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Chapter 3: Matrices

Exercises 3.1

1. Matrix B has 3 rows.

2. Matrix E is 2×3.

3. A, F, and Z have the same order as G.

4. $-D = \begin{bmatrix} -4 & -2 \\ -3 & -5 \end{bmatrix}$

5. $-F = \begin{bmatrix} -1 & -2 & -3 \\ 1 & 0 & -1 \\ -2 & 3 & 4 \end{bmatrix}$

6. Matrix D is 2×2. The zero matrix of that order is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

7. A, C, D, F, G, and Z are square.

8. $-B = \begin{bmatrix} -1 & -1 & -3 & 0 \\ -4 & -2 & -1 & -1 \\ -3 & -2 & 0 & -1 \end{bmatrix}$

9. $a_{23} = 1$

10. $b_{24} = 1$

11. $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ -2 & 1 & 3 \end{bmatrix}$

12. $F^T = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & -3 \\ 3 & 1 & -4 \end{bmatrix}$

13. $A + (-A) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

14. If $a_{3j} = 0$, then $j = 2$ since the only element that is 0 in the 3rd row is in the 2nd column.

15. $C + D = \begin{bmatrix} 5+4 & 3+2 \\ 1+3 & 2+5 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 4 & 7 \end{bmatrix}$

16. $A + F = \begin{bmatrix} 1+1 & 0+2 & -2+3 \\ 3-1 & 2+0 & 1+1 \\ 4+2 & 0-3 & 3-4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 2 & 2 \\ 6 & -3 & -1 \end{bmatrix}$

17. $A - F = \begin{bmatrix} 1-1 & 0-2 & -2-3 \\ 3+1 & 2-0 & 1-1 \\ 4-2 & 0+3 & 3+4 \end{bmatrix} = \begin{bmatrix} 0 & -2 & -5 \\ 4 & 2 & 0 \\ 2 & 3 & 7 \end{bmatrix}$

18. $Z + G^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 5 \\ 1 & 0 & 1 \\ 1 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 0 & 1 \\ 1 & 4 & 0 \end{bmatrix}$

19. $A + A^T = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 6 \end{bmatrix}$

20. $A + F^T = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & -3 \\ 3 & 1 & -4 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & 0 \\ 5 & 2 & -2 \\ 7 & 1 & -1 \end{bmatrix}$

21. Because the orders are different, $D - G$ is undefined.

22. Because the orders are different, $B + F$ is undefined.

23. $3B = 3 \cdot \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 9 & 0 \\ 12 & 6 & 3 & 3 \\ 9 & 6 & 0 & 3 \end{bmatrix}$

24. $4D = 4 \cdot \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 16 & 8 \\ 12 & 20 \end{bmatrix}$

25. $4C + 2D = \begin{bmatrix} 20 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 8 & 4 \\ 6 & 10 \end{bmatrix} = \begin{bmatrix} 28 & 16 \\ 10 & 18 \end{bmatrix}$

26. $8C - 3D = \begin{bmatrix} 40 & 24 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 12 & 6 \\ 9 & 15 \end{bmatrix} = \begin{bmatrix} 28 & 18 \\ -1 & 1 \end{bmatrix}$

27. Because the orders are different, $2A - 3B$ is undefined.

28. $4F + 3G = \begin{bmatrix} 4 & 8 & 12 \\ -4 & 0 & 4 \\ 8 & -12 & -16 \end{bmatrix} + \begin{bmatrix} 6 & 3 & 3 \\ 3 & 0 & 12 \\ 15 & 3 & 0 \end{bmatrix}$

$= \begin{bmatrix} 10 & 11 & 15 \\ -1 & 0 & 16 \\ 23 & -9 & -16 \end{bmatrix}$

29. $\begin{bmatrix} x & 1 & 0 \\ 0 & y & z \\ w & 2 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$

By setting corresponding elements equal we have $x = 3$, $y = 2$, $z = 3$, $w = 4$.

30. $\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 4 & 1 \\ 3 & 1 & y \\ 1 & 0 & w \end{bmatrix}$

By setting corresponding elements equal we have $w = 2$, $x = 4$, $y = 1$, and $z = 1$.

31. $\begin{bmatrix} x & 3 & 2x-1 \\ y & 4 & 4y \end{bmatrix} = \begin{bmatrix} 2x-4 & z & 7 \\ 1 & w+1 & 3y+1 \end{bmatrix}$

$2x - 4 = x$ or $x = 4$, $z = 3$, $y = 1$,
$w + 1 = 4$ or $w = 3$.

