24 inches wide. The box was constructed by cutting a square of x inches by x inches from each corner of the rectangular piece of cardboard and folding the side pieces up.

(b) The polynomial is a 3rd degree polynomial because the length, width, and height of the box are expressions in the variable x.

(c) $$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= (52-2x)(42-2x)x \\ &= 4x^3 - 188x^2 + 2184x\end{aligned}$$

The value of x (to the nearest tenth of an inch) that yields the maximum possible volume of the box may be found by substituting values of x into the volume equation.

x	5	6	7	8	9	10
V	6720	7200	7448	7488	7344	7040

The maximum volume occurs for a value of x between 7 and 8.

x	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9
V	7461.0	7471.9	7480.7	7487.6	7492.5	7495.4	7496.4	7495.5	7492.7

The maximum volume occurs when $x \approx 7.7$ inches.

P.3.79.

$$\sqrt{3}\left(\sqrt{3}\right) = \left(\sqrt{3}\right)^2 = 3$$

P.3.80.

$$\left(\sqrt{6}\right)\left(-\sqrt{6}\right) = -\left(\sqrt{6}\right)^2 = -6$$

P.3.81.

$$-\sqrt{15}\left(\sqrt{15}\right) = -\left(\sqrt{15}\right)^2 = -15$$

P.3.82.

$$-\sqrt{21}\left(-\sqrt{21}\right) = \left(\sqrt{21}\right)^2 = 21$$

P.3.83.

$$2x^2 = 2 \cdot x \cdot x$$
$$x^3 = x \cdot x \cdot x$$
$$4x = 2 \cdot 2 \cdot x$$

The greatest common factor is x.

P.3.84.

$$3x^3 = 3 \cdot x \cdot x \cdot x$$
$$12x^2 = 2 \cdot 2 \cdot 3 \cdot x \cdot x$$
$$42x^3 = 2 \cdot 3 \cdot 7 \cdot x \cdot x \cdot x$$

The greatest common factor is $3x^2$.

P.3.85.

$$x^{10} = x^{10}$$
$$x^{20} = x^{10} \cdot x^{10}$$
$$x^{30} = x^{10} \cdot x^{10} \cdot x^{10}$$

The greatest common factor is x^{10}.

P.3.86.

$$45x^5 = 3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot x \cdot x$$
$$9x^3 = 3 \cdot 3 \cdot x \cdot x \cdot x$$
$$15x^2 = 3 \cdot 5 \cdot x \cdot x$$

The greatest common factor is $3x^2$.

P.3.87.

$$\left(3x^2\right)^3 = 3^3\left(x^2\right)^3 = 27x^6$$

P.3.88.

$$\left(4x^4\right)^2 = 4^2\left(x^4\right)^2 = 16x^8$$

P.3.89.

$$\frac{1}{(h+6)}(h+6) = 1, \quad h \neq -6$$

Multiplicative Inverse Property

P.3.90.

$$(x+3) - (x+3) = 0$$

Additive Inverse Property

P.3.91.

$$\begin{aligned} x(3y) &= (x \cdot 3)y && \text{Associative Property of Multiplication} \\ &= (3x)y && \text{Commutative Property of Multiplication} \end{aligned}$$

P.3.92.

$$\begin{aligned} \tfrac{1}{7}(7 \cdot 12) &= \left(\tfrac{1}{7} \cdot 7\right)12 && \text{Associative Property of Multiplication} \\ &= 1 \cdot 12 && \text{Multiplicative Inverse Property} \\ &= 12 && \text{Multiplicative Identity Property} \end{aligned}$$

P.3.93.

$$x(2x^2 + 3x - 1) = 2x^3 + 3x^2 - x$$

Distributive Property

P.3.94.

$$(x-2)(2x+3) = (x-2)(2x) + (x-2)(3)$$

Distributive Property

Section P.4 Factoring Polynomials

P.4.1.

factoring

P.4.2.

perfect square binomial

P.4.3.

A polynomial is completely factored when each of its factors are prime.

P.4.4.

Four guidelines for factoring polynomials are as follows:

(1) Factor out any common factors using the Distributive Property.

(2) Factor according to one of the special polynomial forms.

(3) Factor as $ax^2 + bx + c = (mx + r)(nx + s)$.

(4) Factor by grouping.

P.4.5.

$$2x^3 - 6x = 2x(x^2 - 3)$$

P.4.6.

$$3z^3 - 6z^2 + 9z = 3z(z^2 - 2z + 3)$$

P.4.7.

$$3x(x-5) + 8(x-5) = (x-5)(3x+8)$$

P.4.8.

$$\begin{aligned}(x+3)^2 - 4(x+3) &= (x+3)[(x+3)-4] \\ &= (x+3)(x-1)\end{aligned}$$

P.4.9.

$$\begin{aligned}x^2 - 81 &= x^2 - 9^2 \\ &= (x+9)(x-9)\end{aligned}$$

P.4.10.

$$\begin{aligned}x^2 - 64 &= x^2 - 8^2 \\ &= (x+8)(x-8)\end{aligned}$$

P.4.11.

$$25y^2 - 4 = (5y)^2 - 2^2 = (5y+2)(5y-2)$$

P.4.12.

$$4y^2 - 49 = (2y)^2 - 7^2 = (2y+7)(2y-7)$$

P.4.13.

$$\begin{aligned}(x-1)^2 - 4 &= (x-1)^2 - (2)^2 \\ &= [(x-1)+2][(x-1)-2] \\ &= (x+1)(x-3)\end{aligned}$$

P.4.14.

$$\begin{aligned}25 - (z+5)^2 &= 5^2 - (z+5)^2 \\ &= [5-(z+5)][5+(z+5)] \\ &= (5-z-5)(5+z+5) \\ &= -z(z+10)\end{aligned}$$

P.4.15.

$$\begin{aligned}81u^4 - 1 &= (9u^2+1)(9u^2-1) \\ &= (9u^2+1)(3u+1)(3u-1)\end{aligned}$$

P.4.16.

$$\begin{aligned}x^4 - 16y^4 &= (x^2+4y^2)(x^2-4y^2) \\ &= (x^2+4y^2)(x+2y)(x-2y)\end{aligned}$$

P.4.17.

$$x^2 - 4x + 4 = x^2 - 2(2)x + 2^2 = (x-2)^2$$

P.4.18.

$$4t^2 + 4t + 1 = (2t)^2 + 2(2t)(1) + 1^2 = (2t+1)^2$$

P.4.19.

$$25z^2 - 30z + 9 = (5z)^2 - 2(5z)(3) + 3^2 = (5z-3)^2$$

P.4.20.

$$\begin{aligned}36y^2 + 84y + 49 &= (6y)^2 + 2(6y)(7) + 7^2 \\ &= (6y+7)^2\end{aligned}$$

P.4.21.

$$\begin{aligned}4y^2 - 12y + 9 &= (2y)^2 - 2(2y)(3) + (3)^2 \\ &= (2y-3)^2\end{aligned}$$

P.4.22.

$$\begin{aligned}9u^2+24uv+16v^2&=(3u)^2+2(3u)(4v)+(4v)^2\\&=(3u+4v)^2\end{aligned}$$

P.4.23.

$$x^3-8=x^3-2^3=(x-2)(x^2+2x+4)$$

P.4.24.

$$x^3+125=x^3+5^3=(x+5)(x^2-5x+25)$$

P.4.25.

$$8t^3-1=(2t)^3-1^3=(2t-1)(4t^2+2t+1)$$

P.4.26.

$$27z^3+1=(3z)^3+1^3=(3z+1)(9z^2-3z+1)$$

P.4.27.

$$27x^3+8=(3x)^3+2^3=(3x+2)(9x^2-6x+4)$$

P.4.28.

$$\begin{aligned}64y^3-125&=(4y)^3-5^3\\&=(4y-5)(16y^2+20y+25)\end{aligned}$$

P.4.29.

$$x^2+x-2=(x+2)(x-1)$$

P.4.30.

$$x^2+5x+6=(x+2)(x+3)$$

P.4.31.

$$s^2-5s+6=(s-3)(s-2)$$

P.4.32.

$$t^2-t-6=(t+2)(t-3)$$

P.4.33.

$$3x^2+10x-8=(3x-2)(x+4)$$

P.4.34.

$$2x^2-3x-27=(2x-9)(x+3)$$

P.4.35.

$$5x^2 + 31x + 6 = (5x + 1)(x + 6)$$

P.4.36.

$$8x^2 + 51x + 18 = (8x + 3)(x + 6)$$

P.4.37.

$$\begin{aligned} -5y^2 - 8y + 4 &= -(5y^2 + 8y - 4) \\ &= -(5y - 2)(y + 2) \end{aligned}$$

P.4.38.

$$\begin{aligned} -6z^2 + 17z + 3 &= -(6z^2 - 17z - 3) \\ &= -(6z + 1)(z - 3) \end{aligned}$$

P.4.39.

$$\begin{aligned} x^3 - x^2 + 2x - 2 &= x^2(x - 1) + 2(x - 1) \\ &= (x - 1)(x^2 + 2) \end{aligned}$$

P.4.40.

$$\begin{aligned} x^3 + 5x^2 - 5x - 25 &= x^2(x + 5) - 5(x + 5) \\ &= (x + 5)(x^2 - 5) \end{aligned}$$

P.4.41.

$$\begin{aligned} 2x^3 - x^2 - 6x + 3 &= x^2(2x - 1) - 3(2x - 1) \\ &= (2x - 1)(x^2 - 3) \end{aligned}$$

P.4.42.

$$\begin{aligned} 3x^3 + x^2 - 15x - 5 &= x^2(3x + 1) - 5(3x + 1) \\ &= (x^2 - 5)(3x + 1) \end{aligned}$$

P.4.43.

$$\begin{aligned} 6 + 2x - 3x^3 - x^4 &= 2(3 + x) - x^3(3 + x) \\ &= (3 + x)(2 - x^3) \end{aligned}$$

P.4.44.

$$\begin{aligned} x^5 + 2x^3 + x^2 + 2 &= x^3(x^2 + 2) + (x^2 + 2) \\ &= (x^2 + 2)(x^3 + 1) \\ &= (x^2 + 2)(x + 1)(x^2 - x + 1) \end{aligned}$$

P.4.45.

$a \cdot c = (2)(9) = 18.$ Rewrite the middle term,

$9x = 6x + 3x,$ because $(6)(3) = 18$ and $6 + 3 = 9.$

$$\begin{aligned} 2x^2 + 9x + 9 &= 2x^2 + 6x + 3x + 9 \\ &= 2x(x+3) + 3(x+3) \\ &= (x+3)(2x+3) \end{aligned}$$

P.4.46.

$a \cdot c = (6)(-2) = -12.$ Rewrite the middle term,

$x = 4x - 3x,$ because $4(-3) = -12$ and $4 + (-3) = 1.$

$$\begin{aligned} 6x^2 + x - 2 &= 6x^2 + 4x - 3x - 2 \\ &= 2x(3x+2) - 1(3x+2) \\ &= (2x-1)(3x+2) \end{aligned}$$

P.4.47.

$a \cdot c = (6)(-15) = -90.$ Rewrite the middle term,

$-x = -10x + 9x,$ because $(-10)(9) = -90$ and $-10 + 9 = -1.$

$$\begin{aligned} 6x^2 - x - 15 &= 6x^2 - 10x + 9x - 15 \\ &= 2x(3x-5) + 3(3x-5) \\ &= (2x+3)(3x-5) \end{aligned}$$

P.4.48.

$a \cdot c = (12)(1) = 12.$ Rewrite the middle term,

$-13x = -12x - x,$ because $(-12)(-1) = 12$ and $-12 - 1 = -13.$

$$\begin{aligned} 12x^2 - 13x + 1 &= 12x^2 - 12x - x + 1 \\ &= 12x(x-1) - 1(x-1) \\ &= (x-1)(12x-1) \end{aligned}$$

P.4.49.

$$6x^2 - 54 = 6(x^2 - 9) = 6(x+3)(x-3)$$

P.4.50.

$$12x^2 - 48 = 12(x^2 - 4) = 12(x+2)(x-2)$$

P.4.51.

$$x^3 - x^2 = x^2\left(x - 1\right)$$

P.4.52.

$$x^3 - 16x = x\left(x^2 - 16\right) = x\left(x + 4\right)\left(x - 4\right)$$

P.4.53.

$$1 - 4x + 4x^2 = \left(1 - 2x\right)^2$$

P.4.54.

$$\begin{aligned} -9x^2 + 6x - 1 &= -\left(9x^2 - 6x + 1\right) \\ &= -\left(9x^2 - 3x - 3x + 1\right) \\ &= -\left[3x\left(3x - 1\right) - \left(3x - 1\right)\right] \\ &= -\left(3x - 1\right)^2 \end{aligned}$$

P.4.55.

$$\begin{aligned} 2x^2 + 4x - 2x^3 &= -2x\left(-x - 2 + x^2\right) \\ &= -2x\left(x^2 - x - 2\right) \\ &= -2x\left(x + 1\right)\left(x - 2\right) \end{aligned}$$

P.4.56.

$$\begin{aligned} 9x^2 + 12x - 3x^3 &= -3x^3 + 9x^2 + 12x \\ &= -3x\left(x^2 - 3x - 4\right) \\ &= -3x\left(x - 4\right)\left(x + 1\right) \end{aligned}$$

P.4.57.

$$\begin{aligned} \left(x^2 + 3\right)^2 - 16x^2 &= \left[\left(x^2 + 3\right) + 4x\right]\left[\left(x^2 + 3\right) - 4x\right] \\ &= \left(x^2 + 4x + 3\right)\left(x^2 - 4x + 3\right) \\ &= \left(x + 3\right)\left(x + 1\right)\left(x - 3\right)\left(x - 1\right) \end{aligned}$$

P.4.58.

$$\begin{aligned} \left(x^2 + 8\right)^2 - 36x^2 &= \left(x^2 + 8\right)^2 - \left(6x\right)^2 \\ &= \left[\left(x^2 + 8\right) - 6x\right]\left[\left(x^2 + 8\right) + 6x\right] \\ &= \left(x^2 - 6x + 8\right)\left(x^2 + 6x + 8\right) \\ &= \left(x - 4\right)\left(x - 2\right)\left(x + 4\right)\left(x + 2\right) \end{aligned}$$

P.4.59.

$$\begin{aligned} 2x^3 + x^2 - 8x - 4 &= x^2(2x+1) - 4(2x+1) \\ &= (2x+1)(x^2-4) \\ &= (2x+1)(x+2)(x-2) \end{aligned}$$

P.4.60.

$$\begin{aligned} 3x^3 + x^2 - 27x - 9 &= x^2(3x+1) - 9(3x+1) \\ &= (3x+1)(x^2-9) \\ &= (3x+1)(x+3)(x-3) \end{aligned}$$

P.4.61.

$$\begin{aligned} 2x(3x+1) + (3x+1)^2 &= (3x+1)\left[2x + (3x+1)\right] \\ &= (3x+1)(5x+1) \end{aligned}$$

P.4.62.

$$\begin{aligned} 4x(2x-1) + (2x-1)^2 &= (2x-1)\left[4x + (2x-1)\right] \\ &= (2x-1)(6x-1) \end{aligned}$$

P.4.63.

$$\begin{aligned} 2(x-2)(x+1)^2 - 3(x-2)^2(x+1) &= (x-2)(x+1)\left[2(x+1) - 3(x-2)\right] \\ &= (x-2)(x+1)\left[2x+2-3x+6\right] \\ &= (x-2)(x+1)(-x+8) \\ &= -(x-2)(x+1)(x-8) \end{aligned}$$

P.4.64.

$$\begin{aligned} 2(x+1)(x-3)^2 - 3(x+1)^2(x-3) &= (x+1)(x-3)\left[2(x-3) - 3(x+1)\right] \\ &= (x+1)(x-3)\left[2x-6-3x-3\right] \\ &= (x+1)(x-3)(-x-9) \\ &= -(x+1)(x-3)(x+9) \end{aligned}$$

P.4.65.

$$\begin{aligned} 5(2x+1)^2(x+1)^2 + (2x+1)(x+1)^3 &= (2x+1)(x+1)^2\left[5(2x+1) + (x+1)\right] \\ &= (2x+1)(x+1)^2(10x+5+x+1) \\ &= (2x+1)(x+1)^2(11x+6) \end{aligned}$$

P.4.66.

$$\begin{aligned}7(3x+2)^2(1-x)^2+(3x+2)(1-x)^3 &= (3x+2)(1-x)^2\left[7(3x+2)+(1-x)\right]\\ &= (3x+2)(1-x)^2(21x+14+1-x)\\ &= (3x+2)(1-x)^2(20x+15)\\ &= 5(3x+2)(1-x)^2(4x+3)\end{aligned}$$

P.4.67.

$$\begin{aligned}16x^2-\tfrac{1}{9} &= (4x)^2-\left(\tfrac{1}{3}\right)^2\\ &= \left(4x+\tfrac{1}{3}\right)\left(4x-\tfrac{1}{3}\right)\end{aligned}$$

P.4.68.

$$\begin{aligned}\tfrac{4}{25}y^2-64 &= \left(\tfrac{2}{5}y\right)^2-(8)^2\\ &= \left(\tfrac{2}{5}y+8\right)\left(\tfrac{2}{5}y-8\right)\end{aligned}$$

P.4.69.

$$\begin{aligned}z^2+z+\tfrac{1}{4} &= z^2+2(z)\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2\\ &= \left(z+\tfrac{1}{2}\right)^2\end{aligned}$$

P.4.70.

$$\begin{aligned}9y^2-\tfrac{3}{2}y+\tfrac{1}{16} &= (3y)^2-2(3y)\left(\tfrac{1}{4}\right)+\left(\tfrac{1}{4}\right)^2\\ &= \left(3y-\tfrac{1}{4}\right)^2\end{aligned}$$

P.4.71.

$$\begin{aligned}y^3+\tfrac{8}{27} &= y^3+\left(\tfrac{2}{3}\right)^3\\ &= \left(y+\tfrac{2}{3}\right)\left(y^2-\tfrac{2}{3}y+\tfrac{4}{9}\right)\end{aligned}$$

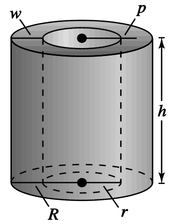
P.4.72.

$$\begin{aligned}x^3-\tfrac{27}{64} &= x^3-\left(\tfrac{3}{4}\right)^3\\ &= \left(x-\tfrac{3}{4}\right)\left(x^2+\tfrac{3}{4}y+\tfrac{9}{16}\right)\end{aligned}$$

P.4.73.

$$\begin{aligned}V &= \pi R^2h-\pi r^2h\\ &= \pi h(R^2-r^2)\\ &= \pi h(R+r)(R-r)\end{aligned}$$

Let $w=$ thickness of the shell and let. $p=$ average radius of the shell.



So, $R = p + \frac{1}{2}w$ and $r = p - \frac{1}{2}w$

$$\begin{aligned} V &= \pi h(R+r)(R-r) \\ &= \pi h\left[\left(p+\frac{1}{2}w\right)+\left(p-\frac{1}{2}w\right)\right]\left[\left(p+\frac{1}{2}w\right)-\left(p-\frac{1}{2}w\right)\right] \\ &= \pi h(2p)(w) \\ &= 2\pi pwh \\ &= 2\pi\ \text{(average radius)(thickness of shell)}\ h \end{aligned}$$

P.4.74.

$$kQx - kx^2 = kx(Q-x)$$

P.4.75.

For $x^2 + bx - 15$ to be factorable, b must equal $m+n$ where $mn = -15$.

Factors of -15	Sum of factors
$(15)(-1)$	$15+(-1)=14$
$(-15)(1)$	$-15+1=-14$
$(3)(-5)$	$3+(-5)=-2$
$(-3)(5)$	$-3+5=2$

The possible b-values are $14,\ -14,\ -2,$ and 2.

P.4.76.

For $x^2 + bx + 24$ to be factorable, b must equal $m+n$ where $mn = 24$.

Factors of 24	Sum of factors
$(1)(24)$	$1+24=25$
$(-1)(-24)$	$-1+(-24)=-25$
$(2)(12)$	$2+12=14$
$(-2)(-12)$	$-2+(-12)=-14$
$(3)(8)$	$3+8=11$
$(-3)(-8)$	$-3+(-8)=-11$
$(4)(6)$	$4+6=10$
$(-4)(-6)$	$-4+(-6)=-10$

The possible b-values are $25,\ -25,\ 14,\ -14,\ 11,\ -11,\ 10,\ -10$.

P.4.77.

For $2x^2+5x+c$ to be factorable, the factors of 2c must add up to 5.

Possible c-values	2c	Factors of 2c that add up to 5
2	4	$(1)(4)=4$ and $1+4=5$
3	6	$(2)(3)=6$ and $2+3=5$
–3	–6	$(6)(-1)=-6$ and $6+(-1)=5$
–7	–14	$(7)(-2)=-14$ and $7+(-2)=5$
–12	–24	$(8)(-3)=-24$ and $8+(-3)=5$

These are a few possible c-values. There are *many* correct answers.

If $c=2$: $2x^2+5x+2=(2x+1)(x+2)$

If $c=3$: $2x^2+5x+3=(2x+3)(x+1)$

If $c=-3$: $2x^2+5x-3=(2x-1)(x+3)$

If $c=-7$: $2x^2+5x-7=(2x+7)(x-1)$

If $c=-12$: $2x^2+5x-12=(2x-3)(x+4)$

P.4.78.

For $3x^2-x+c$ to be factorable, the factors of 3c must add up to -1.

Possible c-values	3c	Factors of 3c must add up to -1
–2	–6	$(2)(-3)=-6$ and $2+(-3)=-1$
–4	–12	$(3)(-4)=-12$ and $3+(-4)=-1$
–10	–30	$(5)(-6)=-30$ and $5+(-6)=-1$

These are a few possible c-values. There are *many* correct answers.

If $c=-2$: $3x^2-x-2=(3x+2)(x-1)$

If $c=-4$: $3x^2-x-4=(3x-4)(x+1)$

If $c=-10$: $3x^2-x-10=(3x+5)(x-2)$

P.4.79.

True. $a^2-b^2=(a+b)(a-b)$

P.4.80.

True. $u^2+2uv+v^2=(u+v)^2$ and $u^2-2uv+v^2=(u-v)^2$

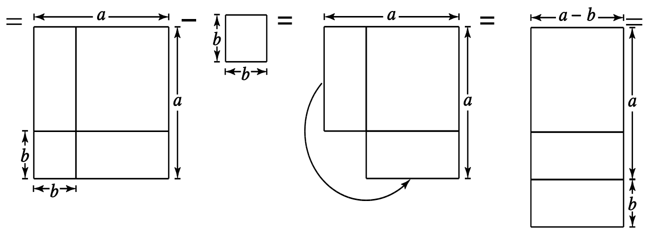
P.4.81.

No, $(3x-6)(x+1)$ is not completely factored because $(3x-6)=3(x-2)$.

So, the completely factored form is $3(x-2)(x+1)$.

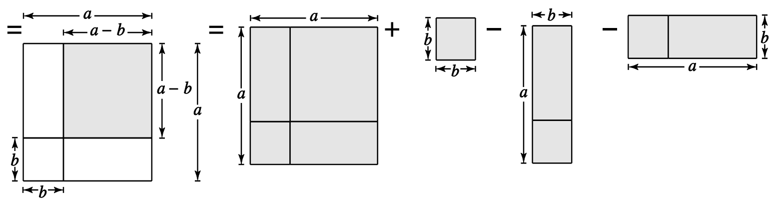
P.4.82.

(a) $a^2 - b^2 =$



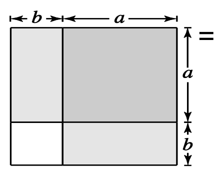
$= (a+b)\ (a-b)$

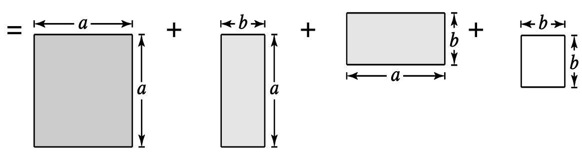
(b) $(a-b)^2$



$= a^2 - 2ab + b^2$

(c) $(a+b)^2$





$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

P.4.83.

3 should be factored out of both binomials to yield

$(3x+6)(3x-9)=3(x+2)(3)(x-3)=9(x+2)(x-3)$.

P.4.84.

$$\begin{aligned} u^6 - v^6 &= \left(u^3\right)^2 - \left(v^3\right)^2 \\ &= \left(u^3 + v^3\right)\left(u^3 - v^3\right) \\ &= \left[\left(u+v\right)\left(u^2 - uv + v^2\right)\right]\left[\left(u-v\right)\left(u^2 + uv + v^2\right)\right] \\ &= \left(u+v\right)\left(u-v\right)\left(u^2 + uv + v^2\right)\left(u^2 - uv + v^2\right) \end{aligned}$$

$$x^6 - 1 = \left(x+1\right)\left(x-1\right)\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)$$

$$x^6 - 64 = x^6 - 2^6 = \left(x+2\right)\left(x-2\right)\left(x^2 + 2x + 4\right)\left(x^2 - 2x + 4\right)$$

P.4.85.

$$\frac{4x}{5} - \frac{5x}{6} = \frac{6\left(4x\right) - 5\left(5x\right)}{5\left(6\right)} = \frac{24x - 25x}{30} = -\frac{x}{30}$$

P.4.86.

$$\frac{3x}{8} + \frac{x}{6} = \frac{6\left(3x\right) + 8x}{8\left(6\right)} = \frac{18x + 8x}{48} = \frac{26x}{48} = \frac{13x}{24}$$

P.4.87.

$$\frac{2x}{15} \cdot \frac{9}{8} = \frac{x}{5} \cdot \frac{3}{4} = \frac{3x}{20}$$

P.4.88.

$$\frac{18x}{11} \div \frac{14x^2}{33} = \frac{18x}{11} \cdot \frac{33}{14x^2} = \frac{9 \cdot 3}{7x} = \frac{27}{7x}$$

P.4.89.

(a) When $x = 1$, $\dfrac{x+1}{x-1} = \dfrac{1+1}{1-1} = \dfrac{2}{0}$. Division by 0 is undefined.

(b) When $x = -1$, $\dfrac{x+1}{x-1} = \dfrac{-1+1}{-1-1} = 0$.

P.4.90.

(a) When $x = 1$, $\dfrac{1-x}{x} = \dfrac{1-1}{1} = 0$.

(b) When $x = 0$, $\dfrac{1-x}{x} = \dfrac{1-0}{0}$. Division by 0 is undefined.

P.4.91.

$$x = x$$
$$x^2 = x \cdot x$$
$$x^3 = x \cdot x \cdot x$$

The least common multiple is x^3.

P.4.92.

$$2x^2 = 2 \cdot x \cdot x$$
$$12x = 2 \cdot 2 \cdot 3 \cdot x$$
$$42x^3 = 2 \cdot 3 \cdot 7 \cdot x \cdot x \cdot x$$

The least common multiple is $84x^3$.

P.4.93.

The least common multiple is $x(x+1)(x-1)$.

P.4.94.

$x^2 - 4 = (x-2)(x+2)$ and $(x - 2x + 8) = (x-4)(x+2)$.

The least common multiple is $(x+2)(x-2)(x-4)$.

SECTION P.5 RATIONAL EXPRESSIONS

P.5.1.

equivalent

P.5.2.

lesser

P.5.3.

A rational expression is in simplest form when its numerator and denominator have no common factors aside from ± 1.

P.5.4.

Values that make the denominator equal to zero are excluded from the domain of a rational expression.

P.5.5.

The domain of $3x^2 - 4x + 7$ is the set of all real numbers.

P.5.6.

The domain of $6x^2 - 9,\ \ x > 0$ is the set of all positive real numbers because the polynomial is restricted to that set.

P.5.7.

The domain of $\dfrac{1}{3-x}$ is the set of all real numbers x such that $x \neq 3$.

P.5.8.

The domain of $\dfrac{1}{x+5}$ is the set of all real numbers x such that $x \neq -5$.

P.5.9.

The domain of $\dfrac{x+6}{3x+2}$ is the set of all real numbers x such that $x \neq -\dfrac{2}{3}$.

P.5.10.

The domain of $\dfrac{x-4}{1-2x}$ is the set of all real numbers x such that $x \neq \dfrac{1}{2}$.

P.5.11.

The domain of $\dfrac{x^2-5x+6}{x^2+6x+8} = \dfrac{(x-2)(x-3)}{(x+4)(x+2)}$ is the set of all real numbers x such that $x \neq -4,\ \ -2$.

P.5.12.

The domain of $\dfrac{x^2-1}{x^2+3x-10}=\dfrac{(x-1)(x+1)}{(x+5)(x-2)}$ is the set of all real numbers x such that $x \neq -5, 2$.

P.5.13.

The domain of $\sqrt{x+7}$ is the set of all real numbers x such that $x \geq -7$.

P.5.14.

The domain of $\sqrt{2x-5}$ is the set of all real numbers x such that $x \geq \dfrac{5}{2}$.

P.5.15.

The domain of $\dfrac{1}{\sqrt{x-3}}$ is the set of all real numbers x such that $x > 3$.

P.5.16.

The domain of $\dfrac{1}{\sqrt{x+2}}$ is the set of all real numbers x such that $x > -2$.

P.5.17.

$$\frac{x-5}{10-2x}=\frac{x-5}{-2(x-5)}$$
$$=-\frac{1}{2},\ x \neq 5$$

P.5.18.

$$\frac{12-4x}{x-3}=\frac{4(3-x)}{x-3}=-4,\ x \neq 3$$

P.5.19.

$$\frac{y^2-16}{y+4}=\frac{(y+4)(y-4)}{y+4}$$
$$=y-4,\ y \neq -4$$

P.5.20.

$$\frac{x^2-25}{5-x}=\frac{(x+5)(x-5)}{-1(x-5)}$$
$$=-(x+5),\ x \neq 5$$

P.5.21.

$$\frac{6y+9y^2}{12y+8}=\frac{3y(3y+2)}{4(3y+2)}=\frac{3y}{4},\ y \neq -\frac{2}{3}$$

P.5.22.

$$\frac{4y-8y^2}{10y-5} = \frac{-4y(2y-1)}{5(2y-1)}$$
$$= -\frac{4y}{5},\quad y \neq \frac{1}{2}$$

P.5.23.

$$\frac{x^2+4x-5}{x^2+8x+15} = \frac{(x+5)(x-1)}{(x+5)(x+3)} = \frac{x-1}{x+3},\quad x \neq -5$$

P.5.24.

$$\frac{x^2+8x-20}{x^2+11x+10} = \frac{(x+10)(x-2)}{(x+10)(x+1)} = \frac{x-2}{x+1},\quad x \neq -10$$

P.5.25.

$$\frac{x^2-x-2}{10-3x-x^2} = \frac{x^2-x-2}{-(x^2+3x-10)}$$
$$= \frac{(x+1)(x-2)}{-(x+5)(x-2)} = -\frac{x+1}{x+5},\quad x \neq 2$$

P.5.26.

$$\frac{4+3x-x^2}{2x^2-7x-4} = \frac{-(x^2-3x-4)}{2x^2-7x-4}$$
$$= \frac{-(x+1)(x-4)}{(2x+1)(x-4)} = -\frac{x+1}{2x+1},\quad x \neq 4$$

P.5.27.

$$\frac{x^2-16}{x^3+x^2-16x-16} = \frac{x^2-16}{x^2(x+1)-16(x+1)}$$
$$= \frac{x^2-16}{(x+1)(x^2-16)}$$
$$= \frac{1}{x+1},\quad x \neq \pm 4$$

P.5.28.

$$\begin{aligned}\frac{x^2-1}{x^3+x^2+9x+9} &= \frac{x^2-1}{x^2(x+1)+9(x+1)}\\ &= \frac{(x-1)(x+1)}{(x+1)(x^2+9)}\\ &= \frac{x-1}{x^2+9},\ x \neq -1\end{aligned}$$

P.5.29.

$$\frac{5}{x-1}\cdot\frac{x-1}{25(x-2)} = \frac{1}{5(x-2)},\quad x \neq 1$$

P.5.30.

$$\begin{aligned}\frac{x+13}{x^3(3-x)}\cdot\frac{x(x-3)}{5} &= \frac{x+13}{x^3(x-3)(-1)}\cdot\frac{x(x-3)}{5}\\ &= \frac{x+13}{-5x^2} = -\frac{x+13}{5x^2},\quad x \neq 3\end{aligned}$$

P.5.31.

$$\begin{aligned}\frac{x^2-4}{12}\div\frac{2-x}{2x+4} &= \frac{x^2-4}{12}\cdot\frac{2x+4}{2-x}\\ &= \frac{(x+2)(x-2)}{12}\cdot\frac{2(x+2)}{-(x-2)}\\ &= -\frac{(x+2)^2}{6},\ x \neq \pm 2\end{aligned}$$

P.5.32.

$$\frac{r}{r-1}\div\frac{r^2}{r^2-1} = \frac{r}{r-1}\cdot\frac{r^2-1}{r^2} = \frac{r(r+1)(r-1)}{r^2(r-1)} = \frac{r+1}{r},\quad r \neq 1$$

P.5.33.

$$\frac{x^2+xy-2y^2}{x^3+x^2y}\cdot\frac{x}{x^2+3xy+2y^2} = \frac{(x+2y)(x-y)}{x^2(x+y)}\cdot\frac{x}{(x+2y)(x+y)} = \frac{x-y}{x(x+y)^2},\quad x \neq -2y$$

P.5.34.

$$\begin{aligned}\frac{x^2-14x+49}{x^2-49}\div\frac{3x-21}{x+7} &= \frac{(x-7)(x-7)}{(x+7)(x-7)}\cdot\frac{x+7}{3(x-7)}\\ &= \frac{1}{3},\quad x \neq \pm 7\end{aligned}$$

P.5.35.

$$\begin{aligned}\frac{1}{3x+2}+\frac{x}{x+1}&=\frac{(1)(x+1)}{(3x+2)(x+1)}+\frac{x(3x+2)}{(3x+2)(x+1)}\\&=\frac{x+1+3x^2+2x}{(3x+2)(x+1)}\\&=\frac{3x^2+3x+1}{(3x+2)(x+1)}\end{aligned}$$

P.5.36.

$$\begin{aligned}\frac{x}{x+4}-\frac{6}{x-1}&=\frac{x(x-1)}{(x+4)(x-1)}-\frac{6(x+4)}{(x+4)(x-1)}\\&=\frac{x^2-x-6x-24}{(x+4)(x-1)}\\&=\frac{x^2-7x-24}{(x+4)(x-1)}\end{aligned}$$

P.5.37.

$$\begin{aligned}\frac{3}{2x+4}-\frac{x}{x+2}&=\frac{3}{2(x+2)}-\frac{x}{x+2}\\&=\frac{3}{2(x+2)}-\frac{2x}{2(x+2)}\\&=\frac{3-2x}{2(x+2)}\end{aligned}$$

P.5.38.

$$\begin{aligned}\frac{2}{x^2-9}+\frac{4}{x+3}&=\frac{2}{(x+3)(x-3)}+\frac{4}{x+3}\\&=\frac{2}{(x+3)(x-3)}+\frac{4(x-3)}{(x+3)(x-3)}\\&=\frac{2+4x-12}{(x+3)(x-3)}\\&=\frac{4x-10}{(x+3)(x-3)}\\&=\frac{2(2x-5)}{(x+3)(x-3)}\end{aligned}$$

P.5.39.

$$-\frac{1}{x}+\frac{2}{x^2+1}+\frac{1}{x^3+x}=\frac{-\left(x^2+1\right)}{x\left(x^2+1\right)}+\frac{2x}{x\left(x^2+1\right)}+\frac{1}{x\left(x^2+1\right)}$$

$$=\frac{-x^2-1+2x+1}{x\left(x^2+1\right)}=\frac{-x^2+2x}{x\left(x^2+1\right)}=\frac{-x\left(x-2\right)}{x\left(x^2+1\right)}$$

$$=-\frac{x-2}{x^2+1}=\frac{2-x}{x^2+1},\ x\neq 0$$

P.5.40.

$$\frac{2}{x+1}+\frac{2}{x-1}+\frac{1}{x^2-1}=\frac{2}{x+1}+\frac{2}{x-1}+\frac{1}{\left(x+1\right)\left(x-1\right)}$$

$$=\frac{2\left(x-1\right)}{\left(x+1\right)\left(x-1\right)}+\frac{2\left(x+1\right)}{\left(x+1\right)\left(x-1\right)}+\frac{1}{\left(x+1\right)\left(x-1\right)}$$

$$=\frac{2x-2+2x+2+1}{\left(x+1\right)\left(x-1\right)}=\frac{4x+1}{\left(x+1\right)\left(x-1\right)}$$

P.5.41.

$$\frac{\left(\frac{x}{2}-1\right)}{\left(x-2\right)}=\frac{\left(\frac{x}{2}-\frac{2}{2}\right)}{\left(\frac{x-2}{1}\right)}=\frac{x-2}{2}\cdot\frac{1}{x-2}=\frac{1}{2},\ x\neq 2$$

P.5.42.

$$\frac{\left(\frac{x}{5}-5\right)}{\left(x+5\right)}=\frac{\left(\frac{x}{5}-\frac{25}{5}\right)}{\left(\frac{x+5}{1}\right)}=\frac{x-25}{5}\cdot\frac{1}{x+5}=\frac{x-25}{5\left(x+5\right)},\ x\neq -5$$

P.5.43.

$$\frac{\left[\frac{x^2}{\left(x+1\right)^2}\right]}{\left[\frac{x}{\left(x+1\right)^3}\right]}=\frac{x^2}{\left(x+1\right)^2}\cdot\frac{\left(x+1\right)^3}{x}=x\left(x+1\right),\ x\neq -1,\ 0$$

P.5.44.

$$\frac{\left(\dfrac{x^2-1}{x}\right)}{\left[\dfrac{(x-1)^2}{x}\right]} = \frac{x^2-1}{x}\cdot\frac{x}{(x-1)^2} = \frac{(x+1)(x-1)}{x}\cdot\frac{x}{(x-1)(x-1)} = \frac{x+1}{x-1},\quad x \neq 0$$

P.5.45.

$$x^2\left(x^2+3\right)^{-4} + \left(x^2+3\right)^3 = \left(x^2+3\right)^{-4}\left[x^2 + \left(x^2+3\right)^7\right] = \frac{x^2+\left(x^2+3\right)^7}{\left(x^2+3\right)^4}$$

P.5.46.

$$\begin{aligned} 2x(x-5)^{-3} - 4x^2(x-5)^{-4} &= -2x(x-5)^{-4}\left[-(x-5)+2x\right] \\ &= -2x(x-5)^{-4}(-x+5+2x) \\ &= \frac{-2x(x+5)}{(x-5)^4} \end{aligned}$$

P.5.47.

$$2x^2(x-1)^{1/2} - 5(x-1)^{-1/2} = (x-1)^{-1/2}\left[2x^2(x-1)^1 - 5\right] = \frac{2x^3-2x^2-5}{(x-1)^{1/2}}$$

P.5.48.

$$\begin{aligned} 4x^3(x+1)^{-3/2} - x(x+1)^{-1/2} &= x(x+1)^{-3/2}\left[4x^2-(x+1)\right] \\ &= x(x+1)^{-3/2}\left(4x^2-x-1\right) \\ &= \frac{x\left(4x^2-x-1\right)}{(x+1)^{3/2}} \end{aligned}$$

P.5.49.

$$\frac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}} = \frac{3x^{1/3}-x^{-2/3}}{3x^{-2/3}}\cdot\frac{x^{2/3}}{x^{2/3}} = \frac{3x^1-x^0}{3x^0} = \frac{3x-1}{3},\quad x \neq 0$$

P.5.50.

$$\frac{-x^3\left(1-x^2\right)^{-1/2}-2x\left(1-x^2\right)^{1/2}}{x^4}=\frac{\dfrac{-x^3}{\left(1-x^2\right)^{1/2}}-2x\left(1-x^2\right)^{1/2}}{x^4}$$

$$=\frac{\dfrac{-x^3}{\left(1-x^2\right)^{1/2}}-\dfrac{2x\left(1-x^2\right)^{1/2}\left(1-x^2\right)^{1/2}}{\left(1-x^2\right)^{1/2}}}{x^4}=\frac{\dfrac{-x^3-2x\left(1-x^2\right)}{\left(1-x^2\right)^{1/2}}}{x^4}$$

$$=\frac{-x^3-2x+2x^3}{\left(1-x^2\right)^{1/2}}\cdot\frac{1}{x^4}=\frac{x^3-2x}{\left(1-x^2\right)^{1/2}}\cdot\frac{1}{x^4}$$

$$=\frac{x\left(x^2-2\right)}{x^4\left(1-x^2\right)^{1/2}}=\frac{x^2-2}{x^3\left(1-x^2\right)^{1/2}}$$

P.5.51.

$$\frac{\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)}{h}=\frac{\left(\dfrac{1}{x+h}-\dfrac{1}{x}\right)}{h}\cdot\frac{x\left(x+h\right)}{x\left(x+h\right)}$$

$$=\frac{x-\left(x+h\right)}{hx\left(x+h\right)}$$

$$=\frac{-h}{hx\left(x+h\right)}$$

$$=-\frac{1}{x\left(x+h\right)},\quad h\neq 0$$

P.5.52.

$$\frac{\left[\dfrac{1}{\left(x+h\right)^2}-\dfrac{1}{x^2}\right]}{h}=\frac{\left[\dfrac{1}{\left(x+h\right)^2}-\dfrac{1}{x^2}\right]}{h}\cdot\frac{x^2\left(x+h\right)^2}{x^2\left(x+h\right)^2}$$

$$=\frac{x^2-\left(x+h\right)^2}{hx^2\left(x+h\right)^2}$$

$$=\frac{x^2-\left(x^2+2xh+h^2\right)}{hx^2\left(x+h\right)^2}$$

$$=\frac{-h\left(2x+h\right)}{hx^2\left(x+h\right)^2}$$

$$=-\frac{2x+h}{x^2\left(x+h\right)^2},\quad h\neq 0$$

P.5.53.

$$\frac{\left(\frac{1}{x+h-4}-\frac{1}{x-4}\right)}{h}=\frac{\left(\frac{1}{x+h-4}-\frac{1}{x-4}\right)}{h}\cdot\frac{(x-4)(x+h-4)}{(x-4)(x+h-4)}$$

$$=\frac{(x-4)-(x+h-4)}{h(x-4)(x+h-4)}$$

$$=\frac{-h}{h(x-4)(x+h-4)}$$

$$=-\frac{1}{(x-4)(x+h-4)},\quad h\neq 0$$

P.5.54.

$$\frac{\left(\frac{x+h}{x+h+1}-\frac{x}{x+1}\right)}{h}=\frac{\left[\frac{(x+h)(x+1)}{(x+h+1)(x+1)}-\frac{x(x+h+1)}{(x+h+1)(x+1)}\right]}{h/1}$$

$$=\left[\frac{(x+h)(x+1)}{(x+h+1)(x+1)}-\frac{x(x+h+1)}{(x+h+1)(x+1)}\right]\cdot\frac{1}{h}$$

$$=\left[\frac{x^2+x+hx+h-x^2-xh-x}{(x+h+1)(x+1)}\right]\cdot\frac{1}{h}$$

$$=\frac{h}{(x+h+1)(x+1)}\cdot\frac{1}{h}=\frac{1}{(x+h+1)(x+1)},\quad h\neq 0$$

P.5.55.

$$\frac{\sqrt{x+2}-\sqrt{x}}{2}=\frac{\sqrt{x+2}-\sqrt{x}}{2}\cdot\frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}+\sqrt{x}}$$

$$=\frac{(x+2)-x}{2\left(\sqrt{x+2}+\sqrt{x}\right)}$$

$$=\frac{1}{\sqrt{x+2}+\sqrt{x}}$$

P.5.56.

$$\frac{\sqrt{z-3}-\sqrt{z}}{-3} = -\frac{\sqrt{z-3}-\sqrt{z}}{3} \cdot \frac{\sqrt{z-3}+\sqrt{z}}{\sqrt{z-3}+\sqrt{z}}$$
$$= -\frac{(z-3)-z}{3\left(\sqrt{z-3}+\sqrt{z}\right)}$$
$$= -\frac{-3}{3\left(\sqrt{z-3}+\sqrt{z}\right)}$$
$$= \frac{1}{\sqrt{z-3}+\sqrt{z}}$$

P.5.57.

$$\frac{\sqrt{t+3}-\sqrt{3}}{t} = \frac{\sqrt{t+3}-\sqrt{3}}{t} \cdot \frac{\sqrt{t+3}+\sqrt{3}}{\sqrt{t+3}+\sqrt{3}}$$
$$= \frac{(t+3)-3}{t\left(\sqrt{t+3}+\sqrt{3}\right)}$$
$$= \frac{t}{t\left(\sqrt{t+3}+\sqrt{3}\right)}$$
$$= \frac{1}{\sqrt{t+3}+\sqrt{3}}, \quad t \neq 0$$

P.5.58.

$$\frac{\sqrt{x+5}-\sqrt{5}}{x} = \frac{\sqrt{x+5}-\sqrt{5}}{x} \cdot \frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$$
$$= \frac{(x+5)-5}{x\left(\sqrt{x+5}+\sqrt{5}\right)}$$
$$= \frac{x}{x\left(\sqrt{x+5}+\sqrt{5}\right)}$$
$$= \frac{1}{\sqrt{x+5}+\sqrt{5}}, \quad x \neq 0$$

P.5.59.

$$\begin{aligned}\frac{\sqrt{x+h+1}-\sqrt{x+1}}{h} &= \frac{\sqrt{x+h+1}-\sqrt{x+1}}{h}\cdot\frac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}\\ &= \frac{(x+h+1)-(x+1)}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}\\ &= \frac{h}{h\left(\sqrt{x+h+1}+\sqrt{x+1}\right)}\\ &= \frac{1}{\sqrt{x+h+1}+\sqrt{x+1}},\quad h\neq 0\end{aligned}$$

P.5.60.

$$\begin{aligned}\frac{\sqrt{x+h-2}-\sqrt{x-2}}{h}\cdot\frac{\sqrt{x+h-2}+\sqrt{x-2}}{\left(\sqrt{x+h-2}+\sqrt{x-2}\right)} &= \frac{x+h-2-x+2}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}\\ &= \frac{h}{h\left(\sqrt{x+h-2}+\sqrt{x-2}\right)}\\ &= \frac{1}{\sqrt{x+h-2}+\sqrt{x-2}},\quad h\neq 0\end{aligned}$$

P.5.61.

$$T = 10\left(\frac{4t^2+16t+75}{t^2+4t+10}\right)$$

t	0	2	4	6	8	10	12	14	16	18	20	22
T	75°	55.9°	48.3°	45°	43.3°	42.3°	41.7°	41.3°	41.1°	40.9°	40.7°	40.6°

T is approaching 40°.

P.5.62.

(a)

Year, t	**Television**	**Digital Videa**
21	3.32	3.01
22	3.16	3.34
23	3.02	3.61
24	2.90	3.82

(b) The values from the models are close to the actual data.

(c) The ratio of the models is

$$\left(\frac{70.46-0.04t}{t}\right)\bigg/\left(\frac{7.404-0.432t}{1-0.074t}\right)$$
$$=\frac{70.46-0.04t}{t}\cdot\frac{1-0.074t}{7.404-0.432t}$$
$$\approx\frac{70.46-5.25404t+0.00296t^2}{7.404t-0.432t^2}$$

(d)

Year, *t*	**Ratio**
21	1.101
22	0.946
23	0.838
24	0.758

Answers will vary. *Sample answer:* The ratios are decreasing and close to 1.

P.5.63.

$$\text{Probability}=\frac{\text{Shaded area}}{\text{Total area}}=\frac{x(x/2)}{x(2x+1)}=\frac{x/2}{2x+1}\cdot\frac{2}{2}=\frac{x}{2(2x+1)},\quad x\neq 0$$

P.5.64.

$$\text{Probability}=\frac{\text{Shaded area}}{\text{Total area}}=\frac{\frac{1}{2}\cdot\frac{4}{x}(x+2)(x+x+4)}{\frac{1}{2}(x+4)\left[(x+2)+\frac{4}{x}(x+2)\right]}$$
$$=\frac{\frac{4(x+2)(2x+4)}{x}}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}=\frac{\frac{4\cdot 2(x+2)^2}{x}}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}$$
$$=\frac{8(x+2)^2}{x}\cdot\frac{1}{(x+4)(x+2)\left(1+\frac{4}{x}\right)}$$
$$=\frac{8(x+2)^2}{(x+4)(x+2)(x+4)}=\frac{8(x+2)}{(x+4)^2},\quad x\neq 0$$

P.5.65.

False. In order for the simplified expression to be equivalent to the original expression, the domain of the simplified expression needs to be restricted. If n is even, $x\neq\pm 1$. If n is odd, $x\neq 1$.

P.5.66.

False. The two expressions are equivalent for all values of x such that $x\neq 1$.

P.5.67.

x	0	1	2	3	4	5	6
$\dfrac{x-3}{x^2-x-6}$	$\dfrac{1}{2}$	$\dfrac{1}{3}$	$\dfrac{1}{4}$	Undef.	$\dfrac{1}{6}$	$\dfrac{1}{7}$	$\dfrac{1}{8}$
$\dfrac{1}{x+2}$	$\dfrac{1}{2}$	$\dfrac{1}{3}$	$\dfrac{1}{4}$	$\dfrac{1}{5}$	$\dfrac{1}{6}$	$\dfrac{1}{7}$	$\dfrac{1}{8}$

The expressions are not equivalent at $x = 3$.

P.5.68.

The bar graph indicates that the oxygen in the pond level drops dramatically for the first week. By the twelfth week, the oxygen levels in the pond are almost at the previous high.

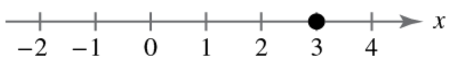
P.5.69.

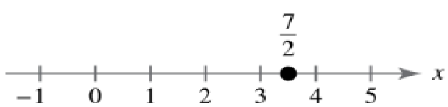
When simplifying fractions, only common factors can be divided out, not terms.

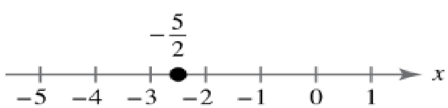
P.5.70.

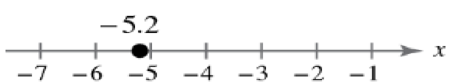
The numerator of the difference should be $x+3-x+1$, not $x+3-x-1$.

P.5.71.

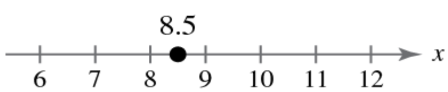
(a) 

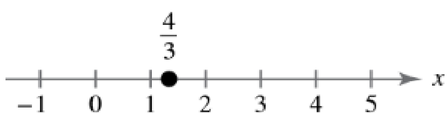
(b) 

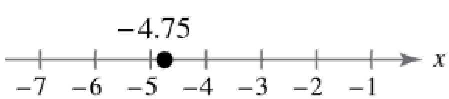
(c) 

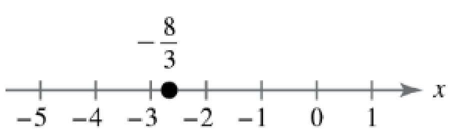
(d) 

P.5.72.

(a) 

(b) 

(c) 

(d) 

P.5.73.

$d(x, 5) = |x - 5|$ and $d(x, 5) \le 3$, so $|x - 5| \le 3$.

P.5.74.

$d(x, -10) = |x + 10|$, and $d(x, -10) \ge 6$, so $|x + 10| \ge 6$.

P.5.75.

$\left(\sqrt{605}\right)^2 = 605$

P.5.76.

$\left(\sqrt{45}\right)^2 = 45$

P.5.77.

$\sqrt{25 + 20} = \sqrt{45} = \sqrt{9 \cdot 5} = 3\sqrt{5}$

P.5.78.

$\sqrt{284 + 321} = \sqrt{605} = \sqrt{5 \cdot 11^2} = 11\sqrt{5}$

P.5.79.

Because $20^2 + 21^2 = 841 = 29^2$, the triangle is a right triangle.

P.5.80.

Because $11^2 + \left(2\sqrt{26}\right)^2 = 225 = 15^2$, he triangle is a right triangle.

Section P.6 The Rectangular Coordinate System and Graphs

P.6.1.

Cartesian

P.6.2.

Midpoint Formula

P.6.3.

The x-axis is the horizontal real number line. Matches (c).

P.6.4.

The y-axis is the vertical real number line. Matches (f).

P.6.5.

The origin is the point of intersection of the vertical and horizontal axes. Matches (a).

P.6.6.

The quadrants are four regions of the coordinate plane. Matches (d).