32. $\begin{bmatrix} x & y & x+3 \\ z & 4 & 4y \end{bmatrix} = \begin{bmatrix} 2x-1 & -1 & w \\ x & 5+y & -4 \end{bmatrix}$

Setting corresponding elements equal gives the following:

$x = 2x-1 \quad y = -1 \quad x+3 = w \quad z = x$

$1 = x \qquad 1+3 = w \qquad z = 1$

$4 = w$

33. $\begin{bmatrix} 3x & 3y \\ 3y & 3z \end{bmatrix} + \begin{bmatrix} 4x & -2y \\ 6y & -8z \end{bmatrix} = \begin{bmatrix} 14 & 4-y \\ 18 & 15 \end{bmatrix}$

Setting corresponding elements equal gives the following:

$3x + 4x = 14 \qquad 3y - 2y = 4 - y$

$7x = 14 \qquad 2y = 4$

$x = 2 \qquad y = 2$

$3y + 6y = 18 \qquad 3z - 8z = 15$

$9y = 18 \qquad -5z = 15$

$y = 2 \qquad z = -3$

34. $\begin{bmatrix} 3x & 12 \\ 12y & 3w \end{bmatrix} - \begin{bmatrix} 8x & 4z \\ -6 & -4w \end{bmatrix} = \begin{bmatrix} 20 & 20 \\ 6 & 14 \end{bmatrix}$

Setting corresponding elements equal gives the following:

$3x - 8x = 20 \qquad 12 - 4z = 20$

$-5x = 20 \qquad -4z = 8$

$x = -4 \qquad z = -2$

$12y - (-6) = 6 \qquad 3w - (-4w) = 14$

$12y + 6 = 6 \qquad 3w + 4w = 14$

$12y = 0 \qquad 7w = 14$

$y = 0 \qquad w = 2$

35. a. $A = \begin{bmatrix} 0.386 & 0.501 & 0.264 & 0.179 \\ 0.602 & 0.828 & 0.671 & 0.316 \end{bmatrix}$

$B = \begin{bmatrix} 0.539 & 4.43 & 2.01 & 1.83 \\ 0.135 & 6.33 & 5.12 & 2.43 \end{bmatrix}$

b. $A + B = \begin{bmatrix} 0.925 & 4.931 & 2.274 & 2.009 \\ 0.737 & 7.158 & 5.791 & 2.746 \end{bmatrix}$

c. $C = \begin{bmatrix} 0.386 & 0.501 & 0.264 & 0.179 \\ 0.539 & 4.43 & 2.01 & 1.83 \end{bmatrix}$

$D = \begin{bmatrix} 0.602 & 0.828 & 0.671 & 0.316 \\ 0.135 & 6.33 & 5.12 & 2.43 \end{bmatrix}$

d. $D - C = \begin{bmatrix} 0.216 & 0.327 & 0.407 & 0.137 \\ -0.404 & 1.9 & 3.12 & 0.6 \end{bmatrix}$

The matrix $D - C$ measure, the increase in the leakage to the oceans from various regions from 2020 to 2050.
The negative entries tell us that the leakage to the oceans has decreased from 2020 to 2050 from the region.

36. a. $B = \begin{bmatrix} 1341.3 & 148.7 & 93.3 \\ 73.8 & 69.4 & 69.2 \\ 1.56 & 2.16 & 3.05 \\ 19.6 & 41.8 & 20.9 \end{bmatrix}$

$A = \begin{bmatrix} 1211.5 & 192.4 & 165.5 \\ 80.8 & 79.0 & 78.4 \\ 1.88 & 1.68 & 1.95 \\ 8.3 & 12.5 & 10.2 \end{bmatrix}$

b. $C = A - B = \begin{bmatrix} -129.8 & 43.7 & 72.2 \\ 7 & 9.6 & 9.2 \\ 0.32 & -0.48 & -1.1 \\ -11.3 & -29.3 & -10.7 \end{bmatrix}$

c. The negative entries in row 3 indicate that the infant mortality rates of all three countries are projected to be reduced, which is a positive change for the countries.

37. a. Capital Expenditures + Gross Operating Costs $= \begin{bmatrix} 11041.7 & 8978.4 & 6461 \\ 8739.8 & 9159.6 & 6877.3 \\ 9798.1 & 9086.7 & 6448.4 \\ 9696.6 & 8926.7 & 6109.5 \end{bmatrix}$

b. Air pollution (R4, C1)

38. a. $M - F = \begin{bmatrix} 413 & 813 & 418 & 1563 & -6136 \\ 417 & 782 & 371 & 1679 & -6607 \\ 418 & 778 & 347 & 1649 & -6987 \\ 414 & 779 & 350 & 1657 & -7314 \end{bmatrix}$

$M - F$ represents the projected differences of male and female populations for different ages and years.

b. The female population is projected to be greater than the male population for those over age 64 in all years 2025 – 2040.

39. a. $A + B = \begin{bmatrix} 825 & 580 & 1560 \\ 810 & 650 & 350 \end{bmatrix}$ **b.** $B - A = \begin{bmatrix} -75 & 20 & -140 \\ 10 & -50 & 50 \end{bmatrix}$

40. Table form.

Republicans	Democrats	Independents	Not Registered	
843	257	135	92	Approve
426	451	127	64	Disapprove
751	92	38	44	No opinion

A 3×4 matrix representing this information is $\begin{bmatrix} 843 & 257 & 135 & 92 \\ 426 & 451 & 127 & 64 \\ 751 & 92 & 38 & 44 \end{bmatrix}$.