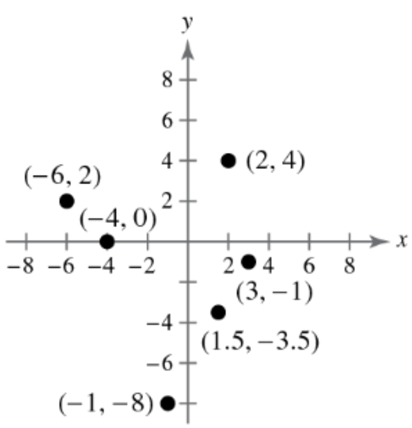
P.6.7.

An x-coordinate is the directed distance from the y-axis. Matches (e).

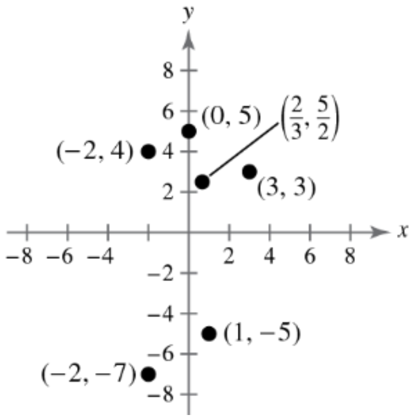
P.6.8.

A y-coordinate is the directed distance from the x-axis. Matches (b).

P.6.9.



P.6.10.



P.6.11.

$(-3, 4)$

P.6.12.

$(-12, 0)$

P.6.13.

$x > 0$ and $y < 0$ in Quadrant IV.

P.6.14.

$x < 0$ and $y < 0$ in Quadrant III.

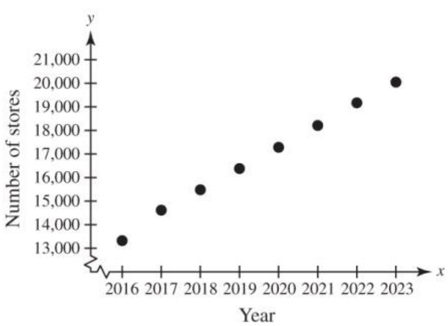
P.6.15.

$x + y = 0, x \neq 0, y \neq 0$ means $x = -y$ or $y = -x$. This occurs in Quadrant II or IV.

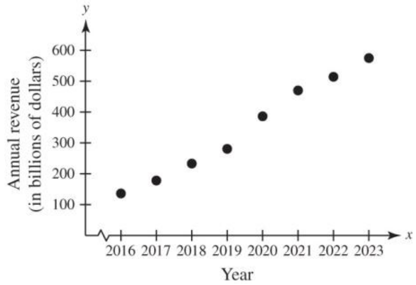
P.6.16.

$(x, y), xy > 0$ means x and y have the same signs. This occurs in Quadrant I or III.

P.6.17.



P.6.18.



P.6.19.

$$\begin{aligned} d &= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \\ &= \sqrt{\left(3 - (-2)\right)^2 + \left(-6 - 6\right)^2} \\ &= \sqrt{(5)^2 + (-12)^2} \\ &= \sqrt{25 + 144} \\ &= 13 \text{ units} \end{aligned}$$

P.6.20.

$$\begin{aligned} d &= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \\ &= \sqrt{(0 - 8)^2 + (20 - 5)^2} \\ &= \sqrt{(-8)^2 + (15)^2} \\ &= \sqrt{64 + 225} \\ &= \sqrt{289} \\ &= 17 \text{ units} \end{aligned}$$

P.6.21.

$$\begin{aligned} d &= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2} \\ &= \sqrt{\left(2 - \frac{1}{2}\right)^2 + \left(-1 - \frac{4}{3}\right)^2} \\ &= \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{7}{3}\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{49}{9}} \\ &= \sqrt{\frac{277}{36}} \\ &= \frac{\sqrt{277}}{6} \text{ units} \end{aligned}$$

P.6.22.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3.9 - 9.5)^2 + (8.2 - (-2.6))^2} \\ &= \sqrt{(-13.4)^2 + (10.8)^2} \\ &= \sqrt{179.56 + 116.64} \\ &= \sqrt{296.2} \end{aligned}$$

P.6.23.

(a) $(1, 0), (13, 5)$

$$\text{Distance} = \sqrt{(13-1)^2 + (5-0)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

$(13, 5), (13, 0)$

$$\text{Distance} = |5 - 0| = |5| = 5$$

$(1, 0), (13, 0)$

$$\text{Distance} = |1 - 13| = |-12| = 12$$

(b) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$

P.6.24.

(a) The distance between $(-1, 1)$ and $(9, 1)$ is 10.

The distance between $(9, 1)$ and $(9, 4)$ is 3.

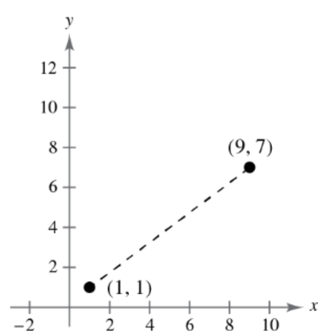
The distance between $(-1, 1)$ and $(9, 4)$ is

$$\sqrt{(9 - (-1))^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}.$$

(b) $10^2 + 3^2 = 109 = \left(\sqrt{109}\right)^2$

P.6.25.

(a)

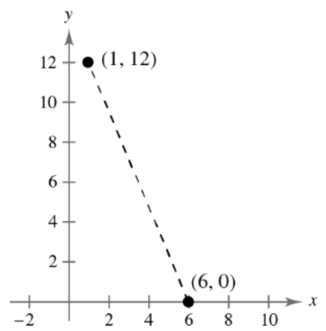


(b) $d = \sqrt{(9-1)^2 + (7-1)^2} = \sqrt{64+36} = 10$

(c) $\left(\dfrac{9+1}{2}, \dfrac{7+1}{2}\right) = (5, 4)$

P.6.26.

(a)

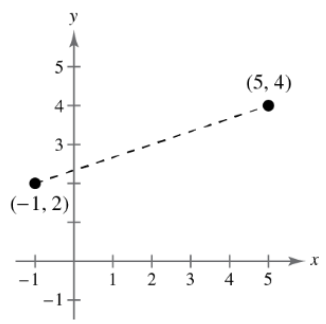


(b) $d = \sqrt{(1-6)^2 + (12-0)^2} = \sqrt{25+144} = 13$

(c) $\left(\dfrac{1+6}{2}, \dfrac{12+0}{2}\right) = \left(\dfrac{7}{2}, 6\right)$

P.6.27.

(a)

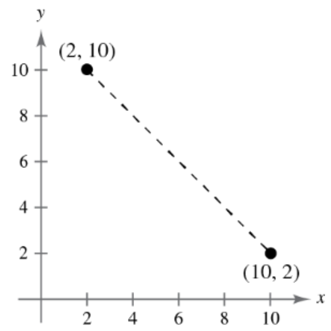


(b) $d = \sqrt{(5+1)^2 + (4-2)^2} + = \sqrt{36+4} = 2\sqrt{10}$

(c) $\left(\dfrac{-1+5}{2}, \dfrac{2+4}{2}\right) = (2, 3)$

P.6.28.

(a)



(b) $d = \sqrt{(2-10)^2 + (10-2)^2}$

$= \sqrt{64+64} = 8\sqrt{2}$

(c) $\left(\dfrac{2+10}{2}, \dfrac{10+2}{2}\right) = (6, 6)$

P.6.29.

$$\begin{aligned} d &= \sqrt{(42-18)^2 + (50-12)^2} \\ &= \sqrt{24^2 + 38^2} \\ &= \sqrt{2020} \\ &= 2\sqrt{505} \\ &\approx 45 \end{aligned}$$

The pass is about 45 yards.

P.6.30.

$$\begin{aligned} d &= \sqrt{120^2 + 150^2} \\ &= \sqrt{36,900} \\ &= 30\sqrt{41} \\ &\approx 192.09 \end{aligned}$$

The plane flies about 192 kilometers.

P.6.31.

$$\begin{aligned} \text{midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{2021 + 2023}{2}, \frac{572.8 + 648.1}{2}\right) \\ &= (2022, 610.45) \end{aligned}$$

In 2022, the sales were about \$610.45 billion.

P.6.32.

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

In this problem, you have $\left(\frac{2022 + 2024}{2}, \frac{8.59 + y_2}{2}\right) = (2023, 14.87)$. So,

$$\begin{aligned} \frac{8.59 + y_2}{2} &= 14.87 \\ 8.59 + y_2 &= 29.74 \\ y_2 &= 21.15. \end{aligned}$$

In 2024, the earnings per share were about \$21.15.

P.6.33.

$$\begin{aligned} (-2 + 2,\ -4 + 5) &= (0, 1) \\ (2 + 2,\ -3 + 5) &= (4, 2) \\ (-1 + 2,\ -1 + 5) &= (1, 4) \end{aligned}$$

P.6.34.

$$\begin{aligned} (-3 + 6, 6 - 3) &= (3, 3) \\ (-5 + 6, 3 - 3) &= (1, 0) \\ (-3 + 6, 0 - 3) &= (3, -3) \\ (-1 + 6, 3 - 3) &= (5, 0) \end{aligned}$$

P.6.35.

True. Because $x < 0$ and $y > 0$, $2x < 0$ and $-3y < 0$, which is located in Quadrant III.

P.6.36.

False. The Midpoint Formula would be used 15 times.

P.6.37.

True. Two sides of the triangle have lengths $\sqrt{149}$ and the third side has a length of $\sqrt{18}$.

P.6.38.

False. The polygon could be a rhombus. For example, consider the points $(4, 0)$, $(0, 6)$, $(-4, 0)$, and $(0, -6)$.

P.6.39.

The y-coordinate of a point on the x-axis is 0. The x-coordinates of a point on the y-axis is 0.

P.6.40.

Because $x_m = \dfrac{x_1 + x_2}{2}$ and $y_m = \dfrac{y_1 + y_2}{2}$ we have:

$$2x_m = x_1 + x_2 \qquad 2y_m = y_1 + y_2$$

$$2x_m - x_1 = x_2 \qquad 2y_m - y_1 = y_2$$

So, $(x_2, y_2) = (2x_m - x_1,\ 2y_m - y_1)$.

P.6.41.

The midpoint of the given line segment is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

The midpoint between (x_1, y_1) and $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is

$$\left(\frac{x_1 + \dfrac{x_1 + x_2}{2}}{2}, \frac{y_1 + \dfrac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right).$$

The midpoint between $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ and (x_2, y_2) is

$$\left(\frac{\dfrac{x_1 + x_2}{2} + x_2}{2}, \frac{\dfrac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

So, the three points are $\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right)$, $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, and

$$\left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

P.6.42.

(a) Because (x_0, y_0) lies in Quadrant II, $(x_0, -y_0)$ must lie in Quadrant III. Matches (ii).

(b) Because (x_0, y_0) lies in Quadrant II, $(-2x_0, y_0)$ must lie in Quadrant I. Matches (iii).

(c) Because (x_0, y_0) lies in Quadrant II, $\left(x_0, \frac{1}{2}y_0\right)$ must lie in Quadrant II. Matches (iv).

(d) Because (x_0, y_0) lies in Quadrant II, $(-x_0, -y_0)$ must lie in Quadrant IV. Matches (i).

P.6.43.

(a) **First Set**

$$d(A, B) = \sqrt{(2-2)^2 + (3-6)^2} = \sqrt{9} = 3$$
$$d(B, C) = \sqrt{(2-6)^2 + (6-3)^2} = \sqrt{16+9} = 5$$
$$d(A, C) = \sqrt{(2-6)^2 + (3-3)^2} = \sqrt{16} = 4$$

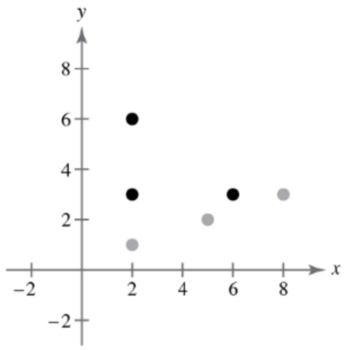
Because $3^2 + 4^2 = 5^2$, A, B, and C are the vertices of a right triangle.

Second Set

$$d(A, B) = \sqrt{(8-5)^2 + (3-2)^2} = \sqrt{10}$$
$$d(B, C) = \sqrt{(5-2)^2 + (2-1)^2} = \sqrt{10}$$
$$d(A, C) = \sqrt{(8-2)^2 + (3-1)^2} = \sqrt{40}$$

A, B, and C are the vertices of an isosceles triangle or are collinear: $\sqrt{10} + \sqrt{10} = 2\sqrt{10} = \sqrt{40}$.

(b)



First set: Not collinear
Second set: Collinear

(c) A set of three points is collinear when the sum of two distances among the points is exactly equal to the third distance.

P.6.44.

(a) distance $= \sqrt{(x_1 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(y_2 - y_1)^2} = |y_2 - y_1|$

(b) distance $= \sqrt{(x_2 - x_1)^2 + (y_1 - y_1)^2}$
$= \sqrt{(x_2 - x_1)^2} = |x_2 - x_1|$

Answers will vary.

P.6.45.

$4x - 6$

(a) $4(-1) - 6 = -4 - 6 = -10$

(b) $4(0) - 6 = 0 - 6 = -6$

P.6.46.

$9 - 7x$

(a) $9 - 7(-3) = 9 + 21 = 30$

(b) $9 - 7(3) = 9 - 21 = -12$

P.6.47.

(a) When $x = -3,\ 2x^3 = 2(-3)^3 = 2(-27) = -54$.

(b) When $x = 0,\ 2x^3 = 2(0)^3 = 0$.

P.6.48.

(a) When $x = 0,\ -3x^{-4} = \dfrac{-3}{x^4} = -\dfrac{3}{0}$. Division by 0 is undefined.

(b) When $x = -2,\ -3x^{-4} = \dfrac{-3}{x^4} = \dfrac{-3}{(-2)^4} = -\dfrac{3}{16}$.

P.6.49.

$(-x)^2 - 2 = x^2 - 2$

P.6.50.

$(-x)^3 - (-x)^2 + 2 = -x^3 - x^2 + 2$

P.6.51.

$-x^2 + (-x)^2 = -x^2 + x^2 = 0$

P.6.52.

$$\begin{aligned} -x^3 + (-x)^3 + (-x)^4 - x^4 &= -x^3 - x^3 + x^4 - x^4 \\ &= -2x^3 \end{aligned}$$

P.6.53.

(a) $$\begin{aligned} P &= R - C \\ &= 135x - (93x + 35{,}000) \\ &= 42x - 35{,}000 \end{aligned}$$

(b) $P = 42(5000) - 35{,}000 = \$175{,}000$

P.6.54.

(a) Time to copy one page: $\frac{1}{50}$ min

(b) Time to copy x pages: $\frac{x}{50}$ min

(c) Time to copy 120 pages: $\frac{120}{50} = \frac{12}{5} = 2.4$ min

(d) Time to copy 10,000 pages: $\frac{10{,}000}{50} = 200$ min

REVIEW EXERCISES FOR CHAPTER P

0P.R.1.

$\left\{11, -14, \ -\frac{8}{9}, \ \frac{5}{2}, \ \sqrt{6}, \ 0.4\right\}$

(a) Natural numbers: 11

(b) Whole numbers: 11

(c) Integers: $11, -14$

(d) Rational numbers: $11, -14, -\frac{8}{9}, \ \frac{5}{2}, \ 0.4$

(e) Irrational numbers: $\sqrt{6}$

0P.R.2.

$\left\{\sqrt{15}, -22, -\frac{10}{3}, \ 0, \ 5.2, \ \frac{3}{7}\right\}$

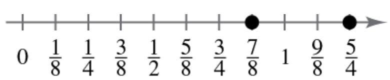
(a) Natural numbers: none

(b) Whole numbers: 0

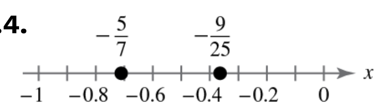
(c) Integers: $-22, \ \ 0$

(d) Rational numbers: $-22, -\frac{10}{3}, \ 0, \ 5.2, \ \frac{3}{7}$

(e) Irrational numbers: $\sqrt{15}$

0P.R.3. 

$$\frac{5}{4} > \frac{7}{8}$$

0P.R.4. 

$$-\frac{9}{25} > -\frac{5}{7}$$

0P.R.5.

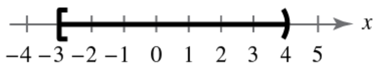
$x \geq 6$ denotes the set of all real numbers greater than or equal to 6.

0P.R.6.

$-4 < x < 4$ denotes the set of all real numbers greater than -4 and less than 4.

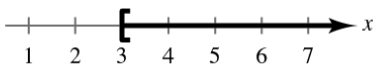
0P.R.7.

The interval $[-3,\ 4)$ denotes the set of all real numbers greater than or equal to -3 and less than 4; $-3 \le x < 4$.



0P.R.8.

The interval $[3,\ \infty)$ denotes the set of all real numbers greater than or equal to 3; $x \ge 3$.



0P.R.9.

$$|\,5\,| = 5$$

0P.R.10.

$$|-16| = 16$$

0P.R.11.

$$|\,8-23\,| = |-15| = 15$$

0P.R.12.

$$|\,14-7\,| = |\,7\,| = 7$$

0P.R.13.

$$-|9| - |-9| = -9 - 9 = -18$$

0P.R.14.

$$|-10| - |-11| = 10 - 11 = -1$$

0P.R.15.

$$d(-74,\ 48) = |48-(-74)| = 122$$

0P.R.16.

$$d(-112,\ -6) = |-6-(-112)| = 106$$

0P.R.17.

$12x - 7$

(a) $12(0) - 7 = -7$

(b) $12(-1)-7=-19$

0P.R.18.

x^2-6x+5

(a) $(-2)^2-6(-2)+5=21$

(b) $(2)^2-6(2)+5=-3$

0P.R.19.

$-x^2+x-1$

(a) $-(1)^2+1-1=-1$

(b) $-(-1)^2+(-1)-1=-3$

0P.R.20.

$\dfrac{x}{x-3}$

(a) $\dfrac{-3}{-3-3}=\dfrac{1}{2}$

(b) $\dfrac{3}{3-3}$ is undefined.

0P.R.21.

$0+(a-5)=a-5$

Illustrates the Additive Identity Property

0P.R.22.

$1\cdot(3x+4)=3x+4$

Illustrates the Multiplicative Identity Property

0P.R.23.

$2x+(3x-10)=(2x+3x)-10$

Illustrates the Associative Property of Addition

0P.R.24.

$4(t+2)=4\cdot t+4\cdot 2$

Illustrates the Distributive Property

0P.R.25.

$$\left(t^2+1\right)+3=3+\left(t^2+1\right)$$

Illustrates the Commutative Property of Addition

0P.R.26.

$$\frac{2}{y+4}\cdot\frac{y+4}{2}=1, \quad y\neq -4$$

Illustrates the Multiplicative Inverse Property

0P.R.27.

$$-6+6=0$$

0P.R.28.

$$(-8)(-4)=32$$

0P.R.29.

$$(-8)(-4)=32$$

0P.R.30.

$$5(20+7)=5(27)=135$$

0P.R.31.

$$\frac{x}{5}+\frac{7x}{12}=\frac{12x}{60}+\frac{35x}{60}=\frac{47x}{60}$$

0P.R.32.

$$\frac{x}{2}-\frac{2x}{5}=\frac{5x}{10}-\frac{4x}{10}=\frac{x}{10}$$

0P.R.33.

$$\frac{3x}{10}\cdot\frac{5}{3}=\frac{x}{2}\cdot\frac{1}{1}=\frac{x}{2}$$

0P.R.34.

$$\frac{9}{x}\div\frac{1}{6}=\frac{9}{x}\cdot\frac{6}{1}=\frac{54}{x}$$

0P.R.35.

$$3x^2\left(4x^3\right)^3=3x^2\left(64x^9\right)=192x^{11}$$

0P.R.36.

$$(3a)^2(6a^3) = 9a^2(6a^3) = 54a^5$$

0P.R.37.

$$(-2z)^3 = -8z^3$$

0P.R.38.

$$\left[(x+2)^2\right]^3 = (x+2)^6$$

0P.R.39.

$$\frac{5y^6}{10y} = \frac{y^{6-1}}{2} = \frac{y^5}{2}, \quad y \neq 0$$

0P.R.40.

$$\frac{36x^5}{9x^{10}} = \frac{4 \cdot 9 \cdot x^5}{9 \cdot x^5 \cdot x^5} = \frac{4}{x^5}$$

0P.R.41.

$$\frac{(8y)^0}{y^2} = \frac{1}{y^2}$$

0P.R.42.

$$\frac{40(b-3)^5}{75(b-3)^2} = \frac{8}{15}(b-3)^{5-2} = \frac{8}{15}(b-3)^3, \quad b \neq 3$$

0P.R.43.

$$\frac{a^2}{b^{-2}} = a^2b^2$$

0P.R.44.

$$(a^2b^4)(3ab^{-2}) = 3a^{2+1}b^{4-2} = 3a^3b^2$$

0P.R.45.

$$\frac{6^2u^3v^{-3}}{12u^{-2}v} = \frac{36u^{3-(-2)}v^{-3-1}}{12} = 3u^5v^{-4} = \frac{3u^5}{v^4}$$

0P.R.46.

$$\frac{3^{-4}m^{-1}n^{-3}}{9^{-2}mn^{-3}} = \frac{9^2n^3}{3^4mmn^3} = \frac{81}{81m^2} = \frac{1}{m^2}$$

0P.R.47.

$$\frac{(5a)^{-2}}{(5a)^2} = (5a)^{-2-2} = (5a)^{-4} = \frac{1}{(5a)^4} = \frac{1}{625a^4}$$

0P.R.48.

$$\frac{4\left(x^{-1}\right)^{-3}}{4^{-2}\left(x^{-1}\right)^{-1}} = \frac{4^{1-(-2)}x^3}{x} = 4^3x^{3-1} = 64x^2$$

0P.R.49.

$$\left(x+y^{-1}\right)^{-1} = \frac{1}{x+y^{-1}} = \frac{1}{x+\frac{1}{y}} = \frac{1}{\frac{xy}{y}+\frac{1}{y}} = \frac{1}{\frac{xy+1}{y}} = 1\cdot\frac{y}{xy+1} = \frac{y}{xy+1}$$

0P.R.50.

$$\left(\frac{x^{-3}}{y}\right)\left(\frac{x}{y}\right)^{-1} = \left(\frac{1}{x^3y}\right)\left(\frac{y}{x}\right) = \frac{1}{x^4}, \quad y \neq 0$$

0P.R.51.

$$274,400,000 = 2.744 \times 10^8$$

0P.R.52.

$$0.3048 = 3.048 \times 10^{-1}$$

0P.R.53.

$$4.84 \times 10^8 = 484,000,000$$

0P.R.54.

$$2.74 \times 10^{-3} = 0.00274$$

0P.R.55.

$$\sqrt[3]{27^2} = \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$$

0P.R.56.

$$\sqrt{49^3} = \left(\sqrt{49}\right)^3 = (7)^3 = 343$$

0P.R.57.

$$\sqrt[3]{\frac{64}{125}} = \sqrt[3]{\frac{4^3}{5^3}} = \frac{4}{5}$$

0P.R.58.

$$\sqrt{\frac{81}{100}} = \sqrt{\frac{9^2}{10^2}} = \frac{9}{10}$$

0P.R.59.

$$\left(\sqrt[3]{216}\right)^3 = \left(\sqrt[3]{6^3}\right)^3 = (6)^3 = 216$$

0P.R.60.

$$\sqrt[4]{32^4} = \left(\sqrt[4]{32}\right)^4 = 32$$

0P.R.61.

$$\sqrt[3]{\frac{2x^3}{27}} = \sqrt[3]{\frac{2x^3}{3^3}} = \frac{x}{3} \quad \sqrt[3]{2}$$

0P.R.62.

$$\sqrt[5]{64x^6} = \sqrt[5]{(2x)^6} = 2x\sqrt[5]{2x}$$

0P.R.63.

$$\begin{aligned}\sqrt{12x^3} + \sqrt{3x} &= \sqrt{4x^2 \cdot 3x} + \sqrt{3x}\\ &= 2x\sqrt{3x} + \sqrt{3x}\\ &= (2x+1)\sqrt{3x}\end{aligned}$$

0P.R.64.

$$\begin{aligned}\sqrt{27x^3} - \sqrt{3x^3} &= \sqrt{9x^2 \cdot 3x} - \sqrt{x^2 \cdot 3x}\\ &= 3x\sqrt{3x} - x\sqrt{3x}\\ &= (3x - x)\sqrt{3x}\\ &= 2x\sqrt{3x}\end{aligned}$$

0P.R.65.

$$\sqrt{8x^3} + \sqrt{2x} = 2x\sqrt{2x} + \sqrt{2x} = (2x+1)\sqrt{2x}$$

0P.R.66.

$$\begin{aligned}\sqrt{18x^5}-\sqrt{8x^3}&=3x^2\sqrt{2x}-2x\sqrt{2x}\\&=\left(3x^2-2x\right)\sqrt{2x}\\&=x\sqrt{2x}\left(3x-2\right)\end{aligned}$$

0P.R.67.

$$\frac{3}{4\sqrt{3}}=\frac{3}{4\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{4\left(3\right)}=\frac{\sqrt{3}}{4}$$

0P.R.68.

$$\frac{12}{\sqrt[3]{4}}=\frac{12}{\sqrt[3]{4}}\cdot\frac{\sqrt[3]{4^2}}{\sqrt[3]{4^2}}=\frac{12\sqrt[3]{16}}{4}=3\sqrt[3]{16}=3\sqrt[3]{8\cdot 2}=6\sqrt[3]{2}$$

0P.R.69.

$$\frac{1}{2-\sqrt{3}}=\frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}=\frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}=\frac{2+\sqrt{3}}{4-3}=\frac{2+\sqrt{3}}{1}=2+\sqrt{3}$$

0P.R.70.

$$\frac{1}{\sqrt{5}+1}=\frac{1}{\sqrt{5}+1}\cdot\frac{\sqrt{5}-1}{\sqrt{5}-1}=\frac{\sqrt{5}-1}{5-1}=\frac{\sqrt{5}-1}{4}$$

0P.R.71.

$$\frac{\sqrt{7}+1}{2}=\frac{\sqrt{7}+1}{2}\cdot\frac{\sqrt{7}-1}{\sqrt{7}-1}=\frac{\left(\sqrt{7}\right)^2-1^2}{2\left(\sqrt{7}-1\right)}=\frac{7-1}{2\left(\sqrt{7}-1\right)}=\frac{6}{2\left(\sqrt{7}-1\right)}=\frac{3}{\sqrt{7}-1}$$

0P.R.72.

$$\frac{\sqrt{2}-\sqrt{11}}{3}=\frac{\sqrt{2}-\sqrt{11}}{3}\cdot\frac{\sqrt{2}+\sqrt{11}}{\sqrt{2}+\sqrt{11}}=\frac{2-11}{\sqrt{2}+\sqrt{11}}=-\frac{9}{\left(\sqrt{2}+\sqrt{11}\right)}=-\frac{3}{\sqrt{2}+\sqrt{11}}$$

0P.R.73.

$$16^{3/2}=\sqrt{16^3}=\left(\sqrt{16}\right)^3=\left(4\right)^3=64$$

0P.R.74.

$$\left(\frac{64}{125}\right)^{-2/3} = \left(\frac{125}{64}\right)^{2/3} = \left[\left(\frac{125}{64}\right)^{1/3}\right]^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$$

0P.R.75.

$$\sqrt{\left(6x^2\right)^2} = 6x^2$$

0P.R.76.

$$\sqrt{\sqrt{128}} = \sqrt{\sqrt{2^7}} = 2^{7/4} = 2\left(2^{3/4}\right) = 2\sqrt[4]{8}$$

0P.R.77.

Standard form: $-11x^2 + 3$

Degree: 2

Leading coefficient: -11

0P.R.78.

Standard form: $-5x^5 + 3x^3 + x - 4$

Degree: 5

Leading coefficient: -5

0P.R.79.

Standard form: $-12x^2 - 4$

Degree: 2

Leading coefficient: -12

0P.R.80.

Standard form: $-7x^2 + 12x + 6$

Degree: 2

Leading coefficient: -7

0P.R.81.

$$\begin{aligned} -\left(3x^2 + 2x\right) + \left(1 - 5x\right) &= -3x^2 - 2x + 1 - 5x \\ &= -3x^2 - 7x + 1 \end{aligned}$$

0P.R.82.

$$\begin{aligned} 8y - \left[2y^2 - \left(3y - 8\right)\right] &= 8y - 2y^2 + \left(3y - 8\right) \\ &= -2y^2 + 11y - 8 \end{aligned}$$

0P.R.83.

$$\begin{aligned} 2x\left(x^2 - 5x + 6\right) &= \left(2x\right)\left(x^2\right) + \left(2x\right)\left(-5x\right) + \left(2x\right)\left(6\right) \\ &= 2x^3 - 10x^2 + 12x \end{aligned}$$

0P.R.84.

$$\left(3x^3 - 1.5x^2 + 4\right)\left(-3x\right) = \left(3x\right)\left(-3x\right) - 1.5x^2\left(-3x\right) + 4\left(-3x\right) = -9x^4 + 4.5x^3 - 12x$$

0P.R.85.

$$\begin{aligned} \left(3x - 6\right)\left(5x + 1\right) &= 15x^2 + 3x - 30x - 6 \\ &= 15x^2 - 27x - 6 \end{aligned}$$

0P.R.86.

$$\begin{aligned} (x + 2)(x^2 - 2) &= x^3 - 2x + 2x^2 - 4 \\ &= x^3 + 2x^2 - 2x - 4 \end{aligned}$$

0P.R.87.

$$\left(6x + 5\right)\left(6x - 5\right) = \left(6x\right)^2 - 5^2 = 36x^2 - 25$$

0P.R.88.

$$\begin{aligned} \left(3\sqrt{5} + 2x\right)\left(3\sqrt{5} - 2x\right) &= \left(3\sqrt{5}\right)^2 - \left(2x\right)^2 \\ &= 45 - 4x^2 \end{aligned}$$

0P.R.89.

$$\begin{aligned} \left(2x - 3\right)^2 &= \left(2x\right)^2 - 2\left(2x\right)\left(3\right) + 3^3 \\ &= 4x^2 - 12x + 9 \end{aligned}$$

0P.R.90.

$$\begin{aligned} \left(x - 4\right)^3 &= x^3 - 3x^2\left(4\right) + 3x\left(4\right)^2 - 4^3 \\ &= x^3 - 12x^2 + 48x - 64 \end{aligned}$$

0P.R.91.

$$\begin{array}{rrrrrrrrr} & & & & x^2 & + & x & + & 5 \\ & & \times & & x^2 & - & 7x & - & 2 \\ \hline & & & - & 2x^2 & - & 2x & - & 10 \\ & - & 7x^3 & - & 7x^2 & - & 35x & & \\ x^4 & + & x^3 & + & 5x^2 & & & & \\ \hline x^4 & - & 6x^3 & - & 4x^2 & - & 37x & - & 10 \end{array}$$

0P.R.92.

$$\begin{aligned}\left(6x^4-20x^2-x+3\right)-\left(9x^4-11x^2+16\right)&=6x^4-20x^2-x+3-9x^4+11x^2-16\\&=\left(6x^4-9x^4\right)+\left(-20x^2+11x^2\right)-x+(3-16)\\&=-3x^4-9x^2-x-13\end{aligned}$$

0P.R.93.

$$\begin{aligned}\text{Area}&=\left(x+12\right)\left(x+16\right)\\&=x^2+16x+12x+192\\&=x^2+28x+192\text{ square feet}\end{aligned}$$

0P.R.94.

$$\begin{aligned}\left(x+5\right)\left(x+3\right)&=x^2+3x+5x+15\\&=x^2+8x+15\end{aligned}$$

This illustrates the Distributive Property.

0P.R.95.

$$x^3-x=x\left(x^2-1\right)=x\left(x+1\right)\left(x-1\right)$$

0P.R.96.

$$x\left(x-3\right)+4\left(x-3\right)=\left(x-3\right)\left(x+4\right)$$

0P.R.97.

$$25x^2-49=\left(5x\right)^2-7^2=\left(5x+7\right)\left(5x-7\right)$$

0P.R.98.

$$\begin{aligned}36x^2-81&=9\left(4x^2-9\right)\\&=9\left[\left(2x\right)^2-3^2\right]\\&=9\left(2x+3\right)\left(2x-3\right)\end{aligned}$$

0P.R.99.

$$x^3-64=x^3-4^3=\left(x-4\right)\left(x^2+4x+16\right)$$

0P.R.100.

$$8x^3+27=\left(2x+3\right)\left(4x^2-6x+9\right)$$

0P.R.101.

$$2x^2 + 21x + 10 = (2x + 1)(x + 10)$$

0P.R.102.

$$3x^2 + 14x + 8 = (x + 4)(3x + 2)$$

0P.R.103.

$$x^3 - x^2 - 12x = x(x^2 - x - 12) = x(x - 4)(x + 3)$$

0P.R.104.

$$\begin{aligned} 4x^4 - 6x^3 - 10x^2 &= 2x^2\left(2x^2 - 3x - 5\right) \\ &= 2x^2\left(2x - 5\right)\left(x + 1\right) \end{aligned}$$

0P.R.105.

$$\begin{aligned} x^3 + x^2 - 2x - 2 &= x^2\left(x + 1\right) - 2\left(x + 1\right) \\ &= \left(x + 1\right)\left(x^2 - 2\right) \end{aligned}$$

0P.R.106.

$$\begin{aligned} x^3 - 4x^2 + 2x - 8 &= x^2\left(x - 4\right) + 2\left(x - 4\right) \\ &= \left(x^2 + 2\right)\left(x - 4\right) \end{aligned}$$

0P.R.107.

The domain of $\dfrac{1}{x+1}$ is the set of all real numbers x such that $x \neq -1$.

0P.R.108.

The domain of $\dfrac{1}{x^2 + 3x - 18} = \dfrac{1}{\left(x + 6\right)\left(x - 3\right)}$ is all real numbers x such that $x \neq -6,\ 3$.

0P.R.109.

The domain of $\sqrt{x + 2}$ is the set of all real numbers x such that $x \geq -2$.

0P.R.110.

$$5x - 4 \geq 0 \Rightarrow x \geq \tfrac{4}{5}$$

The domain of $\sqrt{5x - 4}$ is all real numbers x such that $x \geq \tfrac{4}{5}$.

0P.R.111.

$$\frac{x^2 - 64}{5\left(3x + 24\right)} = \frac{\left(x + 8\right)\left(x - 8\right)}{5 \cdot 3\left(x + 8\right)} = \frac{x - 8}{15}, \qquad x \neq -8$$

0P.R.112.

$$\frac{x^3+27}{x^2+x-6} = \frac{(x+3)(x^2-3x+9)}{(x+3)(x-2)}$$
$$= \frac{x^2-3x+9}{x-2}, \quad x \neq -3$$

0P.R.113.

$$\frac{x^2-7x+12}{x^2+8x+16}\cdot\frac{x+4}{x^2-9} = \frac{(x-3)(x-4)}{(x+4)^2}\cdot\frac{x+4}{(x+3)(x-3)}$$
$$= \frac{x-4}{(x+4)(x+3)}, \; x \neq 3$$

0P.R.114.

$$\frac{2x-6}{3x+6} \div \frac{3-x}{2x+4} = \frac{2x-6}{3x+6}\cdot\frac{2x+4}{3-x} = \frac{2(x-3)}{3(x+2)}\cdot\frac{2(x+2)}{-(x-3)} = -\frac{4}{3}, \; x \neq -2, \; 3$$

0P.R.115.

$$\frac{2}{x-4}+\frac{6}{x^2-16} = \frac{2}{x-4}+\frac{6}{(x+4)(x-4)}$$
$$= \frac{2(x+4)+6}{(x+4)(x-4)}$$
$$= \frac{2x+8+6}{(x+4)(x-4)}$$
$$= \frac{2x+14}{(x+4)(x-4)}$$

0P.R.116.

$$\frac{3x}{x^2-25}-\frac{2}{x^2+x-20} = \frac{3x}{(x-5)(x+5)}-\frac{2}{(x+5)(x-4)}$$
$$= \frac{3x(x-4)-2(x-5)}{(x-5)(x+5)(x-4)}$$
$$= \frac{3x^2-12x-2x+10}{(x-5)(x+5)(x-4)}$$
$$= \frac{3x^2-14x+10}{(x-5)(x+5)(x-4)}$$

0P.R.117.

$$\frac{\left(\frac{3a}{\frac{a^2}{x}-1}\right)}{\left(\frac{a}{x}-1\right)} = \frac{\left(\frac{3a}{\frac{a^2-x}{x}}\right)}{\left(\frac{a-x}{x}\right)} = \frac{3a}{1}\cdot\frac{x}{a^2-x}\cdot\frac{x}{a-x} = \frac{3ax^2}{\left(a^2-x\right)\left(a-x\right)},\ x \neq 0$$

0P.R.118.

$$\frac{\left(\frac{1}{2x-3}-\frac{1}{2x+3}\right)}{\left(\frac{1}{2x}-\frac{1}{2x+3}\right)} = \frac{\left(\frac{2x+3}{(2x-3)(2x+3)}-\frac{2x-3}{(2x-3)(2x+3)}\right)}{\left(\frac{2x+3}{2x(2x+3)}-\frac{2x}{2x(2x+3)}\right)}$$

$$= \frac{\frac{2x+3-2x+3}{(2x-3)(2x+3)}}{\frac{2x+3-2x}{2x(2x+3)}} = \frac{6}{(2x-3)(2x+3)}\cdot\frac{2x(2x+3)}{3} = \frac{4x}{2x-3},\quad x \neq 0,\ -\frac{3}{2}$$

0P.R.119.

$$\frac{\left[\frac{1}{2(x+h)}-\frac{1}{2x}\right]}{h} = \frac{\frac{x-(x+6)}{2x(x+h)}}{h} = \frac{-h}{2x(x+h)}\cdot\frac{1}{h} = \frac{-1}{2x(x+h)},\quad h \neq 0$$

0P.R.120.

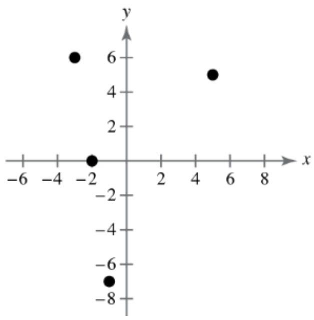
$$\frac{\left(\frac{1}{x+h-3}-\frac{1}{x-3}\right)}{h} = \frac{\frac{x-3}{(x-3)(x+h-3)}-\frac{x+h-3}{(x-3)(x+h-3)}}{\frac{h}{1}}$$

$$= \frac{x-3-x-h+3}{(x-3)(x+h-3)}\cdot\frac{1}{h}$$

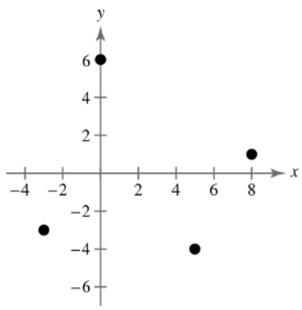
$$= \frac{-h}{h(x-3)(x+h-3)}$$

$$= \frac{-1}{(x-3)(x+h-3)},\quad h \neq 0$$

0P.R.121.



0P.R.122.



0P.R.123.

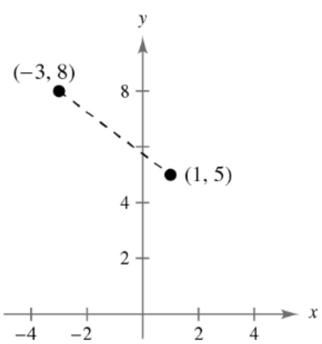
$x > 0$ and $y = -2$ in Quadrant IV.

0P.R.124.

$xy = 4$ means x and y have the same signs. This occurs in Quadrants I and III.

0P.R.125.

(a)

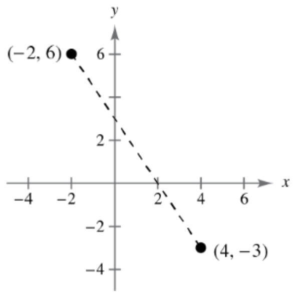


(b) $d = \sqrt{(-3-1)^2 + (8-5)^2} = \sqrt{16+9} = 5$

(c) Midpoint: $\left(\dfrac{-3+1}{2}, \dfrac{8+5}{2}\right) = \left(-1, \dfrac{13}{2}\right)$

0P.R.126.

(a)



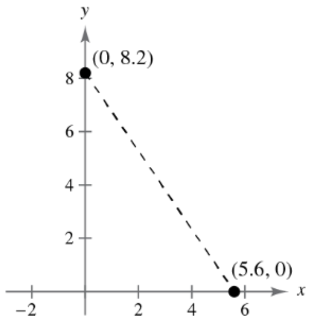
(b) $d = \sqrt{(-2-4)^2 + (6+3)^2}$

$= \sqrt{36+81} = \sqrt{117} = 3\sqrt{13}$

(c) Midpoint: $\left(\dfrac{-2+4}{2}, \dfrac{6-3}{2}\right) = \left(1, \dfrac{3}{2}\right)$

0P.R.127.

(a)



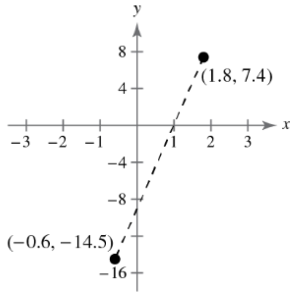
(b) $d = \sqrt{(5.6-0)^2 + (0-8.2)^2}$

$= \sqrt{31.36+67.24} = \sqrt{98.6}$

(c) Midpoint: $\left(\dfrac{0+5.6}{2}, \dfrac{8.2+0}{2}\right) = (2.8, 4.1)$

0P.R.128.

(a)



(b) $d = \sqrt{(1.8-0.6)^2 + (7.4+14.5)^2}$

$= \sqrt{5.76+479.61} = \sqrt{485.37}$

(c) Midpoint: $\left(\frac{1.8-0.6}{2},\ \frac{7.4-14.5}{2}\right)=\left(0.6,\ \ -3.55\right)$

0P.R.129.

$\left(4-4,\ 8-8\right)=\left(0,\ 0\right)$
$\left(6-4,\ 8-8\right)=\left(2,\ 0\right)$
$\left(4-4,\ 3-8\right)=\left(0,\ \ -5\right)$
$\left(6-4,\ 3-8\right)=\left(2,\ \ -5\right)$

0P.R.130.

$\left(0-2,\ 1+3\right)=\left(-2,\ 4\right)$
$\left(3-2,\ 3+3\right)=\left(1,\ 6\right)$
$\left(0-2,\ 5+3\right)=\left(-2,\ \ 8\right)$
$\left(-3-2,\ 3+3\right)=\left(-5,\ \ 6\right)$

0P.R.131.

False. $\left(a+b\right)^2=a^2+2ab+b^2\neq a^2+b^2$

There is also a cross-product term when a binomial sum is squared.

0P.R.132.

False. $x^n-y^n=\left(x-y\right)\left(x^{n-1}+x^{n-2}y+\cdots+y^{n-1}\right)$ for odd values of n, not for all values of n. $x^n-y^n=\left(x^{n/2}+y^{n/2}\right)\left(x^{n/2}-y^{n/2}\right)$ for even values of n.

Problem Solving for Chapter P

0P.PS.1.

(a) Men's

Maximum Volume:

$$V = \frac{4}{3}\pi\left(65\right)^3 \approx 1{,}150{,}347 \text{ mm}^3$$

Minimum Volume:

$$V = \frac{4}{3}\pi\left(55\right)^3 \approx 696{,}910 \text{ mm}^3$$

Women's

Maximum Volume:

$$V = \frac{4}{3}\pi\left(55\right)^3 \approx 696{,}910 \text{ mm}^3$$

Minimum Volume:

$$V = \frac{4}{3}\pi\left(\frac{95}{2}\right)^3 \approx 448{,}921 \text{ mm}^3$$

(b) Men's

Maximum density:

$$\frac{7.26}{696{,}910} \approx 1.04 \times 10^{-5} \text{ kg/mm}^3$$

Minimum density:

$$\frac{7.26}{1{,}150{,}347} \approx 6.31 \times 10^{-6} \text{ kg/mm}^3$$

Women's

Maximum density:

$$\frac{4.00}{448{,}921} \approx 8.91 \times 10^{-6} \text{ kg/mm}^3$$

Minimum density:

$$\frac{4.00}{696{,}910} \approx 5.74 \times 10^{-6} \text{ kg/mm}^3$$

(c) No. The weight would be different. Cork is much lighter than iron so it would have a much smaller density.

0P.PS.2.

Let $a=5$ and $b=-3$. Then $|a-b| = |5-(-3)| = 8$ and $|a| - |b| = |5| - |-3| = 2$.

Thus, $|a-b| > |a| - |b|$.

Let $a=11$ and $b=3$. Then $|a-b| = |11-3| = 8$ and $|a| - |b| = |11| - |3| = 8$.

Thus, $|a-b| = |a| - |b|$.

To prove $|a-b| \ge |a| - |b|$ for all a, b, consider the following cases.

Case 1: $a>0$ and $b<0$

$$|a-b| = |a| + |b| > |a| - |b|$$

Case 2: $a<0$ and $b>0$

$$|a-b| = |a| + |b| > |a| - |b|$$

Case 3: $a>0$ and $b>0$

(i) $a>b$

Then, $|a-b| = |a| - |b|$

(ii) $a<b \Rightarrow |a| < |b|$

Then, $|a-b| = |b| - |a| > |a| - |b|$

Case 4: $a<0$ and $b<0$

(i) $a>b \Rightarrow |b| > |a|$

Then, $|a-b| = |b| - |a| > |a| - |b|$

(ii) $a<b$

Then, $|a-b| = |a| - |b|$

Therefore, $|a-b| \ge |a| - |b|$ for all a, b.

0P.PS.3.

Answers will vary depending on how leap years are handled. One method is to assume that each year has 365.25 days. The number of beats in a year would be

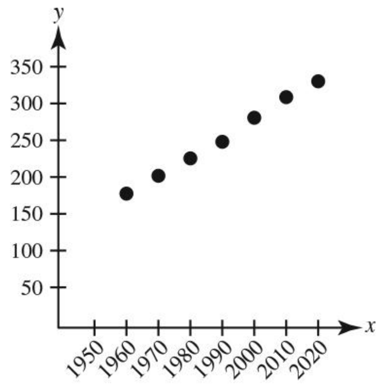
$$\left(365.25\ \frac{\text{days}}{\text{year}}\right)\left(24\ \frac{\text{hours}}{\text{day}}\right)\left(60\ \frac{\text{minutes}}{\text{hour}}\right)\left(70\ \frac{\text{beats}}{\text{minute}}\right) = 36{,}817{,}200 \text{ beats per year.}$$

For a man's average life, there would be $(73.5)(36{,}817{,}200) = 2{,}706{,}064{,}200$ beats.

For a woman's average life, there would be $(79.3)(36{,}817{,}200) = 2{,}919{,}603{,}960$ beats.

0P.PS.4.

(a)



Answers will vary.

(b) 1960–1970 = 203.21 – 179.32 = 23.89 million

1970–1980 = 226.55 – 203.21 = 23.34 million

1980–1990 = 248.71 – 226.55 = 22.19 million

1990–2000 = 281.42 – 248.71 = 32.71 million

2000–2010 = 308.75 – 281.42 = 27.33 million

2010–2020 = 331.45 – 308.75 = 22.70 million

(c) The population increased the most from 1990 to 2000.

The population increased the least from 1980 to 1990.

(d) 1960–1970 = $23.90 \div 179.32$ ≈ 0.133 = 13.3%

1970–1980 = $23.34 \div 203.21$ ≈ 0.115 = 11.5%

1980–1990 = $22.19 \div 226.55$ ≈ 0.098 = 9.8%

1990–2000 = $32.71 \div 248.71$ ≈ 0.132 = 13.2%

2000–2010 = $27.33 \div 281.42$ ≈ 0.097 = 9.7%

2010–2020 = $22.70 \div 308.75$ ≈ 0.074 = 7.4%

(e) Greatest percent: 1960 to 1970

Least percent: 2010 to 2020

0P.PS.5.

$$r = 1 - \left(\frac{3225}{12,000}\right)^{1/4} \approx 0.280 \text{ or } 28\%$$

0P.PS.6.

Planet	Mercury	Venus	Earth	Mars	Jupiter
x	0.387	0.723	1.000	1.524	5.203
$\sqrt{x}$	0.622	0.850	1.000	1.235	2.281
y	0.241	0.615	1.000	1.881	11.862
$\sqrt[3]{y}$	0.622	0.850	1.000	1.234	2.281

The square root of the distance from a planet to the sun is equal to the cube root of the period of the planet.

0P.PS.7.

(a) The distance between (x_1, y_1) and $\dfrac{2x_1 + x_2}{3}, \dfrac{2y_1 + y_2}{3}$ is

$$d = \sqrt{\left(x_1 - \frac{2x_1 + x_2}{3}\right)^2 + \left(y_1 - \frac{2y_1 + y_2}{3}\right)^2}$$

$$= \sqrt{\left(\frac{x_1 - x_2}{3}\right)^2 + \left(y_1 - \frac{2y_1 + y_2}{3}\right)^2}$$

$$= \sqrt{\frac{1}{9}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]}$$

$= \dfrac{1}{3}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ which is $\dfrac{1}{3}$ of the distance between (x_1, y_1) and (x_2, y_2).

The second point of trisection is

$$\left(\frac{\left(\frac{2x_1 + x_2}{3}\right) + x_2}{3}, \frac{\left(\frac{2y_1 + y_2}{3}\right) + y_2}{3}\right) = \left(\frac{x_1 + 2x_2}{3}, \frac{y_1 + 2y_2}{3}\right).$$

(b) (i) $(1, \ -2)$ and $(4, 1)$

The points of trisection are:

$$\left(\frac{2(1) + 4}{3}, \frac{2(-2) + 1}{3}\right) = (2, \ -1)$$

$$\left(\frac{1 + 2(4)}{3}, \frac{-2 + 2(1)}{3}\right) = (3, \ 0)$$

(ii) $(-2,\ -3)$ and $(0,\ 0)$

The points of trisection are:

$$\left(\frac{2(-2)+0}{3},\ \frac{2(-3)+0}{3}\right) = \left(-\frac{4}{3},\ -2\right)$$

$$\left(\frac{-2+2(0)}{3},\ \frac{-3+2(0)}{3}\right) = \left(-\frac{2}{3},\ -1\right)$$

0P.PS.8.

$$y_1 = 2x\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}} \qquad y_2 = \frac{2-3x^2}{\sqrt{1-x^2}}$$

When $x = 0$, $y_1 = 0$. When $x = 0$, $y_2 = 2$.

Thus, $y_1 \neq y_2$.

$$\begin{aligned} y_1 &= \frac{2x\sqrt{1-x^2}}{1} - \frac{x^3}{\sqrt{1-x^2}} \\ &= \frac{2x\sqrt{1-x^2}}{1} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}} - \frac{x^3}{\sqrt{1-x^2}} \\ &= \frac{2x\left(1-x^2\right) - x^3}{\sqrt{1-x^2}} \\ &= \frac{2x - 2x^3 - x^3}{\sqrt{1-x^2}} \\ &= \frac{2x - 3x^3}{\sqrt{1-x^2}} \\ &= \frac{x\left(2-3x^2\right)}{\sqrt{1-x^2}} \end{aligned}$$

Let $y_2 = \dfrac{x\left(2-3x^2\right)}{\sqrt{1-x^2}}$. Then $y_1 = y_2$.

0P.PS.9.

One golf ball: $\dfrac{1.6 \times 10^7}{1.58 \times 10^8} \approx 0.101$ pound

$0.101(16) = 1.616$ ounces

0P.PS.10.

(a) Either graph could be misleading. The scales on the vertical axes make it appear that the rise in profits is either dramatic or small, but the total increase is only 2–6 units.

(b) If the company wanted to gloss over the dip in profits during July, as at a stockholders meeting, the first graph could be used. If the company wished to project an image of rapidly increasing profits, as to potential investors, the second graph could be used.

Solution and Answer Guide

LARSON, ALGEBRA & TRIG, 12E, 2027, 9798214407227;
CHAPTER 1: EQUATIONS, INEQUALITIES, AND MATHEMATICAL MODELING

TABLE OF CONTENTS

Section 1.1 Graphs of Equations

1.1.1.

solution or solution point

1.1.2.

graph

1.1.3.

Three approaches to solve problems mathematically are algebraic, graphical, and numerical.

1.1.4.

Let (x, y) be any point on the circle. The distance between the center (h, k) and (x, y) is the radius r. So, $\sqrt{(x-h)^2+(y-k)^2}=r$.

1.1.5.

(a) $(2, 0): (2)^2 - 3(2) + 2 \stackrel{?}{=} 0$

$$4 - 6 + 2 \stackrel{?}{=} 0$$
$$0 = 0$$

Yes, the point *is* on the graph.