41. a.

$$A = \begin{array}{c} \begin{array}{cc} \text{M} & \text{F} \end{array} \\ \begin{bmatrix} 78.6 & 82.9 \\ 80.2 & 84.1 \\ 81.7 & 85.2 \\ 83.2 & 86.4 \\ 84.5 & 87.4 \end{bmatrix} \end{array} \quad B = \begin{array}{c} \begin{array}{cc} \text{M} & \text{F} \end{array} \\ \begin{bmatrix} 74.0 & 79.8 \\ 76.1 & 81.3 \\ 78.0 & 82.8 \\ 79.8 & 84.1 \\ 81.4 & 85.3 \end{bmatrix} \end{array}$$

b.

$$A - B = \begin{array}{c} \begin{array}{cc} \text{M} & \text{F} \end{array} \\ \begin{bmatrix} 4.6 & 3.1 \\ 4.1 & 2.8 \\ 3.7 & 2.4 \\ 3.4 & 2.3 \\ 3.1 & 2.1 \end{bmatrix} \end{array}$$

c. The life expectancy difference is greatest in 2020.

42. a. $E = \begin{bmatrix} 2336 & 1446 & 6034 & 593 \\ 6693 & 4143 & 17{,}289 & 1699 \end{bmatrix}$

b.

$1.056E = \begin{bmatrix} 2466.8 & 1527 & 6317.9 & 626.2 \\ 7067.8 & 4375 & 18257.2 & 1794.1 \end{bmatrix}$

43. All answers are in quadrillion BTUs.

a. $E - M = \begin{bmatrix} 16.55 & 18.4 & 19.3 \\ 5.99 & 8.6 & 11.59 \\ -12.72 & -12.96 & -13.2 \end{bmatrix}$

This represents the U.S. balance of trade for various energy types for selected years.

b. $\frac{1}{12}C \approx \begin{bmatrix} 3.03 & 2.96 & 2.9 \\ 2.54 & 2.44 & 2.39 \\ 2.58 & 2.77 & 2.97 \end{bmatrix}$

This represents the average monthly U.S. consumption of various energy types for selected years.

c. $C+E-M=\begin{bmatrix}52.93 & 53.86 & 54.07\\ 36.44 & 37.89 & 40.30\\ 18.20 & 20.25 & 22.45\end{bmatrix}$

This represents the total U.S. production of various energy types for selected years.

44. a.

$$0.65\begin{bmatrix}120,000 & 180,000\\ 75,000 & 45,000\\ 105,000 & 174,000\end{bmatrix}=\begin{bmatrix}78,000 & 117,000\\ 48,750 & 29,250\\ 68,250 & 113,100\end{bmatrix}$$

b. $\begin{bmatrix}24,000 & 216,000\\ 15,000 & 54,000\\ 21,000 & 208,800\end{bmatrix}$ (This is 20% of the first column and 120% of the second.)

45. a. $1.05A=\begin{bmatrix}92.40 & 168 & 420 & 21\\ 84 & 168 & 84 & 0\\ 117.60 & 294 & 189 & 0\\ 63 & 294 & 84 & 42\\ 84 & 0 & 420 & 21\end{bmatrix}$

b. $1.10A=\begin{bmatrix}96.80 & 176 & 440 & 22\\ 88 & 176 & 88 & 0\\ 123.20 & 308 & 198 & 0\\ 66 & 308 & 88 & 44\\ 88 & 0 & 440 & 22\end{bmatrix}$

46. a. $0.80\cdot\begin{bmatrix}50,000 & 52,000 & 24,000\\ 30,000 & 104,000 & 10,000\\ 16,000 & 40,000 & 240,000\end{bmatrix}$

$=\begin{bmatrix}40,000 & 41,600 & 19,200\\ 24,000 & 83,200 & 8000\\ 12,800 & 32,000 & 192,000\end{bmatrix}$

b. They paid 100% of the debts due in 30 days and 50% of the debts due in 60 days.

47. a. $\begin{bmatrix}0 & 1 & 1 & 0\\ 1 & 0 & 1 & 1\\ 0 & 0 & 0 & 1\\ 0 & 1 & 1 & 0\end{bmatrix}$

b. $\begin{bmatrix}0 & 1 & 1 & 1\\ 1 & 0 & 1 & 1\\ 0 & 1 & 0 & 1\\ 1 & 1 & 1 & 0\end{bmatrix}$

c. $A+B=\begin{bmatrix}0 & 2 & 2 & 1\\ 2 & 0 & 2 & 2\\ 0 & 1 & 0 & 2\\ 1 & 2 & 2 & 0\end{bmatrix}$

Sum in Row 1 = 5, Row 2 = 6, Row 3 = 3, Row 4 = 5. Person 2 is the most active.