(b) $(-2, 8): (-2)^2 - 3(-2) + 2 \stackrel{?}{=} 8$

$$4 + 6 + 2 \stackrel{?}{=} 8$$
$$12 \neq 8$$

No, the point *is not* on the graph.

1.1.6.

(a) $(0, 2): 2 \stackrel{?}{=} \sqrt{0+4}$

$$2 = 2$$

Yes, the point *is* on the graph.

(b) $(5, 3): 3 \stackrel{?}{=} \sqrt{5+4}$

$$3 \stackrel{?}{=} \sqrt{9}$$
$$3 = 3$$

Yes, the point *is* on the graph.

1.1.7.

(a) $(1, 5): 5 \stackrel{?}{=} 4 - |1 - 2|$

$5 \stackrel{?}{=} 4 - 1$

$5 \neq 3$

No, the point *is not* on the graph.

(b) $(6, 0): 0 \stackrel{?}{=} 4 - |6 - 2|$

$0 \stackrel{?}{=} 4 - 4$

$0 = 0$

Yes, the point *is* on the graph.

1.1.8.

(a) $(6, 0): 2(6)^2 + 5(0)^2 \stackrel{?}{=} 8$

$2(36) + 5(0) \stackrel{?}{=} 8$

$72 + 0 \stackrel{?}{=} 8$

$72 \neq 8$

No, the point *is not* on the graph.

(b) $(0, 4): 2(0)^2 + 5(4)^2 \stackrel{?}{=} 8$

$2(0) + 5(16) \stackrel{?}{=} 8$

$0 + 80 \stackrel{?}{=} 8$

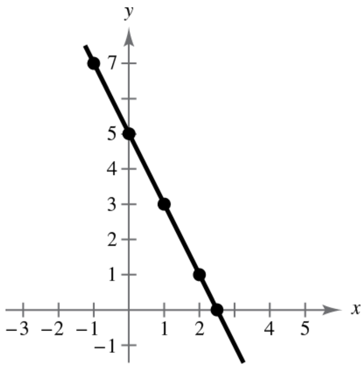
$80 \neq 8$

No, the point *is not* on the graph.

1.1.9.

$y = -2x + 5$

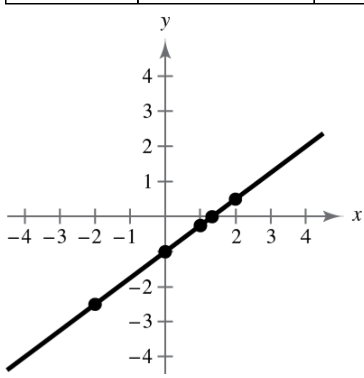
x	-1	0	1	2	$\frac{5}{2}$
y	7	5	3	1	0
(x, y)	$(-1, 7)$	$(0, 5)$	$(1, 3)$	$(2, 1)$	$\left(\frac{5}{2}, 0\right)$



1.1.10.

$y = \frac{3}{4}x - 1$

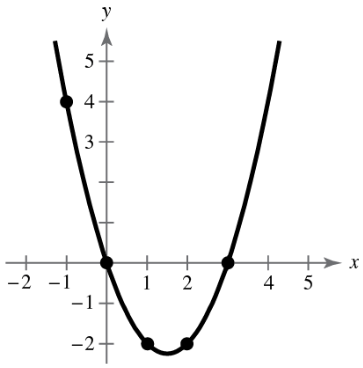
x	-2	0	1	$\frac{4}{3}$	2
y	$-\frac{5}{2}$	-1	$-\frac{1}{4}$	0	$\frac{1}{2}$
(x, y)	$\left(-2, -\frac{5}{2}\right)$	$(0, -1)$	$\left(1, -\frac{1}{4}\right)$	$\left(\frac{4}{3}, 0\right)$	$\left(2, \frac{1}{2}\right)$



1.1.11.

$y = x^2 - 3x$

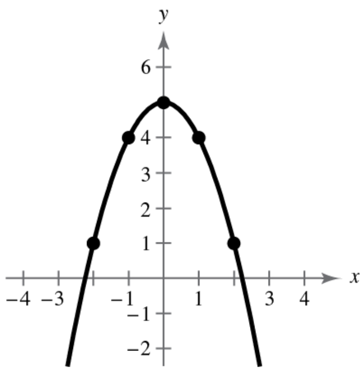
x	-1	0	1	2	3
y	4	0	-2	-2	0
(x, y)	$(-1, 4)$	$(0, 0)$	$(1, -2)$	$(2, -2)$	$(3, 0)$



1.1.12.

$y = 5 - x^2$

x	-2	-1	0	1	2
y	1	4	5	4	1
(x, y)	$(-2, 1)$	$(-1, 4)$	$(0, 5)$	$(1, 4)$	$(2, 1)$



1.1.13.

x-intercept: $(-2, 0)$

y-intercept: $(0, 2)$

1.1.14.

x-intercept: $(4, 0)$

y-intercepts: $(0, \pm 2)$

1.1.15.

x-intercept: $(3, 0)$

y-intercept: $(0, 9)$

1.1.16.

x-intercepts: $(\pm 2, 0), (0, 0)$

y-intercepts: $(0, 16), (0, 0)$

1.1.17.

$x^2 - y = 0$

$(-x)^2 - y = 0 \Rightarrow x^2 - y = 0 \Rightarrow$ y-axis symmetry

$x^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No x-axis symmetry

$(-x)^2 - (-y) = 0 \Rightarrow x^2 + y = 0 \Rightarrow$ No origin symmetry

1.1.18.

$x - y^2 = 0$

$(-x) - y^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No y-axis symmetry

$x - (-y)^2 = 0 \Rightarrow x - y^2 = 0 \Rightarrow$ x-axis symmetry

$(-x) - (-y)^2 = 0 \Rightarrow -x - y^2 = 0 \Rightarrow$ No origin symmetry

1.1.19.

$y = x^3$

$y = (-x)^3 \Rightarrow y = -x^3 \Rightarrow$ No y-axis symmetry

$-y = x^3 \Rightarrow y = -x^3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^3 \Rightarrow -y = -x^3 \Rightarrow y = x^3 \Rightarrow$ Origin symmetry

1.1.20.

$y = x^4 - x^2 + 3$

$y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = x^4 - x^2 + 3 \Rightarrow$ y-axis symmetry

$-y = x^4 - x^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 - (-x)^2 + 3 \Rightarrow y = -x^4 + x^2 - 3 \Rightarrow$ No origin symmetry

1.1.21.

$$y = \frac{x}{x^2 + 1}$$
$$y = \frac{-x}{(-x)^2 + 1} \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \text{No } y\text{-axis symmetry}$$
$$-y = \frac{x}{x^2 + 1} \Rightarrow y = \frac{-x}{x^2 + 1} \Rightarrow \text{No } x\text{-axis symmetry}$$
$$-y = \frac{-x}{(-x)^2 + 1} \Rightarrow -y = \frac{-x}{x^2 + 1} \Rightarrow y = \frac{x}{x^2 + 1} \Rightarrow \text{Origin symmetry}$$

1.1.22.

$$y = \frac{1}{1 + x^2}$$
$$y = \frac{1}{1 + (-x)^2} \Rightarrow y = \frac{1}{1 + x^2} \Rightarrow y\text{-axis symmetry}$$
$$-y = \frac{1}{1 + x^2} \Rightarrow y = \frac{-1}{1 + x^2} \Rightarrow \text{No } x\text{-axis symmetry}$$
$$-y = \frac{1}{1 + (-x)^2} \Rightarrow y = \frac{-1}{1 + x^2} \Rightarrow \text{No origin symmetry}$$

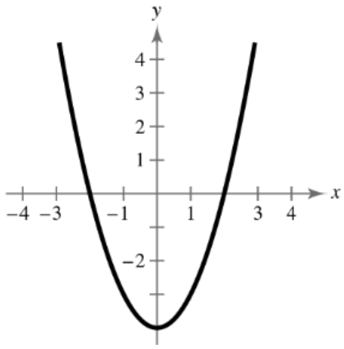
1.1.23.

$$xy^2 + 10 = 0$$
$$(-x)y^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No } y\text{-axis symmetry}$$
$$x(-y)^2 + 10 = 0 \Rightarrow xy^2 + 10 = 0 \Rightarrow x\text{-axis symmetry}$$
$$(-x)(-y)^2 + 10 = 0 \Rightarrow -xy^2 + 10 = 0 \Rightarrow \text{No origin symmetry}$$

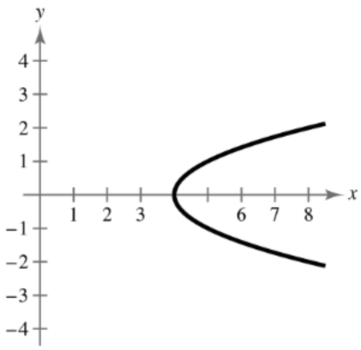
1.1.24.

$$xy = 4$$
$$(-x)y = 4 \Rightarrow xy = -4 \Rightarrow \text{No } y\text{-axis symmetry}$$
$$x(-y) = 4 \Rightarrow xy = -4 \Rightarrow \text{No } x\text{-axis symmetry}$$
$$(-x)(-y) = 4 \Rightarrow xy = 4 \Rightarrow \text{Origin symmetry}$$

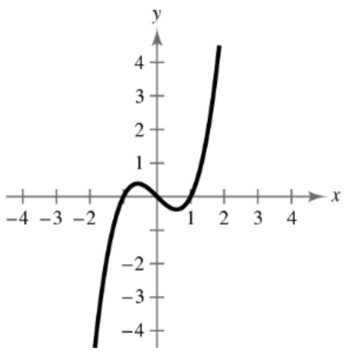
1.1.25.



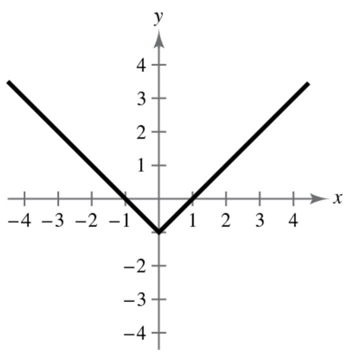
1.1.26.



1.1.27.



1.1.28.



1.1.29.

Center: $(0, 0)$; Radius: 3

$$(x-0)^2+(y-0)^2=3^2$$
$$x^2+y^2=9$$

1.1.30.

Center: $(0, 0)$; Radius: 7

$$(x-0)^2+(y-0)^2=7^2$$
$$x^2+y^2=49$$

1.1.31.

Center: $(-4, 5)$; Radius: 2

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\ [x-(-4)]^2+[y-5]^2&=2^2\\ (x+4)^2+(y-5)^2&=4\end{aligned}$$

1.1.32.

Center: $(1, -3)$; Radius: $\sqrt{11}$

$$\begin{aligned}(x-h)^2+(y-k)^2&=r^2\\ (x-1)^2+[y-(-3)]^2&=\sqrt{11}^2\\ (x-1)^2+(y+3)^2&=11\end{aligned}$$

1.1.33.

Center: $(3, 8)$; Solution point: $(-9, 13)$

$$\begin{aligned}r&=\sqrt{(x-h)^2+(y-k)^2}\\ &=\sqrt{(-9-3)^2+(13-8)^2}\\ &=\sqrt{(-12)^2+(5)^2}\\ &=\sqrt{144+25}\\ &=\sqrt{169}\\ &=13\end{aligned}$$

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-3)^2+(y-8)^2=13^2$$
$$(x-3)^2+(y-8)^2=169$$

1.1.34.

Center: $(-2,-6)$; Solution point: $(1,-10)$

$$\begin{aligned} r &= \sqrt{(x-h)^2+(y-k)^2} \\ &= \sqrt{[1-(-2)]^2+[-10-(-6)]^2} \\ &= \sqrt{(3)^2+(-4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 \\ [x-(-2)]^2+[y-(-6)]^2 &= 5^2 \\ (x+2)^2+(y+6)^2 &= 25 \end{aligned}$$

1.1.35.

Endpoints of a diameter: $(3,2),(-9,-8)$

$$\begin{aligned} r &= \frac{1}{2}\sqrt{(-9-3)^2+(-8-2)^2} \\ &= \frac{1}{2}\sqrt{(-12)^2+(-10)^2} \\ &= \frac{1}{2}\sqrt{144+100} \\ &= \frac{1}{2}\sqrt{244}=\frac{1}{2}(2)\sqrt{61}=\sqrt{61} \end{aligned}$$

$$\begin{aligned} (h,k): \left(\frac{3+(-9)}{2}\cdot\frac{2+(-8)}{2}\right) &= \left(\frac{-6}{2}\cdot\frac{-6}{2}\right) \\ &= (-3,-3) \end{aligned}$$

$$\begin{aligned} (x-h)^2+(y-k)^2 &= r^2 \\ [x-(-3)]^2+[y-(-3)]^2 &= \left(\sqrt{61}\right)^2 \\ (x+3)^2+(y+3)^2 &= 61 \end{aligned}$$

1.1.36.

Endpoints of a diameter: $(11, -5), (3, 15)$

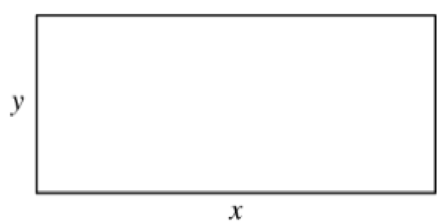
$$\begin{aligned}
r &= \frac{1}{2}\sqrt{(3-11)^2 + [15-(-5)]^2} \\
&= \frac{1}{2}\sqrt{(-8)^2 + (20)^2} \\
&= \frac{1}{2}\sqrt{64+400} \\
&= \frac{1}{2}\sqrt{464} = \frac{1}{2}(4)\sqrt{29} = 2\sqrt{29}
\end{aligned}$$

$$(h, k): \left(\frac{11+3}{2} \cdot \frac{-5+15}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$$

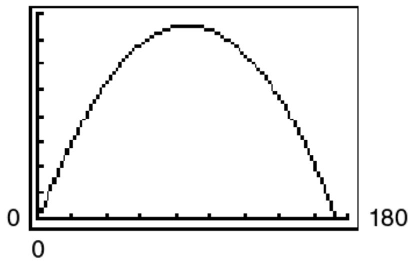
$$\begin{aligned}
(x-h)^2 + (y-k)^2 &= r^2 \\
(x-7)^2 + (y-5)^2 &= \left(2\sqrt{29}\right)^2 \\
(x-7)^2 + (y-5)^2 &= 116
\end{aligned}$$

1.1.37.

(a)



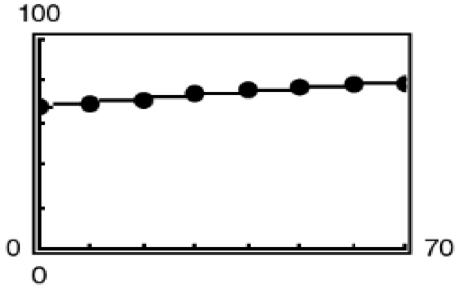
(b)
$$\begin{aligned}
2x + 2y &= \frac{1040}{3} \\
2y &= \frac{1040}{3} - 2x \\
y &= \frac{520}{3} - x \\
A = xy &= x\left(\frac{520}{3} - x\right)
\end{aligned}$$

(c) 

(d) When $x = y = 86\frac{2}{3}$ yards, the area is a maximum of $7511\frac{1}{9}$ square yards.

(e) A regulation NFL playing field is 120 yards long and $53\frac{1}{3}$ yards wide. The actual area is 6400 square yards.

1.1.38.

(a) 

The model fits the data well.
Each data value is close to the graph of the model.

(b) Using the graph or the model $(\text{with } t = 40)$, the life expectancy is approximately 75.2 years.

(c) The model and the line $y = 70.1$ intersect at $t = 11.1$, corresponding to the year 1961.

Algebraically, $\dfrac{68.0 + 0.33(11.1)}{1 + 0.002(11.1)} \approx 70.1.$

(d) The y-intercept is $(0, 68.0)$. In 1950 $(t = 0)$, the life expectancy of a child (at birth) was 68.0 years.

(e) Answers will vary.

1.1.39.

False. The line $y = x$ is symmetric with respect to the origin.

1.1.40.

False. The line $y = 0$ has infinitely many x-intercepts.

1.1.41.

The test is for symmetry with respect to the x-axis. The statement should read: The graph of $x = 3y^2$ is symmetric with respect to the x-axis because replacing y with $-y$ yields an equivalent equation.

1.1.42.

x-axis symmetry:
$$\begin{aligned} x^2 + y^2 &= 1 \\ x^2 + (-y)^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

y-axis symmetry:
$$\begin{aligned} x^2 + y^2 &= 1 \\ (-x)^2 + y^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

Origin symmetry:
$$\begin{aligned} x^2 + y^2 &= 1 \\ (-x)^2 + (-y)^2 &= 1 \\ x^2 + y^2 &= 1 \end{aligned}$$

So, the graph of the equation is symmetric with respect to x-axis, y-axis, and origin.

1.1.43.

$y = ax^2 + bx^3$

(a) $y = a(-x)^2 + b(-x)^3$

$= ax^2 - bx^3$

To be symmetric with respect to the y-axis; a can be any non-zero real number, b must be zero.

Sample answer: $a = 1, b = 0$

(b) $-y = a(-x)^2 + b(-x)^3$

$-y = ax^2 - bx^3$

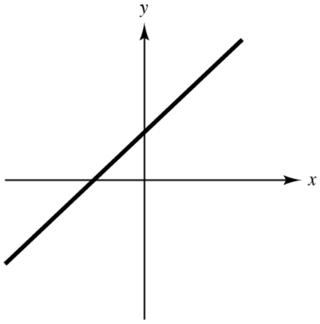
$y = -ax^2 + bx^3$

To be symmetric with respect to the origin; a must be zero, b can be any non-zero real number.

Sample answer: $a = 0, b = 1$

1.1.44.

The line would rise from left to right, passing through Quadrants I, II, and III.



1.1.45.

$$3(7x+1)=3(7x)+3(1)=21x+3$$

1.1.46.

$$5(x-6)=5(x)-5(6)=5x-30$$

1.1.47.

$$6(x-1)+4=6(x)-6(1)=6x-6+4=6x-2$$

1.1.48.

$$\begin{aligned}4(x+2)-12&=4(x)+4(2)=4x+8-12\\&=4x-4\end{aligned}$$

1.1.49.

The least common denominator is $3(4)=12$.

1.1.50.

The least common denominator is 9.

1.1.51.

The least common denominator is $x-4$.

1.1.52.

The least common denominator is $x^2-4=(x+2)(x-2)$.

1.1.53.

$$\begin{aligned} 7\sqrt{72} - 5\sqrt{18} &= 7\sqrt{2 \cdot 36} - 5\sqrt{2 \cdot 9} \\ &= 7(6)\sqrt{2} - 5(3)\sqrt{2} \\ &= (42 - 15)\sqrt{2} \\ &= 27\sqrt{2} \end{aligned}$$

1.1.54.

$$\begin{aligned} -10\sqrt{25y} - \sqrt{y} &= (-10)(5)\left(\sqrt{y} - \sqrt{y}\right) \\ &= -50\left(\sqrt{y} - \sqrt{y}\right) \\ &= -51\sqrt{y} \end{aligned}$$

1.1.55.

$$7^{3/2} \cdot 7^{11/2} = 7^{3/2\ +\ 11/2} = 7^7 = 823{,}543$$

1.1.56.

$$\frac{10^{17/4}}{10^{5/4}} = 10^{17/4\ -\ 5/4} = 10^{12/4} = 10^3 = 1000$$

1.1.57.

$$(9x - 4) + (2x^2 - x + 15) = 2x^2 + 8x + 11$$

1.1.58.

$$\begin{aligned} 4x(11 - x + 3x^2) &= 44x - 4x^2 + 12x^3 \\ &= 12x^3 - 4x^2 + 44x \end{aligned}$$

1.1.59.

$$\begin{aligned} (2x + 9)(x - 7) &= 2x^2 + 9x - 14x - 63 \\ &= 2x^2 - 5x - 63 \end{aligned}$$

1.1.60.

$$\begin{aligned} (3x^2 - 5)(-x^2 + 1) &= -3x^4 + 5x^2 + 3x^2 - 5 \\ &= -3x^4 + 8x^2 - 5 \end{aligned}$$

SECTION 1.2 LINEAR EQUATIONS IN ONE VARIABLE

1.2.1.

equation

1.2.2.

identities; conditional; contradictions

1.2.3.

$ax + b = 0$

1.2.4.

equivalent

1.2.5.

rational

1.2.6.

extraneous

1.2.7.

Yes. $8 = x - 3$ and $x = 11$ are equivalent equations.

1.2.8.

$$4\left(\frac{x}{2} + 1\right) = 4\left(\frac{1}{4}\right) \rightarrow 2x + 4 = 1$$

1.2.9.

The equation $3(x - 1) = 3x - 3$ is an *identity* by the Distributive Property. The equation is true for all real values of *x*.

1.2.10.

The equation $2(x + 1) = 2x - 1$ is a *contradiction*. There are no real values of *x* for which the equation is true.

1.2.11.

The equation $2(x - 1) = 3x + 1$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x = -3$.

1.2.12.

The equation $3(x+2)=3x+2$ is a *contradiction*. There are no real values of x for which the equation is true.

1.2.13.

The equation $2(x+3)-5=2x+1$ is an *identity* by simplification. The equation is true for all real values of x.

1.2.14.

The equation $3(x-1)+2=4x-2$ is a *conditional equation*. The only value in the domain that satisfies the equation is $x=1$.

1.2.15.

$$\begin{aligned}2x+11&=15\\2x+11-11&=15-11\\2x&=4\\\frac{2x}{2}&=\frac{4}{2}\\x&=2\end{aligned}$$

1.2.16.

$$\begin{aligned}7x+2&=23\\7x+2-2&=23-2\\7x&=21\\\frac{7x}{7}&=\frac{21}{7}\\x&=3\end{aligned}$$

1.2.17.

$$\begin{aligned}7-2x&=25\\7-7-2x&=25-7\\-2x&=18\\\frac{-2x}{-2}&=\frac{18}{-2}\\x&=-9\end{aligned}$$

1.2.18.

$$\begin{aligned}
7 - x &= 19 \\
7 - x + x &= 19 + x \\
7 &= 19 + x \\
7 - 19 &= 19 + x - 19 \\
-12 &= x
\end{aligned}$$

1.2.19.

$$\begin{aligned}
3x - 5 &= 2x + 7 \\
3x - 2x - 5 &= 2x - 2x + 7 \\
x - 5 &= 7 \\
x - 5 + 5 &= 7 + 5 \\
x &= 12
\end{aligned}$$

1.2.20.

$$\begin{aligned}
5x + 3 &= 6 - 2x \\
5x + 2x + 3 &= 6 - 2x + 2x \\
7x + 3 &= 6 \\
7x + 3 - 3 &= 6 - 3 \\
7x &= 3 \\
\frac{7x}{7} &= \frac{3}{7} \\
x &= \frac{3}{7}
\end{aligned}$$

1.2.21.

$$\begin{aligned}
4y + 2 - 5y &= 7 - 6y \\
4y - 5y + 2 &= 7 - 6y \\
-y + 2 &= 7 - 6y \\
-y + 6y + 2 &= 7 - 6y + 6y \\
5y + 2 &= 7 \\
5y + 2 - 2 &= 7 - 2 \\
5y &= 5 \\
\frac{5y}{5} &= \frac{5}{5} \\
y &= 1
\end{aligned}$$

1.2.22.

$$\begin{aligned}
5y+1&=8y-5+6y\\
5y+1&=8y+6y-5\\
5y+1&=14y-5\\
5y-5y+1&=14y-5y-5\\
1&=9y-5\\
1+5&=9y-5+5\\
6&=9y\\
\frac{6}{9}&=\frac{9y}{9}\\
\frac{2}{3}&=y
\end{aligned}$$

1.2.23.

$$\begin{aligned}
x-3(2x+3)&=8-5x\\
x-6x-9&=8-5x\\
-5x-9&=8-5x\\
-5x+5x-9&=8-5x+5x\\
-9&\neq 8
\end{aligned}$$

Because $-9=8$ is a contradiction, the equation has no solution.

1.2.24.

$$\begin{aligned}
9x-10&=5x+2(2x-5)\\
9x-10&=5x+4x-10\\
9x-10&=9x-10
\end{aligned}$$

Because the equation is an identity, the solution is the set of all real numbers.

1.2.25.

$$\begin{aligned}
0.25x+0.75(10-x)&=3\\
0.25x+7.5-0.75x&=3\\
-0.50x+7.5&=3\\
-0.50x&=-4.5\\
x&=9
\end{aligned}$$

1.2.26.

$$\begin{aligned}0.60x + 0.40(100 - x) &= 50\\ 0.60x + 40 - 0.40x &= 50\\ 0.20x &= 10\\ x &= 50\end{aligned}$$

1.2.27.

$$\begin{aligned}\frac{3x}{8} - \frac{4x}{3} &= 4\\ (24)\frac{3x}{8} - (24)\frac{4x}{3} &= (24)4\\ 9x - 32x &= 96\\ -23x &= 96\\ x &= -\frac{96}{23}\end{aligned}$$

1.2.28.

$$\begin{aligned}\frac{x}{5} - \frac{x}{2} &= 3 + \frac{3x}{10}\\ (10)\frac{x}{5} - (10)\frac{x}{2} &= (10)3 + (10)\frac{3x}{10}\\ 2x - 5x &= 30 + 3x\\ -6x &= 30\\ x &= -5\end{aligned}$$

1.2.29.

$$\begin{aligned}\frac{5x - 4}{5x + 4} &= \frac{2}{3}\\ 3(5x - 4) &= 2(5x + 4)\\ 15x - 12 &= 10x + 8\\ 5x &= 20\\ x &= 4\end{aligned}$$

1.2.30.

$$\begin{aligned}\frac{10x + 3}{5x + 6} &= \frac{1}{2}\\ 2(10x + 3) &= 1(5x + 6)\\ 20x + 6 &= 5x + 6\\ 15x &= 0\\ x &= 0\end{aligned}$$

1.2.31.

$$\begin{aligned}10-\frac{13}{x}&=4+\frac{5}{x}\\ \frac{10x-13}{x}&=\frac{4x+5}{x}\\ 10x-13&=4x+5\\ 6x&=18\\ x&=3\end{aligned}$$

1.2.32.

$$\begin{aligned}\frac{15}{x}-4&=\frac{6}{x}+3\\ \frac{15}{x}-\frac{6}{x}&=7\\ \frac{9}{x}&=7\\ 9&=7x\\ \frac{9}{7}&=x\end{aligned}$$

1.2.33.

$$\begin{aligned}3&=2+\frac{2}{z+2}\\ 3(z+2)&=\left(2+\frac{2}{z+2}\right)(z+2)\\ 3z+6&=2z+4+2\\ z&=0\end{aligned}$$

1.2.34.

$$\begin{aligned}\frac{1}{x}+\frac{2}{x-5}&=0\\ x(x-5)\frac{1}{x}+x(x-5)\frac{2}{x-5}&=x(x-5)0\\ x-5+2x&=0\\ 3x-5&=0\\ 3x&=5\\ x&=\frac{5}{3}\end{aligned}$$

1.2.35.

$$\begin{aligned}\frac{x}{x+4}+\frac{4}{x+4}+2&=0\\ \frac{x+4}{x+4}+2&=0\\ 1+2&=0\\ 3&\neq 0\end{aligned}$$

Because $3=0$ is a contradiction, the equation has no solution.

1.2.36.

$$\begin{aligned}\frac{7}{2x+1}-\frac{8x}{2x-1}&=-4 &&\text{Multiply each term by } (2x+1)(2x-1).\\ 7(2x-1)-8x(2x+1)&=-4(2x+1)(2x-1)\\ 14x-7-16x^2-8x&=-16x^2+4\\ 6x&=11\\ x&=\frac{11}{6}\end{aligned}$$

1.2.37.

$$\begin{aligned}\frac{2}{(x-4)(x-2)}&=\frac{1}{x-4}+\frac{2}{x-2} &&\text{Multiply each term by } (x-4)(x-2).\\ 2&=1(x-2)+2(x-4)\\ 2&=x-2+2x-8\\ 2&=3x-10\\ 12&=3x\\ 4&=x\end{aligned}$$

A check reveals that $x=4$ yields a denominator of zero. So, $x=4$ is an extraneous solution, and the original equation has no real solution.

1.2.38.

$$\frac{12}{(x-1)(x+3)} = \frac{3}{x-1} + \frac{2}{x+3} \qquad \text{Multiply each term by } (x-1)(x+3).$$

$$(x-1)(x+3)\frac{12}{(x-1)(x+3)} = (x-1)(x+3)\frac{3}{x-1} + (x-1)(x+3)\frac{2}{x+3}$$

$$12 = 3(x+3) + 2(x-1)$$

$$12 = 3x + 9 + 2x - 2$$

$$12 = 5x + 7$$

$$5 = 5x$$

$$x = 1$$

A check reveals that $x = 1$ yields a denominator of zero. So, $x = 1$ is an extraneous solution, and the original equation has no real solution.

1.2.39.

$$\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9} \qquad \text{Multiply each term by } (x+3)(x-3).$$

$$\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{(x+3)(x-3)}$$

$$1(x+3) + 1(x-3) = 10$$

$$2x = 10$$

$$x = 5$$

1.2.40.

$$\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2+x-6}$$

$$\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{(x+3)(x-2)} \qquad \text{Multiply each term by } (x+3)(x-2).$$

$$(x+3) + 3(x-2) = 4$$

$$x + 3 + 3x - 6 = 4$$

$$4x - 3 = 4$$

$$4x = 7$$

$$x = \frac{7}{4}$$

1.2.41.

$$\begin{aligned} y &= 12 - 5x \\ 0 &= 12 - 5x \\ 5x &= 12 \\ x &= \frac{12}{5} \end{aligned} \qquad \begin{aligned} y &= 12 - 5x \\ y &= 12 - 5(0) \\ y &= 12 \end{aligned}$$

The x-intercept is $\left(\frac{12}{5},\ 0\right)$ and the y-intercept is $(0,\ 12)$.

1.2.42.

$$\begin{aligned} y &= 16 - 3x \\ 0 &= 16 - 3x \\ -16 &= -3x \\ \frac{16}{3} &= x \end{aligned} \qquad \begin{aligned} y &= 16 - 3x \\ y &= 16 - 3(0) \\ y &= 16 \end{aligned}$$

The x-intercept is $\left(\frac{16}{3},\ 0\right)$ and the y-intercept is $(0,\ 16)$.

1.2.43.

$$\begin{aligned} y &= -3(2x+1) \\ 0 &= -3(2x+1) \\ 0 &= 2x+1 \\ x &= -\frac{1}{2} \end{aligned} \qquad \begin{aligned} y &= -3(2x+1) \\ y &= -3(2(0)+1) \\ y &= -3 \end{aligned}$$

The x-intercept is $\left(-\frac{1}{2},\ 0\right)$ and the y-intercept is $(0,\ -3)$.

1.2.44.

$$\begin{aligned} y &= 5 - (6 - x) \\ 0 &= 5 - (6 - x) \\ 0 &= -1 + x \\ 1 &= x \end{aligned} \qquad \begin{aligned} y &= 5 - (6 - x) \\ y &= 5 - (6 - 0) \\ y &= -1 \end{aligned}$$

The x-intercept is $(1,\ 0)$ and the y-intercept is $(0,\ -1)$.

1.2.45.

$$\begin{aligned} 2x+3y&=10 \\ 2x+3(0)&=10 \\ 2x&=10 \\ x&=5 \end{aligned} \qquad \begin{aligned} 2x+3y&=10 \\ 2(0)+3y&=10 \\ 3y&=10 \\ y&=\frac{10}{3} \end{aligned}$$

The x-intercept is $(5,\ 0)$ and the y-intercept is $\left(0,\ \frac{10}{3}\right)$.

1.2.46.

$$\begin{aligned} 4x-5y&=12 \\ 4x-5(0)&=12 \\ 4x&=12 \\ x&=3 \end{aligned} \qquad \begin{aligned} 4x-5y&=12 \\ 4(0)-5y&=12 \\ -5y&=12 \\ y&=-\frac{12}{5} \end{aligned}$$

The x-intercept is $(3,\ 0)$ and the y-intercept is $\left(0,\ -\frac{12}{5}\right)$.

1.2.47.

$$\begin{aligned} 4y-0.75x+1.2&=0 \\ 4(0)-0.75x+1.2&=0 \\ -0.75+1.2&=0 \\ x=\frac{1.2}{0.75}&=1.6 \end{aligned} \qquad \begin{aligned} 4y-0.75x+1.2&=0 \\ 4y-0.75(0)+1.2&=0 \\ 4y+1.2&=0 \\ y=\frac{-1.2}{4}&=-0.3 \end{aligned}$$

The x-intercept is $(1.6,\ 0)$ and the y-intercept is $(0,\ -0.3)$.

1.2.48.

$$\begin{aligned} 3y+2.5x-3.4&=0 \\ 3(0)2.5x-3.4&=0 \\ 2.5x&=3.4 \\ x&=\frac{3.4}{2.5} \\ x&=1.36 \end{aligned} \qquad \begin{aligned} 3y+2.5x-3.4&=0 \\ 3y+2.5(0)-3.4&=0 \\ 3y&=3.4 \\ y&=\frac{3.4}{3} \\ y&=1.1\overline{3} \end{aligned}$$

The x-intercept is $(1.36,\ 0)$ and the y-intercept is $(0,\ 1.1\overline{3})$.

1.2.49.

$$\begin{aligned} \frac{2x}{5}+8-3y&=0 \\ 2x+40-15y&=0 \\ 2x+40-15(0)&=0 \\ 2x+40&=0 \\ x&=-20 \end{aligned} \qquad \begin{aligned} 2x+40-15y&=0 \\ 2(0)+40-15y&=0 \\ 40-15y&=0 \\ y&=\frac{40}{15}=\frac{8}{3} \end{aligned}$$

The x-intercept is $(-20,\ 0)$ and the y-intercept is $\left(0,\ \frac{8}{3}\right)$.

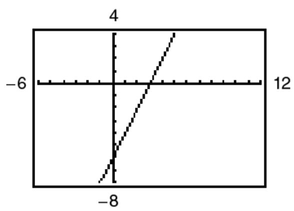
1.2.50.

$$\begin{aligned} \frac{8x}{3}+5-2y&=0 \\ \frac{8x}{3}+5-2(0)&=0 \\ \frac{8x}{3}&=-5 \\ x&=-\frac{15}{8} \end{aligned} \qquad \begin{aligned} \frac{8x}{3}+5-2y&=0 \\ \frac{8(0)}{3}+5-2y&=0 \\ 5-2y&=0 \\ y&=\frac{5}{2} \end{aligned}$$

The x-intercept is $\left(-\frac{15}{8},\ 0\right)$ and the y-intercept is $\left(0,\ \frac{5}{2}\right)$.

1.2.51.

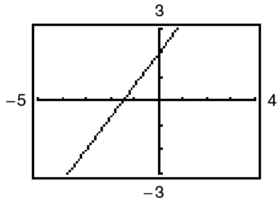
$$y=2(x-1)-4 \qquad \begin{aligned} 0&=2(x-1-4) \\ 0&=2x-2-4 \\ 0&=2x-6 \\ 6&=2x \\ 3&=x \\ x&=3 \end{aligned}$$



The x-intercept is $x=3$. The solution of $0=2(x-1)-4$ and the x-intercept of $y=2(x-1)-4$ are the same. They are both $x=3$. The x-intercept is $(3,\ 0)$.

1.2.52.

$$y = \frac{4}{3}x + 2 \qquad\qquad \begin{aligned} y &= \frac{4}{3}x + 2 \\ -\frac{4}{3}x &= 2 \\ \left(-\frac{3}{4}\right)\left(-\frac{4}{3}x\right) &= \left(-\frac{3}{4}\right)(2) \\ x &= -\frac{3}{2} \end{aligned}$$

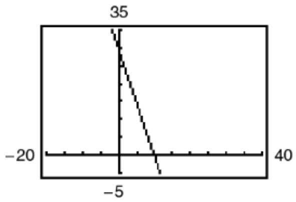


Intercept: $\left(-\frac{3}{2},\ 0\right)$

The solution to $0 = \frac{4}{3}x + 2$ is the same as the x-intercept of $y = \frac{4}{3}x + 2$. They are both $x = -\frac{3}{2}$.

1.2.53.

$$y = 20 - (3x - 10) \qquad\qquad \begin{aligned} 0 &= 20 - (3x - 10) \\ 0 &= 20 - 3x + 10 \\ 0 &= 30 - 3x \\ 3x &= 30 \\ x &= 10 \end{aligned}$$

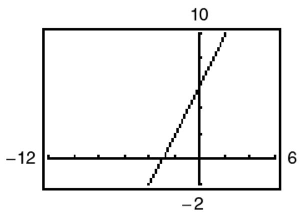


The x-intercept is $x = 10$. The solution of $0 = 20 - (3x - 10)$ and the x-intercept of $y = 20 - (3x - 10)$ are the same. They are both $x = 10$. The x-intercept is $(10,\ 0)$.

1.2.54.

$y = 10 + 2(x - 2)$

$$\begin{aligned} 0 &= 10 + 2(x-2) \\ 0 &= 10 + 2x - 4 \\ 0 &= 6 + 2x \\ -2x &= 6 \\ x &= -3 \end{aligned}$$



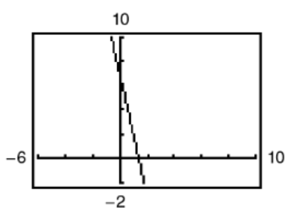
Intercept: $(-3,\ 0)$

The solution to $0 = 10 + 2(x-2)$ is the same as the x-intercept of $y = 10 + 2(x-2)$. They are both $x = -3$.

1.2.55.

$y = -38 + 5(9 - x)$

$$\begin{aligned} 0 &= -38 + 5(9-x) \\ 0 &= -38 + 45 - 5x \\ 0 &= 7 - 5x \\ 5x &= 7 \\ x &= \frac{7}{5} \end{aligned}$$

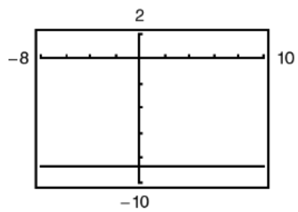


The x-intercept is at $x = \frac{7}{5}$. The solution of $0 = -38 + 5(9-x)$ and the x-intercept of $y = -38 + 5(9-x)$ are the same. They are both $x = \frac{7}{5}$. The x-intercept is $\left(\frac{7}{5},\ 0\right)$.

1.2.56.

$$y = 6x - 6\left(\frac{16}{11} + x\right)$$

$$\begin{aligned} 0 &= 6x - 6\left(\frac{16}{11} + x\right) \\ 0 &= 6x - \frac{96}{11} - 6x \\ 0 &\neq -\frac{96}{11} \end{aligned}$$



There is no x-intercept.

1.2.57.

$$\frac{2}{7.398} - \frac{4.405}{x} = \frac{1}{x} \quad \text{Multiply each side by } 7.398x.$$

$$\begin{aligned} 2x - (4.405)(7.398) &= 7.398 \\ 2x &= (4.405)(7.398) + 7.398 \\ 2x &= (5.405)(7.398) \\ x &= \frac{(5.405)(7.398)}{2} \approx 19.993 \end{aligned}$$

1.2.58.

$$\frac{3}{6.350} - \frac{6}{x} = 18 \quad \text{Multiply each side by } 6.350x.$$

$$\begin{aligned} 3x - 6(6.350) &= 18(6.350)x \\ 3x - 38.1 &= 114.3x \\ -38.1 &= 111.3x \\ -0.342 &\approx x \end{aligned}$$

1.2.59.

$$\begin{aligned} 0.275x + 0.725(500 - x) &= 300 \\ 0.275x + 362.5 - 0.725x &= 300 \\ -0.45x &= -62.5 \\ x &= \frac{62.5}{0.45} \approx 138.889 \end{aligned}$$

1.2.60.

$$\begin{aligned} 2.763 - 4.5(2.1x - 5.1432) &= 6.32x + 5 \\ 2.763 - 9.45x + 23.1444 &= 6.32x + 5 \\ 20.9074 &= 15.77x \\ 1.326 &\approx x \end{aligned}$$

1.2.61.

Let $y = 18$.

$$\begin{aligned} y &= 0.514x - 14.75 \\ 18 &= 0.514x - 14.75 \\ 32.75 &= 0.514x \\ \frac{32.75}{0.514} &= x \\ 63.7 &= x \end{aligned}$$

So, the height of the female is about 63.7 inches.

1.2.62.

Let $y = 23$.

$$\begin{aligned} y &= 0.532x - 17.03 \\ 23 &= 0.532x - 17.03 \\ 40.03 &= 0.532x \\ \frac{40.03}{0.532} &= x \\ 75.2 &= x \end{aligned}$$

The height of the missing man is about 75.2 inches. Because 75.2 in. $\left(\frac{1\text{ ft}}{12\text{ in.}}\right) = 6.27\text{ ft}$ is about 6 feet 3 inches, it is possible the femur belongs to the missing man.

1.2.63.

(a) The y-intercept is about $(0,\ 413)$.

(b) Let $t = 0$.

$$\begin{aligned} y &= 5.62t + 413.1 \\ &= 5.62(0) + 413.1 \\ &= 413.1 \end{aligned}$$

The y-intercept is $(0,\ 413.1)$.

In 2010, the population of Raleigh was about 413,100.

(c) Let $y = 475$.

$$\begin{aligned} 475 &= 5.62t + 413.1 \\ 61.9 &= 5.62t \\ \frac{61.9}{5.62} &= t \\ 11 &\approx t \end{aligned}$$

The population reached 475,000 in 2021.

1.2.64.

(a) The y-intercept is approximately $(0,\ 100)$.

(b) Let $t = 0$.

$$\begin{aligned} y &= -1.80t + 100.3 \\ &= -1.80(0) + 100.3 \\ &= 100.3 \end{aligned}$$

The y-intercept is $(0,\ 100.3)$.

In 2010, the population of Flint was about 100,300.

(c) Let $y = 85.5$.

$$\begin{aligned} 85.5 &= -1.80t + 100.3 \\ -14.8 &= -1.80t \\ \frac{-14.8}{-1.80} &= t \\ 8 &\approx t \end{aligned}$$

The population was 85,500 in 2018.

1.2.65.

Let $c = 10,000$.

$$\begin{aligned} c &= 0.37m + 2600 \\ 10,000 &= 0.37m + 2600 \\ 7400 &= 0.37m \\ \frac{7400}{0.37} &= m \\ m &= 20,000 \end{aligned}$$

So, the number of miles is 20,000.

1.2.66.

Let $y = 1$.

$$\begin{aligned} y &= -0.25t + 8 \\ 1 &= -0.25t + 8 \\ 0.25t &= 7 \\ t &= 28 \text{ hours} \end{aligned}$$

1.2.67.

$$\begin{aligned} x(3 - x) &= 10 \\ 3x - x^2 &= 10 \end{aligned}$$

False. This is a quadratic equation. The equation cannot be written in the form $ax + b = 0$.

1.2.68.

$$\begin{aligned} 2(x + 3) &= 3x + 3 \\ 2x + 6 &= 3x + 3 \\ 3 &= x \end{aligned}$$

False, $x = 3$ is a solution.

1.2.69.

$$\begin{aligned} 3(x - 1) - 2 &= 3x - 6 \\ 3x - 3 - 2 &= 3x - 6 \\ 3x - 5 &= 3x - 6 \\ -5 &\neq -6 \end{aligned}$$

False. The equation $-5 = -6$ is a contradiction, so the original equation has no solution.

1.2.70.

$$\begin{aligned} 2 - \frac{1}{x-2} &= \frac{3}{x-2} \\ (x-2)\left(2 - \frac{1}{x-2}\right) &= (x-2)\left(\frac{3}{x-2}\right) \\ 2(x-2) - 1 &= 3 \\ 2x - 4 - 1 &= 3 \\ 2x - 5 &= 3 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

False. $x = 4$ is a solution.

1.2.71.

$$\frac{3x+2}{5}=7$$

$$\begin{aligned} 3x+2&=35 \quad \text{and} \quad & x+9&=20 \\ 3x&=33 & x&=11 \\ x&=11 \end{aligned}$$

Yes, they are equivalent equations. They both have the solution $x=11$.

1.2.72.

(a) The x-intercept is $(20{,}000,\ 0)$ and the y-intercept is $(0,\ 10{,}000)$. The subsidy is \$0 for an earned income of \$20,000. The subsidy is \$10,000 for an earned income of \$0.

(b) Set one of S or E equal to 0 and solve for the other.

(c) The earned income is \$8000.

(d) Set T equal to 14,000, substitute $10{,}000-\frac{1}{2}E$ for S in the equation $T=E+S$, and solve for E.

1.2.73.

Area of shaded region = Area of outer rectangle − Area of inner rectangle

$$\begin{aligned} A&=2x(2x+6)-x(x+4) \\ &=4x^2+12x-x^2-4x \\ &=3x^2+8x \end{aligned}$$

1.2.74.

Area of shaded region = Area of larger triangle − Area of smaller triangle

$$\begin{aligned} A&=\frac{1}{2}(2x+8)^2-\frac{1}{2}(x+4)^2 \\ &=\frac{1}{2}\left[4x^2+32x+64-(x^2+8x+16)\right] \\ &=\frac{1}{2}(3x^2+24x+48) \\ &=\frac{3}{2}x^2+12x+24 \end{aligned}$$

1.2.75.

$$\begin{aligned}\frac{144}{x} &= \frac{36}{55}\\ 36x &= 144(55)\\ x &= \frac{7920}{36}\\ x &= 220\end{aligned}$$

1.2.76.

$$\begin{aligned}\frac{19}{2} &= \frac{x}{19}\\ 2x &= 19(19)\\ x &= \frac{361}{2}\end{aligned}$$

1.2.77.

$$\begin{aligned}\frac{x}{5} &= \frac{5}{x}\\ x^2 &= 25\\ x &= \pm 5\end{aligned}$$

1.2.78.

$$\begin{aligned}\frac{14}{x} &= \frac{15}{x+1}\\ 14(x+1) &= 15x\\ 14x+14 &= 15x\\ x &= 14\end{aligned}$$

1.2.79.

$$(600)(0.24) = 144$$

So, 144 is 24% of 600.

1.2.80.

$$\begin{aligned}0.55x &= 110\\ x &= \frac{110}{0.55}\\ x &= 200\end{aligned}$$

So, 55% of 200 is 110.

1.2.81.

$$176x = 66.88$$
$$x = \frac{66.88}{176}$$
$$x = 0.38$$

So, 38% of 176 is 66.88.

1.2.82.

$$(16x)\,y = 9x$$
$$y = \frac{9x}{16x}$$
$$y = \frac{9}{16} = 0.5625$$

So, 56.25% of 16x is 9x.

1.2.83.

$$\sqrt[3]{16x^5} = \left(2^4 x^5\right)^{1/3} = 2x\sqrt[3]{2x^2}$$

1.2.84.

$$\sqrt[5]{\frac{x^8 z^4}{32}} = \frac{x}{2}\sqrt[5]{x^3 z^4}$$

1.2.85.

$$\sqrt[6]{x^3} = x^{3/6} = x^{1/2} = \sqrt{x}$$

1.2.86.

$$\sqrt[6]{(x+1)^4} = (x+1)^{4/6} = (x+1)^{2/3}$$

Section 1.3 Modeling with Linear Equations

1.3.1.

mathematical modeling

1.3.2.

verbal model; algebraic equation

1.3.3.

A hidden equality is a statement that two algebraic expressions are equal.

1.3.4.

Answers will vary. *Sample* answers:

Addition: sum, plus, increased by, more than, total of Subtraction: difference, minus, less than, decreased by, subtracted from, reduced by

Multiplication: product, multiplied by, twice, times, percent of.

Division: quotient, divided by, ratio, per

Equality: equals, equal to, is, are, was, will be, represents

1.3.5.

$y+2$

The sum of a number and 2

A number increased by 2

1.3.6.

$x-8$

The difference of a number and 8

A number decreased by 8

1.3.7.

$\frac{t}{6}$

A number divided by 6

1.3.8.

$\frac{1}{3}u$

The product of $\frac{1}{3}$ and a number

1.3.9.

$$\frac{z-2}{3}$$

A number decreased by 2, then divided by 3

1.3.10.

$$\frac{x+9}{5}$$

A number increased by 9, then divided by 5

1.3.11.

$$-2(d+5)$$

The product of -2 and a number increased by 5

1.3.12.

$$10y(y-3)$$

The product of 10, a number, and 3 less than the same number

1.3.13.

Verbal Model: $\text{Product} = (\text{first odd integer}) \times (\text{second odd integer})$

Labels: Product $= P$, first odd integer $= 2n-1$, second odd integer $= 2n-1+2 = 2n+1$

Equation: $P = (2n-1)(2n+1) = 4n^2 - 1$

1.3.14.

Verbal Model: $(\text{Sum}) = (\text{first even number})^2 + (\text{second even number})^2$

Labels: $\text{Sum} = S$, first even number $= 2n$, second even number $= 2n+2$

Equation: $S = (2n)^2 + (2n+2)^2 = 4n^2 + 4n^2 + 8n + 4 = 8n^2 + 8n + 4$

1.3.15.

Verbal Model: $(\text{Distance}) = (\text{rate}) \cdot (\text{time})$

Labels: $\text{Distance} = d$, rate $= 55$ mph, time $= t$

Equation: $d = 55t$

1.3.16.

Verbal Model: $(\text{time}) = (\text{distance}) \div (\text{rate})$

Labels: time $= t$, distance $= 900$ km, rate $= r$

Equation: $t = \dfrac{900}{r}$

1.3.17.

Verbal Model: $(\text{Amount of acid}) = 20\% \cdot (\text{amount of solution})$

Labels: Amount of acid $(\text{in gallons}) = A$, amount of solution $(\text{in gallons}) = x$

Equation: $A = 0.20x$

1.3.18.

Verbal Model: $(\text{Sale price}) = (\text{list price}) - (\text{discount})$

Labels: Sale price $= S$, list price $= L$, discount $= 0.33L$

Equation: $S = L - 0.33L = 0.67L$

1.3.19.

Verbal Model: $(\text{Total cost}) = (\text{unit cost})(\text{number of units}) + (\text{fixed cost})$

Labels: Total cost $= C$, unit cost $= \$40$, number of units $= x$, fixed cost $= \$2500$

Equation: $C = 2500 + 40x$

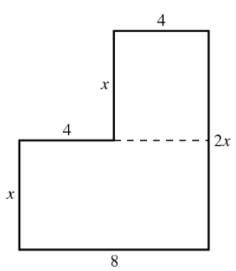
1.3.20.

Verbal Model: $(\text{Revenue}) - (\text{price})(\text{number of units})$

Labels: Revenue $= R$, price $= \$12.99$, number of units $= x$

Equation: $R = 12.99x$

1.3.21.



Area = Area of top rectangle + Area of bottom rectangle

$A = 4x + 8x = 12x$

1.3.22.

$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$A = \frac{1}{2}\left(\frac{2}{3}b + 1\right) = \frac{1}{3}b^2 + \frac{1}{2}b$$

1.3.23.

Verbal Model: $\text{Sum} = (\text{first number}) + (\text{second number})$

Labels: $\text{Sum} = 525$, first number $= n$, second number $= n + 1$

Equation:
$$525 = n + (n + 1)$$
$$525 = 2n + 1$$
$$524 = 2n$$
$$n = 262$$

Answer: First number $= n = 262$, second number $= n + 1 = 263$

1.3.24.

Verbal Model: $\text{Sum} = (\text{first number}) + (\text{second number}) + (\text{third number})$

Labels: $\text{Sum} = 804$, first number $= n$, second number $= n + 1$, third number $= n + 2$

Equation:
$$804 = n + n + 1 + n + 2$$
$$804 = 3n + 3$$
$$801 = 3n$$
$$267 = n$$

Answer: $n = 267$, $n + 1 = 268$ (second number), and $n + 2 = 269$ (third number)

1.3.25.

Verbal Model: $\text{Difference} = (\text{one number}) - (\text{another number})$

Labels: Difference $= 148$, one number $= 5x$, another number $= x$

Equation:
$$148 = 5x - x$$
$$148 = 4x$$
$$x = 37$$
$$5x = 185$$

Answer: The two numbers are 37 and 185.

1.3.26.

Verbal Model: Difference $=$ (number) $-$ (one-fifth of number)

Labels: Difference $= 76$, number $= n$, one-fifth of number $= \frac{1}{5}n$

Equation:

$$76 = n - \frac{1}{5}n$$
$$76 = \frac{4}{5}n$$
$$95 = n$$

Answer: The numbers are 95 and $\frac{1}{5} \cdot 95 = 19$.

1.3.27.

Verbal Model: Product = (smaller number) × (larger number)
= (smaller number)^2 -5

Labels: Smaller number $= n$, larger number $= n + 1$

Equation:

$$n(n+1) = n^2 - 5$$
$$n^2 + n = n^2 - 5$$
$$n = -5$$

Answer: Smaller number $= n = -5$, larger number $= n + 1 = -4$

1.3.28.