48. a. $A=\begin{bmatrix}0 & 1 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 1\\ 1 & 0 & 1 & 0 & 1\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}$

b. $B=\begin{bmatrix}0 & 0 & 1 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 1\\ 0 & 1 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$

c. $A+B=\begin{bmatrix}0 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0 & 1\\ 0 & 2 & 0 & 0 & 2\\ 1 & 1 & 1 & 0 & 2\\ 0 & 1 & 0 & 0 & 0\end{bmatrix}$

Sum in Row 1 = 4, Row 2 = 1, Row 3 = 4, Row 4 = 5, Row 5 = 1.
Player 4 is the top-ranked player.

49. a. $A+B=\begin{bmatrix}50+30 & 30+45\\ 36+22 & 44+62\end{bmatrix}=\begin{bmatrix}80 & 75\\ 58 & 106\end{bmatrix}$

b. $A+B+C=\begin{bmatrix}80+96 & 75+52\\ 58+81 & 106+37\end{bmatrix}$

$=\begin{bmatrix}176 & 127\\ 139 & 143\end{bmatrix}$

c. $A-D=\begin{bmatrix}50-40 & 30-26\\ 36-29 & 44-42\end{bmatrix}=\begin{bmatrix}10 & 4\\ 7 & 2\end{bmatrix}$

d. $B-D=\begin{bmatrix}30-40 & 45-26\\ 22-29 & 62-42\end{bmatrix}=\begin{bmatrix}-10 & 19\\ -7 & 20\end{bmatrix}$

Shortage; take from inventory.

50.

$$\begin{array}{c} \\ \text{WHN 1} \\ \text{WHN 2} \\ \text{WHN 3} \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 23 & 23 & 15 & 26 & 20 & 34 \\ 24 & 23 & 16 & 28 & 21 & 33 \\ 18 & 17 & 15 & 18 & 17 & 22 \end{bmatrix} \end{array}$$

51. **a**. 3,4,5,6
b. 1

52. Subtract 0.01 from each entry in column 2 and column 5.
Subtract 0.02 from each entry in column 7.

$$\text{New } E = \begin{bmatrix} 1.00 & 0.94 & 0.95 & 0.95 & 0.94 & 0.85 & 0.83 & 0.85 & 0.85 \\ 0.85 & 0.99 & 0.95 & 0.95 & 0.94 & 0.95 & 0.83 & 0.85 & 0.85 \\ 0.85 & 0.84 & 1.00 & 0.95 & 0.94 & 0.95 & 0.93 & 0.85 & 0.85 \\ 0.85 & 0.84 & 0.85 & 1.00 & 0.94 & 0.95 & 0.93 & 0.95 & 0.85 \\ 0.85 & 0.84 & 0.85 & 0.85 & 0.99 & 0.95 & 0.93 & 0.95 & 0.95 \\ 0.95 & 0.84 & 0.85 & 0.85 & 0.84 & 1.00 & 0.93 & 0.95 & 0.95 \\ 0.95 & 0.94 & 0.85 & 0.85 & 0.84 & 0.85 & 0.98 & 0.95 & 0.95 \\ 0.95 & 0.94 & 0.95 & 0.85 & 0.84 & 0.85 & 0.83 & 1.00 & 0.95 \\ 0.95 & 0.94 & 0.95 & 0.95 & 0.84 & 0.85 & 0.83 & 0.85 & 1.00 \end{bmatrix}$$

53. For each row: (C1 + C2 + C3 + C4) ÷ 4 = average efficiency

W1: 0.9625	W2: 0.9375	W3: 0.9125
W4: 0.8875	W5: 0.85	W6: 0.875
W7: 0.90	W8: 0.925	W9: 0.95

W5 is least efficient. He works best at center 5.

54. Work Centers

$$\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ [0.88 & 0.90 & 0.92 & 0.94 & 0.96 & 0.93 & 0.91 & 0.89 & 0.87] \end{array}$$

Number 5; Number 9

Exercises 3.2

1. $$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\cdot 4 + 2\cdot 5 + 3\cdot 6 \end{bmatrix} = [32]$$

(not 32)

2. $$\begin{bmatrix} 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2\cdot 0 + 0\cdot 1 + 3(-3) \end{bmatrix} = [-9]$$

3. $$\begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 1\cdot 3 + 2\cdot 4 & 1\cdot 5 + 2\cdot 6 \end{bmatrix}$$
$$= \begin{bmatrix} 11 & 17 \end{bmatrix}$$

4. $$\begin{bmatrix} 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 3\cdot 1 + 0\cdot 4 & 3\cdot 2 + 0\cdot 5 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 6 \end{bmatrix}$$

5. $$CD = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}\cdot\begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 20+9 & 10+15 \\ 4+6 & 2+10 \end{bmatrix}$$
$$= \begin{bmatrix} 29 & 25 \\ 10 & 12 \end{bmatrix}$$

6. $$DC = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}\cdot\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 20+2 & 12+4 \\ 15+5 & 9+10 \end{bmatrix}$$
$$= \begin{bmatrix} 22 & 16 \\ 20 & 19 \end{bmatrix}$$