Verbal Model: Difference = (reciprocal of smaller number)
$-$(reciprocal of larger number)
$= \frac{1}{4}$(reciprocal of smaller number)

Labels: Smaller number $= n$, larger number $= n + 1$, difference $= \frac{1}{4n}$

Equation:

$$\frac{1}{4n} = \frac{1}{n} - \frac{1}{n+1} \qquad \text{Multiply each side by } 4n(n+1).$$

$$\begin{aligned} 4n(n+1)\frac{1}{4n} &= 4n(n+1)\frac{1}{n} - 4n(n+1)\frac{1}{n+1} \\ n+1 &= 4(n+1) - 4n \\ n+1 &= 4n+4-4n \\ n &= 3 \end{aligned}$$

Answer: The numbers are 3 and $n+1=4$.

1.3.29.

Verbal Model: $(\text{first paycheck}) + (\text{second paycheck}) = \text{total}$

Labels: second paycheck $= x$, first paycheck $= 0.85x$, total $= \$1125$

Equation:

$$\begin{aligned} 0.85x + x &= 1125 \\ 1.85x &= 1125 \\ x &\approx 608.11 \\ 0.85x &\approx 516.89 \end{aligned}$$

Answer: The first salesperson's weekly paycheck is \$516.89 and the second salesperson's weekly paycheck is \$608.11.

1.3.30.

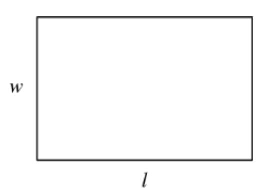
Let P be the price of the ticket.

$$\begin{aligned} P(1-0.165) &= 116.90 \\ P(0.835) &= 116.90 \\ P &= \frac{116.90}{0.835} = 140 \end{aligned}$$

The original list price is \$140.00.

1.3.31.

(a)



(b) $l = 1.5w$

$$\begin{aligned} P &= 2l + 2w \\ &= 2(1.5w) + 2w \\ &= 5w \end{aligned}$$

(c) $25 = 5w$
$5 = w$

Width: $w = 5$ meters
Length: $l = 1.5w = 7.5$ meters
Dimensions: 7.5 meters $\times$ 5 meters

1.3.32.

Let x be the length of the field. The width is $\frac{2}{3}x$, and the perimeter is 400.

$$2x + 2\left(\frac{2}{3}\right)x = 400$$
$$\frac{6}{3}x + \frac{4}{3}x = 400$$
$$\frac{10}{3}x = 400$$
$$x = \frac{3(400)}{10} = 120$$

The dimensions are 120 yards by 80 yards.

1.3.33.

Verbal Model: $\text{Average} = \dfrac{(\text{test } \#1) + (\text{test } \#2) + (\text{test } \#3) + (\text{test } \#4)}{4}$

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = x

Equation: $90 = \dfrac{87 + 92 + 84 + x}{4}$

Answer: You must score 97 or better on test #4 to earn an A for the course.

1.3.34.

Verbal Model: $\text{Average} = \dfrac{(\text{test } \#1) + (\text{test } \#2) + (\text{test } \#3) + (\text{test } \#4)}{5}$

Labels: Average = 90, test #1 = 87, test #2 = 92, test #3 = 84, test #4 = x

Equation:

$$\begin{aligned}90 &= \frac{87+92+84+x}{5}\\ 450 &= 87+92+84+x\\ 450 &= 263+x\\ 187 &= x\end{aligned}$$

Answer: You must score 187 out of 200 on the last test to get an A in the course.

1.3.35.

$$\text{Rate} = \frac{\text{distance}}{\text{time}} = \frac{50\ \text{kilometers}}{\frac{1}{2}\ \text{hour}} = 100\ \text{kilometers/hour}$$

$$\text{Total time} = \frac{\text{total distance}}{\text{rate}} = \frac{500\ \text{kilometers}}{100\ \text{kilometers/hour}} = 5\ \text{hours}$$

The entire trip takes 5 hours.

1.3.36.

Verbal Model: $(\text{Distance}) = (\text{rate})(\text{time}_1 + \text{time}_2)$

Labels: $\text{Distance} = 2 \cdot 200 = 400$ miles, rate $= 2$,

$$\text{time}_1 = \frac{\text{distance}}{\text{rate}_1} = \frac{200}{55}\ \text{hours},$$

$$\text{time}_2 = \frac{\text{distance}}{\text{rate}_2} = \frac{200}{40}\ \text{hours}$$

Equation:

$$\begin{aligned}400 &= r\left(\frac{200}{55}+\frac{200}{40}\right)\\ 400 &= r\left(\frac{1600}{440}+\frac{2200}{440}\right) = \frac{3800}{440}r\\ 46.3 &\approx r\end{aligned}$$

The average speed for the round trip was approximately 46.3 miles per hour.

1.3.37.

Verbal Model: $$\frac{(\text{Height of building})}{(\text{Length of building's shadow})} = \frac{(\text{Height of post})}{(\text{Length of post's shadow})}$$

Labels: Height of building $= x$ (feet)
Length of building's shadow $= 105$ (feet)
Height of post $= 3 \cdot 12 = 36$(inches)
Length of post's shadow $= 4$(inches)

Equation: $$\frac{x}{105} = \frac{36}{4}$$
$$x = 945$$

One Liberty Place is 945 feet tall.

1.3.38.

Verbal Model: $$\frac{(\text{height of tree})}{(\text{length of tree's shadow})} = \frac{(\text{height of lamppost})}{(\text{length of lamppost's shadow})}$$

Labels: height of tree $= h$, height of tree's shadow $= 8$ meters,
height of lamppost $= 2$ meters, height of lamppost's shadow
$= 0.75$ meter

Equation: $$\frac{h}{8} = \frac{2}{0.75}$$
$$h = \frac{8(2)}{0.75} = 21\frac{1}{3}$$

The tree is $21\frac{1}{3}$ meters tall.

1.3.39.

Verbal Model: $\boxed{\text{Interest from } 4\tfrac{1}{2}\%} + \boxed{\text{Interest from } 5\%} = \boxed{\text{Total interest}}$

Labels: Amount invested at $4\frac{1}{2}\% = x$ dollars
Amount invested at $5\% = 12{,}000 - x$ dollars
Interest from $4\frac{1}{2}\% = x(0.045)$ dollars
Interest from $5\% = (12{,}000 - x)(0.05)$ dollars
Total annual interest $= 580$ dollars

Equation:

$$\begin{aligned} 0.045x + 0.05(12{,}000 - x) &= 580 \\ 0.045x + 600 - 0.05x &= 580 \\ -0.005x &= -20 \\ x &= 4000 \end{aligned}$$

So, \$4000 was invested at $4\frac{1}{2}\%$ and $\$12{,}000 - \$4000 = \$8000$ was invested at 5%.

1.3.40.

Verbal Model: $\boxed{\text{Interest from } 3\%} + \boxed{\text{Interest from } 4\frac{1}{2}\%} = \boxed{\text{Total interest}}$

Labels: Amount invested at $3\% = x$

Amount invested at $4\frac{1}{2}\% = 25{,}000 - x$

Equation:

$$\begin{aligned} 0.03x + 0.045(25{,}000 - x) &= 900 \\ 0.03x + 1125 - 0.045x &= 900 \\ -0.015x &= -225 \\ x &= 15{,}000 \end{aligned}$$

So, \$15,000 was invested at 3% and $\$25{,}000 - \$15{,}000 = \$10{,}000$ was invested at $4\frac{1}{2}\%$.

1.3.41.

Verbal Model: $(\text{Profit from dogwood trees}) + (\text{profit from red maple trees})$
$= (\text{total profit})$

Labels: Inventory of dogwood trees = x,
inventory of red maple trees = $40{,}000 - x$,
profit from dogwood trees = $0.25x$,
profit from red maple trees = $0.17(40{,}000 - x)$,
total profit = $0.20(40{,}000) = 8000$

Equation:

$$\begin{aligned} 0.25x + 0.17(40{,}000) &= 8000 \\ 0.25x + 6800 - 0.17x &= 8000 \\ 0.08x &= 1200 \\ x &= 15{,}000 \end{aligned}$$

The amount invested in dogwood trees was \$15,000 and the amount invested in red maple trees was $\$40{,}000 - \$15{,}000 = \$25{,}000$.

1.3.42.

Verbal Model: $(\text{Profit from all - electric}) + (\text{profit from hybrid vehicles})$
$= (\text{total profit})$

Labels: Inventory of all-electric $= x$, inventory of hybrid vehicles $= 600{,}000 - x$, profit from all - electric $= 0.24x$, profit from hybrid vehicles $= 0.28(600{,}000 - x)$, total profit $= 0.25(600{,}000) = 150{,}000$

Equation:

$$\begin{aligned} 0.24x + 0.28(600{,}000 - x) &= 150{,}000 \\ 0.24x + 168{,}000 - 0.28x &= 150{,}000 \\ -0.04x &= -18{,}000 \\ x &= 450{,}000 \end{aligned}$$

The amount invested in all-electric was $\$450{,}000$ and the amount invested in hybrid vehicles was $\$600{,}000 - \$450{,}000 = \$150{,}000$.

1.3.43.

Verbal Model: $\begin{pmatrix}\text{Price per pound}\\\text{of peanuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{peanuts}\end{pmatrix} + \begin{pmatrix}\text{price per pound}\\\text{of walnuts}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{walnuts}\end{pmatrix}$
$= \begin{pmatrix}\text{price per pound}\\\text{of nut mixture}\end{pmatrix}\begin{pmatrix}\text{pounds of}\\\text{nut mixture}\end{pmatrix}$

Labels: Price per pound of peanuts $= \$1.49$, pounds of peanuts $= x$, price per pound of peanuts $= \$2.69$, pounds of walnuts $= 100 - x$, price per pound of nut mixture $= \$2.21$, pounds of nut mixture $= 100$

Equation:

$$\begin{aligned} 1.49x + 2.69(100 - x) &= 2.21(100) \\ 1.49x + 2.69 - 2.69x &= 2.21 \\ -1.2x &= -48 \\ x &= 40 \end{aligned}$$

There were 40 pounds of peanuts and $100 - 40 = 60$ pounds of walnuts in the mixture.

1.3.44.

Verbal Model: $\boxed{\text{Amount of gasoline in mixture}} + \boxed{\text{Amount of gasoline to add}} = \boxed{\text{Amount of gasoline in final mixture}}$

Labels: Amount of gasoline in mixture $= \frac{32}{33}(2)$ (gallons)

Amount of gasoline to add $= x$ (gallons)

Amount of gasoline in final mixture $= \frac{50}{51}(2+x)$ (gallons)

Equation:

$$\frac{64}{33} + x = \frac{50}{51}(2+x)$$
$$\frac{64}{33} + x = \frac{100}{51} + \frac{50}{51}x$$
$$3264 + 1683x = 3300 + 1650x$$
$$33x = 36$$
$$x \approx 1.09$$

The forester should add about 1.09 gallons of gasoline to the mixture.

1.3.45.

$$A = \frac{1}{2}bh$$
$$2A = bh$$
$$\frac{2A}{b} = h$$

1.3.46.

$$V = l\,w\,h$$

$$\frac{V}{w\,h} = l$$

1.3.47.

$$A = P + Prt$$
$$A - P = Prt$$
$$\frac{A-P}{Pt} = r$$

1.3.48.

$$\begin{aligned} A &= \frac{1}{2}(a+b)h \\ 2A &= (a+b)h \\ \frac{2A}{h} &= a+b \\ b &= \frac{2A}{h} - a \end{aligned}$$

1.3.49.

$$C = \frac{5}{9}(F-32)$$

$$\begin{aligned} &= \frac{5}{9}(28.8-32) \\ &= \frac{5}{9}(-3.2) \\ &\approx -1.8 \end{aligned}$$

The temperature is about -1.8°C.

1.3.50.

$$F = \frac{9}{5}C + 32$$

$$\begin{aligned} &= \frac{9}{5}(27) + 32 \\ &= 48.6 + 32 \\ &= 80.6 \end{aligned}$$

The temperature is 80.6°F.

1.3.51.

$$V = \frac{4}{3}\pi r^3$$

$$\begin{aligned} 5.96 &= \frac{4}{3}\pi r^3 \\ 17.88 &= 4\pi r^3 \\ \frac{17.88}{4\pi} &= r^3 \\ r &= \sqrt[3]{\frac{4.47}{\pi}} \approx 1.12 \text{ inches} \end{aligned}$$

1.3.52.

$$V = \pi r^2 h$$

$$h = \frac{V}{\pi r^2} = \frac{603.2}{\pi (2)^2} \approx 48 \text{ feet}$$

1.3.53.

True. The expression $\dfrac{x^3}{(x-4)^2}$ can be described as

x cubed divided by the square of the difference of x and 4.

1.3.54.

Area of circle: $A = \pi r^2 = \pi(2)^2 = 4\pi \approx 12.56 \text{ in.}^2$

Area of square: $A = s^2 = (4)^2 = 16 \text{ in.}^2$

True. $12.56 \text{ in.}^2 < 16 \text{ in.}^2$, so the area of the circle is less than the area of the square.

1.3.55.

One possible interpretation: $\dfrac{5}{3n}$

Another possible interpretation: $\dfrac{5}{n} \cdot 3 = \dfrac{15}{n}$

The phrase "the quotient of 5 and a number" indicates the variable is in the denominator.

So, the expression $\dfrac{3n}{5}$ is not a possible interpretation.

1.3.56.

(a) *Verbal Model:* $\dfrac{\text{Height of building}}{\text{Length of building's shadow}} = \dfrac{\text{Height of post}}{\text{Length of post's shadow}}$

(b) *Equation:* $\dfrac{x}{30} = \dfrac{4}{3}$

1.3.57.

$$\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right) = x^2 - x\sqrt{3} + x\sqrt{3} - 3 = x^2 - 3$$

1.3.58.

$$\left(x+3\sqrt{2}\right)\left(x-3\sqrt{2}\right)=x^2-\left(3\sqrt{2}\right)^2=x^2-18$$

1.3.59.

$$\begin{aligned}\left(x-3+\sqrt{7}\right)\left(x-3-\sqrt{7}\right)&=\left(x-3\right)^2-\left(\sqrt{7}\right)^2\\&=x^2-6x+9-7\\&=x^2-6x+2\end{aligned}$$

1.3.60.

$$\begin{aligned}\left(x+\sqrt{2}+2\right)\left(x+\sqrt{2}-2\right)&=\left(x+\sqrt{2}\right)^2-4\\&=x^2+2\sqrt{2}x+2-4\\&=x^2+2\sqrt{2}x-2\end{aligned}$$

1.3.61.

$$4x^2+4x+1=\left(2x+1\right)^2$$

1.3.62.

$$x^2-22x+121=\left(x-11\right)\left(x-11\right)=\left(x-11\right)^2$$

1.3.63.

$$u^3+27v^3=\left(u+3v\right)\left(u^2-3uv+9v^2\right)$$

1.3.64.

$$\left(x+2\right)^3-y^3=\left(x+2-y\right)\left[\left(x+2\right)^2+\left(x+2\right)y+y^2\right]$$

1.3.65.

$$2x^2+9x+4=\left(2x+1\right)\left(x+4\right)$$

1.3.66.

$$2x^2-3x-5=\left(x+1\right)\left(2x-5\right)$$

1.3.67.

$$\frac{-3+\sqrt{3^2-4\left(-9\right)}}{2}=\frac{-3+\sqrt{45}}{2}=\frac{-3+3\sqrt{5}}{2}$$

1.3.68.

$$\begin{aligned}\frac{-2-\sqrt{2^2-4(3)(-10)}}{2(3)} &= \frac{-2-\sqrt{124}}{6}\\ &= \frac{-2-2\sqrt{31}}{6}\\ &= \frac{-1-\sqrt{31}}{3}\end{aligned}$$

1.3.69.

$$\begin{aligned}\left[3\left(2-\sqrt{2}\right)-6\right]^2-18 &= \left[3(2)-3\sqrt{2}-6\right]^2-18\\ &= \left(-3\sqrt{2}\right)^2-18\\ &= 9(2)-18\\ &= 0\end{aligned}$$

1.3.70.

$$\left(-1+\sqrt{7}\right)^2+2\left(-1+\sqrt{7}\right)-6=\left(1-2\sqrt{7}+7\right)-2+2\sqrt{7}-6=0$$

1.3.71.

$$9.46\times10^{12}=9,460,000,000,000$$

1.3.72.

$$9.02\times10^{-6}=0.00000902$$

1.3.73.

$$-3.75\times10^{-4}=-0.000375$$

1.3.74.

$$1.83\times10^{8}=183,000,000$$

Section 1.4 Quadratic Equations and Applications

1.4.1.

quadratic equation

1.4.2.

second-degree polynomial

1.4.3.

discriminant

1.4.4.

Pythagorean Theorem

1.4.5.

Four methods to solve a quadratic equation are: factoring, extracting square roots, completing the square, and using the Quadratic Formula.

1.4.6.

The height of an object that is falling is given by the equation $s = -16t^2 + v_0t + s_0$, where s is the height, v_0 is the initial velocity, and s_0 is the initial height.

1.4.7.

$$6x^2 + 3x = 0$$
$$3x(2x+1) = 0$$

$$3x = 0 \quad \text{or} \quad 2x + 1 = 0$$
$$x = 0 \quad \text{or} \quad x = -\frac{1}{2}$$

1.4.8.

$$8x^2 - 2x = 0$$
$$2x(4x-1) = 0$$
$$8x = 0 \quad \text{or} \quad 4x - 1 = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{1}{4}$$

1.4.9.

$$\begin{aligned} 3+5x-2x^2 &= 0 \\ (3-x)(1+2x) &= 0 \end{aligned}$$

$$\begin{aligned} 3-x=0 \quad &\text{or} \quad 1+2x=0 \\ x=3 \quad &\text{or} \quad x=-\tfrac{1}{2} \end{aligned}$$

1.4.10.

$$\begin{aligned} x^2+6x+9 &= 0 \\ (x+3)(x+3) &= 0 \\ x+3 &= 0 \\ x &= -3 \end{aligned}$$

1.4.11.

$$\begin{aligned} x^2+10x+25 &= 0 \\ (x+5)(x+5) &= 0 \\ x+5 &= 0 \\ x &= -5 \end{aligned}$$

1.4.12.

$$\begin{aligned} 4x^2+12x+9 &= 0 \\ (2x+3)(2x+3) &= 0 \end{aligned}$$

$$2x+3=0 \quad \Rightarrow \quad x=-\frac{3}{2}$$

1.4.13.

$$\begin{aligned} 16x^2-9 &= 0 \\ (4x+3)(4x+3) &= 0 \\ 4x+3=0 &\Rightarrow x=-\tfrac{3}{4} \\ 4x-3=0 &\Rightarrow x=\tfrac{3}{4} \end{aligned}$$

1.4.14.

$$\begin{aligned} x^2-2x-8 &= 0 \\ (x-4)(x+2) &= 0 \end{aligned}$$

$$\begin{aligned} x-4=0 \quad &\text{or} \quad x+2=0 \\ x=4 \quad &\text{or} \quad x=-2 \end{aligned}$$

1.4.15.

$$2x^2 = 19x + 33$$
$$2x^2 - 19x - 33 = 0$$
$$(2x+3)(x-11) = 0$$

$$2x + 3 = 0 \quad \Rightarrow \quad x = -\frac{3}{2}$$
$$x - 11 = 0 \quad \Rightarrow \quad x = 11$$

1.4.16.

$$-x^2 + 4x = 3$$
$$-x^2 + 4x - 3 = 0$$
$$(-1)(-x^2 + 4x - 3) = (-1)(0)$$
$$x^2 - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x - 1 = 0 \Rightarrow x = 1$$

1.4.17.

$$\frac{3}{4}x^2 + 8x + 20 = 0$$
$$4\left(\frac{3}{4}x^2 + 8x + 20\right) = 4(0)$$
$$3x^2 + 32x + 80 = 0$$
$$(3x+20)(x+4) = 0$$

$$3x + 20 = 0 \quad \text{or} \quad x + 4 = 0$$
$$x = -\frac{20}{3} \quad \text{or} \quad x = -4$$

1.4.18.

$$\frac{1}{8}x^2 - x - 16 = 0$$
$$x^2 - 8x - 128 = 0$$
$$(x-16)(x+8) = 0$$

$$x - 16 = 0 \quad \Rightarrow \quad x = 16$$
$$x + 8 = 0 \quad \Rightarrow \quad x = -8$$

1.4.19.

$$\begin{aligned} x^2 &= 49 \\ x &= \pm 7 \end{aligned}$$

1.4.20.

$$\begin{aligned} x^2 &= 144 \\ x &= \pm 12 \end{aligned}$$

1.4.21.

$$\begin{aligned} x^2 &= 19 \\ x &= \pm\sqrt{19} \\ x &\approx \pm 4.36 \end{aligned}$$

1.4.22.

$$\begin{aligned} x^2 &= 43 \\ x &= \pm\sqrt{43} \\ x &\approx 6.56 \end{aligned}$$

1.4.23.

$$\begin{aligned} 3x^2 &= 81 \\ x^2 &= 27 \\ x &= \pm 3\sqrt{3} \\ &\approx \pm 5.20 \end{aligned}$$

1.4.24.

$$\begin{aligned} 9x^2 &= 36 \\ x^2 &= 4 \\ x &= \pm\sqrt{4} = \pm 2 \end{aligned}$$

1.4.25.

$$\begin{aligned} (x-4)^2 &= 49 \\ x-4 &= \pm 7 \\ x &= 4 \pm 7 \\ x &= 11 \text{ or } x = -3 \end{aligned}$$

1.4.26.

$$
\begin{aligned}
(x-5)^2 &= 25 \\
x-5 &= \pm 5 \\
x &= 5 \pm 5 \\
x &= 0 \quad \text{or} \quad x = 10
\end{aligned}
$$

1.4.27.

$$
\begin{aligned}
(x+2)^2 &= 14 \\
x+2 &= \pm\sqrt{14} \\
x &= -2 \pm \sqrt{14} \\
&\approx 1.74,\quad -5.74
\end{aligned}
$$

1.4.28.

$$
\begin{aligned}
(x+9)^2 &= 24 \\
x+9 &= \pm\sqrt{24} \\
x &= -9 \pm 2\sqrt{6} \\
&\approx -4.10,\quad -13.90
\end{aligned}
$$

1.4.29.

$$
\begin{aligned}
(2x-1)^2 &= 18 \\
2x-1 &= \pm\sqrt{18} \\
2x &= 1 \pm 3\sqrt{2} \\
x &= \frac{1 \pm 3\sqrt{2}}{2} \\
&\approx 2.62,\quad -1.62
\end{aligned}
$$

1.4.30.

$$
\begin{aligned}
(4x+7)^2 &= 44 \\
4x+7 &= \pm\sqrt{44} \\
4x &= -7 \pm 2\sqrt{11} \\
x &= \frac{-7 \pm 2\sqrt{11}}{4} = -\frac{7}{4} \pm \frac{\sqrt{11}}{2} \\
&\approx -0.09,\quad -3.41
\end{aligned}
$$

1.4.31.

$$(x-7)^2 = (x+3)^2$$
$$x-7 = \pm(x+3)$$

$$x-7 = x+3 \quad \text{or} \quad x-7 = -x-3$$
$$-7 \neq 3 \quad \text{or} \quad 2x = 4$$
$$x = 2$$

The only solution of the equation is $x = 2$.

1.4.32.

$$(x+5)^2 = (x+4)^2$$
$$x+5 = \pm(x+4)$$

$$x+5 = +(x+4) \quad \text{or} \quad x+5 = -(x+4)$$
$$5 \neq 4 \quad \text{or} \quad x+5 = -x-4$$
$$2x = -9$$
$$x = -\frac{9}{2}$$

The only solution of the equation is $x = -\frac{9}{2}$.

1.4.33.

$$x^2 + 4x - 32 = 0$$
$$x^2 + 4x = 32$$
$$x^2 + 4x + 2^2 = 32 + 2^2$$
$$(x+2)^2 = 36$$
$$x + 2 = \pm 6$$
$$x = -2 \pm 6$$
$$x = 4 \quad \text{or} \quad x = -8$$

1.4.34.

$$x^2 - 2x - 3 = 0$$
$$x^2 - 2x = 3$$
$$x^2 - 2x + (-1)^2 = 3 + (1)^2$$

$$
\begin{aligned}
(x-1)^2 &= 4\\
x-1 &= \pm\sqrt{4}\\
x &= 1 \pm 2\\
x &= 3 \quad \text{or} \quad x = -1
\end{aligned}
$$

1.4.35.

$$
\begin{aligned}
x^2+4x+2 &= 0\\
x^2+4x &= -2\\
x^2+4x+2^2 &= -2+2^2\\
(x+2)^2 &= 2\\
x+2 &= \pm\sqrt{2}\\
x &= -2 \pm \sqrt{2}
\end{aligned}
$$

1.4.36.

$$
\begin{aligned}
x^2+8x+14 &= 0\\
x^2+8x &= -14\\
x^2+8x+4^2 &= -14+16\\
(x+4)^2 &= 2\\
x+4 &= \pm\sqrt{2}\\
x &= -4 \pm \sqrt{2}
\end{aligned}
$$

1.4.37.

$$
\begin{aligned}
6x^2-12x &= -3\\
x^2-2x &= -\frac{1}{2}\\
x^2-2x+1^2 &= -\frac{1}{2}+1^2\\
(x-1)^2 &= \frac{1}{2}\\
x-1 &= \pm\sqrt{\frac{1}{2}}\\
x &= 1\pm\sqrt{\frac{1}{2}}\\
x &= 1\pm\frac{\sqrt{2}}{2}
\end{aligned}
$$

1.4.38.

$$\begin{aligned}
4x^2 - 4x &= 1\\
x^2 - x &= \frac{1}{4}\\
x^2 - x + \left(-\frac{1}{2}\right)^2 &= \frac{1}{4} + \left(-\frac{1}{2}\right)^2\\
\left(x - \frac{1}{2}\right)^2 &= \frac{1}{2}\\
x - \frac{1}{2} &= \pm\frac{\sqrt{2}}{2}\\
x &= \frac{1}{2} \pm \frac{\sqrt{2}}{2}\\
x &= \frac{1 \pm \sqrt{2}}{2}
\end{aligned}$$

1.4.39.

$$\begin{aligned}
7 + 2x - x^2 &= 0\\
-x^2 + 2x + 7 &= 0\\
x^2 - 2x - 7 &= 0\\
x^2 - 2x &= 7\\
x^2 - 2x + (-1)^2 &= 7 + (-1)^2\\
(x - 1)^2 &= 8\\
x - 1 &= \pm 2\sqrt{2}\\
x &= 1 \pm 2\sqrt{2}
\end{aligned}$$

1.4.40.

$$\begin{aligned}
-x^2 + x - 1 &= 0\\
x^2 - x + 1 &= 0\\
x^2 - x + \tfrac{1}{4} &= -1 + \tfrac{1}{4}\\
\left(x - \tfrac{1}{2}\right)^2 &= -\tfrac{3}{4}
\end{aligned}$$

No real solution

1.4.41.

$$2x^2 + 5x - 8 = 0$$
$$2x^2 + 5x = 8$$
$$x^2 + \frac{5}{2}x = 4$$
$$x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = 4 + \left(\frac{5}{4}\right)^2$$
$$\left(x + \frac{5}{4}\right)^2 = \frac{89}{16}$$
$$x + \frac{5}{4} = \pm\frac{\sqrt{89}}{4}$$
$$x = -\frac{5}{4} \pm \frac{\sqrt{89}}{4}$$
$$x = \frac{-5 \pm \sqrt{89}}{4}$$

1.4.42.

$$3x^2 - 4x - 7 = 0$$
$$3x^2 - 4x = 7$$
$$x^2 - \frac{4}{3}x = \frac{7}{3}$$
$$x^2 - \frac{4}{3}x + \left(-\frac{2}{3}\right)^2 = \frac{7}{3} + \left(-\frac{2}{3}\right)^2$$
$$\left(x - \frac{2}{3}\right)^2 = \frac{25}{9}$$
$$x - \frac{2}{3} = \pm\frac{5}{3}$$
$$x = \frac{2}{3} \pm \frac{5}{3}$$
$$x = -1 \quad \text{or} \quad x = \frac{7}{3}$$

1.4.43.

$$9x^2 + 12x + 4 = 0$$
$$b^2 - 4ac = (12)^2 - 4(9)(4) = 0$$

One repeated real solution

1.4.44.

$$x^2 + 2x + 4 = 0$$
$$b^2 - 4ac = (2)^2 - 4(1)(4) = 4 - 16 = -12 < 0$$

No real solution

1.4.45.

$$2x^2 - 5x + 5 = 0$$
$$b^2 - 4ac = (-5)^2 - 4(2)(5) = -15 < 0$$

No real solution

1.4.46.

$$-5x^2 - 4x + 1 = 0$$
$$b^2 - 4ac = (-4)^2 - 4(-5)(1) = 16 + 20 = 36 > 0$$

Two real solutions

1.4.47.

$$2x^2 - x - 1 = 0$$
$$b^2 - 4ac = (-1)^2 - 4(2)(-1) = 9 > 0$$

Two real solutions

1.4.48.

$$x^2 - 4x + 4 = 0$$
$$b^2 - 4ac = (-4)^2 - 4(1)(4) = 16 - 16 = 0$$

One repeated solution

1.4.49.

$$2x^2 + x - 1 = 0$$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{-1 \pm 3}{4} = \frac{1}{2},\quad -1 \end{aligned}$$

1.4.50.

$$2x^2 - x - 1 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1+8}}{4} \\ &= \frac{1 \pm 3}{4} = 1, \quad -\frac{1}{2} \end{aligned}$$

1.4.51.

$$16x^2 + 8x - 3 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(16)(-3)}}{2(16)} \\ &= \frac{-8 \pm 16}{32} = \frac{1}{4}, \quad -\frac{3}{4} \end{aligned}$$

1.4.52.

$$25x^2 - 20x + 3 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-20) \pm \sqrt{(-20)^2 - (25)(3)}}{2(25)} \\ &= \frac{20 \pm \sqrt{400 - 300}}{50} \\ &= \frac{20 \pm 10}{50} = \frac{3}{5}, \quad \frac{1}{5} \end{aligned}$$

1.4.53.

$$x^2 + 8x - 4 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-8 \pm \sqrt{8^2 - 4(1)(-4)}}{2(1)} \\ &= \frac{-8 \pm 4\sqrt{5}}{2} = -4 \pm 2\sqrt{5} \end{aligned}$$

1.4.54.

$$9x^2 + 30x + 25 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-30 \pm \sqrt{30^2 - 4(9)(25)}}{2(9)} \\ &= \frac{-30 \pm 0}{18} = -\frac{5}{3} \end{aligned}$$

1.4.55.

$$2x^2 - 7x + 1 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)} \\ &= \frac{7 \pm \sqrt{49 - 8}}{2(2)} \\ &= \frac{7 \pm \sqrt{41}}{4} \\ &= \frac{7}{4} \pm \frac{\sqrt{41}}{4} \end{aligned}$$

1.4.56.

$$36x^2 + 24x - 7 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-24 \pm \sqrt{24^2 - 4(36)(-7)}}{2(36)} \\ &= \frac{-24 \pm \sqrt{576 + 1008}}{72} \\ &= \frac{-24 \pm \sqrt{(144)(11)}}{72} \\ &= -\frac{1}{3} \pm \frac{\sqrt{11}}{6} \end{aligned}$$

1.4.57.

$$2 + 2x - x^2 = 0$$

$$-x^2 + 2x + 2 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)} \\ &= \frac{-2 \pm 2\sqrt{3}}{-2} \\ &= 1 \pm \sqrt{3} \end{aligned}$$

1.4.58.

$$x^2 + 10 + 8x = 0$$

$$x^2 + 8x + 10 = 0$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(8) \pm \sqrt{(8)^2 - 4(1)(10)}}{2(1)} \\ &= \frac{-8 \pm \sqrt{64 - 40}}{2(1)} \end{aligned}$$

$$= \frac{-8 \pm \sqrt{24}}{2}$$
$$= \frac{-8 \pm 2\sqrt{6}}{2}$$
$$= \frac{2\left(-4 \pm \sqrt{6}\right)}{2}$$
$$= -4 \pm \sqrt{6}$$

1.4.59.

$$x^2 + 16 = -12x$$
$$x^2 + 12x + 16 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(12) \pm \sqrt{(12)^2 - 4(1)(16)}}{2(1)}$$
$$= \frac{-12 \pm \sqrt{144 - 64}}{2(1)}$$
$$= \frac{-12 \pm \sqrt{80}}{2}$$
$$= \frac{-12 \pm 4\sqrt{5}}{2}$$
$$= \frac{4\left(-3 \pm \sqrt{5}\right)}{2}$$
$$= 2\left(-3 \pm \sqrt{5}\right) = -6 \pm 2\sqrt{5}$$

1.4.60.

$$4x = 8 - x^2$$
$$x^2 + 4x - 8 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)}$$
$$= \frac{-4 \pm 4\sqrt{3}}{2} = -2 \pm 2\sqrt{3}$$

1.4.61.

$$4x^2 + 6x = 8$$
$$4x^2 + 6x - 8 = 0$$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(6) \pm \sqrt{(6)^2 - 4(4)(-8)}}{2(4)} \\
&= \frac{-6 \pm \sqrt{36 + 128}}{2(4)} \\
&= \frac{-6 \pm \sqrt{164}}{8} \\
&= \frac{-6 \pm 2\sqrt{41}}{8} \\
&= \frac{2\left(-3 \pm \sqrt{41}\right)}{8} \\
&= \frac{-3 \pm \sqrt{41}}{4} \\
&= \frac{-3}{4} \pm \frac{\sqrt{41}}{4}
\end{aligned}$$

1.4.62.

$$16x^2 + 5 = 40x$$
$$16x^2 - 40x + 5 = 0$$

$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-40) \pm \sqrt{(-40)^2 - 4(16)(5)}}{2(16)} \\
&= \frac{40 \pm \sqrt{1600 - 320}}{32} \\
&= \frac{40 \pm \sqrt{1280}}{32} \\
&= \frac{40 \pm 16\sqrt{5}}{32} \\
&= \frac{5}{4} \pm \frac{\sqrt{5}}{2}
\end{aligned}$$

1.4.63.

$$28x - 49x^2 = 4$$
$$-49x^2 + 28x - 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-28 \pm \sqrt{28^2 - 4(-49)(-4)}}{2(-49)}$$
$$= \frac{-28 \pm 0}{-98} = \frac{2}{7}$$

1.4.64.

$$3x + x^2 - 1 = 0$$
$$x^2 + 3x - 1 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(1)(-1)}}{2(1)}$$
$$= \frac{-3 \pm \sqrt{13}}{2} = -\frac{3}{2} \pm \frac{\sqrt{13}}{2}$$

1.4.65.

$$8t = 5 + 2t^2$$
$$-2t^2 + 8t - 5 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-8 \pm \sqrt{8^2 - 4(-2)(-5)}}{2(-2)}$$
$$= \frac{-8 \pm 2\sqrt{6}}{-4} = 2 \pm \frac{\sqrt{6}}{2}$$

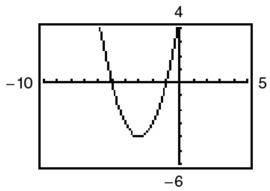
1.4.66.

$$25h^2 + 80h + 61 = 0$$

$$\begin{aligned} h &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-80 \pm \sqrt{80^2 - 4(25)(61)}}{2(25)} \\ &= \frac{-80 \pm \sqrt{6400 - 6100}}{50} \\ &= -\frac{8}{5} \pm \frac{10\sqrt{3}}{50} \\ &= -\frac{8}{5} \pm \frac{\sqrt{3}}{5} \end{aligned}$$

1.4.67.

(a) $y = (x+3)^2 - 4$



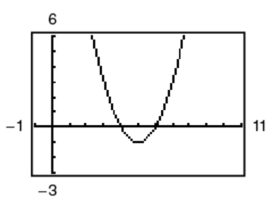
(b) The x-intercepts are $(-1,\ 0)$ and $(-5,\ 0)$.

(c)
$$\begin{aligned} 0 &= (x+3)^2 - 4 \\ 4 &= (x+3)^2 \\ \pm\sqrt{4} &= x + 3 \\ -3 \pm 2 &= x \\ x &= -1 \quad \text{or} \quad x = -5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x+3)^2 - 4$.

1.4.68.

(a) $y = (x-5)^2 - 1$



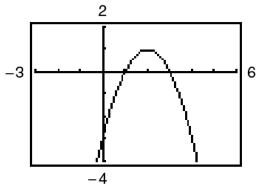
(b) The x-intercepts are $(4,\ 0)$ and $(6,\ 0)$.

(c) $$\begin{aligned} 0 &= (x-5)^2 - 1 \\ (x-5)^2 &= 1 \\ x-5 &= \pm\sqrt{1} \\ x &= 5 \pm 1 = 6,\ 4 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = (x-5)^2 - 1$.

1.4.69.

(a) $y = 1-(x-2)^2$



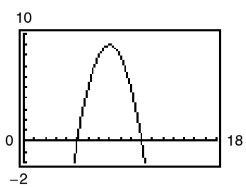
(b) The x-intercepts are $(1,\ 0)$ and $(3,\ 0)$.

(c) $$\begin{aligned} 0 &= 1-(x-2)^2 \\ (x-2)^2 &= 1 \\ x-2 &= \pm 1 \\ x &= 2 \pm 1 \\ x &= 3 \quad \text{or} \quad x = 1 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 1-(x-2)^2$.

1.4.70.

(a) $y = 9-(x-8)^2$



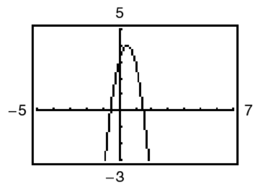
(b) The x-intercepts are $(5,\ 0)$ and $(11,\ 0)$.

(c)
$$\begin{aligned} 0 &= 9-(x-8)^2 \\ (x-8)^2 &= 9 \\ x-8 &= \pm\sqrt{9} \\ x &= 8\pm 3 = 11,\quad 5 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 9-(x-8)^2$.

1.4.71.

(a) $y = -4x^2 + 4x + 3$



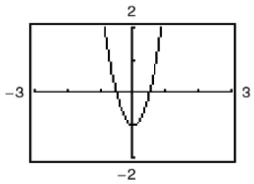
(b) The x-intercepts are $\left(-\frac{1}{2},\ 0\right)$ and $\left(\frac{3}{2},\ 0\right)$.

(c)
$$\begin{aligned} 0 &= -4x^2 + 4x + 3 \\ 4x^2 - 4x &= 3 \\ 4\left(x^2 - x\right) &= 3 \\ x^2 - x &= \frac{3}{4} \\ x^2 - x + \left(\frac{1}{2}\right)^2 &= \frac{3}{4} + \left(\frac{1}{2}\right)^2 \\ \left(x - \frac{1}{2}\right)^2 &= 1 \\ x - \frac{1}{2} &= \pm\sqrt{1} \\ x &= \frac{1}{2} \pm\ 1 \\ x &= \frac{3}{2} \quad \text{or} \quad x = -\frac{1}{2} \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation
$0 = -4x^2 + 4x + 3.$

1.4.72.

(a) $y = 4x^2 - 1$



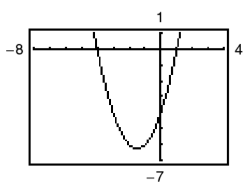
(b) The x-intercepts are $\left(-\frac{1}{2},\ 0\right)$ and $\left(\frac{1}{2},\ 0\right)$.

(c)
$$\begin{aligned} 0 &= 4x^2 - 1 \\ 4x^2 &= 1 \\ x^2 &= \frac{1}{4} \\ x &= \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2} \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = 4x^2 - 1$.

1.4.73.

(a) $y = x^2 + 3x - 4$



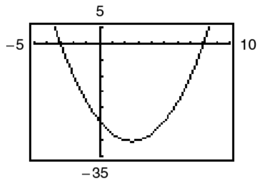
(b) The x-intercepts are $(-4,\ 0)$ and $(1,\ 0)$.

(c)
$$\begin{aligned} 0 &= x^2 + 3x - 4 \\ 0 &= (x+4)(x-1) \\ x + 4 = 0 \quad &\text{or} \quad x - 1 = 0 \\ x = -4 \quad &\text{or} \quad x = 1 \end{aligned}$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 + 3x - 4$.

1.4.74.

(a) $y = x^2 - 5x - 24$



(b) The x-intercepts are $(8,\ 0)$ and $(-3,\ 0)$.

(c)
$$0 = x^2 - 5x - 24$$
$$(x-8)(x+3) = 0$$

$$x - 8 = 0 \Rightarrow x = 8$$
$$x + 3 = 0 \Rightarrow x = -3$$

(d) The x-intercepts of the graphs are solutions of the equation $0 = x^2 - 5x - 24$.

1.4.75.

$x^2 - 2x - 1 = 0$ Complete the square.

$$x^2 - 2x = 1$$
$$x^2 - 2x + 1^2 = 1 + 1^2$$
$$(x-1)^2 = 2$$
$$x - 1 = \pm\sqrt{2}$$
$$x = 1 \pm \sqrt{2}$$

1.4.76.

$14x^2 + 42x = 0$ Factor.

$$14x(x + 3x) = 0$$
$$x(x+3) = 0$$

$$x = 0 \quad \text{or} \quad x + 3 = 0$$
$$x = -3$$

1.4.77.

$(x+2)^2 = 64$ Extract square roots.

$$x + 2 = \pm 8$$

$$\begin{aligned} x+2&=8 \quad \text{or} \quad x+2=-8 \\ x&=6 \quad \text{or} \quad x=-10 \end{aligned}$$

1.4.78.

$$\begin{aligned} x^2-14x+49&=0 \quad \text{Extract square roots.} \\ (x-7)^2&=0 \\ x-7&=0 \\ x&=7 \end{aligned}$$

1.4.79.

$$\begin{aligned} x^2-x-\frac{11}{4}&=0 \quad \text{Complete the square.} \\ x^2-x&=\frac{11}{4} \\ x^2-x+\left(\frac{1}{2}\right)^2&=\frac{11}{4}+\left(\frac{1}{2}\right)^2 \\ \left(x-\frac{1}{2}\right)^2&=\frac{12}{4} \\ x-\frac{1}{2}&=\pm\sqrt{\frac{12}{4}} \\ x&=\frac{1}{2}\pm\sqrt{3} \end{aligned}$$

1.4.80.

$$\begin{aligned} x^2+3x-\frac{3}{4}&=0 \quad \text{Complete the square.} \\ x^2+3x+\left(\frac{3}{2}\right)^2&=\frac{3}{4}+\frac{9}{4} \\ \left(x+\frac{3}{2}\right)^2&=3 \\ x+\frac{3}{2}&=\pm\sqrt{3} \\ x&=-\frac{3}{2}\pm\sqrt{3} \end{aligned}$$

1.4.81.

$$
\begin{aligned}
3x + 4 &= 2x^2 - 7 \quad \text{Quadratic Formula}\\
0 &= 2x^2 - 3x - 11\\
x &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-11)}}{2(2)}\\
&= \frac{3 \pm \sqrt{97}}{4}\\
&= \frac{3}{4} \pm \frac{\sqrt{97}}{4}
\end{aligned}
$$

1.4.82.

$$
\begin{aligned}
(x+1)^2 &= x^2 \quad \text{Extract square roots.}\\
x^2 &= (x+1)^2\\
x &= \pm(x+1)
\end{aligned}
$$

For $x = +(x+1)$:

$0 \neq 1$ No solution

For $x = -(x+1)$:

$$
\begin{aligned}
2x &= -1\\
x &= -\frac{1}{2}
\end{aligned}
$$

1.4.83.

$$
\begin{aligned}
\frac{1}{x^2 - 2x + 5} &= \frac{1}{x^2 - 2x + 1^2 + 5}\\
&= \frac{1}{(x-1)^2 + 4}
\end{aligned}
$$

1.4.84.

$$
\begin{aligned}
\frac{1}{x^2 + 6x + 10} &= \frac{1}{x^2 + 6x + (3)^2 - (3)^2 + 10}\\
&= \frac{1}{x^2 + 6x + 9 + 1}\\
&= \frac{1}{(x+3)^2 + 1}
\end{aligned}
$$

1.4.85.

$$\begin{aligned}\frac{4}{x^2+10x+74} &= \frac{4}{x^2+10x+(5)^2-(5)^2+74}\\ &= \frac{4}{x^2+10x+25+49}\\ &= \frac{4}{(x+5)^2+49}\end{aligned}$$

1.4.86.

$$\begin{aligned}\frac{5}{x^2-18x+162} &= \frac{5}{x^2-18x+\left(9\right)^2-(9)^2+162}\\ &= \frac{5}{x^2-18x+81+81}\\ &= \frac{5}{(x-9)^2+81}\end{aligned}$$

1.4.87.

$$\begin{aligned}\frac{1}{\sqrt{3+2x-x^2}} &= \frac{1}{\sqrt{-1\,(x^2-2x-3)}}\\ &= \frac{1}{\sqrt{-1\left[x^2-2x+(1)^2-(1)^2-3\right]}}\\ &= \frac{1}{\sqrt{-1\,(x^2-2x+1)+4}}\\ &= \frac{1}{\sqrt{4-(x-1)^2}}\end{aligned}$$

1.4.88.

$$\begin{aligned}\frac{1}{\sqrt{9+8x-x^2}} &= \frac{1}{\sqrt{-1\,(x^2-8x-9)}}\\ &= \frac{1}{\sqrt{-1\left[x^2-8x+(4)^2-(4)^2-9\right]}}\\ &= \frac{1}{\sqrt{-1\left[(x^2-8x+16)-25\right]}}\\ &= \frac{1}{\sqrt{25-(x-4)^2}}\end{aligned}$$

1.4.89.

$$\frac{1}{\sqrt{12+4x-x^2}} = \frac{1}{\sqrt{-1(x^2-4x-12)}} = \frac{1}{\sqrt{-1\left[x^2-4x+(2)^2-(2)^2-12\right]}}$$
$$= \frac{1}{\sqrt{-1\left[(x^2-4x+4)-16\right]}} = \frac{1}{\sqrt{16-(x-2)^2}}$$

1.4.90.

$$\frac{1}{\sqrt{16-6x-x^2}} = \frac{1}{\sqrt{16-1\left(x^2+6x\right)}}$$
$$= \frac{1}{\sqrt{16-\left(x^2+6x+3^2\right)+9}}$$
$$= \frac{1}{\sqrt{25-\left(x+3\right)^2}}$$

1.4.91.

(a) $w\left(w+14\right)=1632$

(b) $w^2+14w-1632=0$

$\left(w+48\right)\left(w-34\right)=0$

$w=-48$ or $w=34$

Because the width must be greater than zero, $w=34$ feet and the length is $w+14=48$ feet.

1.4.92.

Total fencing: $4x+3y=100$

Total area: $2xy=350$

$$x=\frac{100-3y}{4}$$
$$2\left(\frac{100-3y}{4}\right)y=350$$
$$\frac{1}{2}\left(100y-3y^2\right)-350=0$$

$$\begin{aligned} 100y - 3y^2 - 700 &= 0 \\ -3y^2 + 100y - 700 &= 0 \\ (3y - 70)(-y + 10) &= 0 \end{aligned}$$

$$3y - 70 = 0 \Rightarrow y = \frac{70}{3}$$

$$-y + 10 = 0 \Rightarrow y = 10$$

For $y = \dfrac{70}{3}$:

$$2x\left(\frac{70}{3}\right) = 350$$

$$x = 7.5$$

For $y = 10$:

$$2x(10) = 350$$

$$x = 17.5$$

There are two solutions: $x = 7.5$ meters and $y = 23\frac{1}{3}$ meters or $x = 17.5$ meters and $y = 10$ meters.

1.4.93.

(a) Volume is 1024 cubic feet.

$$\begin{aligned} V &= l \cdot w \cdot h \\ &= x(x+1)(4) \\ &= 4x^2 + 4x \end{aligned}$$

So,

$$\begin{aligned} 4x^2 + 4x &= 1024 \\ 4x^2 + 4x - 1024 &= 0 \\ 4(x^2 + x - 256) &= 0 \\ x^2 + x - 256 &= 0 \end{aligned}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1) - 256}}{2(1)}$$

$$x \approx 15.51$$

$$x + 1 \approx 16.51$$

So the base of the pool is approximately $15.51 \text{ feet} \times 16.51 \text{ feet}$.

(b) Because 1 cubic foot of water weighs approximately 62.4 pounds,

$$1024 \text{ cubic feet} \cdot \frac{62.4 \text{ pounds}}{1 \text{ cubic foot}} = 63{,}897.6 \text{ pounds.}$$

1.4.94.

Original arrangement: x rows, y seats per row, $xy = 72,\ y = \dfrac{72}{x}$

New arrangement: $(x-2)$ rows, $(y+3)$ seats per row

$$\begin{aligned}
(x-2)(y+3) &= 72\\
(x-2)\left(\frac{72}{x}+3\right) &= 72\\
x(x-2)\left(\frac{72}{x}+3\right) &= 72x\\
(x-2)(72+3x) &= 72x\\
72x + 3x^2 - 144 - 6x &= 72x\\
3x^2 - 6x - 144 &= 0\\
x^2 - 2x - 48 &= 0\\
(x-8)(x+6) &= 0
\end{aligned}$$

$x - 8 = 0 \Rightarrow x = 8$
$x + 6 = 0 \Rightarrow x = -6$

Originally, there were 8 rows of seats with $\dfrac{72}{9} = 9$ seats per row.

1.4.95.

(a) $s = -16t^2 + v_0 t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 984$.

Thus, $s = -16t^2 + 984$.

(b) $s = -16(4)^2 + 984 = 728$ feet

(c)
$$\begin{aligned}
0 &= -16t^2 + 984\\
16t^2 &= 984\\
t^2 &= \frac{984}{16}\\
t &= \sqrt{\frac{984}{16}} = \frac{\sqrt{246}}{2} \approx 7.84
\end{aligned}$$

It will take the coin about 7.84 seconds to strike the ground.

1.4.96.

(a) $s = -16t^2 + v_0 t + s_0$

$s = -16t^2 + 550$

Let $s = 0$ and solve for t.

$$0 = -16t^2 + 550$$
$$16t^2 = 550$$
$$t^2 = \frac{550}{16}$$
$$t = \sqrt{\frac{550}{16}}$$
$$t \approx 5.86$$

The supply package will take about 5.86 seconds to reach the ground.

(b) *Verbal Model:* $(\text{Distance}) = (\text{Rate}) \cdot (\text{Time})$

Labels: Distance $= d$

Rate = 138 miles per hour

Time = $\dfrac{5.86 \text{ seconds}}{3600 \text{ seconds per hour}}$

≈ 0.0016 hour

Equation: $d = (138)(0.0016) \approx 0.22$ mile

The supply package will travel about 0.2 mile.

1.4.97.

(a) $s = -16t^2 + v_0 t + s_0$

Since the object was dropped, $v_0 = 0$, and the initial height is $s_0 = 1815$. Thus, $s = -16t^2 + 1815$.

(b)

Time, t	0	2	4	6	8	10	12
Height, s	1815	1751	1559	1239	791	215	-489

(c) The object reaches the ground between $t = 10$ seconds and $t = 12$ seconds. Numerical approximation will vary, though 10.7 seconds is a reasonable estimate.

(d) $0 = -16t^2 + 1815$

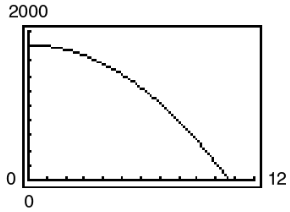
$16t^2 = 1815$

$t^2 = \dfrac{1815}{16}$

$t = \sqrt{\dfrac{1815}{16}} = \dfrac{11\sqrt{15}}{4} \approx 10.65$

It will take the object about 10.65 seconds to reach the ground.

(e)



The zero of the graph is at $t \approx 10.65$ seconds.

1.4.98.

(a) $s = -16t^2 + v_0 t + s_0$

$v_0 = 100 \text{ mph} = \dfrac{(100)(5280)}{3600} = 146\dfrac{2}{3}$ ft/sec

$s_0 = 6 \text{ feet } 3 \text{ inches} = 6\dfrac{1}{4}$ feet

$s = -16t^2 + 146\dfrac{2}{3}t + 6\dfrac{1}{4}$

$s = -16t^2 + 146\dfrac{2}{3}t + 6.25$

(b) When $t = 4: s(4) \approx 336.92$ feet

When $t = 5: s(5) \approx 339.58$ feet

When $t = 6: s(6) \approx 310.25$ feet

During the interval $4 \le t \le 6$, the baseball reached its maximum height.

(c) The ball hits the ground when $s = 0$.

$-16t^2 + 146\frac{2}{3}t + 6\frac{1}{4} = 0$

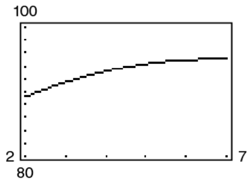
Using the Quadratic Formula,

$$t = \frac{-146\frac{2}{3} \pm \sqrt{\left(146\frac{2}{3}\right)^2 - 4(-16)\left(6\frac{1}{4}\right)}}{2(-16)} \approx \frac{-146\frac{2}{3} \pm 148.02}{-32},$$

$t \approx -0.042$ or $t \approx 9.209$. Time is always positive, so the ball will be in the air for approximately 9.209 seconds.

1.4.99.

(a) $L = -0.270t^2 + 3.59t + 83.1, \quad 2 \le t \le 7$



Minimum at $t = 2$ (89.2% at 2 P.M.)

Maximum at $t \approx 6.65$ (95.0% at 6:38 P.M.)

(b) $L = -0.270t^2 + 3.59t + 83.1$

$$93 = -0.270t^2 + 3.59t + 83.1$$
$$0 = -0.270t^2 + 3.59t - 9.9$$
$$0 = 0.270t^2 - 3.59t + 9.9$$

Using the Quadratic Formula,

$$t = \frac{-(-3.59) \pm \sqrt{(-3.59)^2 - 4(0.270)(9.9)}}{2(0.270)} = \frac{3.59 \pm \sqrt{2.1961}}{0.54},$$

$t \approx 3.9$ and $t \approx 9.4$

Because the domain of the model is $2 \le t \le 7$, $t \approx 3.9$ is the only solution. The patient's blood oxygen level was 93% at approximately 4:00 P.M.

1.4.100.

(a) $150 = 0.45x^2 - 1.65x + 50.75$

$$0 = 0.45x^2 - 1.65x - 99.25$$
$$x = \frac{1.65 \pm \sqrt{(-1.65)^2 - 4(0.45)(-99.25)}}{2(0.45)}$$
$$\approx -13.1, \ 16.8$$

Because $10 \le x \le 25$, choose $16.8^\circ\mathrm{C}$.

$x = 10: \quad 0.45(10)^2 - 1.65(10) + 50.75 = 79.25$

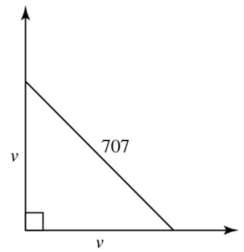
(b) $x = 20: \quad 0.45(20)^2 - 1.65(20) + 50.75 = 197.75$

$197.75 \div 79.25 \approx 2.5$

Oxygen consumption is increased by a factor of approximately 2.5.

1.4.101.

Let v be the speed of the two planes. After 1 hour, the planes have traveled v miles.



So, $2v^2 = 707^2 \Rightarrow v = \dfrac{707}{\sqrt{2}} \approx 500.$

The speed is approximately 500 miles per hour.

1.4.102.

(a) *Model:* $(\text{winch})^2 + (\text{distance to dock})^2 = (\text{length of rope})^2$

Labels: $\text{winch} = 15,\ \text{distance to dock} = x,\ \text{length of rope} = l$

Equation: $15^2 + x^2 = l^2$

(b) When $l = 75:\ 15^2 + x^2 = 75^2$

$$x^2 = 5625 - 225 = 5400$$
$$x = \sqrt{5400} = 30\sqrt{6} \approx 73.5$$

The boat is approximately 73.5 feet from the dock when there is 75 feet of rope out.

1.4.103.

$$-3x^2 + x = -5$$
$$-3x^2 + x + 5 = 0$$

$$b^2 - 4ac = (1)^2 - 4(-3)(5) = 1 + 60 = 61 > 0.$$

True. The quadratic equation has two real solutions.

1.4.104.

False. The product must equal zero for the Zero Factor Property to be used.

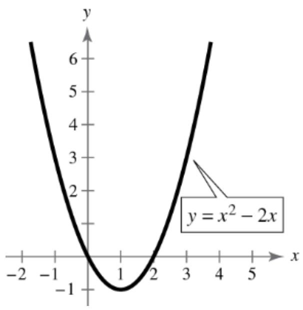
1.4.105.

Yes, the vertex of the parabola would be on the x-axis.

1.4.106.

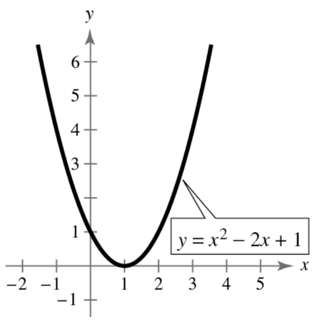
(a) The discriminant is positive because the graph of $y = x^2 - 2x$ has two x-intercepts.

$$b^2 = 4ac = (-2)^2 - 4(1)(0) = 4$$



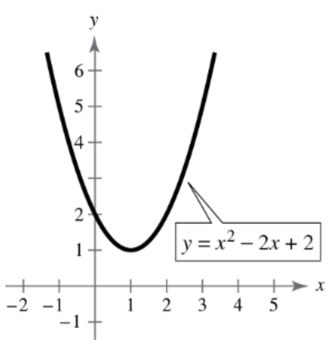
(b) The discriminant is zero because the graph of $y = x^2 - 2x + 1$ has one x-intercept.

$$b^2 = 4ac = (-2)^2 - 4(1)(1) = 0$$



(c) The discriminant is negative because the graph of $y = x^2 - 2x + 2$ has no x-intercepts.

$$b = 4ac = (-2)^2 - 4(1)(2) = -4$$



1.4.107.

The equation in general form is $3x^2 + x - 3 = 0,$ so $a = 3,\ b = 1,\ c = -3.$

1.4.108.

The factors of the quadratic equation should be $(x-2)$ and $(x-4)$.

$(x-2)(x-4) = 0 \Rightarrow x^2 - 6x + 8 = 0$ The factors of the quadratic equation should be $(x-2)$ and $(x-4)$.

$(x-2)(x-4) = 0 \Rightarrow x^2 - 6x + 8 = 0$

1.4.109.

Sample answer:
$$\begin{aligned}(x-0)(x-4) &= 0\\ x(x-5) &= 0\\ x^2 - 4x &= 0\end{aligned}$$

1.4.110.

Sample answer:
$$\begin{aligned}(x-(-2))(x-(-8)) &= 0\\ (x+2)(x+8) &= 0\\ x^2 + 10x + 16 &= 0\end{aligned}$$

1.4.111.

One possible equation is:

$$\begin{aligned}(x-8)(x-14) &= 0\\ x^2 - 22x + 112 &= 0\end{aligned}$$

Any nonzero multiple of this equation would also have these solutions.

1.4.112.

$x = \dfrac{1}{6} \Rightarrow 6x = 1 \Rightarrow 6x - 1$ is a factor.

$x = -\dfrac{2}{5} \Rightarrow 5x = -2 \Rightarrow 5x + 2$ is a factor.

$$\begin{aligned}(6x-1)(5x+2) &= 0\\ 30x^2 + 7x - 2 &= 0\end{aligned}$$

1.4.113.

One possible equation is:

$$\begin{aligned}\left[x-\left(1+\sqrt{2}\right)\right]\left[x-\left(1-\sqrt{2}\right)\right]&=0\\ \left[\left(x-1\right)-\sqrt{2}\right]\left[\left(x-1\right)+\sqrt{2}\right]&=0\\ \left(x-1\right)^2-\left(\sqrt{2}\right)^2&=0\\ x^2-2x+1-2&=0\\ x^2-2x-1&=0\end{aligned}$$

Any nonzero multiple of this equation would also have these solutions.

1.4.114.

$x=-3+\sqrt{5},\ x=-3-\sqrt{5},$ so:

$$\begin{aligned}\left(x-\left(-3+\sqrt{5}\right)\right)\left(x-\left(-3-\sqrt{5}\right)\right)&=0\\ \left(x+3-\sqrt{5}\right)\left(x+3+\sqrt{5}\right)&=0\\ x^2+6x+4&=0\end{aligned}$$

1.4.115.

$$\left(x^2+5x+11\right)\ +\ \left(2x^2-13x+16\right)\ =\ 3x^2-8x+27$$

1.4.116.

$$\left(7x^2-8x+4\right)+\left(9x^3+3x^2+x\right)=9x^3+10x^2-7x+4$$

1.4.117.

$$\left(12x^2-15\right)-\left(x^2-19x-5\right)=12x^2-15-x^2+19x+5=11x^2+19x-10$$

1.4.118.

$$\left(x^2-3x-2\right)-\left(x^2-2\right)-\left(x-3\right)=x^2-3x-2-x^2+2-x+3=-4x+3$$

1.4.119.

$$\left(x+6\right)\left(3x-5\right)=3x^2+18x-5x-30=3x^2+13x-30$$

1.4.120.

$$\left(3x+13\right)\left(4x-7\right)=12x^2+52x-21x-91=12x^2+31x-91$$

1.4.121.

$$(2x-9)(2x+9)=(2x)^2-9^2=4x^2-81$$

1.4.122.

$$(4x+1)^2=(4x+1)(4x+1)=(4x)^2+2(4x)(1)+1^2=16x^2+8x+1$$

1.4.123.

$$(2x^2-y)(3x^2+4y)=6x^4-3x^2y+8x^2y-4y^2=6x^4+5x^2y-4y^2$$

1.4.124.

$$(4x^3+7y^2)(3x^3-y^2)=12x^6+21x^3y^2-4x^3y^2-7y^4=12x^6+17x^3y^2-7y^4$$

1.4.125.

$$\begin{aligned}\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)&=\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2\\&=3-2=1\end{aligned}$$

1.4.126.

$$\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)=\left(\sqrt{5}\right)^2-1^2=5-1=4$$

1.4.127.

$$\begin{aligned}&\left(7\sqrt{2}-4+\sqrt{5}\right)\left(7\sqrt{2}+4-\sqrt{5}\right)\\&=\left[7\sqrt{2}-\left(4-\sqrt{5}\right)\right]\left[7\sqrt{2}+\left(4-\sqrt{5}\right)\right]\\&=\left(7\sqrt{2}\right)^2-\left(4-\sqrt{5}\right)^2\\&=98-\left(16-8\sqrt{5}+5\right)\\&=77+8\sqrt{5}\end{aligned}$$

1.4.128.

$$\begin{aligned}\left(2\sqrt{3}+3\sqrt{2}\right)\left(2\sqrt{3}-3\sqrt{2}\right)&=\left(2\sqrt{3}\right)^2-\left(3\sqrt{2}\right)^2\\&=12-18=-6\end{aligned}$$

1.4.129.

$$\frac{12}{5\sqrt{3}}=\frac{12}{5\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{12\sqrt{3}}{5\cdot 3}=\frac{4\sqrt{3}}{5}$$

1.4.130.

$$\frac{4}{\sqrt{10}-2}\cdot\frac{\sqrt{10}+2}{\sqrt{10}+2}=\frac{4\left(\sqrt{10}+2\right)}{10-4}=\frac{4\sqrt{10}+8}{6}$$
$$=\frac{2\sqrt{10}+4}{3}$$

1.4.131.

$$\frac{3}{8+\sqrt{11}}=\frac{3}{8+\sqrt{11}}\cdot\frac{8-\sqrt{11}}{8-\sqrt{11}}$$
$$=\frac{3\left(8-\sqrt{11}\right)}{64-11}$$
$$=\frac{24-3\sqrt{11}}{53}$$

1.4.132.

$$\frac{14}{3\sqrt{10}-1}=\frac{14}{3\sqrt{10}-1}\cdot\frac{3\sqrt{10}+1}{3\sqrt{10}+1}$$
$$=\frac{14\left(3\sqrt{10}+1\right)}{90-1}$$
$$=\frac{14+42\sqrt{10}}{89}$$

SECTION 1.5 COMPLEX NUMBERS

1.5.1.

$\sqrt{-1}$; -1

1.5.2.

principal square

1.5.3.

(a) ii (b) iii (c) i

1.5.4.

To multiply two complex numbers, $(a+bi)(c+di)$, the FOIL Method can be used;

$$\begin{aligned}(a+bi)(c+di) &= ac+adi+bci+bdi^2\\ &= (ac-bd)+(ad+bc)i.\end{aligned}$$

1.5.5.

The additive inverse of $2-4i$ is $-2+4i$.

1.5.6.

The complex conjugate of $2-4i$ is $2+4i$.

1.5.7.

$$\begin{aligned}(5+i)+(2+3i) &= 5+i+2+3i\\ &= 7+4i\end{aligned}$$

1.5.8.

$$(13-2i)+(-5+6i) = 8+4i$$

1.5.9.

$$(9-i)-(8-i) = 1$$

1.5.10.

$$\begin{aligned}(3+2i)-(6+13i) &= 3+2i-6-13i\\ &= -3-11i\end{aligned}$$

1.5.11.

$$13i-(14-7i) + (2-11i) = -14 + 2 + (13 + 7-11)i = -12 + 9i$$

1.5.12.

$$(25+6i)+(-10+11i)-(17-15i)=(25-10-17)+(6+11+15)i=-2+32i$$

1.5.13.

$$\begin{aligned}(1+i)(3-2i)&=3-2i+3i-2i^2\\&=3+i+2=5+i\end{aligned}$$

1.5.14.

$$\begin{aligned}(7-2i)(3-5i)&=21-35i-6i+10i^2\\&=21-41i-10\\&=11-41i\end{aligned}$$

1.5.15.

$$\begin{aligned}12i(1-9i)&=12i-108i^2\\&=12i+108\\&=108+12i\end{aligned}$$

1.5.16.

$$\begin{aligned}-8i(9+4i)&=-72i-32i^2\\&=32-72i\end{aligned}$$

1.5.17.

$$\begin{aligned}(\sqrt{2}+3i)(\sqrt{2}-3i)&=2-9i^2\\&=2+9=11\end{aligned}$$

1.5.18.

$$\begin{aligned}(4+\sqrt{7}i)(4-\sqrt{7}i)&=16-7i^2\\&=16+7=23\end{aligned}$$

1.5.19.

$$\begin{aligned}(6+7i)^2&=36+84i+49i^2\\&=36+84i-49\\&=-13+84i\end{aligned}$$

1.5.20.

$$\begin{aligned}(5-4i)^2 &= 25-40i+16i^2\\ &= 25-40i-16\\ &= 9-40i\end{aligned}$$

1.5.21.

The complex conjugate of $9+2i$ is $9-2i$.

$$\begin{aligned}(9+2i)(9-2i) &= 81-4i^2\\ &= 81+4\\ &= 85\end{aligned}$$

1.5.22.

The complex conjugate of $8-10i$ is $8+10i$.

$$\begin{aligned}(8-10i)(8+10i) &= 64-100i^2\\ &= 64+100\\ &= 164\end{aligned}$$

1.5.23.

The complex conjugate of $-1-\sqrt{5}i$ is $-1+\sqrt{5}i$.

$$\begin{aligned}\left(-1-\sqrt{5}i\right)\left(-1+\sqrt{5}i\right) &= 1-5i^2\\ &= 1+5=6\end{aligned}$$

1.5.24.

The complex conjugate of $-3+\sqrt{2}i$ is $-3-\sqrt{2}i$.

$$\begin{aligned}\left(-3+\sqrt{2}i\right)\left(-3-\sqrt{2}i\right) &= 9-2i^2\\ &= 9+2\\ &= 11\end{aligned}$$

1.5.25.

The complex conjugate of $\sqrt{-20}=2\sqrt{5}i$ is $-2\sqrt{5}i$.

$$\left(2\sqrt{5}i\right)\left(-2\sqrt{5}i\right)=-20i^2=20$$

1.5.26.

The complex conjugate of $\sqrt{-15}=\sqrt{15}i$ is $-\sqrt{15}i$.

$$\left(\sqrt{15}i\right)\left(-\sqrt{15}i\right)=-15i^2=15$$

1.5.27.

The complex conjugate of $1-\sqrt{-6}$ is $1+\sqrt{-6}$.

$$\left(1-\sqrt{-6}\right)\left(1+\sqrt{-6}\right)=1-\left(-6\right)=7$$

1.5.28.

The complex conjugate of $1+\sqrt{-8}$ is $1-\sqrt{-8}$.

$$\left(1+\sqrt{-8}\right)\left(1-\sqrt{-8}\right)=1-\left(-8\right)=9$$

1.5.29.

$$\begin{aligned}\frac{5+i}{5-i}\cdot\frac{(5+i)}{(5+i)}&=\frac{25+10i+i^2}{25-i^2}\\&=\frac{24+10i}{26}=\frac{12}{13}+\frac{5}{13}i\end{aligned}$$

1.5.30.

$$\begin{aligned}\frac{6-7i}{1-2i}\cdot\frac{1+2i}{1+2i}&=\frac{6+12i-7i-14i^2}{1-4i^2}\\&=\frac{20+5i}{5}=4+i\end{aligned}$$

1.5.31.

$$\frac{9-4i}{i}\cdot\frac{-i}{-i}=\frac{-9i+4i^2}{-i^2}=-4-9i$$

1.5.32.

$$\frac{8+16i}{2i}\cdot\frac{-2i}{-2i}=\frac{-16i-32i^2}{-4i^2}=8-4i$$

1.5.33.

$$\begin{aligned}z_1&=5+2i\\z_2&=3-4i\end{aligned}$$

$$\begin{aligned}\frac{1}{z} &= \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{5+2i} + \frac{1}{3-4i}\\ &= \frac{(3-4i)+(5+2i)}{(5+2i)(3-4i)}\\ &= \frac{8-2i}{23-14i}\\ z &= \frac{23-14i}{8-2i}\left(\frac{8+2i}{8+2i}\right)\\ &= \frac{212-66i}{68} \approx 3.118 - 0.971i\end{aligned}$$

1.5.34.

$$z_1 = 9+16i,\ z_2 = 20-10i$$

$$\begin{aligned}\frac{1}{z} &= \frac{1}{z_1} + \frac{1}{z_2} = \frac{1}{9+16i} + \frac{1}{20-10i}\\ &= \frac{20-10i+9+16i}{(9+16i)(20-10i)} = \frac{29+6i}{340+230i}\\ z &= \left(\frac{340+230i}{29+6i}\right)\left(\frac{29-6i}{29-6i}\right) = \frac{11,240+4630i}{877}\\ &= \frac{11,240}{877} + \frac{4630}{877}i\end{aligned}$$

1.5.35.

$$\begin{aligned}\sqrt{-6}\cdot\sqrt{-2} &= \left(\sqrt{6}i\right)\left(\sqrt{2}i\right) = \sqrt{12}i^2 = \left(2\sqrt{3}\right)(-1)\\ &= -2\sqrt{3}\end{aligned}$$

1.5.36.

$$\begin{aligned}\sqrt{-5}\cdot\sqrt{-10} &= \left(\sqrt{5}i\right)\left(\sqrt{10}i\right)\\ &= \sqrt{50}i^2 = 5\sqrt{2}(-1) = -5\sqrt{2}\end{aligned}$$

1.5.37.

$$\left(\sqrt{-15}\right)^2 = \left(\sqrt{15}i\right)^2 = 15i^2 = -15$$

1.5.38.

$$\left(\sqrt{-75}\right)^2 = \left(\sqrt{75}i\right)^2 = 75i^2 = -75$$

1.5.39.

$$\begin{aligned}\sqrt{-8}+\sqrt{-50} &= \sqrt{8}\,i+\sqrt{50}\,i\\ &= 2\sqrt{2}\,i+5\sqrt{2}\,i\\ &= 7\sqrt{2}\,i\end{aligned}$$

1.5.40.

$$\begin{aligned}\sqrt{-45}-\sqrt{-5} &= \sqrt{45}\,i-\sqrt{5}\,i\\ &= 3\sqrt{5}\,i-\sqrt{5}\,i\\ &= 2\sqrt{5}\,i\end{aligned}$$

1.5.41.

$$\begin{aligned}\left(3+\sqrt{-5}\right)\left(7-\sqrt{-10}\right) &= \left(3+\sqrt{5}i\right)\left(7-\sqrt{10}i\right)\\ &= 21-3\sqrt{10}i+7\sqrt{5}i-\sqrt{50}i^2\\ &= \left(21+\sqrt{50}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i\\ &= \left(21+5\sqrt{2}\right)+\left(7\sqrt{5}-3\sqrt{10}\right)i\end{aligned}$$

1.5.42.

$$\begin{aligned}\left(2-\sqrt{-6}\right)^2 &= \left(2-\sqrt{6}i\right)\left(2-\sqrt{6}i\right)\\ &= 4-2\sqrt{6}i-2\sqrt{6}i+6i^2\\ &= 4-2\sqrt{6}i-2\sqrt{6}i+6\left(-1\right)\\ &= 4-6-4\sqrt{6}i\\ &= -2-4\sqrt{6}i\end{aligned}$$

1.5.43.

$x^2-2x+2=0;\ a=1,\ b=-2,\ c=2$

$$\begin{aligned}x &= \frac{-\left(-2\right)\pm\sqrt{\left(-2\right)^2-4\left(1\right)\left(2\right)}}{2\left(1\right)}\\ &= \frac{2\pm\sqrt{-4}}{2}\\ &= \frac{2\pm 2i}{2}\\ &= 1\pm i\end{aligned}$$

1.5.44.

$x^2 + 6x + 10 = 0;\ a = 1,\ b = 6,\ c = 10$

$$\begin{aligned} x &= \frac{-6 \pm \sqrt{6^2 - 4(1)(10)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{-4}}{2} \\ &= \frac{-6 + 2i}{2} \\ &= -3 \pm i \end{aligned}$$

1.5.45.

$4x^2 + 16x + 21 = 0;\ a = 4,\ b = 16,\ c = 21$

$$\begin{aligned} x &= \frac{-16 \pm \sqrt{(16)^2 - 4(4)(21)}}{2(4)} \\ &= \frac{-16 \pm \sqrt{-80}}{8} \\ &= \frac{-16 \pm \sqrt{80}\ i}{8} \\ &= \frac{-16 \pm 4\sqrt{5}\ i}{8} \\ &= -2 \pm \frac{\sqrt{5}}{2} i \end{aligned}$$

1.5.46.

$16t^2 - 4t + 3 = 0;\ a = 16,\ b = -4,\ c = 3$

$$\begin{aligned} t &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(16)(3)}}{2(16)} \\ &= \frac{4 \pm \sqrt{-176}}{32} \\ &= \frac{4 \pm 4\sqrt{11}i}{32} \\ &= \frac{1}{8} \pm \frac{\sqrt{11}}{8} i \end{aligned}$$

1.5.47.

True.

$$\begin{aligned} x^4 - x^2 + 14 &= 56 \\ \left(-i\sqrt{6}\right)^4 - \left(-i\sqrt{6}\right)^2 + 14 &\stackrel{?}{=} 56 \\ 36 + 6 + 14 &\stackrel{?}{=} 56 \\ 56 &= 56 \end{aligned}$$

1.5.48.

False.

$$\begin{aligned} i^{44} + i^{150} - i^{74} - i^{109} + i^{61} &= \left(i^2\right)^{22} + \left(i^2\right)^{75} - \left(i^2\right)^{37} - \left(i^2\right)^{54} i + \left(i^2\right)^{30} i \\ &= \left(-1\right)^{22} + \left(-1\right)^{75} - \left(-1\right)^{37} - \left(-1\right)^{54} i + \left(-1\right)^{30} i \\ &= 1 - 1 + 1 - i + i = 1 \end{aligned}$$

1.5.49.

$$\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$$

1.5.50.

(i) *D*
(ii) *F*
(iii) *B*
(iv) *E*
(v) *A*
(vi) *C*

1.5.51.

$$\begin{aligned} \left(a_1 + b_1 i\right)\left(a_2 + b_2 i\right) &= a_1 a_2 + a_1 b_2 i + a_2 b_1 i + b_1 b_2 i^2 \\ &= \left(a_1 a_2 - b_1 b_2\right) + \left(a_1 b_2 + a_2 b_1\right) i \end{aligned}$$

The complex conjugate of this product is

$\left(a_1 a_2 - b_1 b_2\right) - \left(a_1 b_2 + a_2 b_1\right) i.$

The product of the complex conjugates is

$$\begin{aligned} \left(a_1 - b_1 i\right)\left(a_2 - b_2 i\right) &= a_1 a_2 - a_1 b_2 i - a_2 b_1 i - b_1 b_2 i^2 \\ &= \left(a_1 a_2 - b_1 b_2\right) - \left(a_1 b_2 + a_2 b_1\right) i. \end{aligned}$$

So, the complex conjugate of the product of two complex numbers is the product of their complex conjugates.

1.5.52.

$$(a_1+b_1i)+(a_2+b_2i)=(a_1+a_2)+(b_1+b_2)i$$

The complex conjugate of this sum is
$(a_1+a_2)-(b_1+b_2)i.$

The sum of the complex conjugates is
$(a_1-b_1i)+(a_2-b_2i)=(a_1+a_2)-(b_1+b_2)i.$

So, the complex conjugate of the sum of two complex numbers is the sum of their complex conjugates.

1.5.53.

$$3x^4-48x^2=3x^2(x^2-16)=3x^2(x-4)(x+4)$$

1.5.54.

$$9x^4-12x^2=3x^2(3x^2-4)$$

1.5.55.

$$\begin{aligned}x^3-3x^2+3x-9&=x^2(x-3)+3(x-3)\\&=(x-3)(x^2+3)\end{aligned}$$

1.5.56.

$$\begin{aligned}x^3-5x^2-2x+10&=x^2(x-5)-2(x-5)\\&=(x-5)(x^2-2)\end{aligned}$$

1.5.57.

$$\begin{aligned}6x^3-27x^2-54x&=3x(2x^2-9x-18)\\&=3x(2x+3)(x-6)\end{aligned}$$

1.5.58.

$$\begin{aligned}12x^3-16x^2-60x&=4x(3x^2-4x-15)\\&=4x(3x+5)(x-3)\end{aligned}$$

1.5.59.

$$\begin{aligned}x^4-3x^2+2&=(x^2-1)(x^2-2)\\&=(x-1)(x+1)(x^2-2)\end{aligned}$$

1.5.60.

$$\begin{aligned} x^4 - 7x^2 + 12 &= \left(x^2 - 3\right)\left(x^2 - 4\right) \\ &= \left(x^2 - 3\right)\left(x - 2\right)\left(x + 2\right) \end{aligned}$$

1.5.61.

$$\begin{aligned} 9x^4 - 37x^2 + 4 &= \left(9x^2 - 1\right)\left(x^2 - 4\right) \\ &= \left(3x + 1\right)\left(3x - 1\right)\left(x + 2\right)\left(x - 2\right) \end{aligned}$$

1.5.62.

$$\begin{aligned} 4x^4 - 37x^2 + 9 &= \left(4x^2 - 1\right)\left(x^2 - 9\right) \\ &= \left(2x + 1\right)\left(2x - 1\right)\left(x + 3\right)\left(x - 3\right) \end{aligned}$$

1.5.63.

$$\begin{aligned} x^5 - x^3 - 4x^2 + 4 &= x^5 - 4x^2 - x^3 + 4 \\ &= x^2\left(x^3 - 4\right) - \left(x^3 - 4\right) \\ &= \left(x^2 - 1\right)\left(x^3 - 4\right) \\ &= \left(x + 1\right)\left(x - 1\right)\left(x^3 - 4\right) \end{aligned}$$

1.5.64.

$$\begin{aligned} x^6 - 2x^4 + x^2 &= x^2\left(x^4 - 2x^2 + 1\right) \\ &= x^2\left(x^2 - 1\right)\left(x^2 - 1\right) \\ &= x^2\left(x + 1\right)\left(x - 1\right)\left(x + 1\right)\left(x - 1\right) \\ &= x^2\left(x + 1\right)^2\left(x - 1\right)^2 \end{aligned}$$

1.5.65.

(a) When $x = 3$, $\sqrt{2x - 5} - \sqrt{x - 3} - 1 = \sqrt{2\left(3\right) - 5} - \sqrt{3 - 3} - 1 = \sqrt{1} - \sqrt{0} - 1 = 1 - 1 = 0.$

(b) When $x = 7$, $\sqrt{2x - 5} + \sqrt{x - 3} - 1 = \sqrt{2\left(7\right) - 5} + \sqrt{7 - 3} - 1 = \sqrt{9} - \sqrt{4} - 1 = 3 - 2 - 1 = 0.$

1.5.66.

(a) When $x = 4$, $\sqrt{5x - 4} + \sqrt{x} - 1 = \sqrt{5\left(4\right) - 4} + \sqrt{4} - 1 = \sqrt{16} + \sqrt{4} - 1 = 5.$

(b) When $x = 1$, $\sqrt{5x - 4} + \sqrt{x} - 1 = \sqrt{5\left(1\right) - 4} + \sqrt{1} - 1 = \sqrt{1} + \sqrt{1} - 1 = 1.$

1.5.67.

(a) When $x = 129$, $(x-4)^{2/3} = (129-4)^{2/3} = 125^{2/3} = 25$.

(b) When $x = -121$, $(x-4)^{2/3} = (-121-4)^{2/3} = (-125)^{2/3} = 25$.

1.5.68.

(a) When $x = 69$, $(x-5)^{3/2} = (69-5)^{3/2} = (64)^{3/2} = 8^3 = 512$.

(b) When $x = 14$, $(x-5)^{3/2} = (14-5)^{3/2} = (9)^{3/2} = 3^3 = 27$.

1.5.69.

(a) When $x = 4$, $\dfrac{2}{x} - \dfrac{3}{x-2} + 1 = \dfrac{2}{4} - \dfrac{3}{4-2} + 1 = \dfrac{1}{2} - \dfrac{3}{2} + 1 = -1 + 1 = 0$.

(b) When $x = -1$, $\dfrac{2}{x} - \dfrac{3}{x-2} + 1 = \dfrac{2}{-1} - \dfrac{3}{-1-2} + 1 = -2 + 1 + 1 = 0$.

1.5.70.

(a) When $x = -4$, $\dfrac{4}{x} - \dfrac{2}{x+3} + 3 = \dfrac{4}{-4} + \dfrac{2}{-4+3} + 3 = -1 - 2 + 3 = 0$.

(b) When $x = -1$, $\dfrac{4}{x} - \dfrac{2}{x+3} + 3 = \dfrac{4}{-1} + \dfrac{2}{-1+3} + 3 = -4 + 1 + 3 = 0$.

1.5.71.

(a) When $x = -3$, $|x^2 - 3x| + 4x - 6 = \left|(-3)^2 - 3(-3)\right| + 4(-3) - 6 = |9+9| - 12 - 6 = 0$.

(b) When $x = 2$, $|x^2 - 3x| + 4x - 6 = \left|(2)^2 - 3(2)\right| + 4(2) - 6 = |4-6| + 8 - 6 = 4$.

1.5.72.

(a) When $x = -3$, $|x^2 + 4x| - 7x - 18 = |9 - 12| - 7(-3) - 18 = |9 - 12| + 21 - 18 = 6$.

(b) When $x = -9$, $|x^2 + 4x| - 7x - 18 = \left|(9)^2 + 4(-9)\right| - 7(-9) - 18 = |18 - 36| + 63 - 18$

$= |45| + 63 - 18 = 45 + 63 - 18 = 90$

SECTION 1.6 OTHER TYPES OF EQUATIONS

1.6.1.

polynomial

1.6.2.

$x(x-3)$

1.6.3.

To eliminate or remove the radical from the equation $\sqrt{x+2} = x$, square each side of the equation to produce the equation $x + 2 = x^2$.

1.6.4.

The equation $x^4 - 2x + 4 = 0$ is *not* of quadratic type.

1.6.5.

$$\begin{aligned} 6x^4 - 54x^2 &= 0 \\ 6x^2\left(x^2 - 9\right) &= 0 \\ 6x^2 = 0 &\Rightarrow x = 0 \\ x^2 - 9 = 0 &\Rightarrow x = \pm 3 \end{aligned}$$

1.6.6.

$$\begin{aligned} 36x^3 - 100x &= 0 \\ 4x\left(9x^2 - 25\right) &= 0 \\ 4x(3x+5)(3x-5) &= 0 \\ 4x = 0 &\Rightarrow x = 0 \\ 3x + 5 = 0 &\Rightarrow x = -\frac{5}{3} \\ 3x - 5 = 0 &\Rightarrow x = \frac{5}{3} \end{aligned}$$

1.6.7.

$$\begin{aligned} 5x^3 + 30x^2 + 45x &= 0 \\ 5x\left(x^2 + 6x + 9\right) &= 0 \\ 5x(x+3)^2 &= 0 \\ 5x = 0 &\Rightarrow x = 0 \\ x + 3 = 0 &\Rightarrow x = -3 \end{aligned}$$

1.6.8.

$$\begin{aligned} 9x^4 - 24x^3 + 16x^2 &= 0 \\ x^2\left(9x^2 - 24x + 16\right) &= 0 \\ x^2\left(3x-4\right)^2 &= 0 \\ x^2 = 0 \Rightarrow x &= 0 \\ 3x - 4 = 0 \Rightarrow x &= \frac{4}{3} \end{aligned}$$

1.6.9.

$$\begin{aligned} x^4 - 81 &= 0 \\ \left(x^2+9\right)\left(x+3\right)\left(x-3\right) &= 0 \\ x^2 + 9 = 0 \Rightarrow x &= \pm 3i \\ x^2 + 3 = 0 \Rightarrow x &= -3 \\ x^2 - 3 = 0 \Rightarrow x &= 3 \end{aligned}$$

1.6.10.

$$\begin{aligned} x^6 - 64 &= 0 \\ \left(x^3-8\right)\left(x^3+8\right) &= 0 \\ \left(x-2\right)\left(x^2+2x+4\right)\left(x+2\right)\left(x^2-2x+4\right) &= 0 \\ x - 2 = 0 \Rightarrow x &= 2 \\ x^2 + 2x + 4 = 0 \Rightarrow x &= -1 \pm \sqrt{3}\, i \\ x + 2 = 0 \Rightarrow x &= -2 \\ x^2 - 2x + 4 = 0 \Rightarrow x &= 1 \pm \sqrt{3}\, i \end{aligned}$$

1.6.11.

$$\begin{aligned} x^3 + 512 &= 0 \\ x^3 + 8^3 &= 0 \\ \left(x+8\right)\left(x^2-8x+64\right) &= 0 \\ x + 8 = 0 \Rightarrow x &= -8 \\ x^2 - 8x + 64 = 0 \Rightarrow x &= 4 \pm 4\sqrt{3}\, i \end{aligned}$$

1.6.12.

$$27x^3 - 343 = 0$$
$$(3x)^3 - 7^3 = 0$$
$$(3x-7)(9x^2 + 21x + 49) = 0$$
$$3x - 7 = 0 \Rightarrow x = \frac{7}{3}$$
$$9x^2 + 21x + 49 = 0 \Rightarrow -\frac{7}{6} \pm \frac{7\sqrt{3}}{6}i$$

1.6.13.

$$x^3 + 2x^2 + 3x + 6 = 0$$
$$x^2(x+2) + 3(x+2) = 0$$
$$(x+2)(x^2+3) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x^2 + 3 = 0 \Rightarrow x = \pm\sqrt{3}\,i$$

1.6.14.

$$x^4 + 2x^3 - 8x - 16 = 0$$
$$x^3(x+2) - 8(x+2) = 0$$
$$(x^3 - 8)(x+2) = 0$$
$$(x-2)(x^2 + 2x + 4)(x+2) = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x^2 + 2x + 4 = 0 \Rightarrow x = -1 \pm \sqrt{3}\,i$$
$$x + 2 = 0 \Rightarrow x = -2$$

1.6.15.

$$x^4 - 4x^2 + 3 = 0$$
$$(x^2)^2 - 4(x^2) + 3 = 0$$

Let $u = x^2$.

$$u^2 - 4u + 3 = 0$$
$$(u-3)(u-1) = 0$$
$$u - 3 = 0 \Rightarrow u = 3$$
$$u - 1 = 0 \Rightarrow u = 1$$

$$\begin{array}{ll} u = 1 & u = 3 \\ x^2 = 1 & x^2 = 3 \\ x = \pm 1 & x = \pm\sqrt{3} \end{array}$$

1.6.16.

$$x^4 - 13x^2 + 36 = 0$$
$$\left(x^2\right)^2 - 13\left(x^2\right) + 36 = 0$$

Let $u = x^2$.

$$u^2 - 13u + 36 = 0$$
$$\left(u-9\right)\left(u-4\right) = 0$$
$$u - 9 = 0 \Rightarrow u = 9$$
$$u - 4 = 0 \Rightarrow u = 4$$

$$u = 9 \qquad u = 4$$
$$x^2 = 9 \qquad x^2 = 4$$
$$x = \pm 3 \qquad x = \pm 2$$

1.6.17.

$$4x^4 - 65x^2 + 16 = 0$$
$$4\left(x^2\right) - 65\left(x^2\right) + 16 = 0$$

Let $u = x^2$.

$$4u - 65u + 16 = 0$$
$$\left(4u-1\right)\left(u-16\right) = 0$$
$$4u - 1 = 0 \Rightarrow u = \frac{1}{4}$$
$$u - 16 = 0 \Rightarrow u = 16$$

$$u = \frac{1}{4} \qquad u = 16$$
$$x^2 = \frac{1}{4} \qquad x^2 = 16$$
$$x = \pm\frac{1}{2} \qquad x = \pm 4$$

1.6.18.

$$36t^4 + 29t^2 - 7 = 0$$
$$36\left(t^2\right)^2 + 29\left(t^2\right) - 7 = 0$$

Let $u = t^2$.

$$36u^2 + 29u - 7 = 0$$
$$(36u - 7)(u + 1) = 0$$
$$36u - 7 = 0 \Rightarrow u = \frac{7}{36}$$
$$u + 1 = 0 \Rightarrow u = -1$$

$$u = \frac{7}{36} \qquad\qquad u = -1$$
$$x^2 = \frac{7}{36} \qquad\qquad x^2 = -1$$
$$x = \pm\frac{\sqrt{7}}{6} \qquad\qquad x = \pm i$$

1.6.19.

$$2x + 9\sqrt{x} = 5$$
$$2x + 9\sqrt{x} - 5 = 0$$
$$2\left(\sqrt{x}\right)^2 + 9\left(\sqrt{x}\right) - 5 = 0$$

Let $u = \sqrt{x}$.

$$2u^2 + 9u - 5 = 0$$
$$(2u - 1)(u + 5) = 0$$
$$2u - 1 = 0 \Rightarrow u = \frac{1}{2}$$
$$u + 5 = 0 \Rightarrow u = -5$$

$$u = \frac{1}{2} \qquad\qquad u = -5 \Rightarrow \sqrt{x} \neq -5$$
$$\sqrt{x} = \frac{1}{2} \qquad\qquad \left(\sqrt{x} = -5 \text{ is not a solution.}\right)$$
$$x = \frac{1}{2}$$

1.6.20.

$$6x - 7\sqrt{x} - 3 = 0$$
$$6\left(\sqrt{x}\right)^2 - 7\left(\sqrt{x}\right) - 3 = 0$$

Let $u = \sqrt{x}$.

$$6u^2 - 7u - 3 = 0$$
$$(3u+1)(2u-2) = 0$$
$$3u + 1 = 0 \Rightarrow u = -\tfrac{1}{3}$$
$$2u - 3 = 0 \Rightarrow u = \tfrac{3}{2}$$

$$u = -\frac{1}{3} \qquad u = \frac{3}{2}$$
$$\sqrt{x} \neq -\frac{1}{3} \qquad \sqrt{x} = \frac{3}{2}$$
$$\left(\sqrt{x} = -\frac{1}{3} \text{ is not a solution.}\right)$$

1.6.21.

$$9t^{2/3} + 24t^{1/3} + 16 = 0$$
$$9\left(t^{1/3}\right)^2 + 24\left(t^{1/3}\right) + 16 = 0$$

Let $u = t^{1/3}$.

$$9u^2 + 24u + 16 = 0$$
$$(3u+4)^2 = 0$$
$$3u + 4 = 0 \Rightarrow u = -\frac{4}{3}$$
$$u = -\frac{4}{3}$$
$$t^{1/3} = -\frac{4}{3}$$
$$t = -\frac{64}{27}$$

1.6.22.

$$3x^{1/3} + 2x^{2/3} = 5$$
$$2x^{2/3} + 3x^{1/3} - 5 = 0$$
$$2\left(x^{1/3}\right)^2 + 3\left(x^{1/3}\right) - 5 = 0$$

Let $u = x^{1/3}$.

$$(2u+5)(u-1) = 0$$
$$2u + 5 = 0 \Rightarrow u = -\frac{5}{2}$$
$$u - 1 = 0 \Rightarrow u = 1$$

$$u = -\frac{5}{2} \qquad\qquad u = 1$$
$$x^{1/3} = -\frac{5}{2} \qquad\qquad x^{1/3} = 1$$
$$x = -\frac{125}{2} \qquad\qquad x = 1$$

1.6.23.

$$\frac{1}{x^2} + \frac{8}{x} + 15 = 0$$
$$\left(\frac{1}{x}\right)^2 + 8\left(\frac{1}{x}\right) + 15 = 0$$
$$\text{Let } u = \frac{1}{x}.$$
$$u^2 + 8u + 15 = 0$$
$$(u+5)(u+3) = 0$$
$$u + 5 = 0 \Rightarrow u = -5$$
$$u + 3 = 0 \Rightarrow u = -3$$

$$u = -5 \qquad\qquad u = -3$$
$$\frac{1}{x} = -5 \qquad\qquad \frac{1}{x} = -3$$
$$x = -\frac{1}{5} \qquad\qquad x = -\frac{1}{3}$$

1.6.24.

$$1 + \frac{3}{x} = -\frac{2}{x^2}$$
$$\frac{2}{x^2} + \frac{3}{x} + 1 = 0$$
$$2\left(\frac{1}{x}\right)^2 + 3\left(\frac{1}{x}\right) + 1 = 0$$

Let $u = \frac{1}{x}$.

$$2u^2 + 3u + 1 = 0$$
$$(2u+1)(u+1) = 0$$
$$2u + 1 = 0 \Rightarrow u = -\frac{1}{2}$$
$$u + 1 = 0 \Rightarrow u = -1$$

$$u = -\frac{1}{2} \qquad\qquad u = -1$$
$$\frac{1}{x} = -\frac{1}{2} \qquad\qquad \frac{1}{x} = -1$$
$$x = -2 \qquad\qquad x = -1$$

1.6.25.

$$2\left(\frac{x}{x+2}\right)^2 - 3\left(\frac{x}{x+2}\right) - 2 = 0$$

Let $u = \dfrac{x}{x+2}$.

$$2u^2 - 3u - 2 = 0$$
$$(2u+1)(u-2) = 0$$
$$2u+1 = 0 \Rightarrow u = -\frac{1}{2}$$
$$u-2 = 0 \Rightarrow u = 2$$

$$u = -\frac{1}{2} \qquad\qquad u = 2$$
$$\frac{x}{x+2} = -\frac{1}{2} \qquad\qquad \frac{x}{x+2} = 2$$
$$x = -\frac{2}{3} \qquad\qquad x = -4$$

1.6.26.

$$6\left(\frac{x}{x+1}\right)^2 + 5\left(\frac{x}{x+1}\right) - 6 = 0$$

Let $u = \dfrac{x}{x+1}$.

$$6u^2 + 5u - 6 = 0$$
$$(3u-2)(2u+3) = 0$$
$$3u-2 = 0 \Rightarrow u = \frac{2}{3}$$
$$2u+3 = 0 \Rightarrow u = -\frac{3}{2}$$

$$u = \frac{2}{3} \qquad\qquad u = -\frac{3}{2}$$
$$\frac{x}{x+1} = \frac{2}{3} \qquad\qquad \frac{x}{x+1} = -\frac{3}{2}$$
$$x = 2 \qquad\qquad x = -\frac{3}{5}$$

1.6.27.

$$\begin{aligned}
\sqrt{5x} - 10 &= 0 \\
\sqrt{5x} &= 10 \\
\left(\sqrt{5x}\right)^2 &= (10)^2 \\
5x &= 100 \\
x &= 20
\end{aligned}$$

1.6.28.

$$\begin{aligned}
\sqrt{3x+1} &= 7 \\
\left(\sqrt{3x+1}\right)^2 &= (7)^2 \\
3x + 1 &= 49 \\
3x &= 48 \\
x &= 16
\end{aligned}$$

1.6.29.

$$\begin{aligned}
4 + \sqrt[3]{2x-9} &= 0 \\
\sqrt[3]{2x-9} &= -4 \\
\left(\sqrt[3]{2x-9}\right)^3 &= (-4)^3 \\
2x - 9 &= -64 \\
2x &= -55 \\
x &= -\frac{55}{2}
\end{aligned}$$

1.6.30.

$$\begin{aligned}
\sqrt[3]{12-x} - 3 &= 0 \\
\sqrt[3]{12-x} &= 3 \\
\left(\sqrt[3]{12-x}\right)^3 &= (3)^3 \\
12 - x &= 27 \\
-x &= 15 \\
x &= -15
\end{aligned}$$

1.6.31.

$$\begin{aligned}
\sqrt{x+8} &= 2+x \\
\left(\sqrt{x+8}\right)^2 &= (2+x)^2 \\
x+8 &= x^2+4x+4 \\
0 &= x^2+3x-4 \\
x^2+3x-4 &= 0 \\
(x+4)(x-1) &= 0 \\
x+4=0 &\Rightarrow x=-4, \text{ extraneous} \\
x-1=0 &\Rightarrow x=1
\end{aligned}$$

1.6.32.

$$\begin{aligned}
2x &= \sqrt{-5x+24}-3 \\
2x+3 &= \sqrt{-5x+24} \\
(2x+3)^2 &= \left(\sqrt{-5x+24}\right)^2 \\
4x^2+12x+9 &= -5x+24 \\
4x^2+17x-15 &= 0 \\
(4x-3)(x+5) &= 0 \\
4x-3=0 &\Rightarrow x=\tfrac{3}{4} \\
x+5 &\Rightarrow x=-5, \text{ extraneous}
\end{aligned}$$

1.6.33.

$$\begin{aligned}
\sqrt{x-3}+1 &= \sqrt{x} \\
\sqrt{x-3} &= \sqrt{x}-1 \\
\left(\sqrt{x-3}\right)^2 &= \left(\sqrt{x}-1\right)^2 \\
x-3 &= x-2\sqrt{x}+1 \\
-4 &= -2\sqrt{x} \\
2 &= \sqrt{x} \\
(2)^2 &= \left(\sqrt{x}\right)^2 \\
4 &= x
\end{aligned}$$

1.6.34.

$$\begin{aligned}
\sqrt{x} + \sqrt{x-24} &= 2\\
\sqrt{x} &= 2 - \sqrt{x-24}\\
\left(\sqrt{x}\right)^2 &= \left(2 - \sqrt{x-24}\right)^2\\
x &= 4 - 4\sqrt{x-24} + x - 24\\
20 &= -4\sqrt{x-24}\\
5 &= -\sqrt{x-24}\\
5^2 &= \left(-\sqrt{x-24}\right)^2\\
25 &= x - 24\\
49 &= x
\end{aligned}$$

$x = 49$ is an extraneous solution, so the equation has no solution.

1.6.35.

$$\begin{aligned}
2\sqrt{x+1} - \sqrt{2x+3} &= 1\\
2\sqrt{x+1} &= 1 + \sqrt{2x+3}\\
\left(2\sqrt{x+1}\right)^2 &= \left(1 + \sqrt{2x+3}\right)^2\\
4(x+1) &= 1 + 2\sqrt{2x+3} + 2x + 3\\
2x &= 2\sqrt{2x+3}\\
x &= \sqrt{2x+3}\\
x^2 &= 2x + 3\\
x^2 - 2x - 3 &= 0\\
(x-3)(x+1) &= 0\\
x - 3 = 0 &\Rightarrow x = 3\\
x + 1 = 0 &\Rightarrow x = -1, \text{ extraneous}
\end{aligned}$$

1.6.36.

$$\begin{aligned}
4\sqrt{x-3} - \sqrt{6x-17} &= 3\\
4\sqrt{x-3} &= 3 + \sqrt{6x-17}\\
\left(4\sqrt{x-3}\right)^2 &= \left(3 + \sqrt{6x-17}\right)^2\\
16(x-3) &= 9 + 6\sqrt{6x-17} + 6x - 17\\
16x - 48 &= 6\sqrt{6x-17} + 6x - 8
\end{aligned}$$

$$10x - 40 = 6\sqrt{6x - 17}$$
$$5x - 20 = 3\sqrt{6x - 17}$$
$$(5x - 20)^2 = \left(3\sqrt{6x - 17}\right)^2$$
$$25x^2 - 200x + 400 = 9(6x - 17)$$
$$25x^2 - 200x + 400 = 54x - 153$$
$$25x^2 - 254x + 553 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-254) \pm \sqrt{(-254)^2 - 4(25)(553)}}{2(25)}$$
$$x = \frac{254 \pm \sqrt{9261}}{50}$$
$$x = \frac{254 + 96}{50} = \frac{350}{50} = 7$$
$$x = \frac{254 - 96}{50} = \frac{158}{50} = \frac{79}{25}, \text{ extraneous}$$

1.6.37.

$$\sqrt{4\sqrt{4x + 9}} = \sqrt{8x + 2}$$
$$\left(\sqrt{4\sqrt{4x + 9}}\right)^2 = \left(\sqrt{8x + 2}\right)^2$$
$$4\sqrt{4x + 9} = 8x + 2$$
$$2\sqrt{4x + 9} = 4x + 1$$
$$\left(2\sqrt{4x + 9}\right)^2 = (4x + 1)^2$$

$$4(4x + 9) = 16x^2 + 8x + 1$$
$$16x + 36 = 16x^2 + 8x + 1$$
$$0 = 16x^2 - 8x - 35$$
$$0 = (4x + 5)(4x - 7)$$
$$4x + 5 = 0 \Rightarrow x = -\frac{5}{4}, \text{ extraneous}$$
$$4x - 7 = 0 \Rightarrow x = \frac{7}{4}$$

1.6.38.

$$\sqrt{16+9\sqrt{x}} = 4+\sqrt{x}$$
$$\left(\sqrt{16+9\sqrt{x}}\right)^2 = \left(4+\sqrt{x}\right)^2$$
$$16+9\sqrt{x} = 16+8\sqrt{x}+x$$
$$\sqrt{x} = x$$
$$\left(\sqrt{x}\right)^2 = \left(x\right)^2$$
$$x = x^2$$
$$x^2 - x = 0$$
$$x\left(x-1\right) = 0$$
$$x = 0$$
$$x-1=0 \Rightarrow x=1$$

1.6.39.

$$\left(x-5\right)^{3/2} = 8$$
$$\left(x-5\right)^3 = 8^2$$
$$x-5 = \sqrt[3]{64}$$
$$x = 5+4 = 9$$

1.6.40.

$$\left(x+2\right)^{2/3} = 9$$
$$\left(x+2\right)^2 = 9^3$$
$$x+2 = \pm\sqrt{729}$$
$$x = -2\pm 27 = -29,\ 25$$

1.6.41.

$$\left(x^2-5\right)^{3/2} = 27$$
$$\left(x^2-5\right)^3 = 27^2$$
$$x^2-5 = \sqrt[3]{27^2}$$
$$x^2 = 5+9$$
$$x^2 = 14$$
$$x = \pm\sqrt{14}$$

1.6.42.

$$\begin{aligned}\left(x^2-x-22\right)^{3/2}&=27\\ x^2-x-22&=27^{2/3}\\ x^2-x-22&=9\\ x^2-x-31&=0\\ x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-31)}}{2(1)}&=\frac{1\pm\sqrt{125}}{2}=\frac{1\pm5\sqrt{5}}{2}\end{aligned}$$

1.6.43.

$$\begin{aligned}3x(x-1)^{1/2}+2(x-1)^{3/2}&=0\\ (x-1)^{1/2}\left[3x+2(x-1)\right]&=0\\ (x-1)^{1/2}(5x-2)&=0\\ (x-1)^{1/2}&=0\Rightarrow x-1=0\Rightarrow x=1\\ 5x-2=0\Rightarrow x&=\frac{2}{5},\ \text{extraneous}\end{aligned}$$

1.6.44.

$$\begin{aligned}4x^2(x-1)^{1/3}+6x(x-1)^{4/3}&=0\\ 2x\left[2x(x-1)^{1/3}+3(x-1)^{4/3}\right]&=0\\ 2x(x-1)^{1/3}\left[2x+3(x-1)\right]&=0\\ 2x(x-1)^{1/3}(5x-3)&=0\\ 2x=0\Rightarrow x&=0\\ x-1=0\Rightarrow x&=1\\ 5x-3=0\Rightarrow x&=\frac{3}{5}\end{aligned}$$

1.6.45.

$$\begin{aligned}\frac{1}{x}-\frac{1}{x+1}&=3\\ x(x+1)\frac{1}{x}-x(x+1)\frac{1}{x+1}&=x(x+1)(3)\\ x+1-x&=3x(x+1)\\ 1&=3x^2+3x\\ 0&=3x^2+3x-1\end{aligned}$$

$$x=\frac{-3\pm\sqrt{(3)^2-4(3)(-1)}}{2(3)}=\frac{-3\pm\sqrt{21}}{6}$$

1.6.46.

$$\begin{aligned}\frac{4}{x+1}-\frac{3}{x+2}&=1\\4(x+2)-3(x+1)&=(x+1)(x+2)\\4x+8-3x&=x^2+3x+2\\x^2+2x-3&=0\\(x-1)(x+3)&=0\\x-1=0&\Rightarrow x=1\\x+3=0&\Rightarrow x=-3\end{aligned}$$

1.6.47.

$$\begin{aligned}3-\frac{14}{x}-\frac{5}{x^2}&=0\\\frac{5}{x^2}+\frac{14}{x}-3&=0\\5\left(\frac{1}{x}\right)^2+14\left(\frac{1}{x}\right)-3&=0\end{aligned}$$

Let $u=\frac{1}{x}$.

$$\begin{aligned}5u^2+14u-3&=0\\(5u-1)(u+3)&=0\\5u-1=0&\Rightarrow u=\frac{1}{5}\\u+3=0&\Rightarrow u=-3\end{aligned}$$

$$\begin{array}{ll}u=\frac{1}{5} & u=-3\\\frac{1}{x}=\frac{1}{5} & \frac{1}{x}=-3\\x=5 & x=-\frac{1}{3}\end{array}$$

1.6.48.

$$5 = \frac{18}{x} + \frac{8}{x^2}$$

$$\frac{8}{x^2} + \frac{18}{x} - 5 = 0$$

$$8\left(\frac{1}{x}\right)^2 + 18\left(\frac{1}{x}\right) - 5 = 0$$

Let $u = \frac{1}{x}$.

$$8u^2 + 18u - 5 = 0$$

$$(4u-1)(2u+5) = 0$$

$$4u - 1 = 0 \Rightarrow u = \frac{1}{4}$$

$$2u + 5 = 0 \Rightarrow u = -\frac{5}{2}$$

$$u = \frac{1}{4} \qquad u = -\frac{5}{2}$$

$$\frac{1}{x} = \frac{1}{4} \qquad \frac{1}{x} = -\frac{5}{2}$$

$$x = 4 \qquad x = -\frac{2}{5}$$

1.6.49.

$$\frac{x+1}{3} - \frac{x+1}{x+2} = 0$$

$$3(x+2)\frac{x+1}{3} - 3(x+2)\frac{x+1}{x+2} = 0$$

$$(x+2)(x+1) - 3(x+1) = 0$$

$$x^2 + 3x + 2 - 3x - 3 = 0$$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$$x + 1 = 0 \Rightarrow x = -1$$

$$x - 1 = 0 \Rightarrow x = 1$$

1.6.50.

$$\begin{aligned}\frac{x}{x^2-4}+\frac{1}{x+2}&=3\\(x+2)(x-2)\frac{x}{x^2-4}+(x+2)(x-2)\frac{1}{x+2}&=3(x+2)(x-2)\\x+x-2&=3x^2-12\\3x^2-2x-10&=0\end{aligned}$$

$$\begin{aligned}x&=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(-10)}}{2(3)}\\&=\frac{2\pm\sqrt{124}}{6}=\frac{2\pm2\sqrt{31}}{6}=\frac{1\pm\sqrt{31}}{3}\end{aligned}$$

1.6.51.

$$\begin{aligned}|2x-5|&=11\\2x-5&=11\Rightarrow x=8\\-(2x-5)&=11\Rightarrow x=-3\end{aligned}$$

1.6.52.

$$\begin{aligned}|3x+2|&=7\\3x+2&=7\Rightarrow x=\frac{5}{3}\\-(3x+2)&=7\\-3x-2&=7\Rightarrow x=-3\end{aligned}$$

1.6.53.

$$|x|=x^2+x-24$$

First equation:

$$\begin{aligned}x&=x^2+x-24\\x^2-24&=0\\x^2&=24\\x&=\pm2\sqrt{6}\end{aligned}$$

Second equation:

$$\begin{aligned}-x&=x^2+x-24\\x^2+2x-24&=0\\(x+6)(x-4)&=0\\x+6&=0\Rightarrow x=-6\\x-4&=0\Rightarrow x=4\end{aligned}$$

Only $x=2\sqrt{6}$ and $x=-6$ are solutions of the original equation. $x=-2\sqrt{6}$ and $x=4$ are extraneous.