Note $CD \neq DC$

7. $DE = \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix}$
$= \begin{bmatrix} 4+10 & 0+2 & 16+0 \\ 3+25 & 0+5 & 12+0 \end{bmatrix}$
$= \begin{bmatrix} 14 & 2 & 16 \\ 28 & 5 & 12 \end{bmatrix}$

8. $CF = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 3 & -4 \end{bmatrix}$
$= \begin{bmatrix} 5+6 & 0-3 & -5+9 & 15-12 \\ 1+4 & 0-2 & -1+6 & 3-8 \end{bmatrix}$
$= \begin{bmatrix} 11 & -3 & 4 & 3 \\ 5 & -2 & 5 & -5 \end{bmatrix}$

9. $AB = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0+6 & 1+0+4 & 3+0+0 & 0+0+2 \\ 3+8+3 & 3+4+2 & 9+2+0 & 0+2+1 \\ 4+0+9 & 4+0+6 & 12+0+0 & 0+0+3 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 3 & 2 \\ 14 & 9 & 11 & 3 \\ 13 & 10 & 12 & 3 \end{bmatrix}$

10. EC is undefined.

11. BA is undefined.

12. $FB^T = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 3 & -4 \end{bmatrix}\begin{bmatrix} 1 & 4 & 3 \\ 1 & 2 & 2 \\ 3 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+0-3+0 & 4+0-1+3 & 3+0-0+3 \\ 2-1+9-0 & 8-2+3-4 & 6-2+0-4 \end{bmatrix} = \begin{bmatrix} -2 & 6 & 6 \\ 10 & 5 & 0 \end{bmatrix}$

13. $EB = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 2 & 1 & 1 \\ 3 & 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1+0+12 & 1+0+8 & 3+0+0 & 0+0+4 \\ 5+4+0 & 5+2+0 & 15+1+0 & 0+1+0 \end{bmatrix} = \begin{bmatrix} 13 & 9 & 3 & 4 \\ 9 & 7 & 16 & 1 \end{bmatrix}$

14. BE is undefined.

15. $EA^T = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1+0+8 & 3+0+4 & 4+0+12 \\ 5+0+0 & 15+2+0 & 20+0+0 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 16 \\ 5 & 17 & 20 \end{bmatrix}$

16. $AE^T = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 1+0+8 & 5+0+0 \\ 3+0+4 & 15+2+0 \\ 4+0+12 & 20+0+0 \end{bmatrix} = \begin{bmatrix} 9 & 5 \\ 7 & 17 \\ 16 & 20 \end{bmatrix}$

17. $A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+0+8 & 0+0+0 & 2+0+6 \\ 3+6+4 & 0+4+0 & 6+2+3 \\ 4+0+12 & 0+0+0 & 8+0+9 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 8 \\ 13 & 4 & 11 \\ 16 & 0 & 17 \end{bmatrix}$

18. $A^3 = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 8 \\ 13 & 4 & 11 \\ 16 & 0 & 17 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 17 \end{bmatrix}$
$= \begin{bmatrix} 9+0+32 & 0+0+0 & 18+0+24 \\ 13+12+44 & 0+8+0 & 26+4+33 \\ 16+0+68 & 0+0+0 & 32+0+51 \end{bmatrix} = \begin{bmatrix} 41 & 0 & 42 \\ 69 & 8 & 63 \\ 84 & 0 & 83 \end{bmatrix}$

19. $C^3 = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \cdot \left(\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \right) = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 25+3 & 15+6 \\ 5+2 & 3+4 \end{bmatrix}$

$= \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 28 & 21 \\ 7 & 7 \end{bmatrix} = \begin{bmatrix} 140+21 & 105+21 \\ 28+14 & 21+14 \end{bmatrix} = \begin{bmatrix} 161 & 126 \\ 42 & 35 \end{bmatrix}$

20. F^2 is undefined.

21. $AA^T = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 10 \\ 5 & 14 & 15 \\ 10 & 15 & 25 \end{bmatrix}$

$(AA^T)^T = \begin{bmatrix} 5 & 5 & 10 \\ 5 & 14 & 15 \\ 10 & 15 & 25 \end{bmatrix}$ $\quad A^T A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 26 & 6 & 17 \\ 6 & 4 & 2 \\ 17 & 2 & 14 \end{bmatrix}$

No, they are not equal.

22. $(CD)E = \left(\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix} \right) \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 29 & 25 \\ 10 & 12 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 154 & 25 & 116 \\ 70 & 12 & 40 \end{bmatrix}$

$C(DE) = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \left(\begin{bmatrix} 4 & 2 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} \right) = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 14 & 2 & 16 \\ 28 & 5 & 12 \end{bmatrix} = \begin{bmatrix} 154 & 25 & 116 \\ 70 & 12 & 40 \end{bmatrix}$

They are equal.

23. No. See problems 5 and 7

24. Yes. In general, $(kA)(B) = k(AB)$.

25. $AB = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \\ -5 & 8 & 2 \\ -7 & 11 & -3 \end{bmatrix} = \begin{bmatrix} -2-25-28 & 4+40+44 & 2+10-12 \\ -1-20-21 & 2+32+33 & 1+8-9 \\ -1+15+14 & 2-24-22 & 1-6+6 \end{bmatrix} = \begin{bmatrix} -55 & 88 & 0 \\ -42 & 67 & 0 \\ 28 & -44 & 1 \end{bmatrix}$

26. $BA = \begin{bmatrix} -1 & 2 & 1 \\ -5 & 8 & 2 \\ -7 & 11 & -3 \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ 1 & -3 & -2 \end{bmatrix} = \begin{bmatrix} -2+2+1 & -5+8-3 & -4+6-2 \\ -10+8+2 & -25+32-6 & -20+24-4 \\ -14+11-3 & -35+44+9 & -28+33+6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -6 & 18 & 11 \end{bmatrix}$

27. $CD = \begin{bmatrix} 3 & 0 & 4 \\ 1 & 7 & -1 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} 4 & 4 & -8 \\ -1 & -1 & 2 \\ -3 & -3 & 6 \end{bmatrix} = \begin{bmatrix} 12+0-12 & 12+0-12 & -24+0+24 \\ 4-7+3 & 4-7+3 & -8+14-6 \\ 12+0-12 & 12+0-12 & -24+0+24 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

28. $DC = \begin{bmatrix} 4 & 4 & -8 \\ -1 & -1 & 2 \\ -3 & -3 & 6 \end{bmatrix} \begin{bmatrix} 3 & 0 & 4 \\ 1 & 7 & -1 \\ 3 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 12+4-24 & 0+28+0 & 16-4-32 \\ -3-1+6 & 0-7+0 & -4+1+8 \\ -9-3+18 & 0-21+0 & -12+3+24 \end{bmatrix} = \begin{bmatrix} -8 & 28 & -20 \\ 2 & -7 & 5 \\ 6 & -21 & 15 \end{bmatrix}$

29. $F^2 = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0+0 & 0+0+0 & 0-4+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$F^3 = \begin{bmatrix} 0 & 0 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & -4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

30. $AZ = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = Z$

31. $AI = A$ See Example 6.

32. $IA = A$

33. $Z(CI) = ZC = Z \qquad (ZC)I = ZI = Z$
Product of zero matrix and A is the zero matrix.

34. $IFZ = Z$

35. Problem 27 shows that a product can be "zero" and neither of the factors is zero.

36. Yes, it is true that multiplication by the zero matrix gives a result of a zero matrix, provided the multiplication is defined.

37. **a.** We will use $\frac{1}{ad-bc}AB$ and $\frac{1}{ad-bc}BA$.

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & -ab+ba \\ cd-dc & -bc+ad \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$BA = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & -bc+ad \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$\frac{1}{ad-bc}AB = \frac{1}{ad-bc}BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

b. For B to exist, we must have

38. $F = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 3 & -4 \end{bmatrix}$ and $F^T = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & 3 \\ 3 & -4 \end{bmatrix}$

a. yes

b. FF^T is 2×2; F^TF is 4×4

c. No, because FF^T and F^TF have different dimensions.

39. $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & -2 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2-2+2 \\ 6-4-4 \\ 4+0-2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$

These values are the solution.

40. $\begin{bmatrix} 3 & 1 & 0 \\ 2 & -2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6-2+0 \\ 4+4+1 \\ 2-2+2 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}$

These values are the solution.

41.

$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1+2+2 \\ 4+0+1 \\ 2+2+1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$

These values are the solution.

42.

$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2+0+2 \\ 6+1+0 \\ 2+2+1 \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 7 \\ 3 \end{bmatrix}$

So, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ is not a solution of $\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ 3 \end{bmatrix}$.

43. $\begin{bmatrix} 0.1 & 0.0 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0.2 & -0.1 & 0.1 & -0.1 \\ 0.1 & -0.1 & 0.1 & -0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & 0 & 10 \\ 0 & 5 & \frac{10}{3} & \frac{5}{3} & -10 \\ -10 & -15 & -\frac{20}{3} & -\frac{35}{3} & 20 \\ 0 & -5 & -\frac{20}{3} & \frac{5}{3} & 10 \\ 10 & 10 & \frac{20}{3} & -\frac{20}{3} & 20 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -0.002 & 0 \\ 0 & 0 & 0.999 & 0.002 & 0 \\ 0 & 0 & 0 & 0.998 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

The graphing calculator should show 1's on the main diagonal and 0's everywhere else (1 may be 0.9999 or 0.998 and 0 may be 0.002 or -0.002).