1.6.54.

$$\left|x^2+6x\right|=3x+18$$

First equation:

$$\begin{aligned} x^2+6x&=3x+18\\ x^2+3x-18&=0\\ (x-3)(x+6)&=0\\ x-3=0&\Rightarrow x=3\\ x+6=0&\Rightarrow x=-6 \end{aligned}$$

Second equation:

$$\begin{aligned} -\left(x^2+6x\right)&=3x+18\\ 0&=x^2+9x+18\\ 0&=(x+3)(x+6)\\ 0&=x+3\Rightarrow x=-3\\ x&=x+6\Rightarrow x=-6 \end{aligned}$$

The solutions of the original equation are $x=\pm 3$ and $x=-6$.

1.6.55.

$$|x+1|=x^2-5$$

First equation:

$$\begin{aligned} x+1&=x^2-5\\ x^2-x-6&=0\\ (x-3)(x+2)&=0\\ x-3=0&\Rightarrow x=3\\ x+2=0&\Rightarrow x=-2 \end{aligned}$$

Second equation:

$$\begin{aligned} -(x+1)&=x^2-5\\ -x-1&=x^2-5\\ x^2+x-4&=0\\ x&=\frac{-1+\sqrt{17}}{2} \end{aligned}$$

Only $x=3$ and $x=\frac{-1-\sqrt{17}}{2}$ are solutions of the original equation. $x=-2$ and $x=\frac{-1+\sqrt{17}}{2}$ are extraneous.

1.6.56.

$$|x-15|=x^2-15x$$

First equation:

$$\begin{aligned} x-15&=x^2-15x\\ x^2-16x+15&=0\\ (x-1)(x-15)&=0\\ x-1=0&\Rightarrow x=1\\ x-15=0&\Rightarrow x=15 \end{aligned}$$

Second equation:

$$\begin{aligned} -(x-15)&=x^2-15x\\ x^2-14x-15&=0\\ (x+1)(x-15)&=0\\ x+1=0&\Rightarrow x=-1\\ x-15=0&\Rightarrow x=15 \end{aligned}$$

Only $x=15$ and $x=-1$ are solutions of the original equation. $x=1$ is extraneous.

1.6.57.

$$\begin{aligned} x^3 - 3x^2 - 1.21x + 3.63 &= 0 \\ x^2(x-3) - 1.21(x-3) &= 0 \\ (x-3)(x^2 - 1.21) &= 0 \\ (x-3)(x+1.1)(x-1.1) &= 0 \\ x &= 3, \pm 1.1 \end{aligned}$$

1.6.58.

$$\begin{aligned} x^4 - 1.7x^3 + x &= 1.7 \\ x^4 - 1.7x^3 + x - 1.7 &= 0 \\ x^3(x-1.7) + (x-1.7) &= 0 \\ (x^3+1)(x-1.7) &= 0 \\ (x+1)(x^2-x+1)(x-1.7) &= 0 \\ x &= -1, 1.7 \end{aligned}$$

1.6.59.

$$7.08x^6 + 4.15x^3 - 9.6 = 0$$

$$a = 7.8, \quad b = 4.15, \quad c = -9.6$$

$$\begin{aligned} x^3 &= \frac{-4.15 \pm \sqrt{(4.15)^2 - 4(7.08)(-9.6)}}{2(7.08)} \\ &= \frac{-4.15 \pm \sqrt{2.89.0945}}{14.16} \end{aligned}$$

$$x = \sqrt[3]{\frac{-4.15 + \sqrt{289.0945}}{14.16}} \approx 0.968$$

$$x = \sqrt[3]{\frac{-4.15 - \sqrt{289.0945}}{14.16}} \approx -1.143$$

1.6.60.

$$5.25x^6 - 0.2x^3 - 1.55 = 0$$

$$\begin{aligned} x^3 &= \frac{0.2 \pm \sqrt{(-0.2)^2 - 4(5.25)(-1.55)}}{2(5.25)} \\ &= \frac{0.2 \pm \sqrt{32.59}}{10.5} \end{aligned}$$

$$x = \sqrt[3]{\frac{0.2 + \sqrt{32.59}}{10.5}} \approx 0.826$$
$$x = \sqrt[3]{\frac{0.2 - \sqrt{32.59}}{10.5}} \approx -0.807$$

1.6.61.

$$1.8x - 6\sqrt{x} - 5.6 = 0 \quad \text{Given equation}$$
$$1.8\left(\sqrt{x}\right)^2 - 6\sqrt{x} - 5.6 = 0$$

Use the Quadratic Formula with $a = 1.8,\ \ b = -6,$ and $c = -5.6.$

$$\sqrt{x} = \frac{6 \pm \sqrt{36 - 4(1.8)(-5.6)}}{2(1.8)} \approx \frac{6 \pm \ 8.7361}{3.6}$$

Considering only the positive value for $\sqrt{x}$, we have

$$\sqrt{x} \approx 4.0934$$
$$x \approx 16.756.$$

1.6.62.

$$5.3x + 3.1 = 9.8\sqrt{x}$$
$$(5.3x + 3.1)^2 = \left(9.8\sqrt{x}\right)^2$$
$$28.09x^2 + 32.86x + 9.61 = 96.04x$$
$$28.09x^2 - 63.18x + 9.61 = 0$$
$$x = \frac{-(-63.18) \pm \sqrt{(-63.18)^2 - 4(28.09)(9.61)}}{2(28.09)}$$
$$= \frac{63.18 \pm \sqrt{2911.9328}}{56.18}$$
$$\approx 2.085,\ 0.164$$

1.6.63.

$$x^2 - \frac{5}{11}x - 3.3 = 0$$
$$x^2 = \frac{(5/11) \pm \sqrt{(5/11)^2 - 4(1)(-3.3)}}{2(1)}$$
$$x \approx -1.603,\ 2.058$$

1.6.64.

$$\frac{4.4}{x} - \frac{5.5}{3.3} = \frac{x}{6.6}$$

$$4.4 - \frac{5}{3}x = \frac{5}{33}x^2$$

$$0 = \frac{5}{33}x^2 + \frac{5}{3}x - 4.4$$

$$x = \frac{(-5/3) \pm \sqrt{(5/3)^2 - 4(5/33)(-4.4)}}{2(5/33)}$$

$$x = -13.2,\ 2.2$$

1.6.65.

$-4,\ 7$

Sample answer: $(x-(-4))(x-7)=0$

$$(x+4)(x-7)=0$$

$$x^2-3x-28=0$$

1.6.66.

$0,\ 2,\ 9$

Sample answer: $(x-0)(x-2)(x-9)=0$

$$x(x-2)(x-9)=0$$

$$x(x^2-11x+18)=0$$

$$x^3-11x^2+18x=0$$

1.6.67.

$\sqrt{3},\ -\sqrt{3},$ and 4

One possible equation is:

$$(x-\sqrt{3})(x-(-\sqrt{3}))(x-4)=0$$

$$(x-\sqrt{3})(x+\sqrt{3})(x-4)=0$$

$$(x^2-3)(x-4)=0$$

$$x^3-4x^2-3x+12=0$$

Any nonzero multiple of this equation would also have these solutions.

1.6.68.

$2\sqrt{7},\ -\sqrt{7}$

$$\left(x-2\sqrt{7}\right)\left(x+\sqrt{7}\right)=0$$
$$x^2+x\sqrt{7}-2x\sqrt{7}-2\left(7\right)=0$$
$$x^2-x\sqrt{7}-14=0$$

Any nonzero multiple of this equation would also have these solutions.

1.6.69.

$i, \quad -i$

Sample answer: $\left(x-i\right)\left(x-\left(-i\right)\right)=0$
$$\left(x-i\right)\left(x+i\right)=0$$
$$x^2-i^2=0$$
$$x^2+1=0$$

1.6.70.

$2i, \quad -2i$

Sample answer: $\left(x-2i\right)\left(x-\left(-2i\right)\right)=0$
$$\left(x-2i\right)\left(x+2i\right)=0$$
$$x^2-4i^2=0$$
$$x^2+4=0$$

1.6.71.

$-1, \quad 1, \quad i, \quad \text{and} -i$

One possible equation is:
$$\left(x-\left(-1\right)\right)\left(x-1\right)\left(x-i\right)\left(x-\left(-i\right)\right)=0$$
$$\left(x+1\right)\left(x-1\right)\left(x-i\right)\left(x+i\right)=0$$
$$\left(x^2-1\right)\left(x^2+1\right)=0$$
$$x^4-1=0$$
Any nonzero multiple of this equation would also have these solutions.

1.6.72.

$4i, \quad -4i, \quad 6, \quad -6$

Sample answer: $\left(x-4i\right)\left(x+4i\right)\left(x-6\right)\left(x+6\right)=0$
$$\left(x^2+16\right)\left(x^2-36\right)=0$$
$$x^4-20x^2-576=0$$

1.6.73.

Labels: Let $x=$ the number of students in the original group. Then $\dfrac{1700}{x}=$ the original cost per student.

When six more students join the group, the cost per student becomes $\frac{1700}{x} - 7.50.$

Model: $(\text{Cost per student}) \cdot (\text{Number of students}) = (\text{Total cost})$

Equation: $\left(\frac{1700}{x} - 7.5\right)(x+6) = 1700$

$$(3400 - 15x)(x+6) = 3400x \quad \text{Multiply each by } 2x \text{ to clear fraction.}$$

$$-15x^2 - 90x + 20,400 = 0$$

$$x = \frac{90 \pm \sqrt{(-90)^2 - 4(-15)(20,400)}}{2(-15)} = \frac{90 \pm 1110}{-30}$$

Using the positive value for x, we conclude that the original number was $x = 34$ students.

1.6.74.

Model: $\begin{pmatrix}\text{Cost per}\\ \text{student}\end{pmatrix} \cdot \begin{pmatrix}\text{Number of}\\ \text{students}\end{pmatrix} = \begin{pmatrix}\text{Monthly}\\ \text{rent}\end{pmatrix}$

Labels: Monthly rent $= x$

Number of students $= 4$

Original cost per student $= \frac{x}{3}$

Cost per student $= \frac{x}{3} - 150$

Equation: $\left(\frac{x}{3} - 150\right)(4) = x$

$$\frac{4x}{3} - 600 = x$$

$$\frac{4x}{3} - x = 600$$

$$\frac{x}{3} = 600$$

$$x = 1800$$

The monthly rent is \$1800.

1.6.75.

Model: $\text{Time} = \frac{\text{Distance}}{\text{Rate}}$

Labels: Let $x =$ average speed of the plane. Then we have a travel time of $t = 145/x$. If the average speed is increased by 40 mph, then

$$t - \frac{12}{60} = \frac{145}{x+40}$$
$$t = \frac{145}{x+40} + \frac{1}{5}.$$

Now, we equate these two equations and solve for x.

Equation: $$\frac{145}{x} = \frac{145}{x+40} + \frac{1}{5}$$
$$145(5)(x+40) = 145(5)x + x(x+40)$$
$$725x + 29{,}000 = 725x + x^2 + 40x$$
$$0 = x^2 + 40x - 29{,}000$$

Using the positive value for x found by the Quadratic Formula, we have $x \approx 151.5$ mph and $x + 40 = 191.5$ mph. The airspeed required to obtain the decrease in travel time is 191.5 miles per hour.

1.6.76.

Model: $(\text{Rate}) \cdot (\text{time}) = (\text{distance})$

Labels: $\text{Distance} = 1080$

$\text{Original time} = t$

$\text{Original rate} = \dfrac{1080}{t}$

$\text{Return time} = t + 2.5$

$\text{Return rate} = \dfrac{1080}{t} - 6$

Equation: $$\left(\frac{1080}{t} - 6\right)(t + 2.5) = 1080$$
$$1080 + \frac{2700}{t} - 6t - 15 = 1080$$
$$\frac{2700}{t} - 6t - 15 = 0$$
$$270 - 0 - 6t^2 - 15t = 0$$
$$2t^2 + 5t - 900 = 0$$
$$(2t + 45)(t - 20) = 0$$
$$2t + 45 = 0 \Rightarrow t = -22.5$$
$$t - 20 = 0 \Rightarrow t = 20$$

The average speed was $\frac{1080}{20} = 54$ miles per hour.

1.6.77.

$$\begin{aligned}
A &= P\left(1+\frac{r}{n}\right)^{nt} \\
2694.58 &= 2500\left(1+\frac{r}{12}\right)^{(12)(5)} \\
\frac{2694.58}{2500} &= \left(1+\frac{r}{12}\right)^{60} \\
1.077832 &= \left(1+\frac{r}{12}\right)^{60} \\
(1.077832)^{1/60} &= 1+\frac{r}{12} \\
\left[(1.0.77832)^{1/60} - 1\right](12) &= r \\
r &\approx 0.015 = 1.5\%
\end{aligned}$$

1.6.78.

$$\begin{aligned}
A &= P\left(1+\frac{r}{n}\right)^{nt} \\
7734.27 &= 6000\left(1+\frac{r}{4}\right)^{(4)(5)} \\
\frac{7734.27}{6000} &= \left(1+\frac{r}{4}\right)^{20} \\
1.289045 &= \left(1+\frac{r}{4}\right)^{20} \\
(1.289045)^{1/20} &= 1+\frac{r}{4} \\
\left[(1.289045)^{1/20}\right](4) &= r \\
r &\approx 0.0511 = 5.1\%
\end{aligned}$$

1.6.79.

$T = 75.82 - 2.11x + 43.51\sqrt{x}, \quad 5 \le x \le 40$

(a) $212 = 75.82 - 2.11x + 43.51\sqrt{x}$

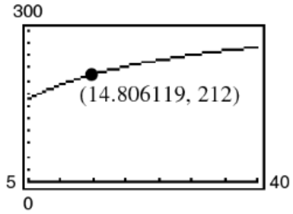
$0 = -2.11x + 43.51\sqrt{x} - 136.18$

By the Quadratic Formula, we have $\sqrt{x} \approx 16.77928 \Rightarrow x \approx 281.333$

$\sqrt{x} \approx 3.84787 \Rightarrow x \approx 14.806.$

Since x is restricted to $5 \le x \le 40$, let $x = 14.806$ pounds per square inch.

(b)



1.6.80.

$$P = \frac{217.77 + 0.819t^2}{1 + 0.0028t^2}, \quad 15 \le t \le 23$$

(a)
$$\begin{aligned} 250 &= \frac{217.77 + 0.8191t^2}{1 + 0.0028t^2} \\ 250 + 0.7t^2 &= 217.77 + 0.8191t^2 \\ 32.23 &= 0.1191t^2 \\ \frac{32.23}{0.1191} &= t^2 \\ 16.45 &\approx t \end{aligned}$$

The total voting age population reached 250 million in 2016.

(b)
$$\begin{aligned} 260 &= \frac{217.77 + 0.8191t^2}{1 + 0.0028t^2} \\ 260 + 0.728t^2 &= 217.77 + 0.8191t^2 \\ 42.23 &= 0.0911t^2 \\ \frac{42.23}{0.0911} &= t^2 \\ 21.53 &\approx t \end{aligned}$$

The total voting age population reached 260 million in 2021.

1.6.81.

$$\frac{1}{t} + \frac{1}{t+3} = \frac{1}{y}$$

$$\begin{aligned}
\frac{1}{t}+\frac{1}{t+3}&=\frac{1}{2}\\
2t(t+3)\frac{1}{t}+2t(t+3)\frac{1}{t+3}&=2t(t+3)\frac{1}{2}\\
2(t+3)+2t&=t(t+3)\\
2t+6+2t&=t^2+3t\\
0&=t^2-t-6\\
0&=(t-3)(t+2)\\
t-3=0&\Rightarrow t=3\\
t+2=0&\Rightarrow t=-2
\end{aligned}$$

Since t represents time, $t = 3$ is the only solution. It takes 3 hours for you working alone to tile the floor.

1.6.82.

$$\frac{1}{t}+\frac{1}{t+2}=\frac{1}{y}$$

$$\begin{aligned}
\frac{1}{t}+\frac{1}{t+2}&=\frac{1}{3}\\
3t(t+2)\left(\frac{1}{t}\right)+3t(t+2)\left(\frac{1}{t+2}\right)&=3t(t+2)\left(\frac{1}{3}\right)\\
3(t+2)+3t&=t(t+2)\\
3t+6+3t&=t^2+2t\\
0&=t^2-4t-6\\
t&=\frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-6)}}{2(1)}\\
&=\frac{4\pm\sqrt{40}}{2}\\
&=\frac{4\pm2\sqrt{10}}{2}\\
&=\frac{2\left(2\pm\sqrt{10}\right)}{2}\\
&=2\pm\sqrt{10}\\
t\approx 5.2 &\text{ or } t\approx -1.2
\end{aligned}$$

Since t represents time, $t \approx 5.2$ is the only solution. It takes approximately 5.2 hours for you working alone to paint the fence.

1.6.83.

$$d = \sqrt{\frac{2U}{k}}$$
$$d^2 = \frac{2U}{k}$$
$$d^2 k = 2U$$
$$\frac{kd^2}{2} = U$$

1.6.84.

$$C = 2\pi\sqrt{\frac{a^2 + b^2}{2}}$$

$$C^2 = \left(2\pi\sqrt{\frac{a^2 + b^2}{2}}\right)^2$$
$$\frac{C^2}{4\pi^2} = \frac{a^2 + b^2}{2}$$
$$\frac{2C^2}{4\pi^2} = a^2 + b^2$$
$$\frac{C^2}{2\pi^2} - b^2 = a^2$$
$$a = \pm\sqrt{\frac{C^2}{2\pi^2} - b^2}$$

1.6.85.

False. See Example 7 on page 123.

1.6.86.

$$\sqrt{x+10} - \sqrt{x-10} = 0$$
$$\sqrt{x+10} = \sqrt{x-10}$$
$$x + 10 = x - 10$$
$$10 \neq -10$$

True. There is no value to satisfy this equation.

1.6.87.

The quadratic equation was not written in general form before the values of a, b, and c were substituted in the Quadratic Formula. As a result, the substitutions in the Quadratic Formula are incorrect.

$$\sqrt{3}x = \sqrt{7x+4}$$
$$3x^2 = 7x+4$$
$$3x^2 - 7x - 4 = 0$$
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-4)}}{2(3)}$$
$$x = \frac{7+\sqrt{97}}{6}$$

1.6.88.

(a) The formula for the volume of the glass cube is V = Length × Width × Height. The volume of the water in the cube is the length × width × height of the water. So, the volume is $x \cdot x \cdot (x-3) = x^2(x-3)$.

(b) Given the equation $x^2(x-3)=320$. The dimensions of the glass cube can be found by solving for x. Then substitute that value into the expression x^3 to find the volume of the cube.

1.6.89.

$$|x-3|=|4x+9|$$

First Equation:

$$\begin{aligned} x-3&=-(4x+9)\\ x-3&=-4x-9\\ 5x&=-6\\ x&=-\frac{6}{5} \end{aligned}$$

Second Equation:

$$\begin{aligned} x-3&=4x+9\\ -3x&=12\\ x&=-4 \end{aligned}$$

Check $x=-\frac{6}{5}$:

$$\begin{aligned} \left|-\frac{6}{5}-3\right| &\stackrel{?}{=} \left|4\left(-\frac{6}{5}\right)+9\right|\\ \left|-\frac{21}{5}\right| &\stackrel{?}{=} \left|\frac{21}{5}\right|\\ \frac{21}{5} &= \frac{21}{5}\ \checkmark \end{aligned}$$

Check $x=-4$:

$$\begin{aligned} |-4-3| &\stackrel{?}{=} |4(-4)+9|\\ |-7| &\stackrel{?}{=} |-7|\\ 7 &= 7\ \checkmark \end{aligned}$$

The solutions are $x=-4$ and $x=-\frac{6}{5}$,

1.6.90.

$$\left|x^2+1\right|=\left|2x^2-1\right|$$

First Equation:

$$x^2+1=-\left(2x^2-1\right)$$
$$x^2-3=-2x^2+1$$
$$3x^2=0$$
$$x=0$$

Second Equation:

$$x^2+1=2x^2-1$$
$$-x^2=-2$$
$$x^2=2$$
$$x=\pm\sqrt{2}$$

Check $x=0$:

$$\left|0^2+1\right|\stackrel{?}{=}\left|2(0)^2-1\right|$$
$$\left|1\right|\stackrel{?}{=}\left|-1\right|$$
$$1=1\checkmark$$

Check $x=-\sqrt{2}$:

$$\left|\left(-\sqrt{2}\right)^2+1\right|\stackrel{?}{=}\left|2\left(-\sqrt{2}\right)^2-1\right|$$
$$\left|3\right|\stackrel{?}{=}\left|3\right|$$
$$3=3$$

Check $x=-\sqrt{2}$:

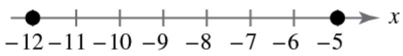
$$\left|\left(-\sqrt{2}\right)^2+1\right|\stackrel{?}{=}\left|2\left(-\sqrt{2}\right)^2-1\right|$$
$$\left|3\right|\stackrel{?}{=}\left|3\right|$$
$$3=3\checkmark$$

Check $x=-\sqrt{2}$:

$$\left|\left(-\sqrt{2}\right)^2+1\right|\stackrel{?}{=}\left|2\left(\sqrt{2}\right)^2-1\right|$$
$$\left|3\right|\stackrel{?}{=}\left|3\right|$$
$$3=3\checkmark$$

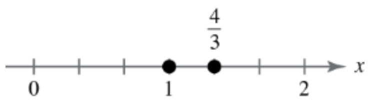
The solutions are $x=0$ and $x=\pm\sqrt{2}$.

1.6.91.



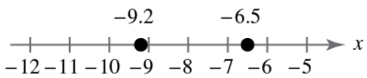
$-5 > -12$

1.6.92.



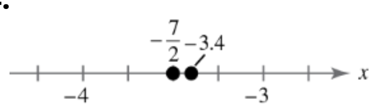
$1 < \frac{4}{3}$

1.6.93.



$-6.5 > -9.2$

1.6.94.



$\frac{-7}{2} < -3.4$

1.6.95.

The inequality $x \le 3$ denotes the set of all real numbers less than or equal to 3.

1.6.96.

The inequality $x > 0$ denotes the set of all real numbers greater than 0.

1.6.97.

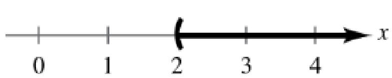
The inequality $-5 < x < 5$ denotes the set of all real numbers greater than –5 and less than 5.

1.6.98.

The inequality $0 < x \le 12$ denotes the set of all positive real numbers less than or equal to 12.

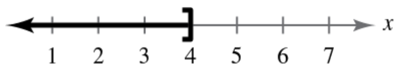
1.6.99.

The interval $(2, \quad \infty)$ denotes the set of all real numbers greater than 2; $x > 2$.



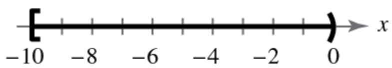
1.6.100.

The interval $(-\infty, \quad 4]$ denotes the set of all real numbers less than or equal to 4; $x \le 4$.



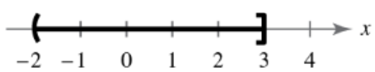
1.6.101.

The interval $[-10, \quad 0)$ denotes the set of all real numbers greater than or equal to -10 and less than 0; $-10 \le x < 0$.



1.6.102.

The interval $(-2, \quad 3]$ denotes the set of all real numbers greater than -2 and less than or equal to 3; $-2 < x \le 3$.



1.6.103.

$$\begin{aligned} 5x - 7 &= 3x + 9 \\ 2x &= 16 \\ x &= 8 \end{aligned}$$

1.6.104.

$$\begin{aligned} 7x - 3 &= 2x + 7 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

1.6.105.

$$\begin{aligned} 1 - \frac{3}{2}x &= x - 4 \\ 5 &= x + \frac{3}{2}x \\ 5 &= \frac{5}{2}x \\ x &= 2 \end{aligned}$$

1.6.106.

$$\begin{aligned} 2 - \frac{5}{3}x &= x - 6 \\ 8 &= x + \frac{5}{3}x \\ 8 &= \frac{8}{3}x \\ x &= 3 \end{aligned}$$

APPENDIX 1.7 LINEAR INEQUALITIES IN ONE VARIABLE

1.7.1.

solution set

1.7.2.

graph

1.7.3.

double

1.7.4.

union

1.7.5.

The inequalities $x - 4 < 5$ and $x > 9$ are not equivalent. The first simplifies to $x - 4 < 5 \Rightarrow x < 9$, which is not equivalent to the second, $x > 9$.

1.7.6.

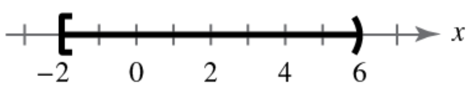
The Transitive Property of Inequalities is as follows:

$a < b$ and $b < c \Rightarrow a < c$.

1.7.7.

Interval: $\left[-2, 6\right)$

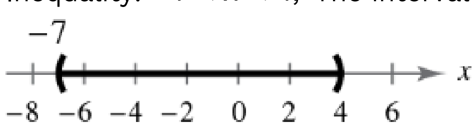
Inequality: $-2 \le x < 6$; The interval is bounded.



1.7.8.

Interval: $\left(-7, 4\right)$

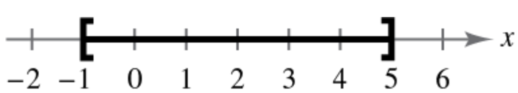
Inequality: $-7 < x < 4$; The interval is bounded.



1.7.9.

Interval: $\left[-1, 5\right]$

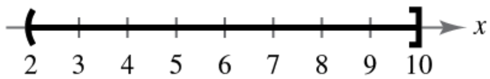
Inequality: $-1 \le x \le 5$; The interval is bounded.



1.7.10.

Interval: $\left(2, 10\right]$

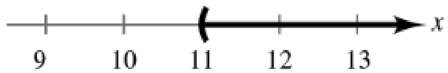
Inequality: $2 < x \le 10$; The interval is bounded.



1.7.11.

Interval: $(11, \infty)$

Inequality: $x > 11$; The interval is unbounded.



1.7.12.

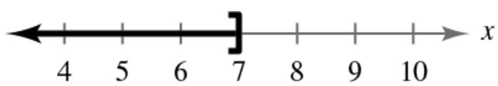
Interval: $[-5, \infty)$

Inequality: $-5 \le x < \infty$ or $x \ge -5$; The interval is unbounded.

1.7.13.

Interval: $(-\infty, 7]$

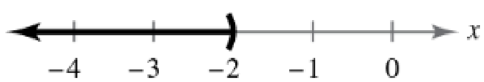
Inequality: $-\infty < x \le 7$ or $x \le 7$; The interval is unbounded.



1.7.14.

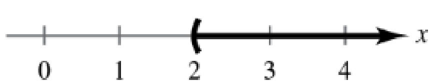
Interval: $(-\infty, -2)$

Inequality: $x < -2$; The interval is unbounded.



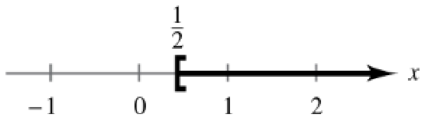
1.7.15.

$$\begin{aligned} 2x + 7 &< 3 + 4x \\ -2x &< -4 \\ x &> 2 \end{aligned}$$



1.7.16.

$$\begin{aligned} 3x + 1 &\ge 2 + x \\ 2x &\ge 1 \\ x &\ge \tfrac{1}{2} \end{aligned}$$

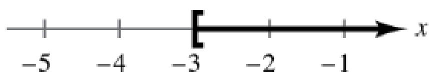


1.7.17.

$$\begin{aligned} 3x - 4 &\geq 4 - 5x \\ 8x &\geq 8 \\ x &\geq 1 \end{aligned}$$

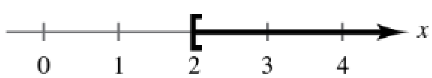
1.7.18.

$$\begin{aligned} 6x - 4 &\leq 2 + 8x \\ -2x &\leq 6 \\ x &\geq -3 \end{aligned}$$



1.7.19.

$$\begin{aligned} \tfrac{1}{2}(8x + 1) &\geq 3x + \tfrac{5}{2} \\ 4x + \tfrac{1}{2} &\geq 3x + \tfrac{5}{2} \\ x &\geq 2 \end{aligned}$$

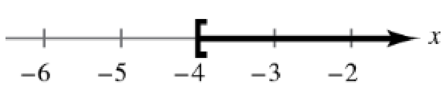


1.7.20.

$$\begin{aligned} 9x - 1 &< \tfrac{3}{4}(16x - 2) \\ 36x - 4 &< 48x - 6 \\ -12x &< -2 \\ x &> \tfrac{1}{6} \end{aligned}$$

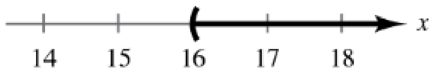
1.7.21.

$$\begin{aligned} 3.6x + 11 &\geq -3.4 \\ 3.6x &\geq 14.4 \\ x &\geq -4 \end{aligned}$$



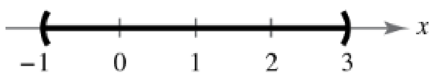
1.7.22.

$$\begin{aligned} 15.6 - 1.3x &< -5.2 \\ -1.3x &< -20.8 \\ x &> 16 \end{aligned}$$



1.7.23.

$$\begin{aligned} 1 < 2x + 3 < 9 \\ -2 < 2x < 6 \\ -1 < x < 3 \end{aligned}$$

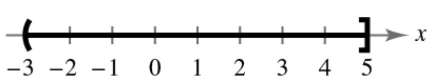


1.7.24.

$$\begin{aligned} -9 \le -2x - 7 < 5 \\ -2 \le -2x < 12 \\ 1 \ge x > -6 \\ -6 < x \le 1 \end{aligned}$$

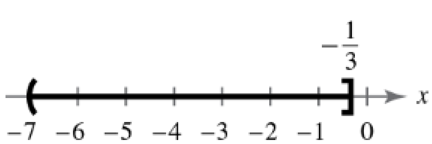
1.7.25.

$$\begin{aligned} -1 \le -(x - 4) < 7 \\ 1 \ge x - 4 > -7 \\ 5 \ge x > -3 \\ -3 < x \le 5 \end{aligned}$$



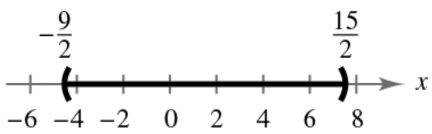
1.7.26.

$$\begin{aligned} 0 < 3(x + 7) \le 20 \\ 0 < x + 7 \le \frac{20}{3} \\ -7 < x \le -\frac{1}{3} \end{aligned}$$



1.7.27.

$$-4 < \frac{2x-3}{3} < 4$$
$$-12 < 2x - 3 < 12$$
$$-9 < 2x < 15$$
$$-\frac{9}{2} < x < \frac{15}{2}$$

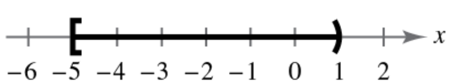


1.7.28.

$$0 \le \frac{x+3}{2} < 5$$
$$0 \le x + 3 < 10$$
$$-3 \le x < 7$$

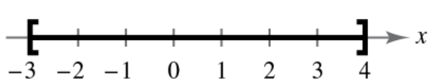
1.7.29.

$$-1 < \frac{-x-2}{3} \le 1$$
$$-3 < -x - 2 \le 3$$
$$-1 < -x \le 5$$
$$1 > x \ge -5$$
$$-5 \le x < 1$$



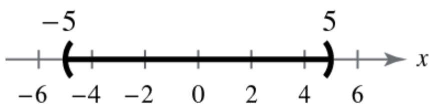
1.7.30.

$$-1 \le \frac{-3x+5}{7} \le 2$$
$$-7 \le -3x + 5 \le 14$$
$$-12 \le -3x \le 9$$
$$4 \ge x \ge -3$$
$$-3 \le x \le 4$$



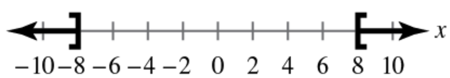
1.7.31.

$$|x| < 5$$
$$-5 < x < 5$$



1.7.32.

$$|x| \geq 8$$
$$x \geq 8 \quad \text{or} \quad x \leq -8$$



1.7.33.

$$|x-5| < -1$$

No solution. The absolute value of a number cannot be less than a negative number.

1.7.34.

$$|x-7| < -5$$

No solution. The absolute value of a number cannot be less than a negative number.

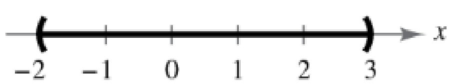
1.7.35.

$$|7-2x| \geq 9$$

$$7-2x \leq -9 \quad \text{or} \quad 7-2x \geq 9$$
$$-2x \leq -16 \qquad\qquad -2x \geq 2$$
$$x \geq 8 \qquad\qquad x \leq -1$$

1.7.36.

$$|1-2x| < 5$$
$$-5 < 1-2x < 5$$
$$-6 < -2x < 4$$
$$3 > x > -2$$
$$-2 < x < 3$$



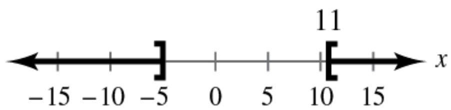
1.7.37.

$$\left|\frac{x-3}{2}\right| \ge 4$$

$$\frac{x-3}{2} \le -4 \quad \text{or} \quad \frac{x-3}{2} \ge 4$$

$$x-3 \le -8 \qquad x-3 \ge 8$$

$$x \le -5 \qquad x \ge 11$$



1.7.38.

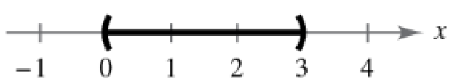
$$\left|1-\frac{2x}{3}\right| < 1$$

$$-1 < 1-\frac{2x}{3} < 1$$

$$-2 < -\frac{2x}{3} < 0$$

$$3 > x > 0$$

$$0 < x < 3$$



1.7.39.

The midpoint of the interval $\left[-3,\ 3\right]$ is 0. The interval represents all real numbers x no more than 3 units from 0.

$$|x-0| \le 3$$

$$|x| \le 3$$

1.7.40.

The graph shows all real numbers more than 3 units from 0.

$$|x-0| > 3$$

$$|x| > 3$$

1.7.41.

All real numbers less than 4 units from -3

$$|x-(-3)| < 4$$

$$|x+3| < 4$$

1.7.42.

All real numbers no more than 7 units from -6

$|x+6| \le 7$

1.7.43.

$\$7.25 \le P \le \$7.75$

1.7.44.

$180 < w < 185.5$

1.7.45.

$r \le 0.08$

1.7.46.

$I \ge \$239{,}000{,}000$

1.7.47.

$$\begin{aligned} 9.00 + 0.75x &> 13.50 \\ 0.75x &> 4.50 \\ x &> 6 \end{aligned}$$

You must produce at least 6 units each hour in order to yield a greater hourly wage at the second job.

1.7.48.

$$\begin{aligned} 10.00 + 1.25x &> 13.75 \\ 1.25x &> 3.75 \\ x &> 3 \end{aligned}$$

You must produce more than 3 units each hour in order to yield a greater hourly wage at the second job.

1.7.49.

$$\begin{aligned} 1000\left(1 + r\left(10\right)\right) &> 2000.00 \\ 1 + 10r &> 2 \\ 10r &> 1 \\ r &> 0.1 \end{aligned}$$

The rate must be greater than 10%.

1.7.50.

The goal is to lose $164 - 128 = 36$ pounds. At $1\frac{1}{2}$ pounds per week, it will take 24 weeks.

$$\begin{aligned} 36 \div 1\tfrac{1}{2} &= 36 \times \frac{2}{3} \\ &= 12 \times 2 \\ &= 24 \end{aligned}$$

1.7.51.

Let x = number of dozen doughnuts sold per day.

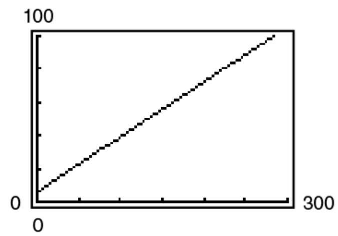
$$\begin{aligned}
\textit{Revenue}: R &= 7.95x\\
\textit{Cost}: C &= 1.45x + 165\\
P &= R - C\\
&= 7.95x - (1.45x + 165)\\
&= 6.50x - 165
\end{aligned}$$

$$\begin{aligned}
400 &\le P \le 1200\\
400 &\le 6.50x - 165 \le 1200\\
565 &\le 6.50x \le 1365\\
86.9 &\le x \le 210
\end{aligned}$$

The daily sales vary between 87 and 210 dozen doughnuts per day.

1.7.52.

(a)

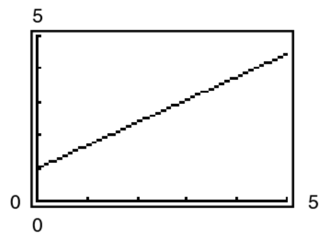


(b) One estimate is $x \le 224$ pounds.

(c)
$$\begin{aligned}
0.33x + 6.20 &\le 80\\
0.33x &\le 73.8\\
x &\le 223.636
\end{aligned}$$

1.7.53.

(a)



(b) From the graph you see that $y \ge 3$ when $x \ge 2.9$.

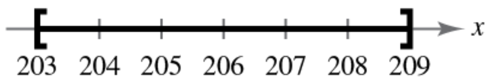
(c) Algebraically:

$$\begin{aligned}
3 &\le 0.692x + 0.988\\
2.012 &\le 0.692x\\
2.91 &\le x\\
x &\ge 2.91
\end{aligned}$$

1.7.54.

(a) $\left|x - 206\right| \le 3$

$$-3 \le x - 206 \le 3$$
$$203 \le x \le 209$$

(b) 

1.7.55.

$1\ \text{oz} = \dfrac{1}{16}$ lb, so $\dfrac{1}{2}$ oz $= \dfrac{1}{32}$ lb.

Because $8.99 \cdot \dfrac{1}{32} = 0.2809375,$ you may be undercharged or overcharged by \$0.28.

1.7.56.

$$24.2 - 0.25 \le s \le 24.2 + 0.25$$
$$23.95 \le s \le 24.45$$

The interval containing the possible side lengths s in centimeters of the square is $\left[23.95,\ 24.45\right]$, so the interval containing the possible areas in square centimeters is $\left[23.95^2,\ 24.45^2\right]$, or $\left[573.6025,\ 597.8025\right]$.

1.7.57.

True. This is the Addition of a Constant Property of Inequalities.

1.7.58.

False. If c is negative, then $ac \ge bc$.

1.7.59.

False. If $-10 \le x \le 8$, then $10 \ge -x$ and $-x \ge -8$.

1.7.60.

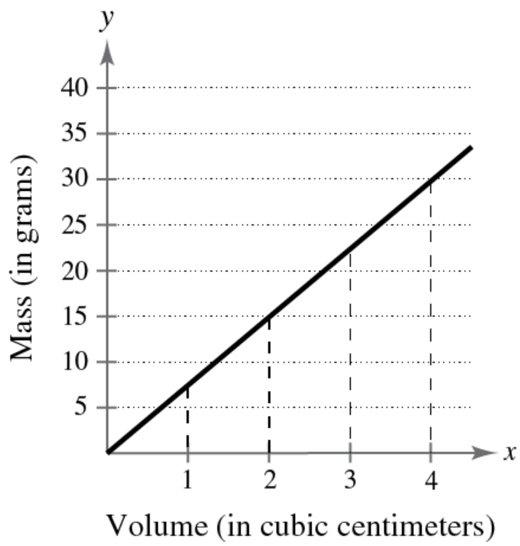
True.

Because $\left|2x - 5\right| \ge 0$, the only solution of $\left|2x - 5\right| \le 0$ is $2x - 5 = 0$, or $x = \dfrac{5}{2}$.

1.7.61.

Answer not unique. *Sample answer*: $x < x + 1$

1.7.62.



(a) When the volume is 2 cubic centimeters, the mass is approximately 15 grams.

(b) When the volume is greater than or equal to 0 cubic centimeters, and less than 4 cubic centimeters, $0 \le x < 4$ the mass is greater than or equal to 0 grams and less than 30 grams, $0 \le y < 30$.

1.7.63.

$|3x - 4|$ is always greater than or equal to 0, so the inequality is true for all real numbers.

1.7.64.

(a) $$\begin{aligned} 500(1+r)^2 &= 500(r+1)^2 \\ &= 500(r^2 + 2r + 1) \\ &= 500r^2 + 1000r + 500 \end{aligned}$$

(b)

r	$2\frac{1}{2}\%$	3%	4%	$4\frac{1}{2}\%$	5%
$500(1+r)^2$	\$525.31	\$530.45	\$540.80	\$546.01	\$551.25

(c) As r increases, the amount increases.

1.7.65.

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x &= 3,\ -2 \end{aligned}$$

1.7.66.

$$\begin{aligned} x^2 - x - 20 &= 0 \\ (x-5)(x+4) &= 0 \\ x &= 5, -4 \end{aligned}$$

1.7.67.

$$\begin{aligned} 4x^2 - 5x &= 6 \\ 4x^2 - 5x - 6 &= 0 \\ (4x+3)(x-2) &= 0 \\ x &= -\tfrac{3}{4}, 2 \end{aligned}$$

1.7.68.

$$\begin{aligned} 2x^2 + 3x &= 5 \\ 2x^2 + 3x - 5 &= 0 \\ (2x+5)(x-1) &= 0 \\ x &= -\tfrac{5}{2}, 1 \end{aligned}$$

1.7.69.

$$\begin{aligned} 2x^3 - 3x^2 &= 32x - 48 \\ 2x^3 - 3x^2 - 32x + 48 &= 0 \\ x^2(2x-3) - 16(2x-3) &= 0 \\ (x^2 - 16)(2x-3) &= 0 \\ (x-4)(x+4)(2x-3) &= 0 \\ x &= \pm 4, \tfrac{3}{2} \end{aligned}$$

1.7.70.

$$\begin{aligned} 3x^3 - x^2 &= 12x - 4 \\ 3x^3 - x^2 - 12x + 4 &= 0 \\ 3x^3 - 12x - x^2 + 4 &= 0 \\ 3x(x^2-4) - (x^2-4) &= 0 \\ (x^2-4)(3x-1) &= 0 \\ x &= \pm 2, \tfrac{1}{3} \end{aligned}$$

1.7.71.

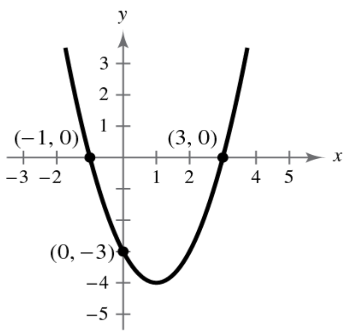
$$\begin{aligned} \frac{2x-7}{x-5} &= 3 \\ 2x - 7 &= 3x - 15 \\ 8 &= x \end{aligned}$$

1.7.72.

$$\frac{1}{x-3} = \frac{3}{2x+1}$$
$$2x+1 = 3(x-3)$$
$$2x+1 = 3x-9$$
$$10 = x$$

1.7.73.

$$y = x^2 - 2x - 3 = (x-3)(x+1)$$
$$y = (-x)^2 - 2(-x) - 3 \Rightarrow y = x^2 + 2x - 3 \Rightarrow \text{No } y\text{-axis symmetry}$$
$$-y = x^2 - 2x - 3 \Rightarrow y = -x^2 + 2x + 3 \Rightarrow \text{No } x\text{-axis symmetry}$$
$$-y = (-x)^2 - 2(-x) - 3 \Rightarrow -y = x^2 + 2x - 3 = y = -x^2 - 2x + 3 \Rightarrow \text{No origin symmetry}$$

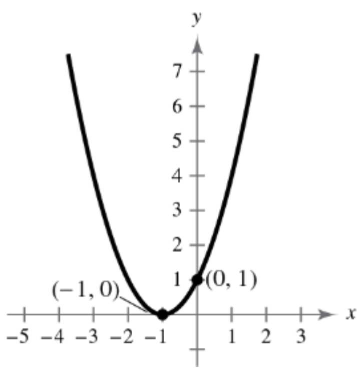


x-intercepts: $(-1, 0)(3, 0)$

y-intercept: $(0, -3)$

1.7.74.

$$y = x^2 + 2x + 1 = (x+1)^2$$
$$y = (-x)^2 + 2(-x) + 1 \Rightarrow y = x^2 - 2x + 1 \Rightarrow \text{No } y\text{-axis symmetry}$$
$$-y = x^2 + 2x + 1 \Rightarrow y = -x^2 - 2x - 1 \Rightarrow \text{No } x\text{-axis symmetry}$$
$$-y = (-x)^2 + 2(-x) + 1 \Rightarrow -y = x^2 - 2x + 1 \Rightarrow y = -x^2 + 2x - 1 \Rightarrow \text{No origin symmetry}$$



x-intercept: $(-1, 0)$

y-intercept: $(0, 1)$

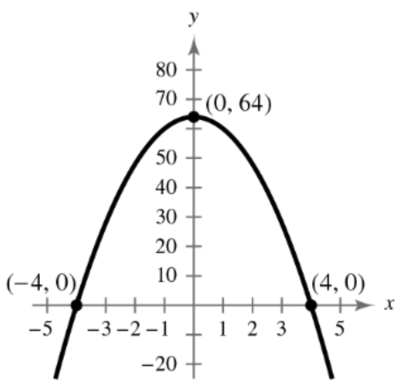
1.7.75.

$y = 64 - 4x^2$

$y = 64 - 4(-x)^2 \Rightarrow y = 64 - 4x^2 \Rightarrow$ y-axis symmetry

$-y = 64 - 4x^2 \Rightarrow y = -64 + 4x^2 \Rightarrow$ No x-axis symmetry

$-y = 64 - 4(-x)^2 \Rightarrow -y = 64 - 4x^2 \Rightarrow y = -64 + 4x^2 \Rightarrow$ No origin symmetry



x-intercepts: $(\pm 4, 0)$

y-intercept: $(0, 64)$

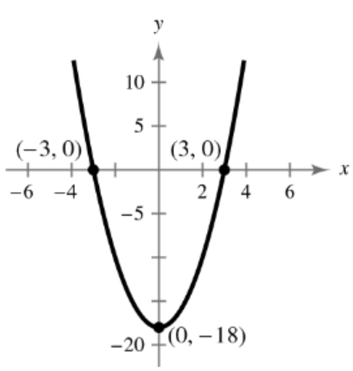
1.7.76.

$y = 2x^2 - 18$

$y = 2(-x)^2 - 18 \Rightarrow y = 2x^2 - 18 \Rightarrow$ y-axis symmetry

$-y = 2x^2 - 18 \Rightarrow y = -2x^2 + 18 \Rightarrow$ No x-axis symmetry

$-y = 2(-x)^2 - 18 \Rightarrow -y = 2x^2 - 18 \Rightarrow y = -2x^2 + 18 \Rightarrow$ No origin symmetry



x-intercepts: $(\pm 3, 0)$

y-intercept: $(0, -18)$

SECTION 1.8 OTHER TYPES OF INEQUALITIES

1.8.1.

positive; negative

1.8.2.

zeros; undefined values

1.8.3.

The key numbers of the inequality are -2 and 5.

1.8.4.

No. $4(4-4)=0$, which is not less than 0.

1.8.5.

$x^2 - 3 < 0$

(a) $x = 3$

$$(3)^2 - 3 \stackrel{?}{<} 0$$
$$6 \not< 0$$

No, $x = 3$ *is not* a solution.

(b) $x = 0$

$$(0)^2 - 3 \stackrel{?}{<} 0$$
$$-3 < 0$$

Yes, $x = 0$ *is* a solution.

(c) $x = \frac{3}{2}$

$$\left(\frac{3}{2}\right)^2 - 3 \stackrel{?}{<} 0$$
$$-\frac{3}{4} < 0$$

Yes, $x = \frac{3}{2}$ *is* a solution.

(d) $x = -5$

$$(-5)^2 - 3 \stackrel{?}{<} 0$$
$$22 \not< 0$$

No, $x = -5$ *is not* a solution.

1.8.6.

$x^2 - 2x - 8 \geq 0$

(a) $x = -2$

$$(-2)^2 - 2(-2) - 8 \overset{?}{\geq} 0$$
$$0 \geq 0$$

Yes, $x = -2$ *is* a solution.

(b) $x = 0$

$$(0)^2 - 2(0) - 8 \overset{?}{\geq} 0$$
$$-8 \not\geq 0$$

No, $x = 0$ *is not* a solution.

(c) $x = -4$

$$(-4)^2 - 2(-4) - 8 \overset{?}{\geq} 0$$
$$16 + 8 - 8 \overset{?}{\geq} 0$$
$$16 \geq 0$$

Yes, $x = -4$ *is* a solution.

(d) $x = 1$

$$(1)^2 - 2(1) - 8 \overset{?}{\geq} 0$$
$$1 - 2 - 8 \overset{?}{\geq} 0$$
$$-9 \not\geq 0$$

No, $x = 1$ *is not* a solution.

1.8.7.

$\frac{x+2}{x-4} \geq 3$

(a) $x = 5$

$$\frac{5+2}{5-4} \overset{?}{\geq} 3$$
$$7 \geq 3$$

Yes, $x = 5$ *is* a solution.

(b) $x = 4$

$$\frac{4+2}{4-4} \overset{?}{\geq} 3$$

$\frac{6}{0}$ is undefined.

No, $x = 4$ *is not* a solution.

(c) $x = -\frac{9}{2}$

$$\frac{-\frac{9}{2}+2}{-\frac{9}{2}-4} \stackrel{?}{\geq} 3$$

$$\frac{5}{17} \not\geq 3$$

No, $x = -\frac{9}{2}$ *is not* a solution.

(d) $x = \frac{9}{2}$

$$\frac{\frac{9}{2}+2}{\frac{9}{2}-4} \stackrel{?}{\geq} 3$$

$$13 \geq 3$$

Yes, $x = \frac{9}{2}$ *is* a solution.

1.8.8.

$$\frac{3x^2}{x^2+4} < 1$$

(a) $x = -2$

$$\frac{3(-2)^2}{(-2)^2+4} \stackrel{?}{<} 1$$

$$\frac{12}{8} \not< 1$$

No, $x = -2$ *is not* a solution.

(b) $x = -1$

$$\frac{3(-1)^2}{(-1)^2+4} \stackrel{?}{<} 1$$

$$\frac{3}{5} < 1$$

Yes, $x = -1$ *is* a solution.

(c) $x = 0$

$$\frac{3(0)^2}{(0)^2 + 4} \stackrel{?}{<} 1$$

$$0 < 1$$

Yes, $x = 0$ *is* a solution.

(d) $x = 3$

$$\frac{3(3)^2}{(3)^2 + 4} \stackrel{?}{<} 1$$

$$\frac{27}{13} \not< 1$$

No, $x = 3$ *is not* a solution.

1.8.9.

$$x^2 - 3x - 18 = (x+3)(x-6)$$

$$x + 3 = 0 \Rightarrow x = -3$$

$$x - 6 = 0 \Rightarrow x = 6$$

The key numbers are -3 and 6.

1.8.10.

$$9x^3 - 25x^2 = 0$$

$$x^2(9x - 25) = 0$$

$$x^2 = 0 \Rightarrow x = 0$$

$$9x - 25 = 0 \Rightarrow x = \tfrac{25}{9}$$

The key numbers are 0 and $\dfrac{25}{9}$.

1.8.11.

$$\frac{1}{x-5} + 1 = \frac{1 + 1(x-5)}{x-5}$$

$$= \frac{x-4}{x-5}$$

$$x - 4 = 0 \Rightarrow x = 4$$

$$x - 5 = 0 \Rightarrow x = 5$$

The key numbers are 4 and 5.

1.8.12.

$$\frac{x}{x+2}-\frac{2}{x-1}=\frac{x(x-1)-2(x+2)}{(x+2)(x-1)}$$
$$=\frac{x^2-x-2x-4}{(x+2)(x-1)}$$
$$=\frac{(x-4)(x+1)}{(x+2)(x-1)}$$

$(x-4)(x+1)=0$

$x-4=0 \Rightarrow x=4$

$x+1=0 \Rightarrow x=-1$

$(x+2)(x-1)=0$

$x+2=0 \Rightarrow x=-2$

$x-1=0 \Rightarrow x=1$

The key numbers are $-2, -1, 1,$ and 4.

1.8.13.

$2x^2+4x<0$

$2x(x+2)<0$

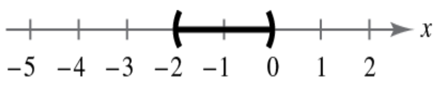
Key numbers: $x=0, -2$

Test intervals: $(-\infty, -2)$, $(-2, 0)$, $(0, \infty)$

Test: Is $2x(x+2)<0$?

Interval	x-Value	Value of $2x(x+2)$	Conclusion
$(-\infty, -2)$	-3	6	Positive
$(-2, 0)$	-1	-2	Negative
$(0, \infty)$	1	3	Positive

Solution set: $(-2, 0)$



1.8.14.

$3x^2-9x\ge 0$

$3x(x-3)\ge 0$

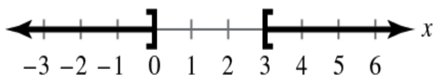
Key numbers: $x=0, 3$

Test intervals: $(-\infty, 0)$, $(0, 3)$, $(3, \infty)$

Test: Is $3x(x-3) > 0$?

Interval	x-Value	Value of $3x(x-3)$	Conclusion
$(-\infty, 0)$	-1	12	Positive
$(0, 3)$	1	-6	Negative
$(3, \infty)$	4	12	Positive

Solution set: $(-\infty, 0] \cup [3, \infty)$



1.8.15.

$$\begin{aligned} x^2 &< 9 \\ x^2 - 9 &< 0 \\ (x+3)(x-3) &< 0 \end{aligned}$$

Key numbers: $x = \pm 3$

Test intervals: $(-\infty, -3)$, $(-3, 3)$, $(3, \infty)$

Test: Is $(x+3)(x-3) < 0$?

Interval	x-Value	Value of $x^2 - 9$	Conclusion
$(-\infty, -3)$	-4	7	Positive
$(-3, 3)$	0	-9	Negative
$(3, \infty)$	4	7	Positive

Solution set: $(-3, 3)$

1.8.16.

$$\begin{aligned} x^2 &\le 25 \\ x^2 - 25 &\le 0 \\ (x+5)(x-5) &\le 0 \end{aligned}$$

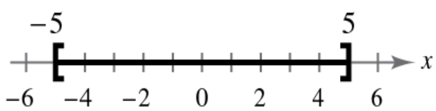
Key numbers: $x = \pm 5$

Test intervals: $(-\infty, -5)$, $(-5, 5)$, $(5, \infty)$

Test: Is $(x+5)(x-5) \le 0$?

Interval	x-Value	Value of $x^2 - 25$	Conclusion
$(-\infty, -5)$	-6	11	Positive
$(-5, 5)$	0	-25	Negative
$(5, \infty)$	6	11	Positive

Solution set: $[-5, 5]$



1.8.17.

$$
\begin{aligned}
(x+2)^2 &\le 25 \\
x^2 + 4x + 4 &\le 25 \\
x^2 + 4x - 21 &\le 0 \\
(x+7)(x-3) &\le 0
\end{aligned}
$$

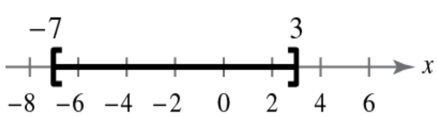
Key numbers: $x = -7, \quad x = 3$

Test intervals: $(-\infty, -7), (-7, 3), (3, \infty)$

Test: Is $(x+7)(x-3) \le 0$?

Interval	x-Value	Value of $(x+7)(x-3)$	Conclusion
$(-\infty, -7)$	-8	$(-1)(-11) = 11$	Positive
$(-7, 3)$	0	$(7)(-3) = -21$	Negative
$(3, \infty)$	4	$(11)(1) = 11$	Positive

Solution set: $[-7, 3]$



1.8.18.

$$
\begin{aligned}
(x-3)^2 &\ge 1 \\
x^2 - 6x + 8 &\ge 0 \\
(x-2)(x-4) &\ge 0
\end{aligned}
$$

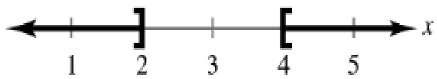
Key numbers: $x = 2, \quad x = 4$

Test intervals: $(-\infty, 2) \Rightarrow (x-2)(x-4) > 0$

$(2, 4) \Rightarrow (x-2)(x-4) < 0$

$(4, \infty) \Rightarrow (x-2)(x-4) > 0$

Solution set: $(-\infty, 2] \cup [4, \infty)$



1.8.19.

$$x^2 + 6x + 1 \geq -7$$
$$x^2 + 6x + 8 \geq 0$$
$$(x+2)(x+4) \geq 0$$

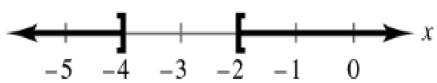
Key numbers: $x = -2,\ x = -4$

Test Intervals: $(-\infty, -4),\ (-4, -2),\ (-2, \infty)$

Test: Is $(x+2)(x+4) > 0$?

Interval	x-Value	Value of $(x+2)(x+4)$	Conclusion
$(-\infty, -4)$	-6	8	Positive
$(-4, -2)$	-3	-1	Negative
$(-2, \infty)$	0	8	Positive

Solution set: $(-\infty, -4] \cup [-2, \infty)$



1.8.20.

$$x^2 - 8x + 2 < 11$$
$$x^2 - 8x - 9 < 0$$
$$(x-9)(x+1) < 0$$

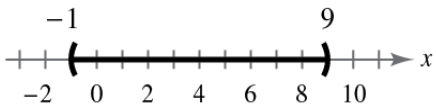
Key numbers: $x = -1,\ x = 9$

Test intervals: $(-\infty, -1) \Rightarrow (x-9)(x+1) > 0$

$(-1, 9) \Rightarrow (x-9)(x+1) < 0$

$(9, \infty) \Rightarrow (x-9)(x+1) > 0$

Solution set: $(-1, 9)$



1.8.21.

$$x^2 + x < 6$$
$$x^2 + x - 6 < 0$$
$$(x+3)(x-2) < 0$$

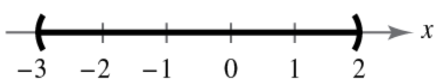
Key numbers: $x = -3,\ x = 2$

Test intervals: $(-\infty, -3), (-3, 2), (2, \infty)$

Test: Is $(x+3)(x-2) < 0$?

Interval	x-Value	Value of $(x+3)(x-2)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-6) = 6$	Positive
$(-3, 2)$	0	$(3)(-2) = -6$	Negative
$(2, \infty)$	3	$(6)(1) = 6$	Positive

Solution set: $(-3, 2)$



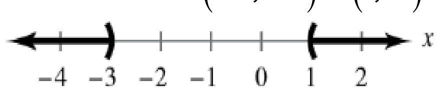
1.8.22.

$$x^2 + 2x > 3$$
$$x^2 + 2x - 3 > 0$$
$$(x+3)(x-1) > 0$$

Key numbers: $x = -3,\ x = 1$

Test intervals: $(-\infty, -3) \Rightarrow (x+3)(x-1) > 0$
$(-3, 1) \Rightarrow (x+3)(x-1) < 0$
$(1, \infty) \Rightarrow (x+3)(x-1) > 0$

Solution set: $(-\infty, -3) \cup (1, \infty)$



1.8.23.

$$x^2 < 3 - 2x$$
$$x^2 + 2x - 3 < 0$$
$$(x+3)(x-1) < 0$$

Key numbers: $x = -3,\ x = 1$

Test intervals: $(-\infty, -3),\ (-3, 1),\ (1, \infty)$

Test: Is $(x+3)(x-1) < 0$?

Interval	x-Value	Value of $(x+3)(x-1)$	Conclusion
$(-\infty, -3)$	-4	$(-1)(-5) = 5$	Positive
$(-3, 1)$	0	$(3)(-1) = -3$	Negative
$(1, \infty)$	2	$(5)(1) = 5$	Positive

Solution set: $(-3,\ 1)$

1.8.24.

$$x^2 > 2x + 8$$
$$x^2 - 2x - 8 > 0$$
$$(x-4)(x+2) > 0$$

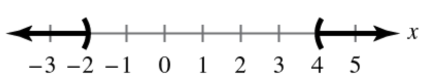
Key numbers: $x = -2,\ x = 4$

Test intervals: $(-\infty, -2),\ (-2, 4),\ (4, \infty)$

Test: Is $(x-4)(x+2) > 0$?

Interval	x-Value	Value of $(x-4)(x+2)$	Conclusion
$(-\infty, -2)$	-3	$(-7)(-1) = 7$	Positive
$(-2, 4)$	0	$(-4)(2) = -8$	Negative
$(4, \infty)$	5	$(1)(7) = 7$	Positive

Solution set: $(-\infty, -2) \cup (4, \infty)$



1.8.25.

$$3x^2 - 11x > 20$$
$$3x^2 - 11x - 20 > 0$$
$$(3x+4)(x-5) > 0$$

Key numbers: $x = 5,\ x = -\frac{4}{3}$

Test intervals: $\left(-\infty,\ -\frac{4}{3}\right),\ \left(-\frac{4}{3},\ 5\right),\ (5,\ \infty)$

Test: Is $(3x+4)(x-5) > 0$?

Interval	x-Value	Value of $(3x+4)(x-5)$	Conclusion
$\left(-\infty, -\frac{4}{3}\right)$	-3	$(-5)(-8) = 40$	Positive
$\left(-\frac{4}{3}, 5\right)$	0	$(4)(-5) = -20$	Negative
$(5, \infty)$	6	$(22)(1) = 22$	Positive

Solution set: $\left(-\infty, -\frac{4}{3}\right) \cup (5, \infty)$

1.8.26.

$$-2x^2 + 6x + 15 \le 0$$
$$2x^2 - 6x - 15 \ge 0$$
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-15)}}{2(2)}$$
$$= \frac{6 \pm \sqrt{156}}{4}$$
$$= \frac{6 \pm 2\sqrt{39}}{4}$$
$$= \frac{3}{2} \pm \frac{\sqrt{39}}{2}$$

Key numbers: $x = \frac{3}{2} - \frac{\sqrt{39}}{2},\ x = \frac{3}{2} + \frac{\sqrt{39}}{2}$

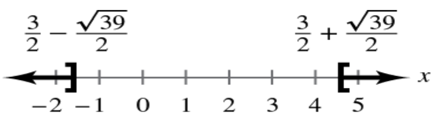
Test intervals:

$$\left(-\infty,\ \frac{3}{2}-\frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2+6x+15<0$$

$$\left(\frac{3}{2}-\frac{\sqrt{39}}{2},\ \frac{3}{2}+\frac{\sqrt{39}}{2}\right) \Rightarrow -2x^2+6x+15>0$$

$$\left(\frac{3}{2}+\frac{\sqrt{39}}{2},\ \infty\right) \Rightarrow -2x^2+6x+15<0$$

Solution set: $\left(-\infty,\ \frac{3}{2}-\frac{\sqrt{39}}{2}\right] \cup \left[\frac{3}{2}+\frac{\sqrt{39}}{2},\ \infty\right)$



1.8.27.

$$\begin{aligned} x^3-3x^2-x+3&>0\\ x^2(x-3)-(x-3)&>0\\ (x-3)(x^2-1)&>0\\ (x-3)(x+1)(x-1)&>0 \end{aligned}$$

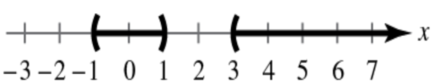
Key numbers: $x=-1,\ x=1,\ x=3$

Test intervals: $(-\infty,-1),\ (-1,1),\ (1,3),(3,\infty)$

Test: Is $(x-3)(x+1)(x-1)>0$?

Interval	x-Value	Value of $(x-3)(x+1)(x-1)$	Conclusion
$(-\infty,-1)$	-2	$(-5)(-1)(-3)=-15$	Negative
$(-1,1)$	0	$(-3)(1)(-1)=3$	Positive
$(1,3)$	2	$(-1)(3)(1)=-3$	Negative
$(3,\infty)$	4	$(1)(5)(3)=15$	Positive

Solution set: $(-1,1)\cup(3,\infty)$



1.8.28.

$$x^3 + 2x^2 - 4x - 8 \le 0$$
$$x^2(x+2) - 4(x+2) \le 0$$
$$(x+2)(x^2-4) \le 0$$
$$(x+2)^2(x-2) \le 0$$

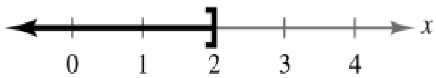
Key numbers: $x = -2,\ x = 2$

Test intervals: $(-\infty, -2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(-2, 2) \Rightarrow x^3 + 2x^2 - 4x - 8 < 0$

$(2, \infty) \Rightarrow x^3 + 2x^2 - 4x - 8 > 0$

Solution set: $(-\infty, 2]$



1.8.29.

$$-x^3 + 7x^2 + 9x > 63$$
$$x^3 - 7x^2 - 9x < -63$$
$$x^3 - 7x^2 - 9x + 63 < 0$$
$$x^2(x-7) - 9(x-7) < 0$$
$$(x-7)(x^2-9) < 0$$
$$(x-7)(x+3)(x-3) < 0$$

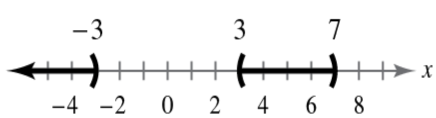
Key numbers: $x = -3,\ x = 3,\ x = 7$

Test intervals: $(-\infty, -3),\ (-3, 3),\ (3, 7),\ (7, \infty)$

Test: Is $(x-7)(x+3)(x-3) < 0$?

Interval	x-Value	Value of $(x-7)(x+3)(x-3)$	Conclusion
$(-\infty, -3)$	-4	$(-11)(-1)(-7) = -77$	Negative
$(-3, 3)$	0	$(-7)(3)(-3) = 63$	Positive
$(3, 7)$	4	$(-3)(7)(1) = -21$	Negative
$(7, \infty)$	8	$(1)(11)(5) = 55$	Positive

Solution set: $(-\infty, -3) \cup (3, 7)$



1.8.30.

$$\begin{aligned}2x^3+13x^2-8x &\geq 52\\ 2x^3+13x^2-8x-52 &\geq 0\\ x^2(2x+13)-4(2x+13) &\geq 0\\ (2x+13)(x^2-4) &\geq 0\\ (2x+13)(x+2)(x-2) &\geq 0\end{aligned}$$

Key numbers: $x=-\frac{13}{2},\ x=-2,\ x=2$

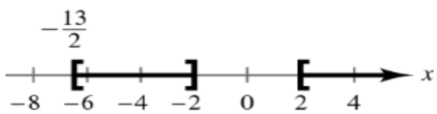
Test intervals: $\left(-\infty,-\frac{13}{2}\right)\Rightarrow 2x^3+13x^2-8x-52<0$

$\left(-\frac{13}{2},-2\right)\Rightarrow 2x^3+13x^2-8x-52>0$

$(-2,2)\Rightarrow 2x^3+13x^2-8x-52<0$

$(2,\infty)\Rightarrow 2x^3+13x^2-8x-52>0$

Solution set: $\left[-\frac{13}{2},-2\right],[2,\infty)$



1.8.31.

$$\begin{aligned}4x^3-6x^2 &< 0\\ 2x^2(2x-3) &< 0\end{aligned}$$

Key numbers: $x=0,\ \ x=\frac{3}{2}$

Test intervals: $(-\infty,0)\Rightarrow 2x^2(2x-3)<0$

$\left(0,\frac{3}{2}\right)\Rightarrow 2\Rightarrow 2x^2(2x-3)<0$

$\left(\frac{3}{2},\infty\right)\Rightarrow 2x^2(2x-3)>0$

Solution set: $(-\infty,0)\cup\left(0,\frac{3}{2}\right)$

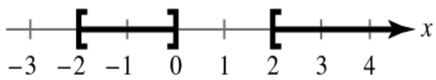
1.8.32.

$$\begin{aligned}x^3-4x &\geq 0\\ x(x+2)(x-2) &\geq 0\end{aligned}$$

Key numbers: $x=0,\ \ x=\pm 2$

Test intervals: $(-\infty, -2) \Rightarrow x(x+2)(x-2) < 0$
$(-2, 0) \Rightarrow x(x+2)(x-2) > 0$
$(0, 2) \Rightarrow x(x+2)(x-2) < 0$
$(2, \infty) \Rightarrow x(x+2)(x-2) > 0$

Solution set: $[-2, 0] \cup [2, \infty)$



1.8.33.

$4x^2 - 4x + 1 \le 0$

$(2x-1)^2 \le 0$

Key number: $x = \frac{1}{2}$

Test Interval	x-Value	Polynomial Value	Conclusion
$\left(-\infty, \frac{1}{2}\right)$	$x = 0$	$[2(0)-1]^2 = 1$	Positive
$\left(\frac{1}{2}, \infty\right)$	$x = 1$	$[2(1)-1]^2 = 1$	Positive

The solution set consists of the single real number $\frac{1}{2}$.

1.8.34.

$x^2 + 3x + 8 > 0$

Using the Quadratic Formula you can determine the key numbers are

$x = -\dfrac{3}{2} \pm \dfrac{\sqrt{23}}{2}i.$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 + 3(0) + 8 = 8$	Positive

The solution set is the set of all real numbers.

1.8.35.

$x^2 - 6x + 12 \le 0$

Using the Quadratic Formula, you can determine that the key numbers are

$x = 3 \pm \sqrt{3}i.$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, \infty)$	$x = 0$	$(0)^2 - 6(0) + 12 = 12$	Positive

The solution set is empty, that is there are no real solutions.

1.8.36.

$$x^2 - 8x + 16 > 0$$
$$(x-4)^2 > 0$$

Key number: $x = 4$

Test Interval	x-Value	Polynomial Value	Conclusion
$(-\infty, 4)$	$x = 0$	$(0-4)^2 = 16$	Positive
$(4, \infty)$	$x = 5$	$(5-4)^2 = 1$	Positive

The solution set consists of all real numbers except $x = 4$, or $(-\infty, 4) \cup (4, \infty)$.

1.8.37.

$$\frac{4x-1}{x} > 0$$

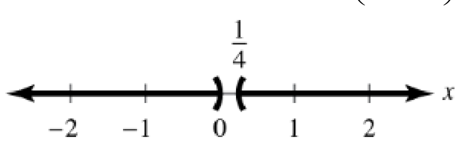
Key numbers: $x = 0,\ x = \frac{1}{4}$

Test intervals: $(-\infty, 0), \left(0, \frac{1}{4}\right), \left(\frac{1}{4}, \infty\right)$

Test: Is $\frac{4x-1}{x} > 0$?

Interval	x-Value	Value of $\frac{4x-1}{x}$	Conclusion
$(-\infty, 0)$	-1	$\frac{-5}{-1} = 5$	Positive
$\left(0, \frac{1}{4}\right)$	$\frac{1}{8}$	$\frac{-\frac{1}{2}}{\frac{1}{8}} = -4$	Negative
$\left(\frac{1}{4}, \infty\right)$	1	$\frac{3}{1} = 3$	Positive

Solution set: $(-\infty, 0) \cup \left(\frac{1}{4}, \infty\right)$



1.8.38.

$$\frac{x-1}{x} < 0$$

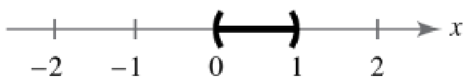
Key numbers: $0, 1$

Test intervals: $(-\infty, 0), (0, 1), (1, \infty)$

Test: Is $\frac{x-1}{x} < 0$?

Interval	x-Value	Value of $\frac{x-1}{x}$	Conclusion
$(-\infty,\ 0)$	-1	$\frac{-1-1}{-1} = 2$	Positive
$(0, 1)$	$\frac{1}{2}$	$\frac{\frac{1}{2}-1}{\frac{1}{2}} = -1$	Negative
$(1, \infty)$	2	$\frac{2-1}{2} = \frac{1}{2}$	Positive

Solution set: $(0,\ 1)$



1.8.39.

$$\frac{3x+5}{x-1} < 2$$
$$\frac{3x+5}{x-1} - 2 < 0$$
$$\frac{3x+5-2(x-1)}{x-1} < 0$$
$$\frac{x+7}{x-1} < 0$$

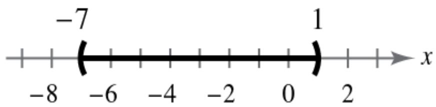
Key numbers: $x = -7,\ x = 1$

Test intervals: $(-\infty,\ -7),\ (-7,\ 1),\ (1,\ \infty)$

Test: Is $\frac{x+7}{x-1} < 0$?

Interval	x-Value	Value of $\frac{x+7}{x-1}$	Conclusion
$(-\infty,\ -7)$	-8	$\frac{-1}{-9} = \frac{1}{9}$	Positive
$(-7,\ 1)$	0	$\frac{0+7}{0-1} = -7$	Negative
$(1,\ \infty)$	2	$\frac{2+9}{2-1} = 11$	Positive

Solution set: $(-7,\ 1)$



1.8.40.

$$\frac{x+12}{x+2} \geq 3$$

$$\frac{x+12}{x+2} - 3 \geq 0$$

$$\frac{x+12-3(x+2)}{x+2} \geq 0$$

$$\frac{6-2x}{x+2} \geq 0$$

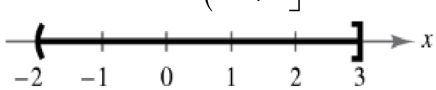
Key numbers: $x = -2,\ x = 3$

Test intervals: $(-\infty, -2),\ (-2, 3),\ (-3, \infty)$

Test: Is $\dfrac{6-2x}{x+2} > 0$?

Interval	x-Value	Value of $\dfrac{6-2x}{x+2}$	Conclusion
$(-\infty, -2)$	-3	$\dfrac{6-2(-3)}{(-3)+2} = -12$	Negative
$(-2, 3)$	0	$\dfrac{6-0}{0+2} = 3$	Positive
$(3, \infty)$	4	$\dfrac{6-8}{4+2} = \dfrac{1}{3}$	Negative

Solution set: $(-2, 3]$



1.8.41.

$$\frac{2}{x+5} > \frac{1}{x-3}$$

$$\frac{2}{x+5} - \frac{1}{x-3} > 0$$

$$\frac{2(x-3)-1(x+5)}{(x+5)(x-3)} > 0$$

$$\frac{x-11}{(x+5)(x-3)} > 0$$

Key numbers: $x = -5,\ x = 3,\ x = 11$

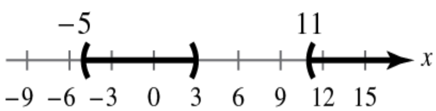
Test intervals: $\left(-\infty, -5\right) \Rightarrow \dfrac{x-11}{\left(x+5\right)\left(x-3\right)} < 0$

$\left(-5, 3\right) \Rightarrow \dfrac{x-11}{\left(x+5\right)\left(x-3\right)} > 0$

$\left(3, 11\right) \Rightarrow \dfrac{x-11}{\left(x+5\right)\left(x-3\right)} < 0$

$\left(11, \infty\right) \Rightarrow \dfrac{x-11}{\left(x+5\right)\left(x-3\right)} > 0$

Solution set: $\left(-5, 3\right) \cup \left(11, \infty\right)$



1.8.42.

$$\frac{5}{x-6} > \frac{3}{x+2}$$

$$\frac{5\left(x+2\right)-3\left(x-6\right)}{\left(x-6\right)\left(x+2\right)} > 0$$

$$\frac{2x+28}{\left(x-6\right)\left(x+2\right)} > 0$$

Key numbers: $x = -14,\ \ x = -2,\ \ x = 6$

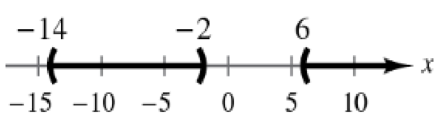
Test intervals: $\left(-\infty, -14\right) \Rightarrow \dfrac{2x+28}{\left(x-6\right)\left(x+2\right)} < 0$

$\left(-14, -2\right) \Rightarrow \dfrac{2x+28}{\left(x-6\right)\left(x+2\right)} > 0$

$\left(-2, 6\right) \Rightarrow \dfrac{2x+28}{\left(x-6\right)\left(x+2\right)} < 0$

$\left(6, \infty\right) \Rightarrow \dfrac{2x+28}{\left(x-6\right)\left(x+2\right)} > 0$

Solution intervals: $\left(-14, -2\right) \cup \left(6, \infty\right)$



1.8.43.

$$\frac{x^2+2x}{x^2-9} \le 0$$

$$\frac{x(x+2)}{(x+3)(x-3)} \le 0$$

Key numbers: $x = 0, \quad x = -2, \quad x = \pm 3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} > 0$

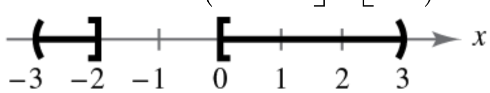
$(-3, -2) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} < 0$

$(-2, 0) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} < 0$

$(3, \infty) \Rightarrow \dfrac{x(x+2)}{(x+3)(x-3)} > 0$

Solution set: $(-3, -2] \cup [0, 3)$



1.8.44.

$$\frac{x^2+x-6}{x^2-4x} \ge 0$$

$$\frac{(x+3)(x-2)}{x(x-4)} \ge 0$$

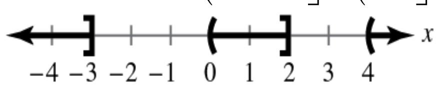
Key numbers: $-3, 0, 2, 4$

Test intervals: $(-\infty, -3), \ (-3, 0), (0, 2), (2, 4), (4, \infty)$

Test: Is $\dfrac{(x+3)(x-2)}{x(x-4)} \ge 0$?

Interval	x-Value	Value of $\dfrac{(x+3)(x-2)}{x(x-4)}$	Conclusion
$(-\infty, -3)$	-4	$\dfrac{(-1)(-6)}{(-4)(-8)} = \dfrac{3}{16}$	Positive
$(-3, 0)$	-1	$\dfrac{(2)(-3)}{(-1)(-5)} = -\dfrac{6}{5}$	Negative
$(0, 2)$	1	$\dfrac{(4)(-1)}{(1)(-3)} = \dfrac{4}{3}$	Positive
$(2, 4)$	3	$\dfrac{(6)(1)}{(3)(-1)} = -2$	Negative
$(4, \infty)$	5	$\dfrac{(8)(3)}{(5)(1)} = \dfrac{24}{5}$	Positive

Solution set: $(-\infty, -3] \cup (0, 2] \cup (4, \infty)$



1.8.45.

$0.3x^2 + 6.26 < 10.8$

$0.3x^2 + 4.54 < 0$

Key numbers: $x \approx \pm 3.89$

Test intervals: $(-\infty, -3.89), (-3.89, 3.89), (3.89, \infty)$

Solution set: $(-3.89, 3.89)$

1.8.46.

$-1.3x^2 + 3.78 > 2.12$

$-1.3x^2 + 1.66 > 0$

Key numbers: $x \approx \pm 1.13$

Test intervals: $(-\infty, -1.13), (-1.13, 1.13), (1.13, \infty)$

Solution set: $(-1.13, 1.13)$

1.8.47.

$$12.5x + 1.6 > 0.5x^2$$
$$-0.5x^2 + 12.5x + 1.6 > 0$$

Key numbers: $x \approx -0.13,\ x \approx 25.13$

Test intervals: $(-\infty, -0.13), (-0.13, 25.13), (25.13, \infty)$

Solution set: $(-0.13, 25.13)$

1.8.48.

$$1.2x^2 + 4.8x + 3.1 < 5.3$$
$$1.2x^2 + 4.8x - 2.2 < 0$$

Key numbers: $x \approx -4.42,\ x \approx 0.42$

Test intervals: $(-\infty, -4.42), (-4.42, 0.42), (0.42, \infty)$

Solution set: $(-4.42, 0.42)$

1.8.49.

$$\frac{1}{2.3x - 5.2} > 3.4$$
$$\frac{1}{2.3x - 5.2} - 3.4 > 0$$
$$\frac{1 - 3.4(2.3x - 5.2)}{2.3x - 5.2} > 0$$
$$\frac{-7.82x + 18.68}{2.3x - 5.2} > 0$$

Key numbers: $x \approx 2.39,\ \ x \approx 2.26$

Test intervals: $(-\infty, 2.26), (2.26, 2.39), (2.39, \infty)$

Solution set: $(2.26, 2.39)$

1.8.50.

$$\frac{2}{3.1x - 3.7} > 5.8$$
$$\frac{2 - 5.8(3.1x - 3.7)}{3.1x - 3.7} > 0$$
$$\frac{23.46 - 17.98x}{3.1x - 3.7} > 0$$

Key numbers: $x \approx 1.19,\ \ x \approx 1.30$

Test intervals: $(-\infty, 1.19) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

$(1.19, 1.30) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} > 0$

$(1.30, \infty) \Rightarrow \dfrac{23.46 - 17.98x}{3.1x - 3.7} < 0$

Solution set: $(1.19, 1.30)$

1.8.51.

$s = -16t^2 + v_0t + s_0 = -16t^2 + 160t$

(a)
$$\begin{aligned} -16t^2 + 160t &= 0 \\ -16t(t - 10) &= 0 \\ t = 0,\ t &= 10 \end{aligned}$$

It will be back on the ground in 10 seconds.

(b)
$$\begin{aligned} -16t^2 + 160t &> 384 \\ -16t^2 + 160t - 384 &> 0 \\ -16(t^2 - 10t + 24) &> 0 \\ t^2 - 10t + 24 &< 0 \\ (t - 4)(t - 6) &< 0 \end{aligned}$$

Key numbers: $t = 4,\ t = 6$

Test intervals: $(-\infty, 4), (4, 6), (6, \infty)$

Solution set: 4 seconds < t < 6 seconds

1.8.52.

$s = -16t^2 + v_0t + s_0 = -16t^2 + 128t$

(a)
$$\begin{aligned} -16t^2 + 128t &= 0 \\ -16t(t - 8) &= 0 \\ -16t = 0 &\Rightarrow t = 0 \\ t - 8 = 0 &\Rightarrow t = 8 \end{aligned}$$

It will be back on the ground in 8 seconds.

(b)
$$\begin{aligned} -16t^2 + 128t &< 128 \\ -16t^2 + 128t - 128 &< 0 \\ -16(t^2 - 8t + 8) &< 0 \\ t^2 - 8t + 8 &> 0 \end{aligned}$$

Key numbers: $t = 4 - 2\sqrt{2},\ t = 4 + 2\sqrt{2}$

Test intervals: $(-\infty, 4 - 2\sqrt{2}), (4 - 2\sqrt{2}, 4 + 2\sqrt{2}), (4 + 2\sqrt{2}, \infty)$

Solution set: 0 seconds $\le t < 4-2\sqrt{2}$ seconds and
$4+2\sqrt{2}$ seconds $< t \le 8$ seconds

1.8.53.

$R = x(75-0.0005x)$ and $C = 30x + 250,000$

$$\begin{aligned}P &= R - C\\ &= (75x-0.0005x^2)-(30x+250,000)\\ &= -0.0005x^2+45x-250,000\end{aligned}$$

$$\begin{aligned}P &\ge 750,000\\ -0.0005x^2+45x-250,000 &\ge 750,000\\ -0.0005x^2+45x-1,000,000 &\ge 0\end{aligned}$$

Key numbers: $x = 40,000$, $x = 50,000$ (These were obtained by using the Quadratic Formula.)

Test intervals: $(0, 40,000), (40,000, 50,000), (50,000, \infty)$

The solution set is $[40,000, 50,000]$ or $40,000 \le x \le 50,000$. The price per unit is

$$p = \frac{R}{x} = 75 - 0.0005x.$$

For $x = 40,000$, $p = \$55$. For $x = 50,000$, $p = \$50$. So, for $40,000 \le x \le 50,000$, $\$50.00 \le p \le \$55.00$.

1.8.54.

$R = x(50-0.0002x)$ and $C = 12x + 150,000$

$$\begin{aligned}P &= R - C\\ &= (50x-0.0002x^2)-(12x+150,000)\\ &= -0.0002x^2+38x-150,000\end{aligned}$$

$$\begin{aligned}P &\ge 1,650,000\\ -0.0002x^2+38x-150,000 &\ge 1,650,000\\ -0.0002x^2+38x-1,800,000 &\ge 0\end{aligned}$$

Key numbers: $x = 90,000$ and $x = 100,000$

Test intervals: $(0, 90,000)$, $(90,000, 100,000)$, $(100,000, \infty)$

The solution set is $[90,000, 100,000]$ or $90,000 \le x \le 100,000$. The price per unit is $p = \dfrac{R}{x} = 50 - 0.0002x$.

For $x = 90,000$, $p = \$32$. For $x = 100,000$, $p = \$30$. So, for $90,000 \le x \le 100,000$, $\$30 \le p \le \$32$.

1.8.55.

$$4 - x^2 \ge 0$$
$$(2 + x)(2 - x) \ge 0$$

Key numbers: $x = \pm 2$

Test intervals: $(-\infty, -2) \Rightarrow 4 - x^2 < 0$

$(-2, 2) \Rightarrow 4 - x^2 > 0$

$(2, \infty) \Rightarrow 4 - x^2 < 0$

Domain: $[-2, 2]$

1.8.56.

The domain of $\sqrt{x^2 - 9}$ can be found by solving the inequality:

$$x^2 - 9 \ge 0$$
$$(x + 3)(x - 3) \ge 0$$

Key numbers: $x = -3,\ x = 3$

Test intervals: $(-\infty, -3) \Rightarrow (x + 3)(x - 3) > 0$

$(-3, 3) \Rightarrow (x + 3)(x - 3) < 0$

$(3, \infty) \Rightarrow (x + 3)(x - 3) > 0$

Domain: $(-\infty, -3] \cup [3, \infty)$

1.8.57.

$$x^2 - 9x + 20 \ge 0$$
$$(x - 4)(x - 5) \ge 0$$

Key numbers: $x = 4,\ x = 5$

Test intervals: $(-\infty, 4), (4, 5), (5, \infty)$

Interval	x-Value	Value of $(x-4)(x-5)$	Conclusion
$(-\infty, 4)$	0	$(-4)(-5) = 20$	Positive
$(4, 5)$	$\frac{9}{2}$	$\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4}$	Negative
$(5, \infty)$	6	$(2)(1) = 2$	Positive

Domain: $(-\infty, 4] \cup [5, \infty)$

1.8.58.

The domain of $\sqrt{49-x^2}$ can be found by solving the inequality:

$$49-x^2 \geq 0$$
$$x^2-49 \leq 0$$
$$(x+7)(x-7) \leq 0$$

Key numbers: $x=-7,\ x=7$

Test intervals: $(-\infty, -7) \Rightarrow (x+7)(x-7) > 0$

$(-7, 7) \Rightarrow (x+7)(x-7) < 0$

$(7, \infty) \Rightarrow (x+7)(x-7) > 0$

Domain: $[-7,\ 7]$

1.8.59.

$$\frac{x}{x^2-2x-35} \geq 0$$
$$\frac{x}{(x+5)(x-7)} \geq 0$$

Key numbers: $x=0,\ x=-5,\ x=7$

Test intervals: $(-\infty, -5) \Rightarrow \dfrac{x}{(x+5)(x-7)} < 0$

$(-5, 0) \Rightarrow \dfrac{x}{(x+5)(x-7)} > 0$

$(0, 7) \Rightarrow \dfrac{x}{(x+5)(x-7)} < 0$

$(7, \infty) \Rightarrow \dfrac{x}{(x+5)(x-7)} > 0$

Domain: $(-5, 0] \cup (7, \infty)$

1.8.60.

$$\frac{x}{x^2-9} \geq 0$$
$$\frac{x}{(x+3)(x-3)} \geq 0$$

Key numbers: $x=-3,\ x=0,\ x=3$

Test intervals: $(-\infty, -3) \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$

$(-3, 0) \Rightarrow \dfrac{x}{(x+3)(x-3)} > 0$

$(0, 3) \Rightarrow \dfrac{x}{(x+3)(x-3)} < 0$

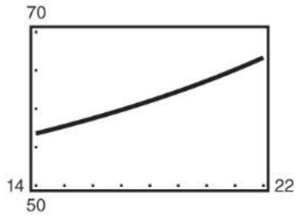
$(3, \infty) \Rightarrow \dfrac{x}{(x+3)(x-3)} > 0$

Domain: $(-3, 0] \cup (3, \infty)$

1.8.61.

$$S = \frac{47.84 - 0.61t}{1 - 0.022t}, \quad 14 \le t \le 22$$

(a)



(b) $S = \dfrac{47.84 - 0.61t}{1 - 0.022t}$

$$\frac{47.84 - 0.61t}{1 - 0.022t} < 60$$

$$\frac{47.84 - 0.61t}{1 - 0.022t} - 60 < 0$$

$$\frac{-12.16 + 0.71t}{1 - 0.022t} < 0$$

Key numbers: $17.1, \; 45.5$

Use the domain of the model to create test intervals.

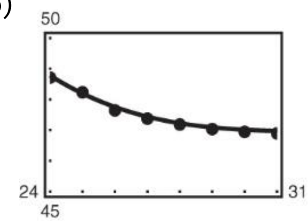
Test Interval	t-Value	Value of expression	Conclusion
$(14, 17.1)$	15	-2.25	Negative
$(17.1, 22)$	18	1.03	Positive

The mean salary was less than \$60,000 for $t < 17.1$, or from 2014 until 2017.

(c) *Sample answer:* No. For $t < 30$, the model rapidly increases, and then becomes negative at about $t = 45.4$.

1.8.62.

(a) and (b)



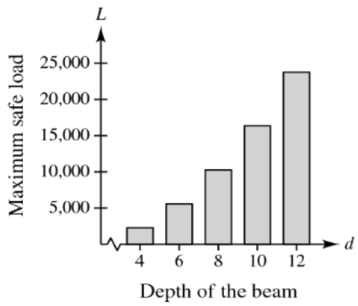
$N = -0.00366t^3 + 0.3474t^2 - 11.017t + 163.68$; The model fits the data well.

(c) Using technology, the number of students enrolled in public elementary and secondary schools was less than 47.00 million for $t > 30.2$, or from about 2030 on.

1.8.63.

(a)

d	4	6	8	10	12
Load	2223.9	5593.9	10,312	16,378	23,792



(b) $2000 \le 168.5d^2 - 472.1$

$2472.1 \le 168.5d^2$

$14.67 \le d^2$

$3.83 \le d$

The minimum depth is 3.83 inches.

1.8.64.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{2}$$

$$2R_1 = 2R + RR_1$$

$$2R_1 = R(2 + R_1)$$

$$\frac{2R_1}{2 + R_1} = R$$

Because $R \ge 1$,

$$\frac{2R_1}{2+R_1} \geq 1$$
$$\frac{2R_1}{2+R_1} - 1 \geq 0$$
$$\frac{R_1 - 2}{2+R_1} \geq 0.$$

Because $R_1 > 0$, the only key number is $R_1 = 2$. The inequality is satisfied when $R_1 \geq 2$ ohms.

1.8.65.

$$2L + 2W = 100 \Rightarrow W = 50 - L$$
$$LW \geq 500$$
$$L(50 - L) \geq 500$$
$$-L^2 + 50L - 500 \geq 0$$

By the Quadratic Formula you have:

Key numbers: $L = 25 \pm 5\sqrt{5}$

Test: Is $-L^2 + 50L - 500 \geq 0$?

Solution set: $25 - 5\sqrt{5} \leq L \leq 25 + 5\sqrt{5}$

$13.8 \text{ meters} \leq L \leq 36.2 \text{ meters}$

1.8.66.

$$2L + 2W = 440 \Rightarrow W = 220 - L$$
$$LW \geq 8000$$
$$L(220 - L) \geq 8000$$
$$-L^2 + 220L - 8000 \geq 0$$

By the Quadratic Formula we have:

Key numbers: $L = 110 \pm 10\sqrt{41}$

Test: Is $-L^2 + 220L - 8000 \geq 0$?

Solution set: $110 - 10\sqrt{41} \leq L \leq 110 + 10\sqrt{41}$

$45.97 \text{ feet} \leq L \leq 174.03 \text{ feet}$

1.8.67.

False.

There are four test intervals. The test intervals are $(-\infty, -3)$, $(-3, 1)$, $(1, 4)$, and $(4, \infty)$.

1.8.68.

True.

The y-values are greater than zero for all values of x.

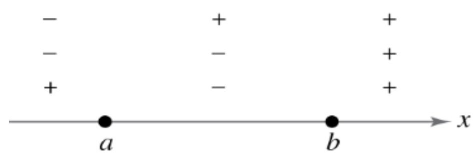
1.8.69.

$\frac{1}{x}$ is undefined when $x = 0$. The correct solution set is $(0, \infty)$.

1.8.70.

(a) $x = a,\ x = b$

(b)



(c) The real zeros of the polynomial.

1.8.71.

$x^2 + bx + 9 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(1)(9) \geq 0$$
$$b^2 - 36 \geq 0$$

Key numbers: $b = -6,\ b = 6$

Test intervals: $(-\infty, -6) \Rightarrow b^2 - 36 > 0$

$(-6, 6) \Rightarrow b^2 - 36 < 0$

$(6, \infty) \Rightarrow b^2 - 36 > 0$

Solution set: $(-\infty, -6] \cup [6, \infty)$

(b) $b^2 - 4ac \geq 0$

Key numbers: $b = -2\sqrt{ac},\ b = 2\sqrt{ac}$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

1.8.72.

$x^2 + bx - 4 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$$b^2 - 4(1)(-4) \geq 0$$
$$b^2 + 16 \geq 0$$

Key numbers: none

Test intervals: $(-\infty, \infty) \Rightarrow b^2 + 16 > 0$

Solution set: $(-\infty, \infty)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c < 0$, b can be any real number.

1.8.73.

$3x^2 + bx + 10 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(3)(10) \geq 0$

$b^2 - 120 \geq 0$

Key numbers: $b = -2\sqrt{30}, b = 2\sqrt{30}$

Test intervals: $\left(-\infty, -2\sqrt{30}\right) \Rightarrow b^2 - 120 > 0$

$\left(-2\sqrt{30}, 2\sqrt{30}\right) \Rightarrow b^2 - 120 < 0$

$\left(2\sqrt{30}, \infty\right) \Rightarrow b^2 - 120 > 0$

Solution set: $\left(-\infty, -2\sqrt{30}\right] \cup \left[2\sqrt{30}, \infty\right)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

1.8.74.

$2x^2 + bx + 5 = 0$

(a) To have at least one real solution, $b^2 - 4ac \geq 0$.

$b^2 - 4(2)(5) \geq 0$

$b^2 - 40 \geq 0$

Key numbers: $b = -2\sqrt{10}, \ b = 2\sqrt{10}$

Test intervals: $\left(-\infty, -2\sqrt{10}\right) \Rightarrow b^2 - 40 > 0$

$\left(-2\sqrt{10}, 2\sqrt{10}\right) \Rightarrow b^2 - 40 < 0$

$\left(2\sqrt{10}, \infty\right) \Rightarrow b^2 - 40 > 0$

Solution set: $\left(-\infty, -2\sqrt{10}\right] \cup \left[2\sqrt{10}, \infty\right)$

(b) $b^2 - 4ac \geq 0$

Similar to part (a), if $a > 0$ and $c > 0$, $b \leq -2\sqrt{ac}$ or $b \geq 2\sqrt{ac}$.

1.8.75.

$$\frac{5-7}{12-18} = \frac{-2}{-6} = \frac{1}{3}$$

1.8.76.

$$\frac{16-6}{6-11} = \frac{10}{-5} = -2$$

1.8.77.

$$\frac{3-3}{4-0}=\frac{0}{4}=0$$

1.8.78.

$$\frac{1-(-1)}{(9-9)}=\frac{2}{0}\rightarrow\text{Division by 0 is undefined.}$$

1.8.79.

x-intercept: $(-1, 0)$

y-intercept: $(0, 1)$

1.8.80.

x-intercept: $(-2, 0)$

y-intercept: $(0, -4)$

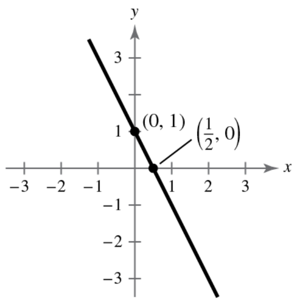
1.8.81.

$2x+y=1$

$2(-x)+y=1\Rightarrow -2x+y=1\Rightarrow$ No y-axis symmetry

$2x+(-y)=1\Rightarrow 2x-y=1\Rightarrow$ No x-axis symmetry

$2(-x)+(-y)=1\Rightarrow -2x-y=1\Rightarrow$ No origin symmetry



x-intercept: $\left(\frac{1}{2}, 0\right)$

y-intercept: $(0, 1)$

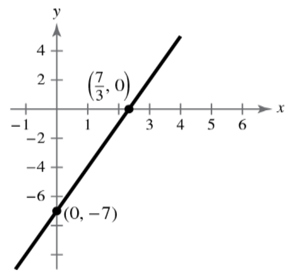
1.8.82.

$3x-y=7$

$3(-x)-y=7\Rightarrow -3x-y=7\Rightarrow$ No y-axis symmetry

$3x-(-y)=7\Rightarrow 3x+y=7\Rightarrow$ No x-axis symmetry

$3(-x)-(-y)=7\Rightarrow -3x+y=7\Rightarrow$ No origin symmetry



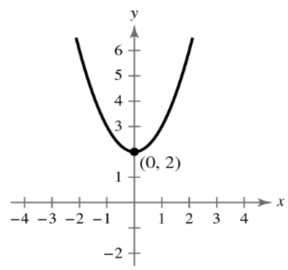
1.8.83.

$y = x^2 + 2$

$y = (-x)^2 + 2 \Rightarrow y = x^2 + 2 \Rightarrow y\text{-axis symmetry}$

$-y = x^2 + 2 \Rightarrow y = -x^2 - 2 \Rightarrow \text{No } x\text{-axis symmetry}$

$-y = (-x)^2 + 2 \Rightarrow y = -x^2 - 2 \Rightarrow \text{No origin symmetry}$



No x-intercepts
y-intercept: $(0, 2)$

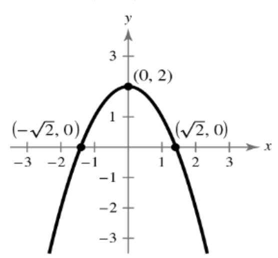
1.8.84.

$y = 2 - x^2$

$y = 2 - (-x)^2 \Rightarrow y = 2 - x^2 \Rightarrow y\text{-axis symmetry}$

$-y = 2 - x^2 \Rightarrow y = -2 + x^2 \Rightarrow \text{No } x\text{-axis symmetry}$

$-y = 2 - (-x)^2 \Rightarrow y = -2 + x^2 \Rightarrow \text{No origin symmetry}$



x-intercepts: $\left(\pm\sqrt{2}, 0\right)$

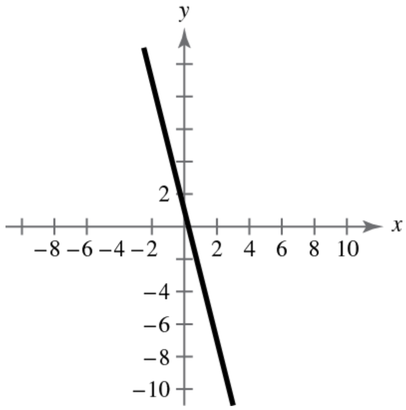
y-intercept: $(0, 2)$

Review Exercises for Chapter 1

1.R.1.

$y = -4x + 1$

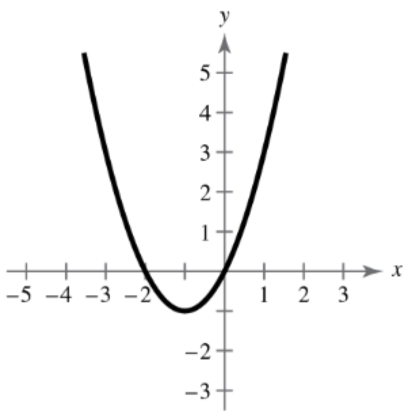
x	-2	-1	0	1	2
y	9	5	1	-3	-7



1.R.2.

$y = x^2 + 2x$

x	-3	-2	-1	0	1
y	3	0	-1	0	3



1.R.3.

x-intercepts: $(-4,\ 0),\ (2,\ 0)$

y-intercept: $(0,\ -2)$

1.R.4.

x-intercepts: $(1,\ 0),\ (5,\ 0)$

y-intercept: $(0,\ 5)$

1.R.5.

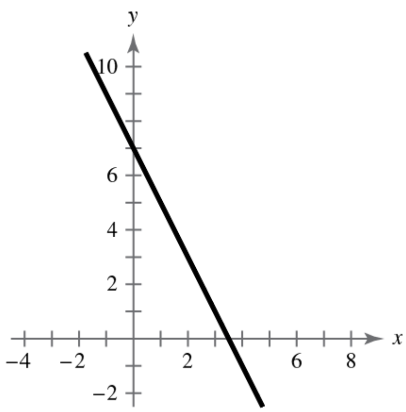
$y = -3x + 7$

Intercepts: $\left(\frac{7}{3},\ 0\right),\ (0, 7)$

$y = -3(-x) + 7 \Rightarrow y = 3x + 7 \Rightarrow$ No y-axis symmetry

$-y = -3x + 7 \Rightarrow y = 3x - 7 \Rightarrow$ No x-axis symmetry

$-y = -3(-x) + 7 \Rightarrow y = -3x - 7 \Rightarrow$ No origin symmetry



1.R.6.