44. $\frac{1}{12}\begin{bmatrix} 3 & 15 & -9 & -15 & 6 \\ 3 & -9 & 3 & 9 & -6 \\ -1 & -17 & 3 & 23 & -4 \\ 0 & 12 & 0 & -12 & 0 \\ -1 & -5 & 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 & 0 & 1 & 3 \\ 1 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 0 & 3 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 12 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 12 \end{bmatrix}$

$$=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix}$$

45. $\begin{bmatrix}0.88&0\\0&0.85\end{bmatrix}\begin{bmatrix}36{,}000&42{,}000\\50{,}000&56{,}000\end{bmatrix}=\begin{bmatrix}31{,}680&36{,}960\\42{,}500&47{,}600\end{bmatrix}$

The product is the price the dealer pays for the cars.

46. Revenue $=\begin{bmatrix}200&40&50&30\end{bmatrix}\begin{bmatrix}\text{Given matrix}\end{bmatrix}$

$$\begin{bmatrix}200&40&50&30\end{bmatrix}\begin{bmatrix}40&63&18\\85&56&42\\6&18&8\\7&10&8\end{bmatrix}=\begin{bmatrix}11{,}910&16{,}040&5{,}920\end{bmatrix}$$

So, the revenue is as follows:
Atlanta: \$11,910,000
Chicago \$16,040,000
New York: \$ 5,920,000

47. a. $A=\begin{bmatrix}89.52&93.97&98.15\\33.86&35.47&36.93\\6.66&6.72&6.74\end{bmatrix}$ and $B=\begin{bmatrix}87.9&0&0\\0&84.8&0\\0&0&81.1\end{bmatrix}$

b. $AB\approx\begin{bmatrix}7868.81&7968.66&7959.97\\2976.29&3007.86&2995.02\\585.41&569.86&546.61\end{bmatrix}$

c. The 1-1 entry is the trillions of BTUs used in single-family households in 2025. The 2-3 entry is the millions of BTUs predicted to be used in multi-family households in 2035.

d. Using AB as stored in a calculator, $\begin{bmatrix}1&1&1\end{bmatrix}AB\approx\begin{bmatrix}11{,}430.51&11{,}546.38&11{,}501.60\end{bmatrix}$. Left to right, each entry gives the millions of BTUs of delivered energy consumed by all households in 2025, 2030, and 2035.

48. $AB=\begin{bmatrix}0.162&0.047\\0.119&0.086\\0.066&0.065\\0.298&0.582\\0.202&0.215\end{bmatrix}\begin{bmatrix}57{,}850{,}000&0\\0&9{,}322{,}000{,}000\end{bmatrix}=\begin{bmatrix}9{,}371{,}700&438{,}134{,}000\\6{,}884{,}150&801{,}692{,}000\\3{,}818{,}100&605{,}930{,}000\\17{,}239{,}300&5{,}425{,}404{,}000\\11{,}685{,}700&2{,}004{,}230{,}000\end{bmatrix}$

49. a. $\begin{bmatrix}0.5&0.5\end{bmatrix}\begin{bmatrix}0.8&0.2\\0.3&0.7\end{bmatrix}=\begin{bmatrix}0.55&0.45\end{bmatrix}$ After 5 years, M has 55% and S has 45% of the population.

b. 10 years: $(PD)D=PD^2$; 15 years: PD^3

c. 60% in M and 40% in S. The population proportions are stable.

50. a. $GA = \begin{bmatrix} 0.20 & 0 & 0 \\ 0 & 0.07 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 12.5 & 250 \\ 11.8 & 215 \\ 9.8 & 190 \end{bmatrix} = \begin{bmatrix} 2.500 & 50.00 \\ 0.826 & 15.05 \\ 0 & 0 \end{bmatrix}$

b. $(I+G)A = \begin{bmatrix} 1.20 & 0 & 0 \\ 0 & 1.07 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = \begin{bmatrix} 15 & 300 \\ 12.626 & 230.05 \\ 9.8 & 190 \end{bmatrix}$

51.

W	a	s	t	e	blank	n	o	t	blank	w	a	n	t	blank	n	o	t
23	1	19	20	5	27	14	15	20	27	23	1	14	20	27	14	15	20

$$\begin{bmatrix} 5 & 9 \\ 6 & 11 \end{bmatrix} \begin{bmatrix} 23 & 19 & 5 & 14 & 20 & 23 & 14 & 27 & 15 \\ 1 & 20 & 27 & 15 & 27 & 1 & 20 & 14 & 20 \end{bmatrix} = \begin{bmatrix} 124 & 275 & 268 & 205 & 343 & 124 & 250 & 261 & 255 \\ 149 & 334 & 327 & 249 & 417 & 149 & 304 & 316 & 310 \end{bmatrix}$$

So 124, 149, 275, 334, 268, 327, 205, 249, 343, 417, 124, 149, 250, 304, 261, 316, 255, 310 is the message sent.

52.

T	o	blank	b	e	blank	o	r	blank	n	o	t	blank	t	o	blank	b	e
20	15	27	2	5	27	15	18	27	14	15	20	27	20	15	27	2	5

$$\begin{bmatrix} 5 & 9 \\ 6 & 11 \end{bmatrix} \begin{bmatrix} 20 & 27 & 5 & 15 & 27 & 15 & 27 & 15 & 2 \\ 15 & 2 & 27 & 18 & 14 & 20 & 20 & 27 & 5 \end{bmatrix} = \begin{bmatrix} 235 & 153 & 268 & 237 & 261 & 255 & 315 & 318 & 55 \\ 285 & 184 & 327 & 288 & 316 & 310 & 382 & 387 & 67 \end{bmatrix}$$

Thus, 235, 285, 153, 184, 268, 327, 237, 288, 261, 316, 255, 310, 315, 382, 318, 387, 55, 67 is sent.

53. a. $B = \begin{bmatrix} \frac{3}{4} & \frac{2}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{5} & \frac{3}{4} \end{bmatrix}; D = \begin{bmatrix} 22 & 30 \\ 12 & 20 \\ 8 & 11 \end{bmatrix}; BD = \begin{bmatrix} 23.3 & 33.25 \\ 18.7 & 27.75 \end{bmatrix}$ Houston needs 23,300 gallons of black crude and 18,700 gallons of gold. Gulfport needs 32,250 gallons of black and 27,750 gallons of gold.

b. $PBD = \begin{bmatrix} 66.56 & 96.89 \end{bmatrix}$; Houston's cost = \$66,560; Gulfport's cost = \$96,890

54. a. $\begin{bmatrix} 8.50 & 5.50 & 12.00 \\ 6.00 & 4.00 & 15.00 \end{bmatrix} \begin{bmatrix} 1200 & 1500 & 1500 & 2000 \\ 1200 & 1000 & 1000 & 1500 \\ 500 & 500 & 400 & 1000 \end{bmatrix} = \begin{bmatrix} 22,800 & 24,250 & 23,050 & 37,250 \\ 19,500 & 20,500 & 19,000 & 33,000 \end{bmatrix}$ \$22,800

b. $\left(\begin{bmatrix} 8.50 & 5.50 & 12.00 \\ 6.00 & 4.00 & 15.00 \end{bmatrix} \begin{bmatrix} 1200 & 1500 & 1500 & 2000 \\ 1200 & 1000 & 1000 & 1500 \\ 500 & 500 & 400 & 1000 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 22,800 & 24,250 & 23,050 & 37,250 \\ 19,500 & 20,500 & 19,000 & 33,000 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 107,350 \\ 92,000 \end{bmatrix}$

CDT represents the total costs for Labor and Materials for the entire year.

55. a. $B = \begin{bmatrix} 0.7 & 8.5 & 10.2 & 1.1 & 5.6 & 3.6 \\ 0.5 & 0.2 & 6.1 & 1.3 & 0.2 & 1.0 \\ 2.2 & 0.4 & 8.8 & 1.2 & 1.2 & 4.8 \\ 251.8 & 63.4 & 81.6 & 35.2 & 54.3 & 144.2 \\ 30.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 788.9 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ **b .** $A = \begin{bmatrix} 1.11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.95 \end{bmatrix}$

56.
$$\begin{bmatrix} 0.7 & 8.5 & 10.2 & 1.1 & 5.6 & 3.6 \\ 0.5 & 0.2 & 6.1 & 1.3 & 0.2 & 1.0 \\ 2.2 & 0.4 & 8.8 & 1.2 & 1.2 & 4.8 \\ 251.8 & 63.4 & 81.6 & 35.2 & 54.3 & 144.2 \\ 30.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 788.9 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1.20 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.03 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.20 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.05 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.97 \end{bmatrix}$$
$$= \begin{bmatrix} 0.840 & 8.755 & 10.710 & 1.320 & 5.880 & 3.492 \\ 0.600 & 0.206 & 6.405 & 1.560 & 0.210 & 0.970 \\ 2.640 & 0.412 & 9.240 & 1.440 & 1.260 & 4.656 \\ 302.160 & 65.302 & 85.680 & 42.240 & 57.015 & 139.874 \\ 36.000 & 1.030 & 1.050 & 1.200 & 1.050 & 0.970 \\ 946.68 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercises 3.3

1. $\left[\begin{array}{ccc|c} 1 & -2 & -1 & -7 \\ 3 & 1 & 2 & 0 \\ 4 & 2 & 2 & 1 \end{array}\right] \xrightarrow{-3R_1 + R_2 \to R_2}$
$\left[\begin{array}{ccc|c} 1 & -2 & -1 & -7 \\ 0 & 7 & 5 & 21 \\ 4 & 2 & 2 & 1 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 1 & -2 & -1 & -7 \\ 3 & 1 & 2 & 0 \\ 4 & 2 & 2 & 1 \end{array}\right] \xrightarrow[-4R_1 + R_3 \to R_3]{} \left[\begin{array}{ccc|c} -1 & 2 & 1 & 7 \\ 3 & 1 & 2 & 0 \\ 0 & 10 & 6 & 29 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 2 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 1 & -2 & 0 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right]$

5. $x = 2,\ y = \frac{1}{2},\ z = -5$