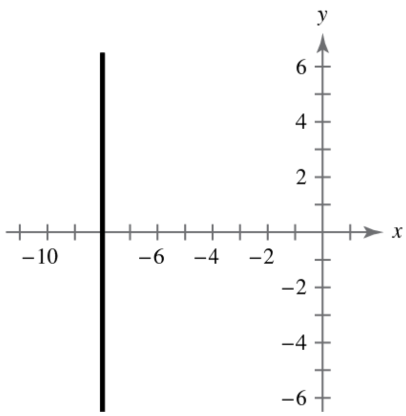
$x = -8$

Intercept: $(-8,\ 0)$, No y-intercept.

$-x = -8 \Rightarrow x = 8 \Rightarrow$ No y-axis symmetry

$x = -8 \Rightarrow x$-axis symmetry

$-x = -8 \Rightarrow x = 8 \Rightarrow$ No origin symmetry



1.R.7.

$x = y^2 - 5$

Intercepts: $(-5,\ 0)$, $(0,\ \pm\sqrt{5})$

$-x = y^2 - 5 \Rightarrow x = -y^2 + 5 \Rightarrow$ No y-axis symmetry

$x = (-y^2) - 5 \Rightarrow x = y^2 - 5 \Rightarrow x$-axis symmetry

$-x = (-y)^2 - 5 \Rightarrow x = -y^2 + 5 \Rightarrow$ No origin symmetry

1.R.8.

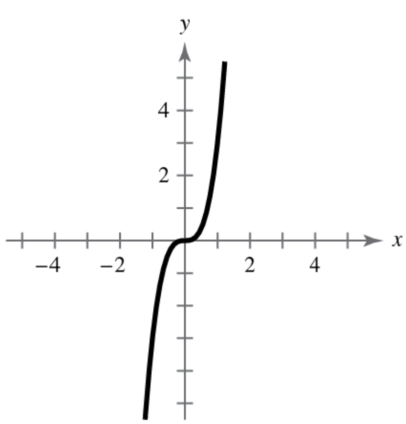
$y = 3x^3$

Intercepts: $(0,\ 0)$

$y = 3(-x)^3 \Rightarrow y = -3x^3 \Rightarrow$ No y-axis symmetry

$-y = 3x^3 \Rightarrow y = -3x^3 \Rightarrow$ No x-axis symmetry

$-y = 3(-x)^3 \Rightarrow y = 3x^3 \Rightarrow$ Original symmetry



1.R.9.

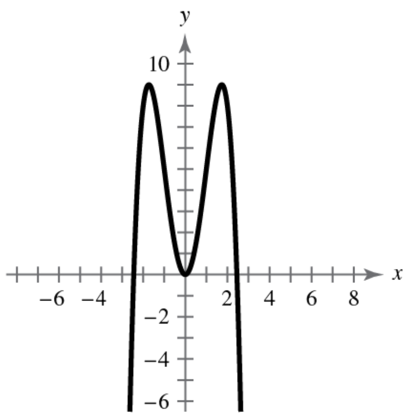
$y = -x^4 + 6x^2$

Intercept: $(0,\ 0)$

$y = -(-x)^4 + 6(-x)^2 \Rightarrow y = -x^4 + 6x^2 \Rightarrow$ y-axis symmetry

$-y = -x^4 + 6x^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No x-axis symmetry

$-y = -(-x)^4 + 6(-x)^2 \Rightarrow y = x^4 - 6x^2 \Rightarrow$ No origin symmetry



1.R.10.

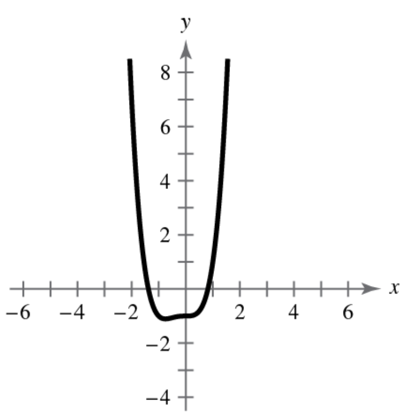
$y = x^4 + x^3 - 1$

Intercept: $(0,\ -1)$

$y = (-x)^4 + (-x)^3 - 1 \Rightarrow y = x^4 - x^3 - 1 \Rightarrow$ y-axis symmetry

$-y = x^4 + x^3 - 1 \Rightarrow y = -x^4 - x^3 + 1 \Rightarrow$ No x-axis symmetry

$-y = (-x)^4 + (-x)^3 - 1 \Rightarrow y = x^4 - x^3 - 1 \Rightarrow$ No origin symmetry



1.R.11.

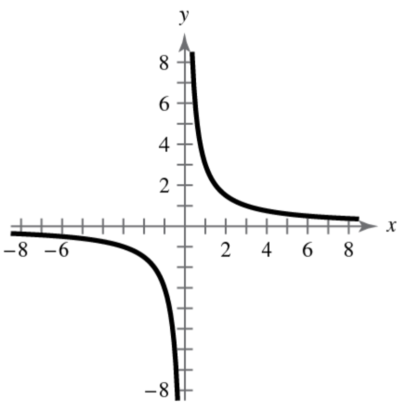
$$y = \frac{3}{x}$$

Intercept: None

$y = \frac{3}{-x} \Rightarrow y = -\frac{3}{x} \Rightarrow$ No y-axis symmetry

$-y = \frac{3}{x} \Rightarrow y = -\frac{3}{x} \Rightarrow$ No x-axis symmetry

$-y = \frac{3}{-x} \Rightarrow y = \frac{3}{x} \Rightarrow$ origin symmetry



1.R.12.

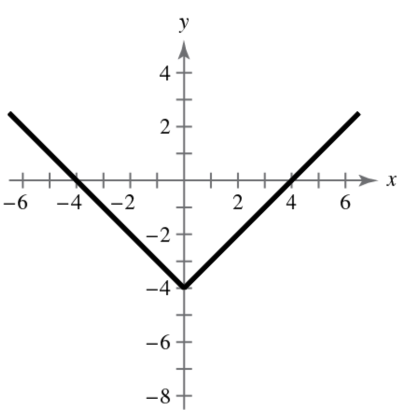
$$y = |x| - 4$$

Intercepts: $(\pm 4,\ 0)$, $(0,\ -4)$

$y = |-x| - 4 \Rightarrow y = |x| - 4 \Rightarrow y$-axis symmetry

$-y = |x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No x-axis symmetry

$-y = |-x| - 4 \Rightarrow y = -|x| + 4 \Rightarrow$ No origin symmetry



1.R.13.

Center: $(0, 0)$

Radius: 4

Standard form: $x^2 + y^2 = 4^2$

$$x^2 + y^2 = 16$$

1.R.14.

Center: $(-1, 2)$

Radius: 5

Standard form: $(x-(-1))^2 + (y-2)^2 = 5^2$

$$(x+1)^2 + (y-2)^2 = 25$$

1.R.15.

Endpoints of a diameter: $(0,\ 0)$ and $(4,\ -6)$

Center: $\left(\dfrac{0+4}{2},\ \dfrac{0+(-6)}{2}\right) = (2,\ -3)$

Radius: $r = \sqrt{(2-0)^2 + (-3-0)^2} = \sqrt{4+9}$

$$= \sqrt{13}$$

Standard form: $(x-2)^2 + (y-(-3))^2 = \left(\sqrt{13}\right)^2$

$$(x-2)^2 + (y+3)^2 = 13$$

1.R.16.

Endpoints of a diameter: $(-2,\ -3)$ and $(4,\ -10)$

Center: $\left(\dfrac{-2+4}{2},\ \dfrac{-3+(-10)}{2}\right) = \left(1,\ -\dfrac{13}{2}\right)$

Radius: $r = \sqrt{(1-(-2))^2 + \left(-\dfrac{13}{2}-(-3)\right)^2} = \sqrt{9+\dfrac{49}{4}} = \sqrt{\dfrac{85}{4}}$

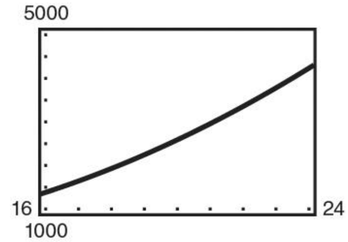
Standard form: $(x-1)^2 + \left(y-\left(-\dfrac{13}{2}\right)\right)^2 = \left(\sqrt{\dfrac{85}{4}}\right)^2$

$$(x-1)^2 + \left(y+\frac{13}{2}\right)^2 = \frac{85}{4}$$

1.R.17.

$S = -13.788t^2 - 194.18t + 963.2, 16 \le t \le 24$

(a)



(b) From the graph, sales were \$3 billion when $t \approx 21.09$, or the year 2021.

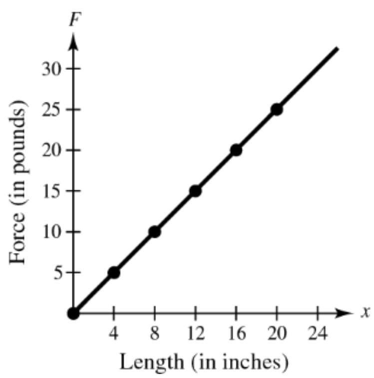
1.R.18.

$F = \dfrac{5}{4}x,\ \ 0 \le x \le 20$

(a)

x	0	4	8	12	16	20
F	0	5	10	15	20	25

(b)



(c) When $x = 10$, $F = \dfrac{50}{4} = 12.5$ pounds.

1.R.19.

$$\begin{aligned} 2(x-2) &= 2x - 4 \\ 2x - 4 &= 2x - 4 \\ 0 &= 0 \quad \text{Identity} \end{aligned}$$

All real numbers are solutions.

1.R.20.

$$\begin{aligned} 2(x+3) &= 2x-2 \\ 2x+6 &= 2x-2 \\ 6 &= -2 \quad \text{Contradiction} \end{aligned}$$

No solution

1.R.21.

$$\begin{aligned} 3(x-2)+2x &= 2(x+3) \\ 3x-6+2x &= 2x+6 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

Conditional equation

1.R.22.

$$\begin{aligned} 5(x-1)-2x &= 3x-5 \\ 5x-5-2x &= 3x-5 \\ 3x-5 &= 3x-5 \\ 0 &= 0 \quad \text{Identity} \end{aligned}$$

All real numbers are solutions.

1.R.23.

$$\begin{aligned} 8x-5 &= 3x+20 \\ 5x &= 25 \\ x &= 5 \end{aligned}$$

1.R.24.

$$\begin{aligned} 7x+3 &= 3x-17 \\ 4x &= -20 \\ x &= -5 \end{aligned}$$

1.R.25.

$$\begin{aligned} 2(x+5)-7 &= x+9 \\ 2x+10-7 &= x+9 \\ 2x+3 &= x+9 \\ x &= 6 \end{aligned}$$

1.R.26.

$$\begin{aligned}
7(x-4) &= 1-(x+9)\\
7x-28 &= 1-x-9\\
7x-28 &= -x-8\\
8x &= 20\\
x &= \frac{5}{2}
\end{aligned}$$

1.R.27.

$$\begin{aligned}
\frac{4x-3}{6}+\frac{x}{4} &= x-2\\
2(4x-3)+3x &= 12x-24\\
8x-6+3x &= 12x-24\\
-x &= -18\\
x &= 18
\end{aligned}$$

1.R.28.

$$\begin{aligned}
3+\frac{2}{x-5} &= \frac{2x}{x-5}\\
\frac{3(x-5)+2}{x-5} &= \frac{2x}{x-5}\\
\frac{3x-15+2}{x-5} &= \frac{2x}{x-5}\\
\frac{3x-13}{x-5} &= \frac{2x}{x-5}\\
(x-5)\frac{3x-13}{x-5} &= \frac{2x}{x-5}(x-5)\\
3x-13 &= 2x\\
x &= 3
\end{aligned}$$

1.R.29.

$y = 3x-1$

x-intercept: $0 = 3x-1 \Rightarrow x = \frac{1}{3}$

y-intercept: $y = 3(0)-1 \Rightarrow y = -1$

The x-intercept is $\left(\frac{1}{3},\ 0\right)$ and the y-intercept is $(0,\ -1)$.

1.R.30.

$$\begin{aligned} y &= -5x+6 \\ 0 &= -5x+6 \\ -6 &= -5x \\ \frac{6}{5} &= x \end{aligned} \qquad \begin{aligned} y &= -5x+6 \\ y &= -5(0)+6 \\ y &= 6 \end{aligned}$$

The x-intercept is $\left(\frac{6}{5},\ 0\right)$ and the y-intercept is $(0,\ 6)$.

1.R.31.

$$y = 2(x-4)$$

x-intercept: $0 = 2(x-4) \Rightarrow x = 4$

y-intercept: $y = 2(0-4) \Rightarrow y = -8$

The x-intercept is $(4,\ 0)$ and the y-intercept is $(0,\ -8)$.

1.R.32.

$$\begin{aligned} y &= 4(7x+1) \\ 0 &= 4(7x+1) \\ 0 &= 28x+4 \\ -4 &= 28x \\ -\frac{1}{7} &= x \end{aligned} \qquad \begin{aligned} y &= 4(7x+1) \\ y &= 4\left[7(0)+1\right] \\ y &= 4 \end{aligned}$$

The x-*intercept* is $\left(-\frac{1}{7},\ 0\right)$ and the y-intercept is $(0,\ 4)$.

1.R.33.

$$\begin{aligned} 244.92 &= 2(3.14)(3)^2 + 2(3.14)(3)h \\ 244.92 &= 56.52 + 18.84h \\ 188.40 &= 18.84h \\ 10 &= h \end{aligned}$$

The *height* is 10 inches.

1.R.34.

$$C = \frac{5}{9}F - \frac{160}{9}$$

$$\frac{5}{9}F = C + \frac{160}{9}$$

$$F = \frac{9}{5}\left(C + \frac{160}{9}\right)$$

For $C = 100°$, $F = \frac{9}{5}\left(100 + \frac{160}{9}\right) = 212°\text{F}$.

1.R.35.

Let x be the revenue for 2023.

The revenue for 2024 is $x + 0.086x$.

The total revenue is 32.8.

$$x + (x + 0.086x) = 32.8$$

$$2.086x = 32.8$$

$$x = \frac{32.8}{2.086} \approx 15.7$$

Revenue for 2023: $\$15.7$ billion

Revenue for 2024: $15.7 + 0.086(15.7) \approx \$17.1$ billion

1.R.36.

Model: $(\text{Original price}) = \dfrac{(\text{sale price})}{(1 - \text{discount rate})}$

Labels: Original price $= x$

Discount rate $= 0.2$

Sale price $= 340$

Equation: $x = \dfrac{340}{1 - 0.2}$

$x = 425$

The original price was $425.

1.R.37.

Let $x =$ the total investment required.

Each person's share is $\frac{x}{9}$. If three more people invest, each person's share is

$\frac{x}{12} + 2500.$ Since this is $2500 less than the original cost, we have:

$$\frac{x}{9} = \frac{x}{12} + 2500$$
$$\frac{x}{9} - \frac{x}{12} = 2500$$
$$\frac{8x - 6x}{72} = 2500$$
$$\frac{2x}{72} = 2500$$
$$x = 90{,}000$$

The total investment to start the business is $90,000.

1.R.38.

	Rate	Time	Distance
To work	r	$\frac{56}{r}$	56
From work	$r+8$	$\frac{56}{r+8}$	56

$$\text{Time} = \frac{\text{distance}}{\text{rate}}$$

Time to work = time from work + 10 minutes

$$\frac{56}{r} = \frac{56}{r+8} + \frac{1}{6} \qquad \text{Convert minutes to portion of an hour.}$$
$$6(r+8)(56) = 6r(56) + r(r+8)$$
$$336r + 2688 = 336r + r^2 + 8r$$
$$0 = r^2 + 8r - 2688$$
$$0 = (r-48)(r+56)$$

Using the positive value for r, we have $r = 48$ miles per hour. The average speed on the trip home was $r + 8 = 56$ miles per hour.

1.R.39.

Let $x =$ the number of liters of pure antifreeze.

$$30\% \text{ of } (10 - x) + 100\% \text{ of } x = 50\% \text{ of } 10$$
$$0.30(10 - x) + 1.00x = 0.50(10)$$
$$3 - 0.30x + 1.00x = 5$$
$$0.70x = 2$$
$$x = \frac{2}{0.70} = \frac{20}{7} = 2.857 \text{ liters}$$

1.R.40.

Model: $\left(\text{Interest from } 4\frac{1}{2}\%\right)+\left(\text{Interest from } 5\frac{1}{2}\%\right)=\left(\text{Total interest}\right)$

Labels:

Amount invested at $4\frac{1}{2}\% = x$, amount invested at $5\frac{1}{2}\% = 6000 - x$

Interest from $4\frac{1}{2}\% = x(0.045)(1)$, interest from $5\frac{1}{2}\% = (6000-x)(0.055)(1)$, total interest = \$3.05

Equation:
$$\begin{aligned} 0.045x + 0.055(6000 - x) &= 305 \\ 0.045x + 330 - 0.055x &= 305 \\ -0.01x &= -25 \\ x &= 2500 \end{aligned}$$

The amount invested at $4\frac{1}{2}\%$ was \$2500 and the amount invested at $5\frac{1}{2}\%$ was $6000 - 2500 = \$3500$.

1.R.41.

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ 3V &= \pi r^2 h \\ \frac{3V}{\pi r^2} &= h \end{aligned}$$

1.R.42.

$$\begin{aligned} E &= \frac{1}{2}mv^2 \\ mv^2 &= 2E \\ m &= \frac{2E}{v^2} \end{aligned}$$

1.R.43.

$$\begin{aligned} 15 + x - 2x^2 &= 0 \\ 0 &= 2x^2 - x - 15 \\ 0 &= (2x+5)(x-3) \\ 2x + 5 = 0 &\Rightarrow x = -\frac{5}{2} \\ x - 3 = 0 &\Rightarrow x = 3 \end{aligned}$$

1.R.44.

$$\begin{aligned} 2x^2 - x - 28 &= 0 \\ (2x+7)(x-4) &= 0 \\ 2x + 7 = 0 &\Rightarrow x = -\frac{7}{2} \\ x - 4 = 0 &\Rightarrow x = 4 \end{aligned}$$

1.R.45.

$$\begin{aligned} 6 &= 3x^2 \\ 2 &= x^2 \\ \pm\sqrt{2} &= x \end{aligned}$$

1.R.46.

$$\begin{aligned} 16x^2 &= 25 \\ 16x^2 - 25 &= 0 \\ (4x-5)(4x+5) &= 0 \\ 4x - 5 = 0 &\Rightarrow x = \frac{5}{4} \\ 4x + 5 = 0 &\Rightarrow x = -\frac{5}{4} \end{aligned}$$

1.R.47.

$$\begin{aligned} (x+13)^2 &= 25 \\ x + 13 &= \pm 5 \\ x &= -13 \pm 5 \\ x &= -18 \text{ or } x = -8 \end{aligned}$$

1.R.48.

$$\begin{aligned} (x-5)^2 &= 30 \\ x - 5 &= \pm\sqrt{30} \\ x &= 5 \pm \sqrt{30} \end{aligned}$$

1.R.49.

$$\begin{aligned} x^2 + 12x + 25 &= 0 \\ x &= \frac{-12 \pm \sqrt{12^2 - 4(1)(25)}}{2(1)} \\ &= \frac{-12 \pm 2\sqrt{11}}{2} \\ &= -6 \pm \sqrt{11} \end{aligned}$$

1.R.50.

$$9x^2 - 12x = 14$$
$$9x^2 - 12x - 14 = 0$$
$$x = \frac{-12 \pm \sqrt{(-12)^2 - 4(9)(-14)}}{2(9)}$$
$$= \frac{-12 \pm 18\sqrt{2}}{18}$$
$$= \frac{2}{3} \pm \sqrt{2}$$

1.R.51.

$$-2x^2 - 5x + 27 = 0$$
$$2x^2 + 5x - 27 = 0$$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(-27)}}{2(2)}$$
$$= \frac{-5 \pm \sqrt{241}}{4}$$

1.R.52.

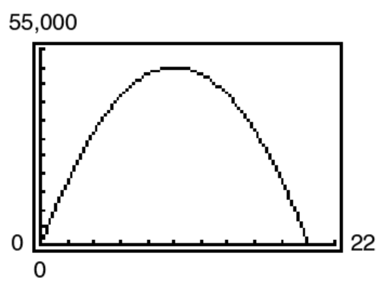
$$-20 - 3x + 3x^2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(3)(-20)}}{2(3)}$$
$$= \frac{3 \pm \sqrt{249}}{6} = \frac{1}{2} \pm \frac{\sqrt{249}}{6}$$

1.R.53.

$M = 500x(20 - x)$

(a) $500x(20 - x) = 0$ when $x = 0$ feet and $x = 20$ feet.

(b)



(c) The bending moment is greatest when $x = 10$ feet.

1.R.54.

(a) $h(t) = -16t^2 + 30t + 5.8$

(b) $h(1) = -16 \cdot 1^2 + 30 \cdot 1 + 5.8 = 19.8$ feet

(c) $-16t^2 + 30t + 5.8 = 6.2$

$-16t^2 + 30t - 0.4 - 0$

1.R.55.

$$\sqrt{-18}\,\sqrt{-6} = \left(\sqrt{18}i\right)\left(\sqrt{6}i\right) = \sqrt{108}\left(i^2\right) = -6\sqrt{3}$$

1.R.56.

$$\sqrt{-27} + \sqrt{-3} = 3\sqrt{3}i + \sqrt{3}i = 4\sqrt{3}i$$

1.R.57.

$$\begin{aligned}\left(5+\sqrt{-10}\right)\left(10-\sqrt{-5}\right) &= \left(5+\sqrt{10}i\right)\left(10-\sqrt{5}i\right)\\ &= 50 + 10\sqrt{10}i - 5\sqrt{5}i - \sqrt{50}\left(i^2\right)\\ &= 50 + \sqrt{50}i + 10\sqrt{10}i - 5\sqrt{5}i\\ &= 50 + 5\sqrt{2} + \left(10\sqrt{10} - 5\sqrt{5}\right)i\end{aligned}$$

1.R.58.

$$\begin{aligned}\left(6-\sqrt{-2}\right)^2 &= \left(6-\sqrt{2}i\right)\left(6-\sqrt{2}i\right)\\ &= 36 - 6\sqrt{2}i - 6\sqrt{2}i + \left(\sqrt{2}i\right)^2\\ &= 36 - 2 - 6\sqrt{2}i - 6\sqrt{2}i\\ &= 34 - 12\sqrt{2}i\end{aligned}$$

1.R.59.

$$(6-4i) + (-9+i) = (6+(-9)) + (-4i+i) = -3-3i$$

1.R.60.

$$(7-2i) - (3-8i) = (7-3) + (-2i+8i) = 4+6i$$

1.R.61.

$$\begin{aligned}-3i(-2+5i) &= 6i - 15i^2\\ &= 6i - 15(-1)\\ &= 15 + 6i\end{aligned}$$

1.R.62.

$$\begin{aligned}(4+i)(3-10i) &= 12-40i+3i-10i^2\\ &= 12-37i-10(-1)\\ &= 22-37i\end{aligned}$$

1.R.63.

$$\begin{aligned}(1+7i)(1-7i) &= 1-49i^2\\ &= 1-49(-1)\\ &= 1+49\\ &= 50\end{aligned}$$

1.R.64.

$$\begin{aligned}(5-9i)^2 &= 25-90i+81i^2\\ &= 25-90i+81(-1)\\ &= 25-81-90i\\ &= -56-90i\end{aligned}$$

1.R.65.

$$\begin{aligned}\frac{4}{1-2i} &= \frac{4}{1-2i}\cdot\frac{1+2i}{1+2i}\\ &= \frac{4+8i}{1-4i^2}\\ &= \frac{4+8i}{5}\\ &= \frac{4}{5}+\frac{8}{5}i\end{aligned}$$

1.R.66.

$$\begin{aligned}\frac{6-5i}{i} &= \frac{6-5i}{i}\cdot\frac{-i}{-i}\\ &= \frac{-6i+5i^2}{-i^2}\\ &= -5-6i\end{aligned}$$

1.R.67.

$$\begin{aligned}\frac{3+2i}{5+i} &= \frac{3+2i}{5+i}\cdot\frac{5-i}{5-i}\\ &= \frac{15-3i+10i-2i^2}{25-i^2}\\ &= \frac{17+7i}{26}\\ &= \frac{17}{26}+\frac{7i}{26}\end{aligned}$$

1.R.68.

$$\frac{7i}{(3+2i)^2} = \frac{7i}{9+12i+4i^2}$$

$$= \frac{7i}{9+12i+4(-1)}$$

$$= \frac{7i}{5+12i}$$

$$= \frac{7i}{5+12i} \cdot \frac{5-12i}{5-12i}$$

$$= \frac{35i-84i^2}{25-144i^2}$$

$$= \frac{35i-84(-1)}{25+144}$$

$$= \frac{84+35i}{169}$$

$$= \frac{84}{169} + \frac{35}{169}i$$

1.R.69.

$$x^2 - 2x + 10 = 0$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{-36}}{2}$$

$$= \frac{2 \pm 6i}{2}$$

$$= 1 \pm 3i$$

1.R.70.

$$x^2+6x+34=0$$

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{-6\pm\sqrt{(6)^2-4(1)(34)}}{2(1)}\\&=\frac{-6\pm\sqrt{-100}}{2}\\&=\frac{-6\pm 10i}{2}\\&=-3\pm 5i\end{aligned}$$

1.R.71.

$$4x^2+4x+7=0$$

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{-4\pm\sqrt{(4)^2-4(4)(7)}}{2(4)}\\&=\frac{-4\pm\sqrt{-96}}{8}\\&=\frac{-4\pm 4\sqrt{6}i}{8}\\&=-\frac{1}{2}\pm\frac{\sqrt{6}}{2}i\end{aligned}$$

1.R.72.

$$6x^2+3x+27=0$$

$$\begin{aligned}x&=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\\&=\frac{-3\pm\sqrt{3^2-4(6)(27)}}{2(6)}\\&=\frac{-3\pm\sqrt{-639}}{12}\\&=\frac{-3\pm 3i\sqrt{71}}{12}=-\frac{1}{4}\pm\frac{\sqrt{71}}{4}i\end{aligned}$$

1.R.73.

$$\begin{aligned} 5x^4 - 12x^3 &= 0 \\ x^3(5x - 12) &= 0 \\ x^3 = 0 \text{ or } 5x - 12 &= 0 \\ x = 0 \text{ or } \quad x &= \frac{12}{5} \end{aligned}$$

1.R.74.

$$\begin{aligned} 4x^3 - 6x^2 &= 0 \\ x^2(4x - 6) &= 0 \\ x^2 = 0 &\Rightarrow x = 0 \\ 4x - 6 = 0 &\Rightarrow x = \frac{3}{2} \end{aligned}$$

1.R.75.

$$\begin{aligned} x^3 - 7x^2 + 4x &= 28 \\ x^2(x - 7) + 4(x - 7) &= 0 \\ (x - 7)(x^2 + 4) &= 0 \\ x - 7 = 0 &\Rightarrow x = 7 \\ x^2 + 4 = 0 &\Rightarrow x^2 = -4 \\ x &= \pm\sqrt{-4} = \pm 2i \end{aligned}$$

1.R.76.

$$\begin{aligned} 9x^4 + 27x^3 - 4x^2 &= 12x \\ 9x^3(x + 3) - 4x(x + 3) &= 0 \\ (x + 3)(9x^3 - 4x) &= 0 \\ (x + 3)(x)(9x^2 - 4) &= 0 \\ x + 3 = 0 &\Rightarrow x = -3 \\ x &= 0 \\ 9x^2 - 4 = 0 &\Rightarrow x = \pm\frac{2}{3} \end{aligned}$$

1.R.77.

$$x^6 - 7x^3 - 8 = 0$$
$$\left(x^3\right)^2 - 7\left(x^3\right) - 8 = 0$$
$$u^2 - 7u - 8 = 0$$
$$\left(u - 8\right)\left(u + 1\right) = 0$$
$$u - 8 = 0$$
$$x^3 - 8 = 0$$
$$\left(x - 2\right)\left(x^2 + 2x + 4\right) = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x^2 + 2x + 4 = 0 \Rightarrow x = -1 \pm \sqrt{3}i$$
$$u + 1 = 0$$
$$x^3 + 1 = 0$$
$$\left(x + 1\right)\left(x^2 - x + 1\right) = 0$$
$$x + 1 = 0 \Rightarrow x = -1$$
$$x^2 - x + 1 = 0 \Rightarrow x = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

1.R.78.

$$x^4 - 13x^2 - 48 = 0$$
$$\left(x^2\right)^2 - 13\left(x^2\right) - 48 = 0$$
Let $u = x^2$.
$$u^2 - 13u - 48 = 0$$
$$\left(u - 16\right)\left(u + 3\right) = 0$$
$$u - 16 = 0 \Rightarrow u = 16$$
$$u + 3 = 0 \Rightarrow u = -3$$

$$u = 16 \qquad u = -3$$
$$x^2 = 16 \qquad x^2 = -3$$
$$x = \pm 4 \qquad x = \pm\sqrt{3}\,i$$

1.R.79.

$$\begin{aligned}
\sqrt{2x+3} &= 2+x \\
2x+3 &= (2+x)^2 \\
2x+3 &= 4+4x+x^2 \\
x^2+2x+1 &= 0 \\
(x+1)^2 &= 0 \\
x &= -1
\end{aligned}$$

1.R.79.

$$\begin{aligned}
\sqrt{2x+3} &= 2+x \\
2x+3 &= (2+x)^2 \\
2x+3 &= 4+4x+x^2 \\
x^2+2x+1 &= 0 \\
(x+1)^2 &= 0 \\
x &= -1
\end{aligned}$$

1.R.80.

$$\begin{aligned}
5\sqrt{x}-\sqrt{x-1} &= 6 \\
5\sqrt{x} &= 6+\sqrt{x-1} \\
25x &= 36+12\sqrt{x-1}+x-1 \\
24x-35 &= 12\sqrt{x-1} \\
576x^2-1680x+1225 &= 144(x-1) \\
576x^2-1824x+1369 &= 0
\end{aligned}$$

$$x = \frac{-(-1824)\pm\sqrt{(-1824)^2-4(576)(1369)}}{2(576)} = \frac{1824\pm\sqrt{172,800}}{1152} = \frac{1824\pm 240\sqrt{3}}{1152}$$

$$x = \frac{38+5\sqrt{3}}{24}$$

$$x = \frac{38-5\sqrt{3}}{24}, \text{ extraneous}$$

1.R.81.

$$
\begin{aligned}
(x-1)^{2/3} - 25 &= 0 \\
(x-1)^{2/3} &= 25 \\
(x-1)^{2} &= 25^{3} \\
x - 1 &= \pm\sqrt{25^{3}} \\
x &= 1 \pm 125 \\
x &= 126 \text{ or } x = -124
\end{aligned}
$$

1.R.82.

$$
\begin{aligned}
(x+2)^{3/4} &= 27 \\
x + 2 &= 27^{4/3} \\
x + 2 &= 81 \\
x &= 79
\end{aligned}
$$

1.R.83.

$$
\begin{aligned}
\frac{5}{x} &= 1 + \frac{3}{x+2} \\
5(x+2) &= 1(x)(x+2) + 3x \\
5x + 10 &= x^2 + 2x + 3x \\
10 &= x^2 \\
\pm\sqrt{10} &= x
\end{aligned}
$$

1.R.84.

$$
\begin{aligned}
\frac{6}{x} + \frac{8}{x+5} &= 3 \\
x(x+5)\frac{6}{x} + x(x+5)\frac{8}{x+5} &= 3x(x+5) \\
6(x+5) + 8x &= 3x(x+5) \\
14x + 30 &= 3x^2 + 15x \\
0 &= 3x^2 + x - 30 \\
0 &= (3x+10)(x-3) \\
0 &= 3x + 10 \Rightarrow x = -\frac{10}{3} \\
0 &= x - 3 \Rightarrow x = 3
\end{aligned}
$$

1.R.85.

$$|x-5| = 10$$

$$x-5 = -10 \text{ or } x-5 = 10$$

$$x = -5 \qquad x = 15$$

1.R.86.

$$|2x+3| = 7$$

$$|2x+3| = 7 \text{ or } 2x+3 = -7$$

$$2x = 4 \qquad 2x = -10$$

$$x = 2 \qquad x = -5$$

1.R.87.

$$|x^2-3| = 2x$$

$$x^2-3 = 2x \quad \text{or} \quad x^2-3 = -2x$$

$$x^2-2x-3 = 0 \qquad x^2+2x-3 = 0$$

$$(x-3)(x+1) = 0 \qquad (x+3)(x-1) = 0$$

$$x = 3 \text{ or } x = -1 \qquad x = -3 \text{ or } x = 1$$

The only solutions of the original equation are $x = 3$ or $x = 1$. ($x = 3$ and $x = -1$ are extraneous.)

1.R.88.

$$|x^2-6| = x$$

$$x^2-6 = x \quad \text{or} \quad -(x^2-6) = x$$

$$x^2-x-6 = 0 \qquad x^2+x-6 = 0$$

$$(x-3)(x+2) = 0 \qquad (x+3)(x-2) = 0$$

$$x-3 = 0 \Rightarrow x = 3 \qquad x-2 = 0 \Rightarrow x = 2$$

$$x+2 = 0 \Rightarrow x = -2, \text{ extraneous} \qquad x+3 = 0 \Rightarrow x = -3, \text{ extraneous}$$

1.R.89.

$$29.95 = 42 - \sqrt{0.001x+2}$$

$$-12.05 = -\sqrt{0.001x+2}$$

$$\sqrt{0.001x+2} = 12.05$$

$$0.001x+2 = 145.2025$$

$$0.001x = 143.2025$$

$$x = 143{,}202.5$$

$$\approx 143{,}203 \text{ units}$$

1.R.90.

Verbal Model: Total cost = Cost underwater × Distance underwater + Cost overland × Distance overland

Labels: Total cost: $\$\ 1,098,662.40$

Cost overland : $24 per foot

Distance overland in feet : $5280(8-x)$

Cost underwater : $30 per foot

Distance underwater in feet : $5280\sqrt{x^2+(3/4)^2} = 5280\sqrt{\dfrac{16x^2+9}{16}} = 1320\sqrt{16x^2+9}$

Equation:

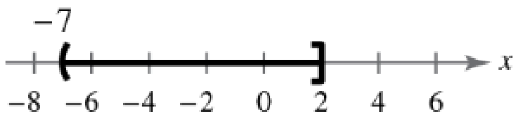
$$\begin{aligned}
1,098,662.40 &= 30\left(1320\sqrt{16x^2+9}\right)+24\left[5280(8-x)\right]\\
1,098,662.40 &= 39,600\sqrt{16x^2+9}+126,720(8-x)\\
1,098,662.40 &= 7920\left[5\sqrt{16x^2+9}+16(8-x)\right]\\
138.72 &= 5\sqrt{16x^2+9}+16(8-x)\\
138.72 &= 5\sqrt{16x^2+9}+128-16x\\
16x+10.72 &= 5\sqrt{16x^2+9}\\
(16x+10.72)^2 &= \left(5\sqrt{16x^2+9}\right)^2\\
256x^2+343.04x+114.9184 &= 25\left(16x^2+9\right)\\
256x^2+343.04x+114.9184 &= 400x^2+225\\
0 &= 144x^2-343.04x+110.0816
\end{aligned}$$

By the Quadratic Formula, $x \approx 2$ or $x \approx 0.382$.

1.R.91.

Interval: $(-7,\ 2]$; The interval is bounded.

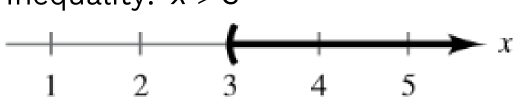
Inequality: $-7 < x \le 2$



1.R.92.

Interval: $(3,\ \infty)$; The interval is unbounded.

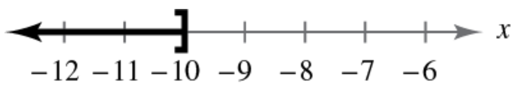
Inequality: $x > 3$



1.R.93.

Interval: $\left(-\infty,\ -10\right]$; The interval is unbounded.

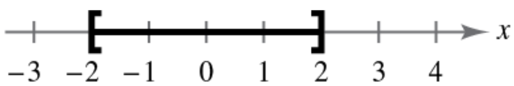
Inequality: $x \le -10$



1.R.94.

Interval: $\left[-2,\ 2\right]$; The interval is bounded.

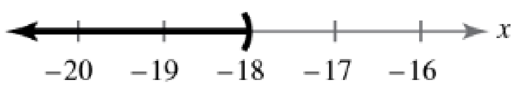
Inequality: $-2 \le x \le 2$



1.R.95.

$$\begin{aligned} 3\left(x+2\right) &< 2x-12 \\ 3x+6 &< 2x-12 \\ x &< -18 \end{aligned}$$

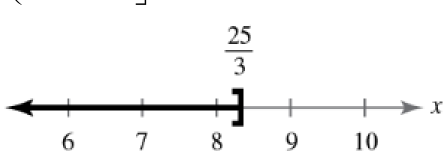
$$\left(-\infty,\ -18\right)$$



1.R.96.

$$\begin{aligned} 2\left(x+5\right) &\ge 5\left(x-3\right) \\ 2x+10 &\ge 5x-15 \\ -3x &\ge -25 \\ x &\le \frac{25}{3} \end{aligned}$$

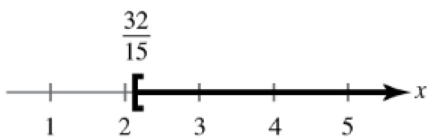
$$\left(-\infty,\ \frac{25}{3}\right]$$



1.R.97.

$$\begin{aligned} 4(5-2x) &\le \tfrac{1}{2}(8-x) \\ 20-8x &\le 4-\tfrac{1}{2}x \\ -\frac{15}{2}x &\le -16 \\ x &\ge \frac{32}{15} \end{aligned}$$

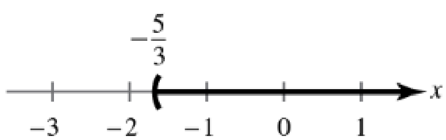
$$\left[\frac{35}{15},\ \infty\right)$$



1.R.98.

$$\begin{aligned} \tfrac{1}{2}(3-x) &> \tfrac{1}{3}(2-3x) \\ 9-3x &> 4-6x \\ 3x &> -5 \\ x &> -\frac{5}{3} \end{aligned}$$

$$\left(-\frac{5}{3},\ \infty\right)$$

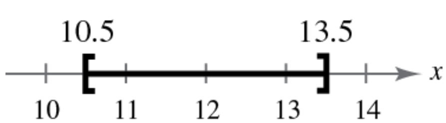


1.R.99.

$3.2 \le 0.4x - 1 \le 4.4$

$4.2 \le 0.4x \le 5.4$

$10.5 \le x \le 13.5$

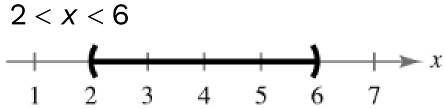


1.R.100.

$1.6 < 0.3x + 1 < 2.8$

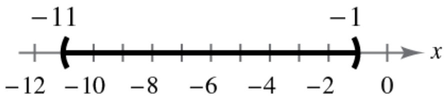
$0.6 < 0.3x < 1.8$

$2 < x < 6$



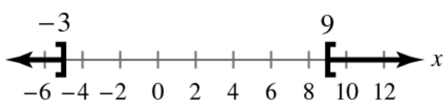
1.R.101.

$$|x+6|<5$$
$$-5<x+6<5$$
$$-11<x<-1$$
$$(-11,\ -1)$$



1.R.102.

$$\frac{2}{3}|3-x|\geq 4$$
$$|3-x|\geq 6$$
$$3-x\leq -6 \quad \text{or} \quad 3-x\geq 6$$
$$-x\leq -9 \quad \text{or} \quad -x\geq 3$$
$$x\geq 9 \quad \text{or} \quad x\leq -3$$
$$x\leq -3 \quad \text{or} \quad x\geq 9$$
$$(-\infty,\ -3]\cup[9,\ \infty)$$



1.R.103.

$$125.33x>92x+1200$$
$$33.33x>1200$$
$$x>36 \text{ units}$$

So, the smallest value of x for which the product returns a profit is 37 units.

1.R.104.

If the side is 19.3 cm, then with the possible error of 0.5 cm we have:

$$18.8\leq \text{side}\leq 19.8$$
$$353.44\text{ cm}^2\leq \text{area}\leq 392.04\text{ cm}^2$$

1.R.105.

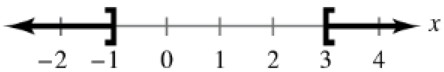
$$x^2-6x-27<0$$
$$(x+3)(x-9)<0$$

Key numbers: $x=-3,\ x=9$

Test intervals: $(-\infty,\ -3),\ (-3,\ 9),\ (9,\ \infty)$

Test: Is $(x+3)(x-9)<0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-3,\ 9)$.



1.R.106.

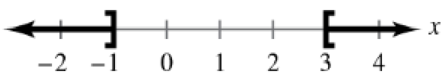
$$x^2-2x\ge 3$$
$$x^2-2x-3\ge 0$$
$$(x-3)(x+1)\ge 0$$

Key numbers: $x=-1,\ \ x=3$

Test intervals: $(-\infty,\ -1),\ (-1,\ 3),\ (3,\ \infty)$

Test: Is $(x-3)(x+1)\ge 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty,\ -1]\cup[3,\ \infty)$.



1.R.107.

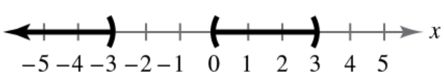
$$5x^3-45x<0$$
$$5x(x^2-9)<0$$
$$5x(x+3)(x-3)<0$$

Key numbers: $x=\pm 3,\ x=0$

Test intervals: $(-\infty,\ -3),\ (-3,\ 0),\ (0,\ 3),\ (3,\ \infty)$

Test: Is $5x(x+3)(x-3)<0$?

By testing an x-value in each test interval in the inequality, the solution set is $(-\infty,\ -3)\cup(0,\ 3)$.



1.R.108.

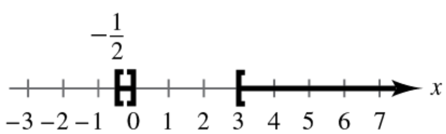
$$2x^3 - 5x^2 - 3x \geq 0$$
$$x\left(2x^2 - 5x - 3\right) \geq 0$$
$$x\left(2x + 1\right)\left(x - 3\right) \geq 0$$

Key numbers: $x = 0 - \frac{1}{2}, \ 3$

Test intervals: $\left(-\infty, \ -\frac{1}{2}\right), \ \left(-\frac{1}{2}, \ 0\right), \ \left(0, \ 3\right), \ \left(3, \ \infty\right)$

Test: Is $x\left(2x+1\right)\left(x-3\right) \geq 0$?

By testing an x-value in each test interval in the inequality, the solution set is $\left[-\frac{1}{2}, \ 0\right] \cup \left[3, \ \infty\right)$.



1.R.109.

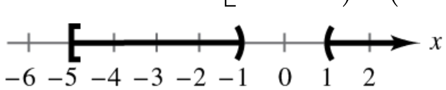
$$\frac{2}{x+1} \leq \frac{3}{x-1}$$
$$\frac{2\left(x-1\right) - 3\left(x+1\right)}{\left(x+1\right)\left(x-1\right)} \leq 0$$
$$\frac{2x - 2 - 3x - 3}{\left(x+1\right)\left(x-1\right)} \leq 0$$
$$\frac{-\left(x+5\right)}{\left(x+1\right)\left(x-1\right)} \leq 0$$

Key numbers: $x = -5, \ x = -1, \ x = 1$

Test intervals: $\left(-5, \ -1\right), \ \left(-1, \ 1\right), \ \left(1, \ \infty\right)$

Test: Is $\frac{-\left(x+5\right)}{\left(x+1\right)\left(x-1\right)} \leq 0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $\left[-5, \ -1\right) \cup \left(1, \ \infty\right)$.



1.R.110.

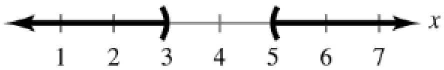
$$\frac{x-5}{3-x}<0$$

Key numbers: $x=5,\ \ x=3$

Test intervals: $(-\infty,\ 3),\ (3,\ 5),\ (5,\ \infty)$

Test: Is $\dfrac{x-5}{3-x}<0$?

By testing an x-value in each test interval in the inequality, we see that the solution set is $(-\infty,\ 3)\cup(5,\ \infty)$.



1.R.111.

$$\begin{aligned}5000(1+r)^2 &> 5500\\(1+r)^2 &> 1.1\\1+r &> 1.0488\\r &> 0.0488\\r &> 4.9\%\end{aligned}$$

1.R.112.

$$\begin{aligned}P &= \frac{1000(1+3t)}{5+t}\\2000 &\le \frac{1000(1+3t)}{5+t}\\2000(5+t) &\le 1000(1+3t)\\10{,}000+2000t &\le 1000+3000t\\-1000t &\le -9000\\t &\ge 9 \text{ days}\end{aligned}$$

At least 9 days are required.

PROBLEM SOLVING FOR CHAPTER 1

1.PS.1.

1.PS.2.

(a) $1+2+3+4+5=15$

$1+2+3+4+5+6+7+8=36$

$1+2+3+4+5+6+7+8+9+10=55$

(b) $1+2+3+\cdots+n=\frac{1}{2}n(n+1)$

When $n=5$: $\frac{1}{2}(5)(6)=15$

When $n=8$: $\frac{1}{2}(8)(9)=36$

When $n=10$: $\frac{1}{2}(10)(11)=55$

(c)
$$\begin{aligned}\frac{1}{2}n(n+1)&=210\\ n(n+1)&=420\\ n^2+n-420&=0\\ (n+21)(n-20)&=0\end{aligned}$$

$n=-21$ or $n=20$

Since n is a natural number, choose $n=20$.

1.PS.3.

$P=0.00256s^2$

(a)
$$\begin{aligned}0.00256s^2&=20\\ s^2&=7812.5\\ s&\approx 88.4 \text{ miles per hour}\end{aligned}$$

(b) $$\begin{aligned} 0.00256s^2 &= 40 \\ s^2 &= 15625 \\ s &= 125 \text{ miles per hour} \end{aligned}$$

No, actually it can survive wind blowing at $\sqrt{2}$ times the speed found in part (a).

(c) The wind speed in the formula is squared, so a small increase in wind speed could have potentially serious effects on a building.

1.PS.4.

$$h = \left(\sqrt{h_0} - \frac{2\pi d^2\sqrt{3}}{lw}t\right)^2$$

$l = 60''$, $w = 30''$, $h_0 = 25''$, $d = 2''$

$$h = \left(5 - \frac{8\pi\sqrt{3}}{1800}t\right)^2 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

(a) $$12.5 = \left(5 - \frac{\pi\sqrt{3}}{225}t\right)^2$$

$$\sqrt{12.5} = 5 - \frac{\pi\sqrt{3}}{225}t$$

$$t = \frac{225}{\pi\sqrt{3}}\left(5 - \sqrt{12.5}\right) \approx 60.6 \text{ seconds}$$

(b) $$0 = \left(\sqrt{12.5} - \frac{\pi\sqrt{3}}{225}t\right)^2$$

$$t = \frac{225\sqrt{12.5}}{\pi\sqrt{3}} \approx 146.2 \text{ seconds}$$

(c) The speed at which the water drains decreases as the amount of the water in the bathtub decreases.

1.PS.5.

(a) If $x^2 + 9 = (x+m)(x+n)$ then $mn = 9$ and $m + n = 0$.

(b) $m + n = 0 \Rightarrow n = -m$

$$m(-m) = 9 \Rightarrow -m^2 = 9 \Rightarrow m^2 = -9$$

There is no integer m such that m^2 equals a negative number. $x^2 + 9$ cannot be factored over the integers.

1.PS.6.

(a) 5, 12, and 13; 8, 15, and 17

7, 24, and 25

(b) $5 \cdot 12 \cdot 13 = 780$ which is divisible by 3, 4, and 5.

$8 \cdot 15 \cdot 17 = 2040$ which is divisible by 3, 4, and 5.

$7 \cdot 24 \cdot 25 = 4200$ which is also divisible by 3, 4, and 5.

(c) Conjecture: If $a^2 + b^2 = c^2$ where a, b, and c are positive integers, then abc is divisible by 60.

1.PS.7.

$$4\sqrt{x} = 2x + k$$

$$2x - 4\sqrt{x} + k = 0 \quad \text{Complete the square.}$$

$$x - 2\sqrt{x} = -\frac{k}{2}$$

$$x - 2\sqrt{x} + 1 = 1 - \frac{k}{2}$$

$$\left(\sqrt{x} - 1\right)^2 = 1 - \frac{k}{2}$$

s	Some k-values
2	−1, 0, 1
1	2 only
0	3, 4, 5

This equation will have two solutions when $1 - \frac{k}{2} > 0$ or when $k < 2$.

This equation will have one solution when $1 - \frac{k}{2} = 0$ or when $k = 2$.

This equation will have no solutions when $1 - \frac{k}{2} < 0$ or when $k > 2$.

1.PS.8.

Equation	$x_1,\ x_2$	$x_1 + x_2$	$x_1 \cdot x_2$
(a) $x^2 - x - 12 = 0$	$4,\ -3$	1	-12
(b) $2x^2 + 5x - 3 = 0$ $(2x-1)(x+3) = 0$	$\frac{1}{2},\ -3$	$-\frac{5}{2}$	$-\frac{3}{2}$
(c) $4x^2 - 9 = 0$ $(2x+3)(2x-3) = 0$	$-\frac{3}{2},\ \frac{3}{2}$	0	$-\frac{9}{4}$
(d) $x^2 - 10x + 34 = 0$ $x = 5 \pm 3i$	$5+3i,\ 5-3i$	10	34

1.PS.9.

(a) $S = \dfrac{-b+\sqrt{b^2-4ac}}{2a} + \dfrac{-b-\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-2a}{2a}$

$= -\dfrac{b}{a}$

(b) $P = \left(\dfrac{-b+\sqrt{b^2-4ac}}{2a}\right)\left(\dfrac{-b-\sqrt{b^2-4ac}}{2a}\right)$

$= \dfrac{b^2-(b^2-4ac)}{4a^2}$

$= \dfrac{4ac}{4a^2}$

$= \dfrac{c}{a}$

1.PS.10.

(a) (i) $\left(\dfrac{-5+5\sqrt{3}i}{2}\right)^3 = 125$

(ii) $\left(\dfrac{-5-5\sqrt{3}i}{2}\right)^3 = 125$

(b) (i) $\left(\dfrac{-3+3\sqrt{3}i}{2}\right)^3 = 27$

(ii) $\left(\dfrac{-3-3\sqrt{3}i}{2}\right)^3 = 27$

(c) (i) The cube roots of 1 are: $1, \dfrac{-1\pm\sqrt{3}i}{2}$.

(ii) The cube roots of 8 are: $2, -1\pm\sqrt{3}i$.

(iii) The cube roots of 64 are: $4, -2\pm2\sqrt{3}i$.

1.PS.11.

(a) $$\begin{aligned} z_m &= \frac{1}{z} \\ &= \frac{1}{1+i} = \frac{1}{1+i}\cdot\frac{1-i}{1-i} \\ &= \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i \end{aligned}$$

(b) $$\begin{aligned} z_m &= \frac{1}{z} \\ &= \frac{1}{3-i} = \frac{1}{3-i}\cdot\frac{3+i}{3+i} \\ &= \frac{3+i}{10} = \frac{3}{10} + \frac{1}{10}i \end{aligned}$$

(c) $$\begin{aligned} z_m &= \frac{1}{z} \\ &= \frac{1}{-2+8i} \\ &= \frac{1}{-2+8i}\cdot\frac{-2-8i}{-2-8i} \\ &= \frac{-2-8i}{68} = -\frac{1}{34} - \frac{2}{17}i \end{aligned}$$

1.PS.12.

$$(a+bi)(a-bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2$$

Since a and b are real numbers, $a^2 + b^2$ is also a real number.

1.PS.13.

(a) $c = 1$

The terms are: $i,\ -1+i,\ -i,\ -1+i,\ -i,\ -1+i,\ -i,\ -1+i,\ -i,\ \ldots.$

The sequence is bounded so $c = i$ *is* in the Mandelbrot Set.

(b) $c = -2$

The terms are: $1+i,\ 1+3i,\ -7+7i,\ 1-97i,\ -9407-1931i,\ \ldots.$

The sequence is unbounded so $c = 1+i$ *is not* in the Mandelbrot Set.

(c) $c = -2$

The terms are: $-2,\ 2,\ 2,\ 2,\ 2,\ \ldots.$

The sequence is bounded so $c = -2$ *is* in the Mandelbrot Set.